Aircraft Propulsion

Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture – 11
Brayton Cycle with Intercooler

Welcome to the class we have seen that how does the Brayton cycle work and how it is useful for aircraft propulsion. We have found out what is the optimum case in which a Brayton cycle should operate and then what is corresponding Net work and efficiency. Then we have thought that we can improve the performance of the Brayton cycle by having certain attachment and we have seen that there can be an attachment of heat exchanger and there can be also an attachment of re-heater.

So those attachments have their own advantages now we are going to see in today's class Brayton cycle with intercooler or Brayton cycle with inter-cooling. Here we will see what is the use of intercooler and how does Brayton cycle's operation changes or how does Brayton cycle's operation change with the attachment of intercooler. We would also see the performance of Brayton cycle with all the attachments whatever we have considered.

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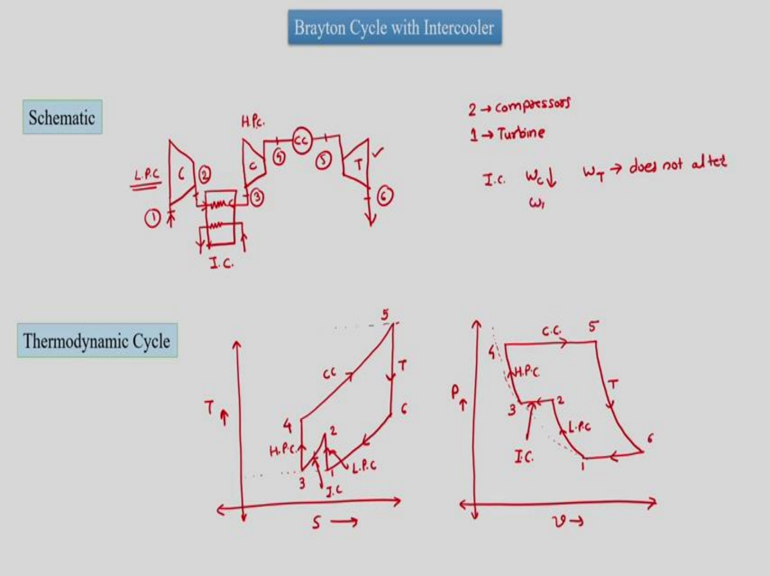

Brayton Cycle with Intercooler

Schematic

2 → compressors
1 → Turbine
I.C. $W_c \downarrow$ W_T → does not alter

Thermodynamic Cycle

So, let us start with Brayton cycle with intercooler now play here we will first draw the schematic and schematics is that there will be one compressor and then this compressor would compress but this is for us now the low pressure compressor. After low pressure compressor before going to high pressure compression this air will get cooled and then it would enter into the high pressure compressor this is high pressure compressor and then there is cooling of this air using some coolant.

And then this is what we are referring here as intercooler. Then from high pressure compressor air will go into the combustion chamber and from combustion chamber air will enter into the turbine. So, here we will have state 1 here, state 2 here, state 3, state 4, state 5 and state 6. So, as what we can see in case of re-heater we had 2 turbines and one compressor in case of intercooler we have with this schematic we have 2 compressors and we have one turbine. This is according to the present schematic.

Then we can draw the thermodynamic cycle for the inter-cooling and this cycle will draw first TS diagram and as per TS diagram state 1 to 2 it is compression which is isentropic so we are at state 1 and state 2 is isentropic compression then state 2 to 3 is intercooler and as what we know in Brayton cycle other heater heat addition or heat rejection processes are isobaric this is also an isobaric cooling.

Since it is cooling temperature will decrease entropy will also decrease and it can come to any temperature down but we will refer it coming down to same temperature as 1. So, the state is 3 temperature is equal to state 1 temperature. Then we will have 3 to 4 as high pressure compression. So, we will have state 4 here then from state 4 to state 5 we have heat addition isobarically.

Then 5 to 6 we have isentropic expansion and 6 to 1 we have heat rejection. So, this is the thermodynamic cycle with inter-cooling for Brayton cycle. Now we will see how PV diagram would be for the inter-cooling process in Brayton cycle. So, here we have 1 to 2 as isentropic compression and 2 to 3 is cooling isobaric. So, this is 3 and practically we will have same temperature this is Isothermal line on the PV diagram.

And 3 to 4 is this, we have; we will see this is isothermal line this is 3 to 4 is isentropic compression 4 to 5 is isobaric heat addition 5 to 6 is isentropic expansion and 6 to 1 is again isobaric heat rejection. So, here we have low pressure compressor here we have high pressure compressor, here we have combustion chamber and turbine. Similarly in the TS diagram here we have low pressure compressor here we have high pressure compressor combustion chamber turbine and this is the place where we are having intercooler.

Here we are having intercooler in 2 to 3 so this is how we will have Brayton cycle diagram with the intercooler. Here as what we can see there is no alteration in the turbine work turbine one does not get altered so far as the temperature 5 is same without or with inter-cooling this 6th is same then we will have same turbine work output inter-cooling just reduces the compressor work.

So, mainly inter-cooling reduces compressor work and it does not alter the turbine work. So, in general W net increases with inter-cooling.

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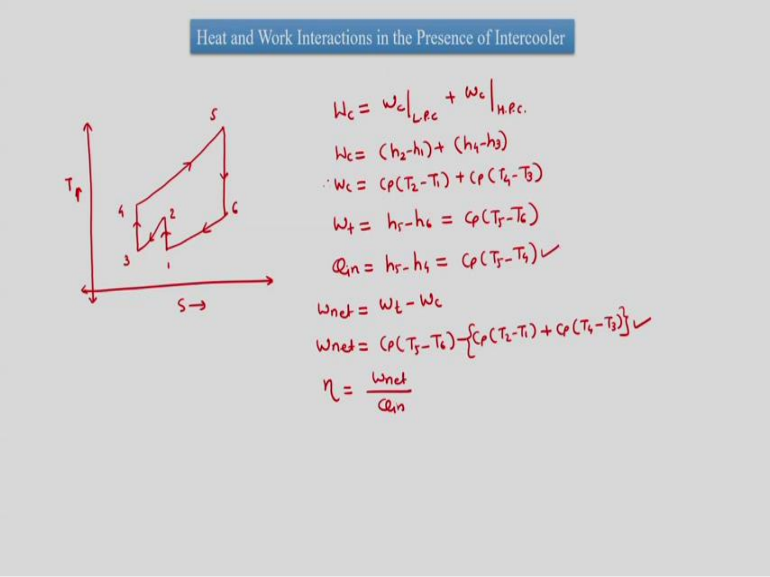

Heat and Work Interactions in the Presence of Intercooler

$$W_c = W_c\big|_{L.P.C} + W_c\big|_{H.P.C.}$$

$$W_c = (h_2-h_1) + (h_4-h_3)$$

$$\therefore W_c = C_p(T_2-T_1) + C_p(T_4-T_3)$$

$$W_t = h_5-h_6 = C_p(T_5-T_6)$$

$$Q_{in} = h_5-h_4 = C_p(T_5-T_4) \checkmark$$

$$W_{net} = W_t - W_c$$

$$W_{net} = C_p(T_5-T_6) - \{C_p(T_2-T_1) + C_p(T_4-T_3)\} \checkmark$$

$$\eta = \frac{W_{net}}{Q_{in}}$$

So, we will see about it then for that let us consider the heat and work interactions in the presence of intercooler. So, for that all sake we will again draw TS diagram for the intercooler T y-axis and S in the x-axis. So, for us 1 to 2 is isentropic compression 2 to 3 is isobaric heat rejection in intercooler, 3 to 4 is isentropic compression, 4 to 5 is heat addition in combustion chamber isobaric 5 to 6 is expansion in turbine, 6 to 1 is again heat rejection. So, considering this we can write down

$$W_c = W_c|_{L.P.C} + W_c|_{H.P.C}$$

$$W_c = (h_2 - h_1) + (h_4 - h_3)$$

$$W_c = C_p(T_2 - T_1) + C_p(T_4 - T_3)$$

$$W_t = h_5 - h_6 = C_p(T_5 - T_6)$$

$$Q_{in} = h_5 - h_4 = C_p(T_5 - T_4)$$

$$W_{net} = W_t - W_c$$

$$\mathrm{W_{net}} = C_p(T_5 - T_6) - \{C_p(T_2 - T_1) + C_p(T_4 - T_3)\}$$

$$\eta = \frac{W_{net}}{Q_{in}}$$

So, this is how we can find out work and heat interactions and the performance for Brayton cycle with intercooler.

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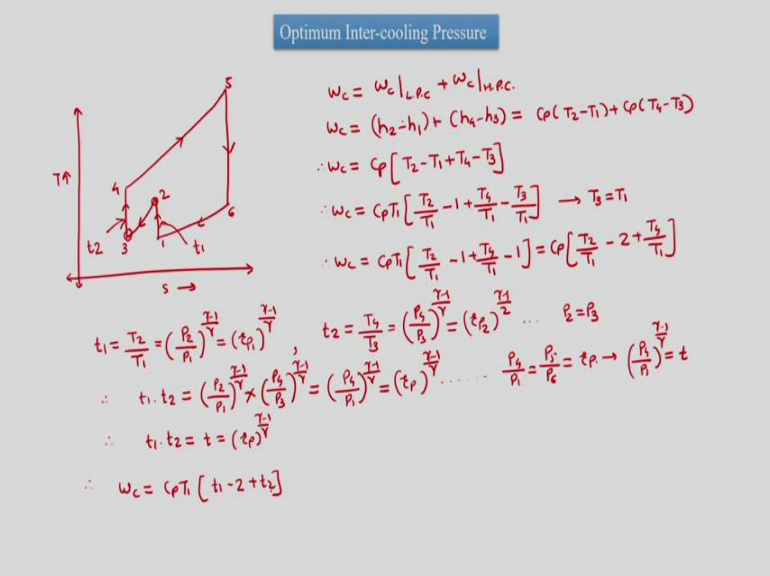

Optimum Inter-cooling Pressure

$$w_c = w_c\big|_{L.P.C} + w_c\big|_{H.P.C.}$$

$$w_c = (h_2 - h_1) + (h_4 - h_3) = C_p(T_2 - T_1) + C_p(T_4 - T_3)$$

$$\therefore w_c = C_p\left[T_2 - T_1 + T_4 - T_3\right]$$

$$\therefore w_c = C_p T_1\left[\frac{T_2}{T_1} - 1 + \frac{T_4}{T_1} - \frac{T_3}{T_1}\right] \longrightarrow T_3 = T_1$$

$$\therefore w_c = C_p T_1\left[\frac{T_2}{T_1} - 1 + \frac{T_4}{T_1} - 1\right] = C_p\left[\frac{T_2}{T_1} - 2 + \frac{T_4}{T_1}\right]$$

$$t_1 = \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (t_{p_1})^{\frac{\gamma-1}{\gamma}}, \quad t_2 = \frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = (t_{p_2})^{\frac{\gamma-1}{2}} \cdots \quad P_2 = P_3$$

$$\therefore t_1 \cdot t_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \times \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (t_p)^{\frac{\gamma-1}{\gamma}} \cdots\cdots \quad \frac{P_4}{P_1} = \frac{P_5}{P_6} = t_p \rightarrow \left(\frac{P_5}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = t$$

$$\therefore t_1 \cdot t_2 = t = (t_p)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore w_c = C_p T_1\left[t_1 - 2 + t_2\right]$$

But we have to argue with a point that we did stop inter-cooling at a state and that state was 3 for us, so we did compression in low-pressure compressor from 1 to 2 and then we did inter-cooling from 2 to 3. So, this state 3 at this moment is in argument and we should find out what is this optimum location for state 3 such that we will have minimum compressor work. So, we will try to find out what is the condition in which we can have minimum compressor work input such that we will have the point 3 accordingly chosen.

So for us we will again write down

$$W_c = W_c|_{L.P.C} + W_c|_{H.P.C}$$

$$W_c = (h_2 - h_1) + (h_4 - h_3)$$

$$W_c = C_p[T_2 - T_1 + T_4 - T_3]$$

$$W_c = C_pT_1\left[\frac{T_2}{T_1} - 1 + \frac{T_4}{T_1} - \frac{T_3}{T_1}\right] \rightarrow T_3 = T_1$$

$$W_c = C_pT_1\left[\frac{T_2}{T_1} - 1 + \frac{T_4}{T_1} - 1\right] = C_p\left[\frac{T_2}{T_1} - 2 + \frac{T_4}{T_1}\right]$$

$$t_1 = \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_{P_1}\right)^{\frac{\gamma-1}{\gamma}}, t_2 = \frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_{P_2}\right)^{\frac{\gamma-1}{\gamma}} \dots P_2 = P_3$$

$$t_1.t_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} * \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} \dots .\frac{P_4}{P_1} = \frac{P_5}{P_6} = r_P \rightarrow \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = t$$

$$t_1.t_2 = t = \left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

$$W_c = C_pT_1[t_1 - 2 + t_2]$$

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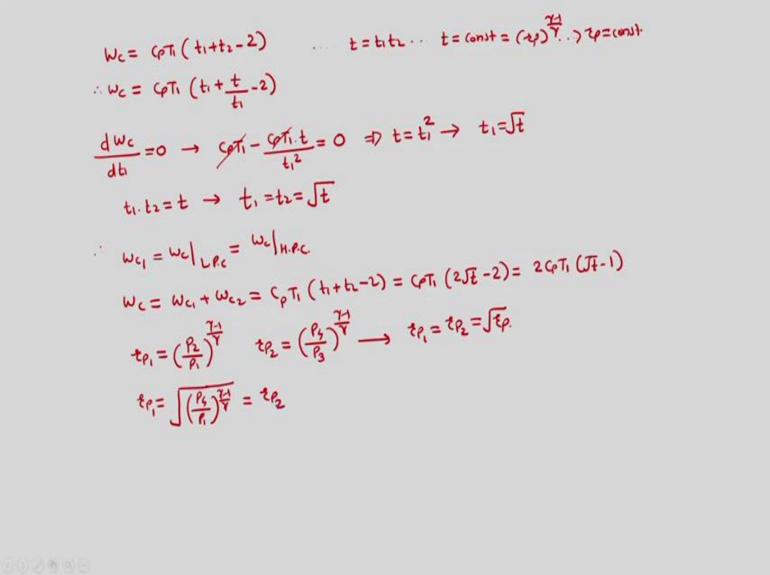

$W_c = C_pT_1(t_1+t_2-2)$ .... $t = t_1t_2$ .. $t = const = (r_p)^{\frac{\gamma-1}{\gamma}}$ .. $r_p = const.$

$\therefore W_c = C_pT_1\left(t_1 + \frac{t}{t_1} - 2\right)$

$\frac{dW_c}{dt_1} = 0 \rightarrow \cancel{C_pT_1} - \frac{\cancel{C_pT_1}.t}{t_1^2} = 0 \Rightarrow t = t_1^2 \rightarrow t_1 = \sqrt{t}$

$t_1.t_2 = t \rightarrow t_1 = t_2 = \sqrt{t}$

$\therefore W_{c_1} = W_c|_{L.P.C} = W_c|_{H.P.C}$

$W_c = W_{c_1} + W_{c_2} = C_pT_1(t_1+t_2-2) = C_pT_1(2\sqrt{t}-2) = 2C_pT_1(\sqrt{t}-1)$

$t_{P_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ $t_{P_2} = \left(\frac{P_5}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \longrightarrow t_{P_1} = t_{P_2} = \sqrt{t_P}.$

$t_{P_1} = \sqrt{\left(\frac{P_5}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} = t_{P_2}$

$$W_c = C_pT_1(t_1 + t_2 - 2) \dots . t = t_1t_2 \dots t = const = \left(r_p\right)^{\frac{\gamma-1}{\gamma}}; r_p = const.$$

$$W_c = C_pT_1\left(t_1 + \frac{t}{t_1} - 2\right)$$

$$\frac{dW_c}{dt_1} = 0 \rightarrow C_pT_1 - C_pT_1.\frac{t}{t_1^2} = 0 \rightarrow t = t_1^2 \rightarrow t_1 = \sqrt{t}$$

$$t_1.t_2 = t \rightarrow t_1 = t_2 = \sqrt{t}$$

$$W_{c1} = W_{c|L.P.C} = W_{c|H.P.C}$$

$$W_c = W_{c1} + W_{c2} = C_pT_1(t_1 + t_2 - 2) = C_pT_1(2\sqrt{t} - 2) = 2C_pT_1(\sqrt{t} - 1)$$

$$r_{p1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} ; r_{p2} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow r_{p1} = r_{p2} = \sqrt{r_p}$$

$$r_{p1} = \sqrt{\left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} = r_{p2}$$

So, this is how we can find out the optimum case.

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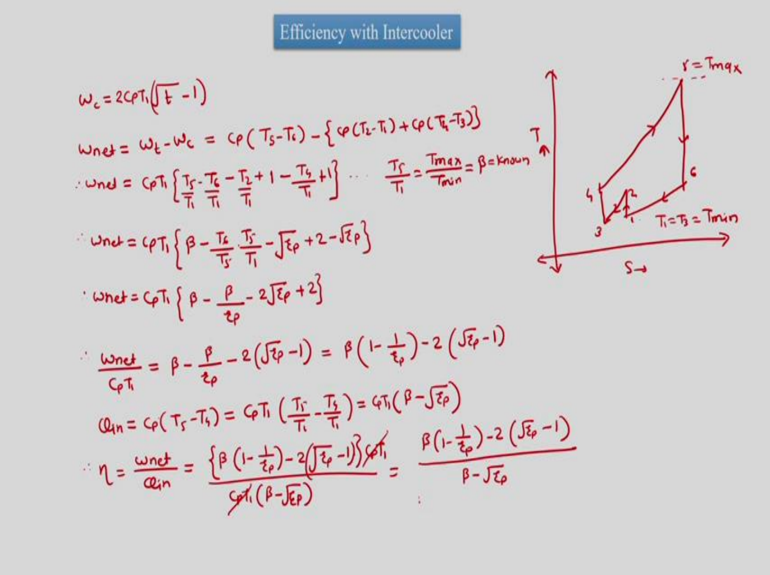

Efficiency with Intercooler

$W_c = 2C_pT_1(\sqrt{t} - 1)$

$W_{net} = W_t - W_c = C_p(T_5 - T_6) - \{C_p(T_2 - T_1) + C_p(T_4 - T_3)\}$

$\therefore W_{net} = C_pT_1\left\{\frac{T_5}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1 - \frac{T_4}{T_1} + 1\right\}$ ... $\frac{T_5}{T_1} = \frac{T_{max}}{T_{min}} = \beta = \text{known}$

$\therefore W_{net} = C_pT_1\left\{\beta - \frac{T_6}{T_5}\frac{T_5}{T_1} - \sqrt{t_p} + 2 - \sqrt{t_p}\right\}$

$\therefore W_{net} = C_pT_1\left\{\beta - \frac{\beta}{t_p} - 2\sqrt{t_p} + 2\right\}$

$\therefore \frac{W_{net}}{C_pT_1} = \beta - \frac{\beta}{t_p} - 2(\sqrt{t_p} - 1) = \beta\left(1 - \frac{1}{t_p}\right) - 2(\sqrt{t_p} - 1)$

$Q_{in} = C_p(T_5 - T_4) = C_pT_1\left(\frac{T_5}{T_1} - \frac{T_4}{T_1}\right) = C_pT_1(\beta - \sqrt{t_p})$

$\therefore \eta = \frac{W_{net}}{Q_{in}} = \frac{\left\{\beta\left(1 - \frac{1}{t_p}\right) - 2(\sqrt{t_p} - 1)\right\}\cancel{C_pT_1}}{\cancel{C_pT_1}(\beta - \sqrt{t_p})} = \frac{\beta\left(1 - \frac{1}{t_p}\right) - 2(\sqrt{t_p} - 1)}{\beta - \sqrt{t_p}}$

Then we will find out what is the optimum work total work and what with the corresponding efficiency. So,

$$W_c = 2C_pT_1(\sqrt{t} - 1)$$

$$W_{net} = W_t - W_c = C_P(T_5 - T_6) - \{C_P(T_2 - T_1) + C_P(T_4 - T_3)\}$$

$$\mathrm{W_{net}} = C_P T_1 \left\{\frac{T_5}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1 + \frac{T_4}{T_1} - 1\right\} \dots \frac{T_5}{T_1} = \frac{T_{max}}{T_{min}} = \beta = known$$

As per the Brayton cycle if we can try to plot it was T on y-axis and S on x-axis, so we have 1 to 2 low-pressure compressor 2 to 3 intercooler 3 to 4 high pressure compressor 4 to 5 as heat addition and then 5 to 6, here $T_5 = T_{max}$ and $T_1 = T_3 = T_{min}$.

$$W_{net} = C_P T_1 \left\{\beta - \frac{T_6}{T_5}\frac{T_5}{T_1} - \sqrt{r_P} + 2 - \sqrt{r_P}\right\}$$

$$W_{net} = C_P T_1 \left\{\beta - \frac{\beta}{r_P} - 2(\sqrt{r_P} - 1)\right\} = \beta\left(1 - \frac{1}{r_P}\right) - 2(\sqrt{r_P} - 1)$$

$$Q_{in} = C_P(T_5 - T_4 = C_p T_1 \left(\frac{T_5}{T_1} - \frac{T_4}{T_1}\right) = C_p T_1(\beta - \sqrt{r_P})$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{\left\{\beta\left(1 - \frac{1}{r_P}\right) - 2(\sqrt{r_P} - 1)\right\}}{(\beta - \sqrt{r_P})} = \frac{\beta\left(1 - \frac{1}{r_P}\right) - 2(\sqrt{r_P} - 1)}{(\beta - \sqrt{r_P})}$$

So, this is the formula for efficiency with intercooler for optimum case. Having said this we can now find out how does efficiency vary with these cases and for that we will see how does net work and efficiency vary with pressure ratio.

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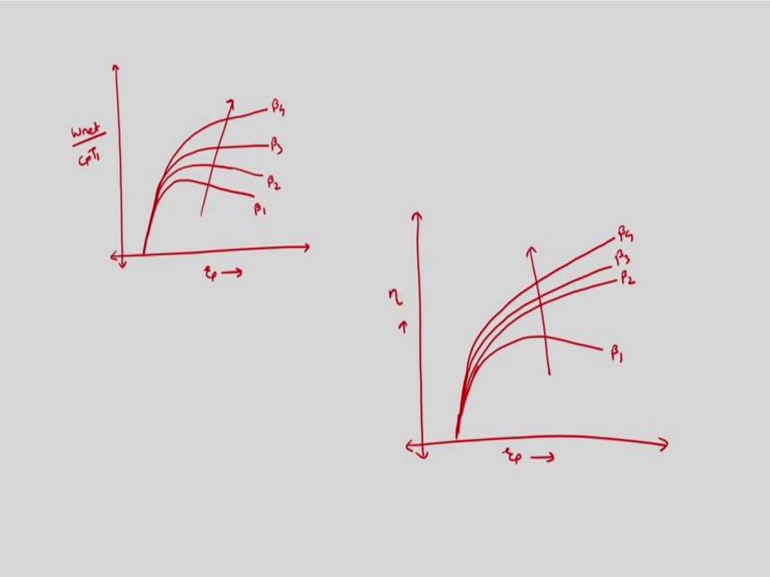

As we have found out the relations for optimum case or for the pressure ratio variation. So, we have $\frac{W_{net}}{C_p T_1}$ on y-axis and pressure ratio on x-axis, so this is a standard variation for a given β_1 for the net work, so β_2, this is β_3 and this is β_4. Now if we see efficiency then this is efficiency with respect to pressure ratio and efficiency would vary as this everything will start from r_{p_1}, so this is β_1, this is β_2, this is β_3, this is β_4 and beta is increasing in this direction. So, this is the variation of efficiency and net work for the intercooler attachment of Brayton cycle.

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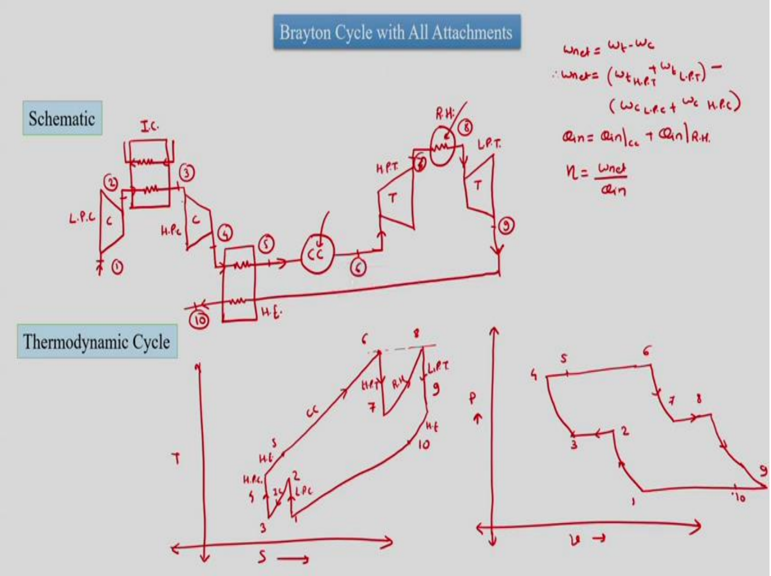

Brayton Cycle with All Attachments

Schematic

Thermodynamic Cycle

$$w_{net} = w_t - w_c$$

$$\therefore w_{net} = (w_{t_{H.P.T}} + w_{t_{L.P.T}}) - (w_{c_{L.P.C}} + w_{c_{H.P.C}})$$

$$Q_{in} = Q_{in}|_{cc} + Q_{in}|_{R.H.}$$

$$\eta = \frac{w_{net}}{Q_{in}}$$

Now let us consider if we have all the attachments in the Brayton cycles practically we mean 3 attachments where we have intercooler, heat exchanger and then the re-heater. So, schematic would be first we have one which is a low pressure compressor so this is a compressor and this is a low pressure compressor from the low pressure compressor air will go into the intercooler. So, we have intercooler attached along with the compressors. So, from low pressure compressor air will go to the intercooler and then high pressure compressor.

From high pressure compressor air will enter into the heat exchanger. So, we draw heat exchanger this is air entry into the heat exchanger and then it will go into the combustion chamber from the combustion chamber air will enter into the first high pressure turbine. So, this is turbine and practically high pressure turbine. Then we will have it entered into the re-heater. This is the arrangement for re-heater where we will have heat addition.

So, this is low pressure turbine and from the low pressure turbine the exhaust will be sent into the heat exchanger and then this is the heat exchanger which has entrance for the air from low-pressure turbine and then also air from the high-pressure compressor. So, suppose this is 1 this is 2 this is 3 this is 4, 5, 6, 7 then we will have air this is re-heater this is 8, 9 and then 10. So, we have such all arrangements.

So now we can see what is the thermodynamic diagram cycle. So, here we have T on y-axis and S entropy on x-axis. So, 1 to 2 is low-pressure compressor, so we have 1 to 2 low-pressure compressor then it is going into the intercooler. So, 2 to 3 is intercooler 3 to 4 is high-pressure compressor after high pressure compressor 5 to 6, 4 to 5 is heat exchanger and then 5 to 6 in the same pressure into the combustion chamber then 6 to 7 into the turbine, 6 to 7 is into the high pressure turbine and from high pressure turbine air is going into the re-heater.

So, re-heater process will take place till 8 and then we have 8 to 9 low-pressure turbine and then from 9 to 10 we have heat rejection into the heat exchanger and 10 to 1 is heated rejection into the atmosphere for open cycle gas turbine power plant. So, we have one here low-pressure compressor here we have intercooler here we have high-pressure compressor it is heat exchanger for the air.

This is combustion chamber this is high pressure turbine this is re-heater this is low pressure turbine and this is heat exchanger for gas. So, these are the different processes taking place in 2 different components. Now we will plot PV diagram for this from PV diagram says that we have 1 to 2 as isentropic compression, 2 to 3 as inter-cooling, 3 to 4 as high pressure compressor which is isentropic compression, 4 to 5 is heat exchanger isobaric heat addition.

5 to 6 is heat addition into the heat exchanger sorry combustion chamber then 6 to 7 is isentropic expansion in high pressure turbine. Then 7 to 8 we have re-heater heat addition in the re-heater which is isobaric, 8 to 9 we have low pressure expansion into the turbine. Then 9 to 10 we have heat exchanger for gases and then we have 10 to 1 as the process 9 is here this is 9 this is 10. So, this is how we have thermodynamic cycle if we have Brayton cycle with all possible attachments.

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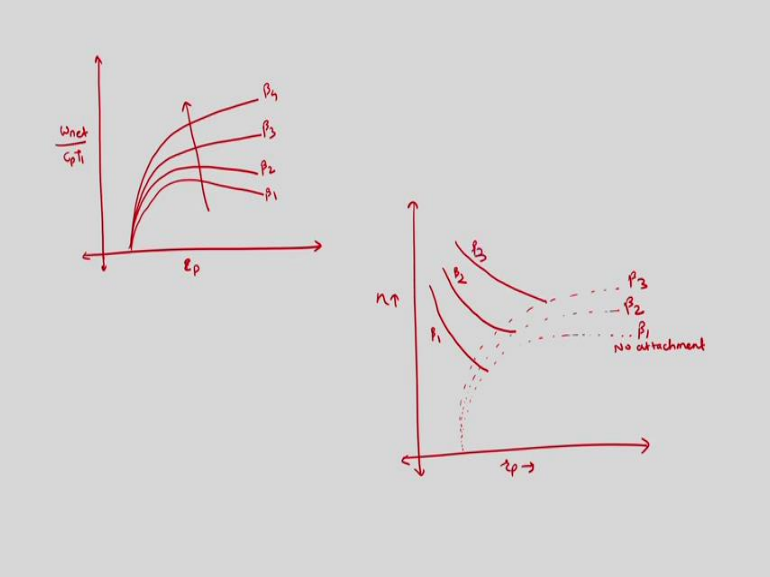

So for these attachments again we can see how does the net work output and net efficiency would vary. So, this is the variation of $\frac{W_{net}}{C_p T_1}$ and then this is r_p it is simple over here to find out we know here

$$W_{net} = W_t - W_c = \left(W_{t_{H.P.T}} + W_{t_{L.P.T}}\right) - \left(W_{c_{L.P.C}} + W_{c_{H.P.C}}\right)$$

$$Q_{in} = Q_{in|_{cc}} + Q_{in|_{R.H}}$$

$$\eta = \frac{W_{net}}{Q_{in}}$$

So there is external heat addition only in 2 processes this is in the heated combustion chamber and this is in the re-heater. So, this if we try to find out and then we can we find out what is the optimum case but we will get same for the reheating we will get a condition we which states that both the turbine show the same work.

If reheating is there both the compressors will do same work if inter-cooling is there and then the temperature at the exit of the heat exchanger should be same as the temperature at the exit of the low-pressure turbine. So, these things whatever we have found out for the individual attachments would continue to be present for the complete set of attachments. So, hence we can find out what is the variation of $\frac{W_{net}}{C_p T_1}$ then this will vary with different betas.

So, this is for β_1, β_2, β_3, β_4 as said this is the increment of beta then we have efficiency calculated varying with pressure ratio first we will find out efficiency which is for no attachment case this is efficiency or no attachment. And for no attachment further we have efficiency for different betas with each attachment. So, this is efficiency for β_2, this is efficiency for β_3 and then we will get no attachment this is β_1.

So, we will have efficiency β_1 here efficiency β_2 here this is efficiency β_3 here. So, this there is no idea no good idea to continue use of all attachments beyond this pressure ratio for β_1. Since it decreases the efficiency as in case of without any attachment, so
with all attachments the efficiency is decreasing with beta and for β_1 with increasing pressure ratio but beyond this point it was below the efficiency without attachment.

So, we can consider this efficiency variation for designing the Brayton cycle with different attachments. So, this is how we will consider Brayton cycle for inter-cooling or considering Brayton cycle for different attachments. thank you.