Aircraft Propulsion
Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture – 12
Gas Turbine Attachment: Examples

Welcome to the class, today we will see some examples on gas turbine attachments. We have seen what do you mean by attachments. We practically mean here that there can be an intercooler there can be a heat exchanger are there can be re-heater. And everything has its own advantage and own limitations. So we are going to see how these attachments are going to alter the performance of a gas turbine.

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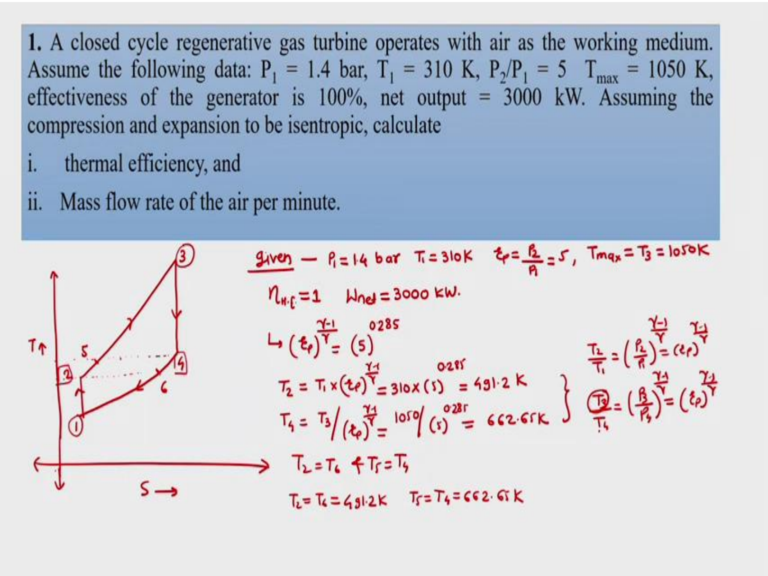

Through examples. For that let us consider the first example it says that there is a closed cycle regenerative gas turbine which operates with air as working medium. So we are given that air is working medium assume the following data P1 is equal to 1.4 bar T1 is equal to 310 Kelvin P2 by P1 is equal to 5 T-max is equal to 1050. Effectiveness of heat exchanger is 100% net power output is 3000 kilowatt.

Here practically one is the compressor inlet and P2 by P1, so the pressure ratio. Assuming the compression expansion as isentropic calculate efficiency and mass flow rate of the air per minute. So we are told that it is a gas turbine which is operating with the regenerator. So, there is a heat exchanger which is attached with the gas turbine. So we will first draw the TS diagram and note down what are the known things given things for us it is 1, 2, 3, 4 but 2 to 5 and 4 to 6 they are the processes in heat exchanger.

So, given things in this example

$$P_1 = 1.4\ bar\ ; T_1 = 310\ K\ ; r_P = \frac{P_2}{P_1} = 5\ ; T_{max} = T_3 = 1050\ K$$

$$\eta_{H.E.} = 1\ ; W_{net} = 3000\ kW$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = (5)^{0.285}$$

$$T_2 = T_1 * (r_P)^{\frac{\gamma-1}{\gamma}} = 310 * (5)^{0.285} = 491.2\ K$$

$$T_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1050}{(5)^{0.285}} = 662.65\ K$$

$$T_2 = T_6 = 491.2\ K\ ; T_5 = T_4 = 662.65\ K$$

Now we can calculate all the necessary things.

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$$W_{net} = 3000\ kW = 3000 \times 10^3 = W_t - W_c$$

$$\therefore W_{net} = 3000 \times 10^3 = \dot{m} C_p (T_3 - T_4) - \dot{m} (T_2 - T_1)$$

$$W_{net} = 3000 \times \cancel{10^3} = \dot{m} \times 1.005 \times \cancel{10^3} [T_3 - T_4 - T_2 + T_1]$$

$$3000 = \dot{m} \times 1.005 \times [1050 - 662.65 - 491.2 + 310]$$

$$\dot{m} \times 207.18 = 3000$$

$$\therefore \dot{m} = 14.48\ kg/s$$

$$\dot{m} = 14.48 \times 60 = 868.80\ kg/min$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{3000 \times 10^3}{\dot{m} C_p (T_3 - T_5)} = \frac{W_t - W_c}{Q_{in}}$$

$$\eta = \frac{\cancel{\dot{m}} [T_3 - T_4) \cancel{C_p} - \cancel{\dot{m}} [T_2 - T_1] \cancel{C_p}}{\cancel{\dot{m}} \cdot (T_3 - T_5) \cdot \cancel{C_p}}$$

$$\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_5)} = \frac{(1050 - 662.25) - (491.2 - 310)}{(1050 - 662.65)} = 53.22\%$$

$$W_{net} = 3000\ kW = 3000 * 10^3 = W_t - W_c$$

$$3000 * 10^3 = \dot{m} C_p [(T_3 - T_4) - (T_2 - T_1)] = \dot{m} * 1.005 * 10^3 * [T_3 - T_4 - T_2 + T_1]$$

$$\dot{m} = 14.48\ kg/s$$

$$\dot{m} = 14.48 * 60 = 868.80\ kg/min$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{3000 * 10^3}{\dot{m}C_p(T_3 - T_5)} = \frac{W_t - W_c}{Q_{in}} = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_5)}$$
$$= \frac{(1050 - 662.25) - (491.2 - 310)}{(1050 - 662.65)} = 53.22\ \%$$

So, this is how we can calculate an example with heat exchanger. So, let us move on the next example.

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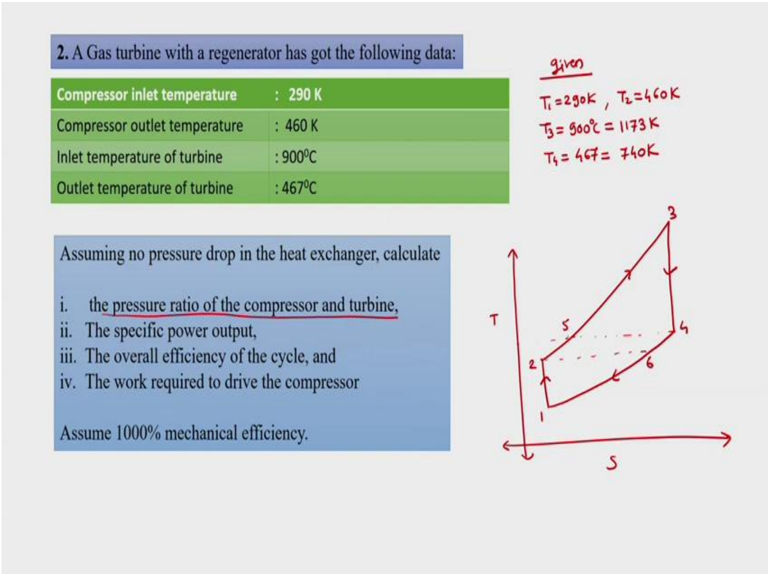

Next example says that there is a gas turbine with regenerator and it has got the following data. Compressor inlet temperature is given as 290 Kelvin, compressor outlet temperature is given as 460 Kelvin, inlet temperature to the turbine is given as 900 degree Celsius outlet temperature to the turbine is given as 467 degree Celsius, assuming low pressure drop in the heat exchanger. Calculate pressure ratio of the compressor and turbine specific power output, overall efficiency of the cycle and work required to drive the compressor.

Here we have to understand that since we are working with an ideal turbine and we are using here static properties for calculation of the cycle parameters we are always neglecting pressure losses. So, the turbine and compressor pressure ratio will be same. So, they are not separate things which we have to calculate but we can cross verify. Here the given things for us first of all again we need to plot same TS diagram for gas turbine power plant with regenerator.

We have perfect regenerator, it is told to us that it is 543 and then it is 6 and given things for us in this example are

$$T_1 = 290\ K\ ; T_2 = 460\ K; T_3 = 900^0C = 1173\ K\ ; T_4 = 467^0C = 740\ K$$

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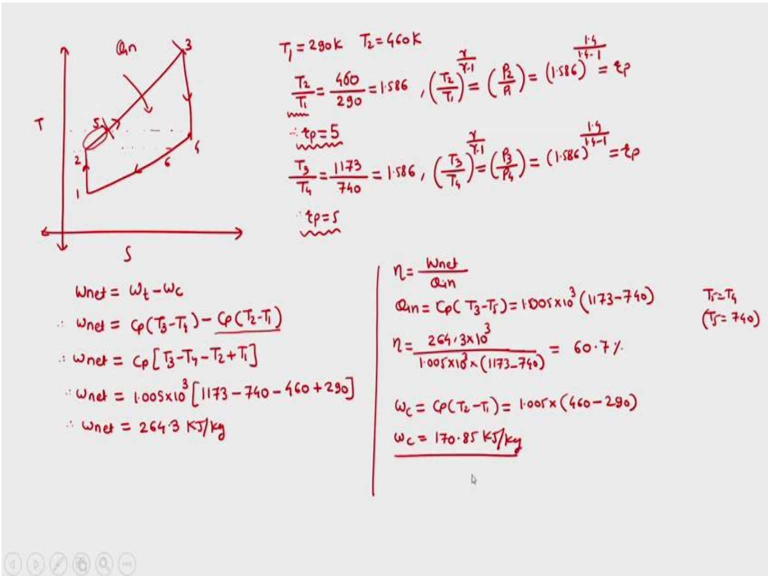

So, we will plot TS diagram for reference we have to keep this point in mind that the regenerator will be ideal if the temperatures are equal we are actually saying it if it has maximum efficiency. So if regenerator works with maximum efficiency with 100% efficiency then the outlet temperature for the heat exchanger in case of air is equal to outlet temperature for the gas in case of turbine. Let us move and then we can say

$$\frac{T_2}{T_1} = \frac{460}{290} = 1.586\ ; \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{P_2}{P_1}\right) = (1.586)^{\frac{1.4}{1.4-1}} = r_P$$

$$r_P = 5$$

$$\frac{T_3}{T_4} = \frac{1173}{740} = 1.586\ ; \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{P_3}{P_4}\right) = (1.586)^{\frac{1.4}{1.4-1}} = r_P$$

$$W_{net} = W_t - W_c = C_p[(T_3 - T_4) - (T_2 - T_1)]$$

$$W_{net} = C_p[T_3 - T_4 - T_2 + T_1]$$

$$W_{net} = 1.005 * 10^3 * [1173 - 740 - 460 - 290]$$

$$W_{net} = 264.3\ kJ/kg$$

$$\eta = \frac{W_{net}}{Q_{in}}$$

$$Q_{in} = C_P(T_3 - T_5) = 1.005 * 10^3 * (1173 - 740)$$

$$\eta = \frac{264.3 * 10^3}{1.005 * 10^3 * (1173 - 740)} = 60.7\ \%$$

$$W_c = C_P(T_2 - T_1) = 1.005 * (460 - 290) = 170.85\ kJ/kg$$

So, this is how we can find out and workout for the cases where we are having heat exchanger.

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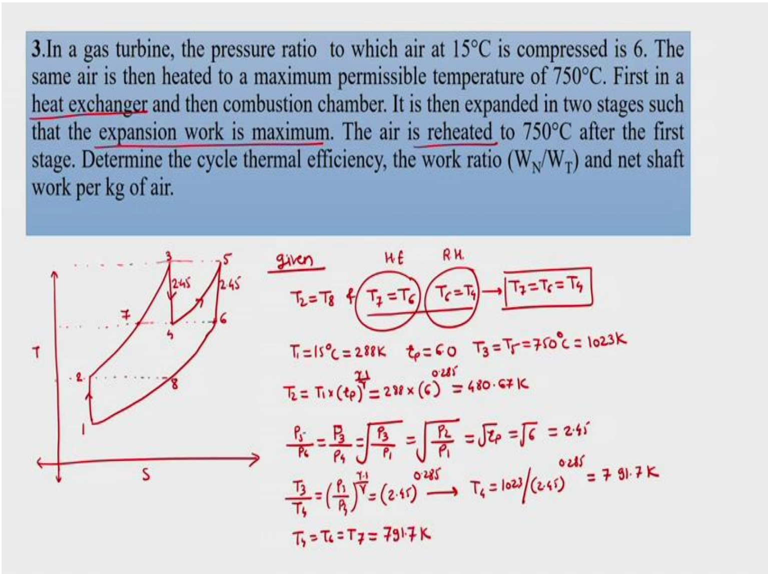

So let us go to the next example, in this example It is told that in gas turbine the pressure ratio to which air at 15 degree Celsius is compressed is 6. The same here is heated to maximum permissible temperature of 750 degree Celsius. First in heat exchanger and then in a combustion chamber so here air is not heated only in the combustion chamber but it is also heated in heat exchanger.

It is then expanded to two stages such that expansion work is maximum. The air is rich reheated to 750 degree Celsius after the first stage. Determine cycle efficiency work ratio and net shaft work per kg of air. So let us see this example has two attachments in it where it says that it is heated in heat exchanger and it is also reheated this example has everything that basically two things it as heat exchanger and it also has the re-heater but we have to consider.

Point to be noted in this case is that the expansion work is maximum in the two stages. And we have seen that it will be maximum, if both the turbines do same work. So and then that outlet temperature of both the turbines has to be same. So, 1 to 2 is compressor, 2 to 3 3 to 4 4 to 5, 5 to 6 then we also have heat exchanger and then that is also we are going to consider as ideal unless otherwise stated for the effectiveness.

We can so what we will have is the temperature and outlet of the turbine will be same as outlet of the heat exchanger. So, we have 7 here and we have the other case we have 8 here. So, we have maximum effectiveness of the heat exchanger this give us

$$T_2 = T_8 \, ; T_7 = T_6 ; T_6 = T_4 \rightarrow T_7 = T_6 = T_4$$

$$T_1 = 15^0 C = 288\ K \, ; r_P = 6 \, ; T_3 = T_5 = 750^0 C = 1023\ K$$

$$T_2 = T_1 * (r_P)^{\frac{\gamma-1}{\gamma}} = 288 * (6)^{0.285} = 480.67\ K$$

$$\frac{P_5}{P_6} = \frac{P_3}{P_4} = \sqrt{\frac{P_3}{P_1}} = \sqrt{\frac{P_2}{P_1}} = \sqrt{r_P} = \sqrt{6} = 2.45$$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = (2.45)^{0.285} \rightarrow T_4 = \frac{1023}{(2.45)^{0.285}} = 791.7\ K$$

$$T_4 = T_6 = T_7 = 791.7\ K$$

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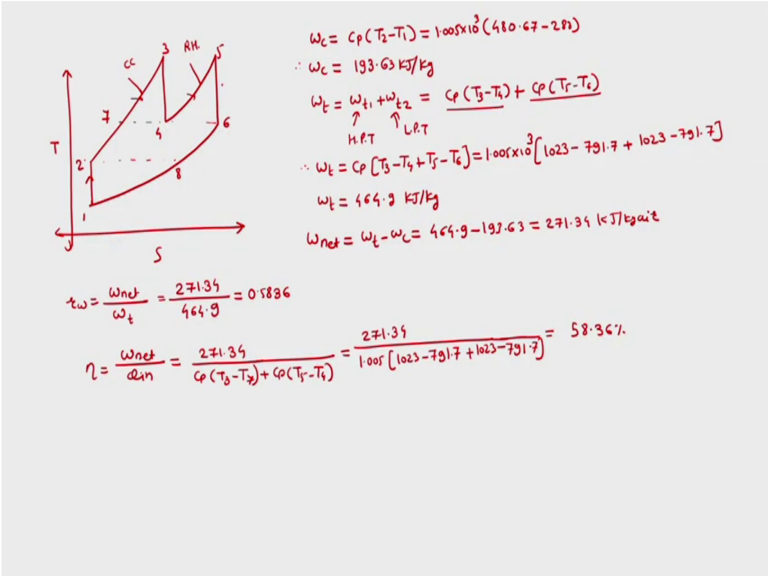

So we move ahead with this given things first we will replot the TS diagram. Such that the numbers will help us 1, 2, 3, 4, 5, 6, 7 and 8 now we can find out compressor work here and then turbine work and then net work for that compressor work it is only one compressor and then there is no mass flow rate is given, so it is specific work output which we can find out

$$W_c = C_P(T_2 - T_1) = 1.005 * 10^3 * (480.67 - 283)$$

$$W_c = 193.63\ kJ/kg$$

$$W_t = W_{t1} + W_{t2} = C_P(T_3 - T_4) + C_P(T_5 - T_6)$$

$$W_t = C_P[T_3 - T_4 + T_5 - T_6] = 1.005 * 10^3 * [1023 - 791.7 + 1023 - 791.7]$$

$$W_t = 464.9\ kJ/kg$$

$$W_{net} = W_t - W_c = 464.9 - 193.63 = 271.34 \frac{kJ}{kg}\ air$$

$$r_w = \frac{W_{net}}{W_t} = \frac{271.34}{464.9} = 0.5836$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{271.34}{C_p(T_3 - T_7) + C_p(T_5 - T_4)} = \frac{271.34}{1.005 * [1023 - 791.7 + 1023 - 791.7]} = 58.36\ \%$$

Now here the take away is that we will have the combustion chamber heat addition and the re-heater heat addition same in all the; if expansion is also ideal and there is no loss in the combustion chamber and there is effectiveness of heat exchanger also 100%.

One more thing what we took it from the theoretical part was the pressure ratio for the maximum net work was rP, suppose is the pressure ratio for compressor in both turbines will have their pressure ratio as the square root of rP will see the next example.

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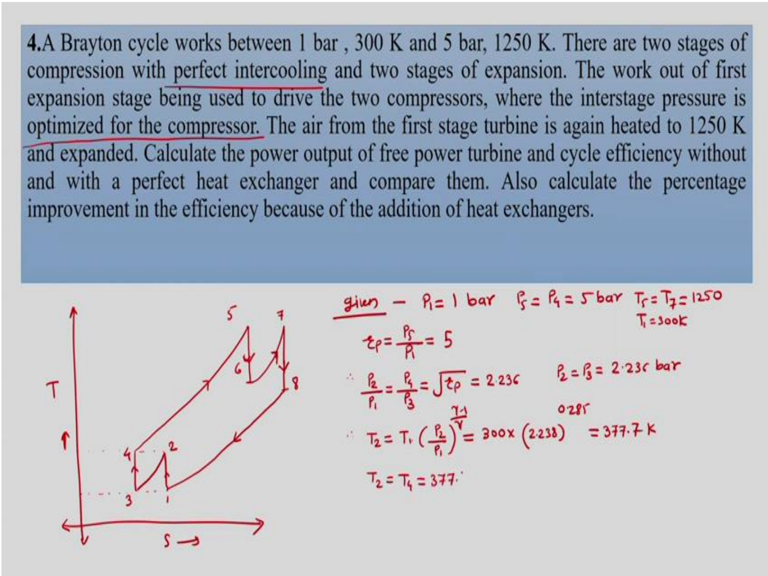

It state that a Brayton cycle works between 1 bar and 300 Kelvin and 5 bar, 1250 Kelvin the two stages of compression with perfect gas intercooling and two stages of expansion are there. The work output of first expansion stage being used to drive the two stage compressor where the interstage pressure is optimised for the compressor air from the first stage turbine is again heated 1250 Kelvin and expanded.

Calculate power output of free turbine and cycle efficiency without and with perfect heat exchanger and compare them. Also calculate the percentage improvement in efficiency because of addition of heat exchangers. So in this example, it should be noted that this example has heat exchanger, intercooler and re-heater. So, it has all the attachments, So in this example, let us first draw the TS diagram.

As per TS diagram we will have first as intercooler, so 1 to 2, 2 to 3 and 3 to 4 so it is compression and intercooler. It is told that intercooling process is optimised. So the temperature at the inlet is also same and temperature at outlet is also same for both the compressors. Since it is told that perfect intercooling is there then we have process of heat addition from 4 to 5 or we might have heat exchanger in the other part of the same example.

5 to 6 and 6 to 7, 7 to 8 it should be noted in the example that it is not told that the two stages of expansion are optimised. So, re-heating process does not have any optimum criteria so the T6 and T8 need not be same pressure ratio. P5 by P6 is not equal to P7 by P8 this should be noted. So, first we will solve without heat exchanger and then we are given here that

$$P_1 = 1\ bar\ ; P_5 = P_4 = 5\ bar\ ; T_5 = T_7 = 1250\ K\ ; T_1 = 300\ K$$

$$r_P = \frac{P_5}{P_1} = 5\ ; \frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt{r_P} = 2.236\ ; P_2 = P_3 = 2.236\ bar$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 * (2.238)^{0.285} = 377.7\ K$$

$$T_2 = T_4 = 377.7\ K$$

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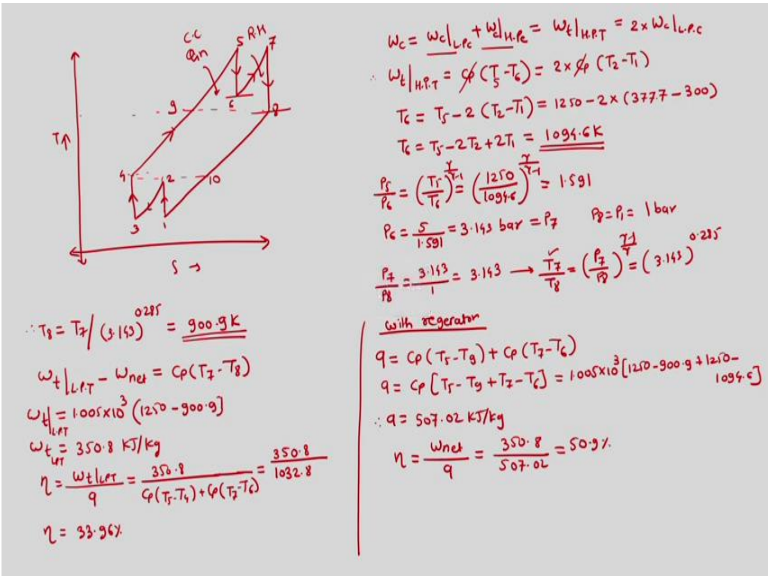

In this example, we are told that both the compressors are driven by the one of the turbines it is the high pressure turbine which is driving both compresses then there is a free turbine or power turbine. And that power turbine is basically giving the power output. So, we actually are not knowing what is the pressure ratio for both the turbines. So first we should be finding out the pressure ratio.

Before that we can find out before that we can find out the temperature at outlet of the high pressure turbine. Since we are told that it is giving the work output as equal to the work required in the process of compression. So, we will equate it. So we have compressor work is equal to compressor which is low pressure compressor plus compressor which is high pressure compressor and that is equal to one of the turbines which is high pressure turbine and then that is Wt HPT.

So, we have;

$$W_C = W_{c|_{LPC}} + W_{c|_{HPC}} = W_{t_{HPT}} = 2 * W_{c_{LPC}}$$

$$W_{t_{HPT}} = C_P(T_5 - T_6) = 2 * C_p * (T_2 - T_1)$$

$$T_6 = T_5 - 2(T_2 - T_1) = 1250 - 2 * (377.7 - 300)$$

$$T_6 = T_5 - 2T_2 + 2T_1 = 1094.6\ K$$

$$\frac{P_5}{P_6} = \left(\frac{T_5}{T_6}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1250}{1094.6}\right)^{\frac{\gamma}{\gamma-1}} = 1.591$$

$$P_6 = \frac{5}{1.591} = 3.143\ bar = P_7\ ; P_8 = P_1 = 1\ bar$$

$$\frac{P_7}{P_8} = 3.143 \rightarrow \frac{T_7}{T_8} = \left(\frac{P_7}{P_8}\right)^{\frac{\gamma-1}{\gamma}} = (3.143)^{0.285}$$

$$T_8 = \frac{T_7}{(3.143)^{0.285}} = 900.9\ K$$

$$W_{t|_{LPT}} - W_{net} = C_p(T_7 - T_8)$$

$$W_{t|_{LPT}} = 1.005 * 10^3 * (1250 - 900.9)$$

$$W_{t|_{LPT}} = 350.8\ kJ/kg$$

$$\eta = \frac{W_{t|_{LPT}}}{q} = \frac{350.8}{C_p(T_5 - T_4) + C_p(T_7 - T_6)} = \frac{350.8}{1032.8} = 33.96\ \%$$

We can now work with regenerator. With regenerator turbine and compressor work will not get altered only the thing which is going to alter is Q. And now with the regenerator we have one point which is 5 rather it is a perfect regenerator. So, perfect regenerator will have point which is 9 here that will have same temperature as 8 if it is perfect regenerator and then it will give us a temperature which is 10 at the exit of the heat exchanger for gas.

With regeneration

$$q = C_p(T_5 - T_9) + C_p(T_7 - T_6) = C_p[T_5 - T_9 + T_7 - T_6]$$
$$= 1.005 * 10^3 * [1250 - 900.9 + 1250 - 1094.6] = 507.02\ kJ/kg$$

$$\eta = \frac{W_{net}}{q} = \frac{350.8}{507.02} = 50.9\ \%$$

So we have to remember a point over here that if the turbines are not optimised in the re-heating process, then the turbine work will not be same and pressure ratio will be same.

So, we have to take care for that but you heat exchanger perfectly optimised and heat exchanger is having 100% effectiveness than the exit temperature for the high pressure turbine will be the same as the exit temperature for the heat exchanger for the air sorry. And then this we can take for solving the example. Thank you.