Aircraft Propulsion
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Lecture – 10
Brayton Cycle With Heat Exchanger Re-Heater

Welcome to the class we have seen what do we mean Brayton cycle and how does an ideal Brayton cycle thermodynamically would look like. I have also done calculation for the ideal Brayton cycle and also for the Brayton cycle with the non ideal components. Now, we are going to see some attachments to the Brayton cycle and today's class we are going to deal among those as heat exchanger and re-heater.

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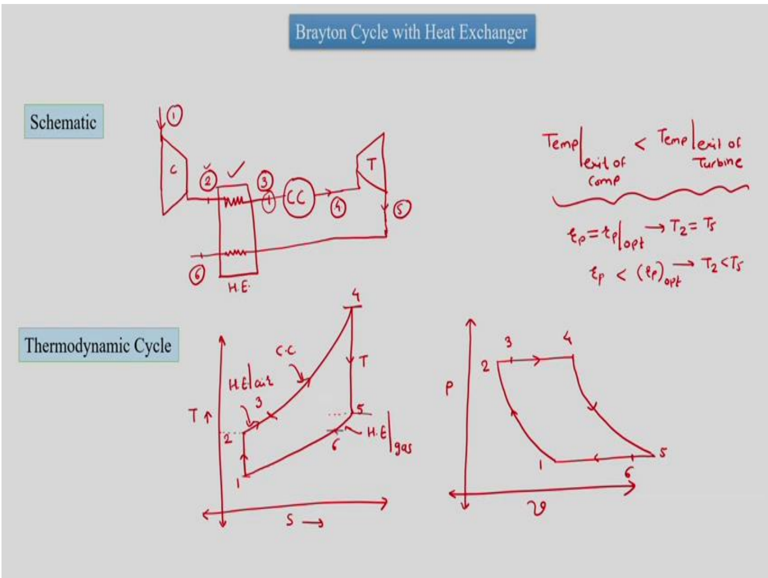

So, let us start practically; let us see first Brayton cycle with heat exchanger. Schematic we have seen it when we were considering different configurations. Here we will first have a compressor as we would have in general; air will come first in to the compressor and then while going in towards the combustion chamber it will get heated. From the state 2 to state 3 then it will appear in to a combustion chamber then it will have combustion process from state 3 to state 4.

At 4 it will enter into a gas turbine. So this is gas turbine and the process ends for the expansion isentropic expansion in state 5. So, then the air the gas at state 5 gives its heat to the air which is going into the combustion chamber and this heat transfer will take place at state 5 and state

6 and this heat transfer unit is called as heat exchanger. So this is turbine, this is how we have seen as an arrangement for schematic for Brayton cycle with heat exchanger. This is the exclusive heat exchanger we are going to see.

Now we will see about the Thermodynamics cycle. So, thermodynamics cycle with heat changer would not get largely altered but we have to show point number 4 and 6. Let us plot T and S then we have 1 to 2 isentropic compression and 2 to 3 is heat transfer in the heat exchanger. 3 to 4, is heat addition in the combustion chamber? We should note that the pressure in the heat exchanger here and pressure in the combustion chamber here is same and both are entertaining constant pressure heat interaction.

4 to 5 is turbine 5 to 6 is heat transfer in heat exchanger on gas side, and this is air side and 6 to 1 it is heat rejection. And again we have pressure at 5, 6 and 1 is same. So, looking at these we can draw PV diagram for Brayton cycle with heat exchanger and then we can show that 1 to 2 is isentropic compression 2 to 3 is heat addition into the heat exchanger 3 to 4 is heat addition into the combustion chamber and then we have 4 to 5 is isentropic expansion and 5 to 6 is the heat addition into the heat rejection into the heat exchanger.

This is how we have the schematic and thermodynamics cycle, the point which we should note here that the temperature at 2 has to be lower than temperature at 5. That means the temperature at the exit of compressor has to be lower than temperature at the exit of turbine to implement the heat exchanger. So, this remains the criteria to implement heat exchanger practically we have seen that when we are working on pressure ratio, which is optimum pressure ratio.

Then in that case we have seen that T2 is equal to T5 as per this diagram. So, now since this remained the constraint so we should work on the pressure ratios which are lower than rP optimum and those pressure ratios will lead to T2 to be less than T5 and then we can implement a heat exchanger. So, this is what we should remember as the constraint to implement heat exchanger.

$$Temp|_{Exit\ of\ compressor} < Temp|_{Exit\ of\ turbine}$$

$$r_p = r_{p_{opt}} \rightarrow T - 2 = T_5$$

$$r_p < r_{p_{opt}} \rightarrow T_2 < T_5$$

The idea of putting heat exchanger is that instead of passing the air from station 2 into the combustion chamber and heating it to 4 we are passing it at the station 3 to reheat it at 4. So practically the mean temperature of heat addition would get increased. Now instead of having air heated instead of having air heated from 2 to 4 it got heated from 3 to 4 so mean temperature of heat addition has increased parallely mean temperature of heat rejection has also decreased.

Since we do not reject the heat from 5 to 1 we reject the heat from 6 to 1. So since mean temperature of heat addition increase and mean temperature of heat rejection would decrease the efficiency of Brayton cycle practically will increase, so, importance of heat exchanger is to increase the efficiency. In this remains the constraint for implementation of heat exchanger.

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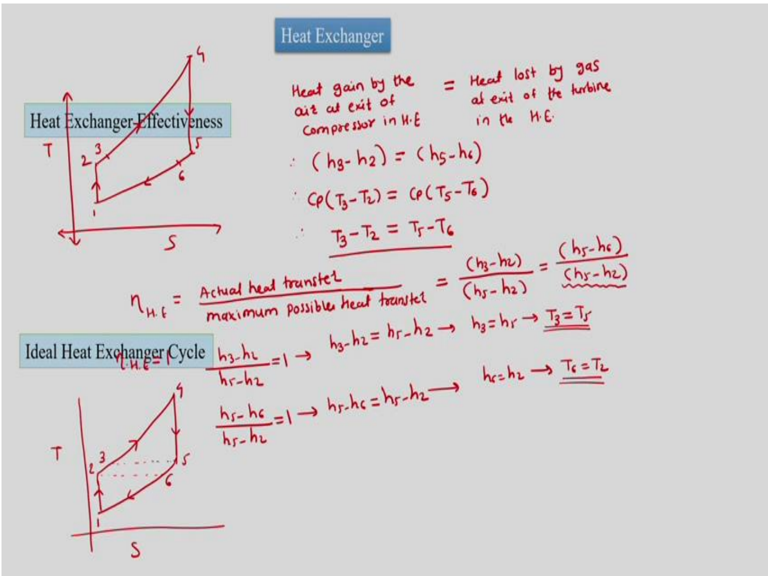

So, now we will again plot TS diagram for the heat exchanger so as to calculate heat exchanger calculation. So, this is TS so 1 to 2, to 3, to 4, to 5, to 6 in the heat exchanger, we should say that amount of heat gained by air at exit of compressor in heat exchanger is equal to heat lost by gas practically we are going to work with air standard. So, this is also this is also air but in general this will be the combustion products which will be giving heat to the air.

So heat lost by the gas at the exit of turbine in the heat exchanger. So, this is

$$\textit{Heat gain by the air at exit of compressor in H.E.} \\ = \textit{Heat lost by gas at exit of the turbine in the H.E.}$$

$$(h_3 - h_2) = (h_5 - h_6)$$

$$C_p(T_3 - T_2) = C_p(T_5 - T_6)$$

$$T_3 - T_2 = T_5 - T_6$$

$$\eta_{H.E} = \frac{Actual\ heat\ transfer}{Maximum\ possible\ heat\ transfer} = \frac{(h_3 - h_2)}{(h_5 - h_2)} = \frac{(h_3 - h_6)}{(h_5 - h_2)}$$

$$\eta_{H.E} = 1$$

$$\frac{(h_3 - h_2)}{(h_5 - h_2)} = 1 \rightarrow (h_3 - h_2) = h_5 - h_2 \rightarrow h_3 = h_5 \rightarrow T_3 = T_5$$

$$\frac{(h_5 - h_6)}{(h_5 - h_2)} = 1 \rightarrow (h_5 - h_6) = (h_5 - h_2) \rightarrow h_5 = h_2 \rightarrow T_6 = T_2$$

So, this is how the ideal heat exchanger based Brayton cycle would look like then let us move ahead and find out the work interaction and heat interaction in the process of heat exchanger.

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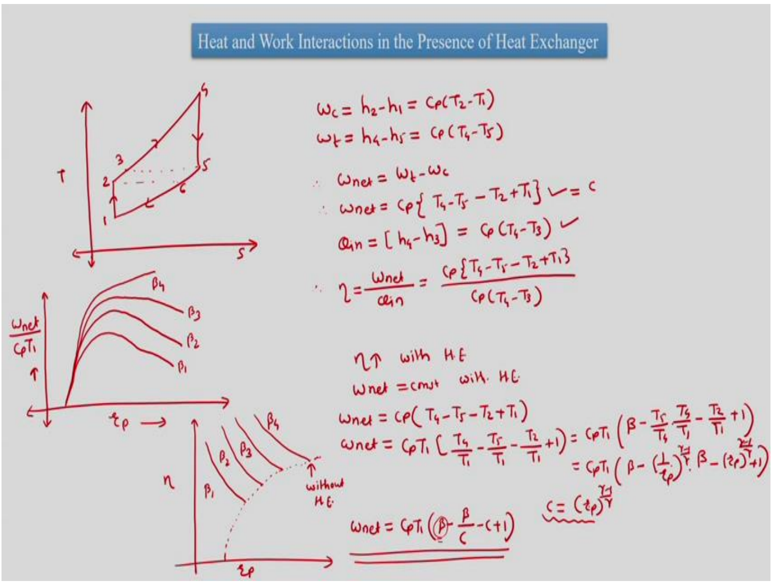

So, here again, we will plot now ideal heat exchanger such that we have TS diagram 1 to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 6 so this is TS diagram. Here

$$W_c = h_2 - h_1 = C_p(T_2 - T_1)$$

$$W_t = h_4 - h_5 = C_p(T_4 - T_5)$$

$$\mathrm{W_{net}} = W_t - W_c$$

$$\mathrm{W_{net}} = \frac{C_p\{T_4 - T_5 - T_2 + T_1\}}{C_p(T_4 - T_3)}$$

$$W_{net} = C_p(T_4 - T_5 - T_2 + T_1)$$

$$W_{net} = C_p T_1\left(\beta - \frac{T_5}{T_4} - \frac{T_4}{T} - \frac{T_2}{T_1} + 1\right) = C_p T_1\left(\beta - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}\beta - (r_p)^{\frac{\gamma-1}{\gamma}} + 1\right)$$

$$C = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$W_{net} = C_p T_1\left(\beta - \frac{\beta}{C} - C + 1\right)$$

So this would be simplified formula for W net and we can use this formula for analysing the plot. Or we can rather use this to plot W_{net} versus other non dimensional $\frac{W_{net}}{C_p T_1}$ versus r_p so we can plot it for different $r_p's$ obviously we mean here that we are plotting with respect your rp the C contains r_p and we are working with constant beta and constant T1 and we get plot like this, this is the plot for different because betas this is betas, this is β_1, this is β_2, this is β_3 and this is β_4.

Parallely if we plot efficiency versus r_p then we can see that first we will plot the efficiency. for heat exchanger without attachment this is heat exchanger for Brayton cycle without heat exchanger. This is without heat exchanger. We have seen this plot earlier and this is efficiency variation with respect to r_p. But now this efficiency would have beta in it. The heat exchanger, the Brayton cycle efficiency is independent of β.

It is only function of rp without heat exchanger, but now with exchanger it becomes function of beta. So, we have to plot it for different beta's so in this case efficiency is like this. So this efficiency for $\beta_1, \beta_2, \beta_3, \beta_4$ so at higher $r_p's$ efficiency would decrease. At lower $r_p's$ only we can use the heat exchange process that limiting r_p depends upon beta. This is the understanding from the Brayton cycle with heat exchanger.

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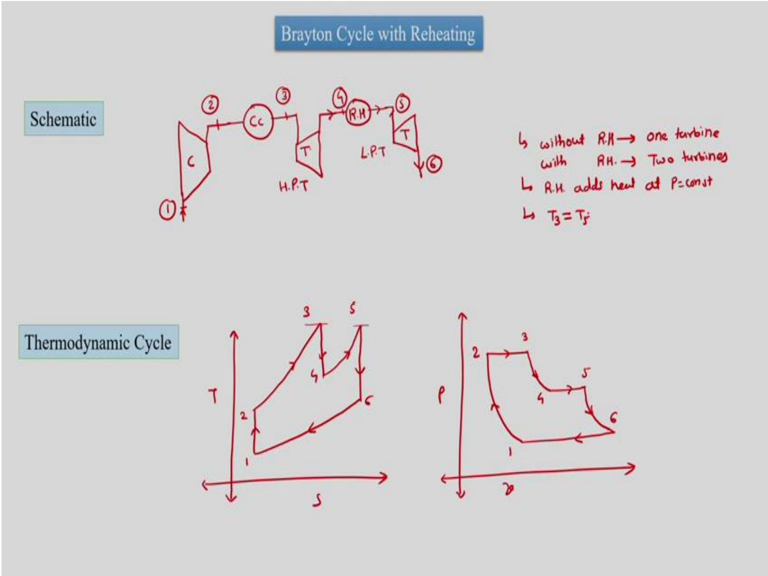

So, let us move ahead and see Brayton cycle with reheat, so we will first plot this schematic. Schematic is as expected, except that we are having an attachment operator, we will have air, entering in 1 and leaving at station 2 in the compressor then we have combustion chamber from 2 to 3 then we have turbine which is called as high pressure turbine. The exit of high pressure turbine is given to a again heat transfer unit that is called as re-heater and after heating air will pass into again a turbine and that turbine is low pressure turbine then air would go out.

So, this is the schematic of Brayton cycle with different components where an extra component of re-heater is added. Now we have to draw thermodynamic cycle and thermodynamic cycle which is at this moment will plot as TS diagram is 1 to 2 is compression process, 2 to 3 is heat addition in the combustion chamber, 3 to 4 is high pressure turbine, 4 to 5 is re-heater and 5 to 6 is low pressure turbine and 6 to 1 is the heat rejection process.

Now we will also plot the PV diagram for this and PV diagram will be 1 to 2 isentropic compression, 2 to 3 is heat addition in the combustion chamber, 3 to 4 is high pressure turbine expansion 4 to 5 is heat addition re-heater and 5 to 6 is expansion in low pressure turbine and then 6 to 1 is heat rejection. So this is how we have different processes in the Brayton cycle with re-heater attachment as what we see here.

We if would not have implemented the re-heater then we will have expansion in only one turbine without re-heater we have only one turbine. But with re-heater we will have many turbines and practically as per this diagram we are saying that we have 2 turbines. So, this is the new attachment this is the new understanding further one should know one more thing that the re-heater is heating the gas or air and then that the temperature at the exit of re-heater in general would be same as the temperature at the exit of combustion chamber.

Unless otherwise prescribed or required we would have this T 5 different T 3 but otherwise T 5 is equal to T 3 re-heater also does the constant pressure heat addition so this is one thing second re-heater adds heat at pressure constant process for isobaric process and in general T 3 is equal to T 5.

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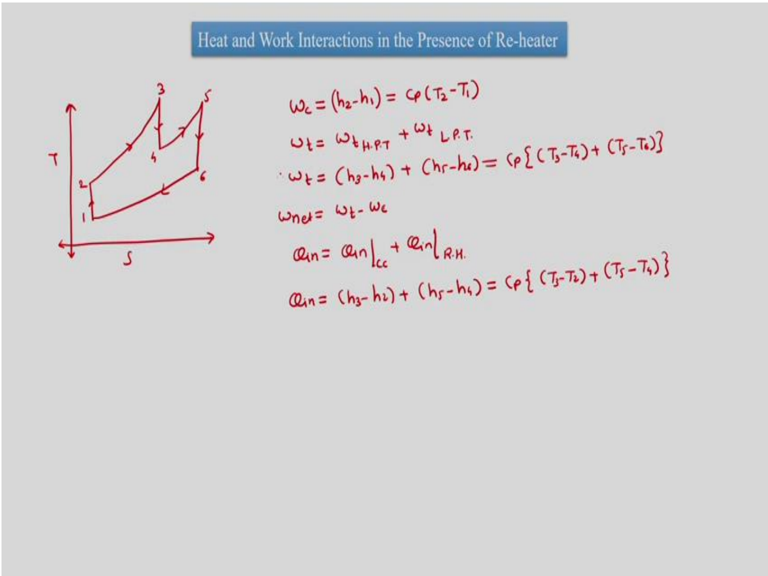

So, let us see work and heat interactions in the re-heating process or Brayton cycle with re-heater, here we can say

$$\mathrm{W_c} = (h_2 - h_1) = C_p(T_2 - T_1)$$

$$W_t = W_{t\,(H.P.T)} + W_{t\,(L.P.T)}$$

$$W_t = (h_3 - h_4) + (h_5 - h_6) = C_p\{(T_3 - T_4) + (T_5 - T_4)\}$$

$$W_{net} = W_t - W_c$$

$$Q_{in} = Q_{in}|_{cc} + Q_{in}|_{R.H}$$

$$Q_{in} = (h_3 - h_2) + (h_5 - h_4) = C_p\{(T_3 - T_2) + (T_5 - T_4)\}$$

Here since heat addition and net work both get altered due to the re heater we cannot make a generalized comment at this stage about increment or the decrement in either.

But it is sure that W_{net} would increase in the presence of re-heater since the two turbines are added and then they are going to give us more net work. However, since we are having Q in which is also having two component, so comment about efficiency alone is not visible at this state, but we can comment about W net. W net increases with the re-heater arrangement and this is the major reason for which re-heater is attached into the Brayton cycle.

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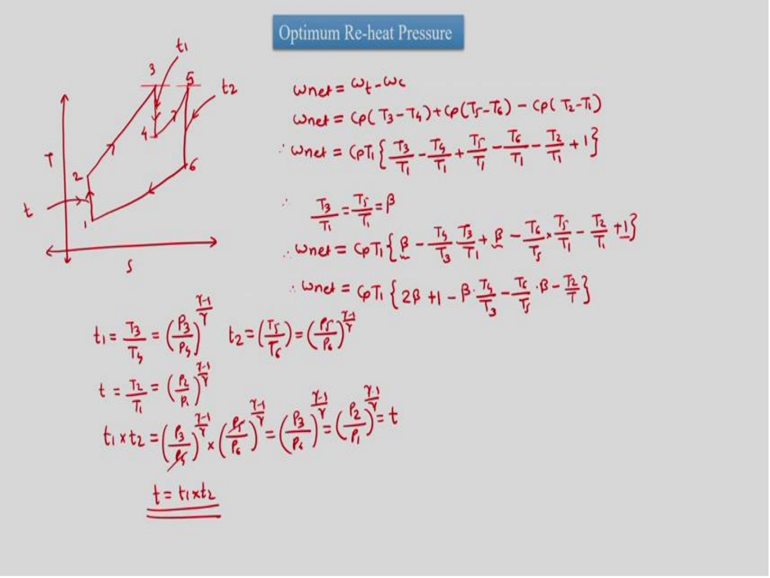

So, let us workout further for optimum reheat pressure in here that we have seen that we do reheating and process operating start at the exit of the high pressure turbine and every end the process at 0.4 but why this 0. 4 is here is 0.4 can be here or this 0.4 can be very close to 0.3 like this. There are different ways the location of 0.4 is possible. So, let us assume at this moment this is the location of 0.4 and we will find out what is the optimum location of 0.4. So this is 5 and this is 6 that is what we mean by saying that optimum reheat pressure.

And optimum reheat pressure means the pressure at which W net will be maximum then we will find out

$$W_{net} = W_t - W - c = C_p T_1 \left\{ \frac{T_3}{T_1} - \frac{T_4}{T_1} + \frac{T_5}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1 \right\}$$

$$\frac{T_3}{T_1} = \frac{T_5}{T_1} = \beta$$

$$\frac{T_3}{T_1} = \frac{T_5}{T_1} = \beta$$

$$W_{net} = C_p T_1 \left\{ \beta - \frac{T_4}{T_3}\frac{T_3}{T_1} + \beta - \frac{T_6}{T_5}\frac{T_5}{T_1} - \frac{T_2}{T_1} + 1 \right\}$$

$$W_{net} = C_p T_1 \left\{ 2\beta + 1 - \beta.\frac{T_4}{T_3} - \frac{T_6}{T_5}\beta - \frac{T_2}{T_1} \right\}$$

$$t_1 = \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} ; t_2 = \left(\frac{T_5}{T_6}\right) = \left(\frac{P_5}{P_6}\right)^{\frac{\gamma-1}{\gamma}}$$

$$t = \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$t_1 * t_2 = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} * \left(\frac{P_5}{P_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_3}{P_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = t$$

$$t = t_1 * t_2$$

So this we should know we should use this for deriving the optimum case. So, using this we can further we should remember this expression. This expression and will rewrite this expression.

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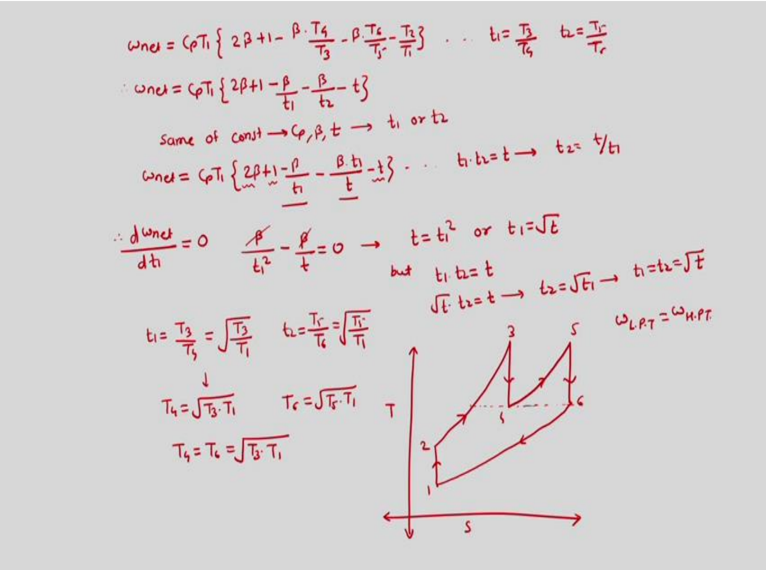

$$W_{net} = C_pT_1\left\{2\beta + 1 - \beta.\frac{T_4}{T_3} - \frac{T_6}{T_5}\beta - \frac{T_2}{T_1}\right\}$$

$$t_1 = \frac{T_3}{T_4}\ ; t_2 = \frac{T_5}{T_6}$$

$$W_{net} = C_pT_1\left\{2\beta + 1 - \frac{\beta}{t_1} - \frac{\beta}{t_2} - t\right\}$$

$$W_{net} = C_pT_1\left\{2\beta + 1 - \frac{\beta}{t_1} - \frac{\beta}{t_2} - t\right\}$$

So we are not altering compressor work, we are not altering the maximum and minimum temperature ratio.

We are not altering the working medium. So we are just altering the pressure at which we would be doing that reheat. So here we are just altering the exhaust temperature to the high pressure turbine for that also will replace this t2 by t1 and we will say that

$$\frac{dW_{net}}{dt_1} = 0\ ; \frac{\beta}{t_1^2} - \frac{\beta}{t} = 0 \rightarrow t = t_1^2\ or\ t_1 = \sqrt{t}$$

$$but\ t_1.t_2 = t$$

$$\sqrt{t}.t_2 = t \rightarrow t_2 = \sqrt{t_1} \rightarrow t_1 = t_2 = \sqrt{t}$$

$$t_1 = \frac{T_3}{T_4} = \sqrt{\frac{T_3}{T_1}}; t_2 = \frac{T_5}{T_6} = \sqrt{\frac{T_5}{T_1}}$$

$$T_4 = \sqrt{T_3.T_1}\ ; T_6 = \sqrt{T_5.T_1}$$

$$T_4 = T_6 = \sqrt{T_3.T_1}$$

So we are having a diagram which is TS diagram again special for optimum case and for the diagram we should have T 4 is equal to T 6. So these 2 temperatures should be same. So, the exit of temperature by in both turbines has to be same and practically both the turbine pressure ratios are also same and since both the turbines have same pressure ratio.

And exit and inlet temperature are same so we mean that

$$W_{L.P.T} = W_{H.P.T}$$

Both the turbines and also doing same amount of work in case of optimum Brayton cycle with Brayton cycle with heat exchange with re-heater in optimum case. Now we can find out efficiency here below note down this formula.

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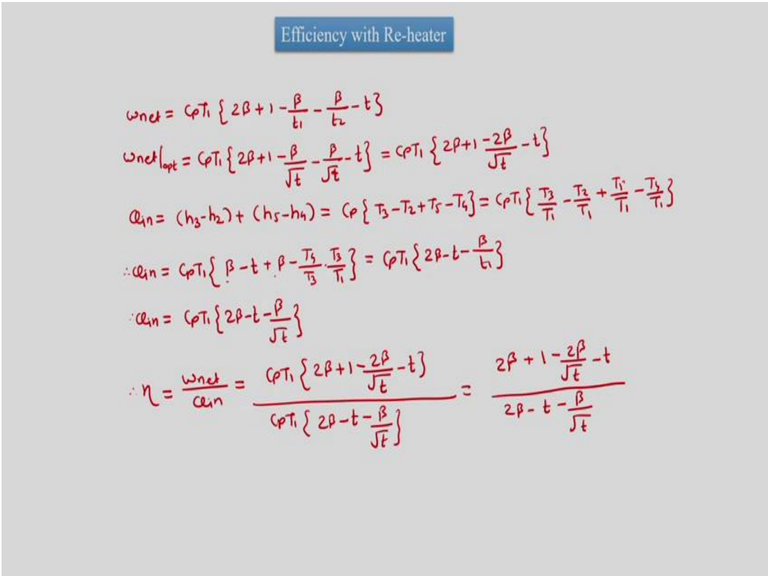

And we can work out with efficiency that formula was

$$W_{net} = C_p T_1 \left\{ 2\beta + 1 - \frac{\beta}{t_1} - \frac{\beta}{t_2} - t \right\}$$

$$W_{net} = C_p T_1 \left\{ 2\beta + 1 - \frac{\beta}{\sqrt{t}} - \frac{\beta}{\sqrt{t}} - t \right\} = C_p T_1 \left\{ 2\beta + 1 - \frac{2\beta}{\sqrt{t}} - t \right\}$$

$$Q_{in} = (h_3 - h_2) + (h_5 - h_4) = C_p\{T_3 - T_2 + T_5 - T_4\} = C_p T_1 \left\{ \frac{T_3}{T_1} - \frac{T_2}{T_1} + \frac{T_5}{T_1} - \frac{T_4}{T_1} \right\}$$

$$Q_{in} = C_p T_1 \left\{ \beta - t + \beta - \frac{T_4}{T_3}\frac{T_3}{T_1} \right\} = C_p T_1 \left\{ 2\beta - t - \frac{\beta}{t_1} \right\}$$

$$Q_{in} = C_p T_1 \left\{ 2\beta - t - \frac{\beta}{\sqrt{t}} \right\}$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{2\beta + 1 - \dfrac{2\beta}{\sqrt{t}} - t}{2\beta - t - \dfrac{\beta}{\sqrt{t}}}$$

This is the formula for maximum efficiency in case of Brayton cycle with re-heater attachment. So this we can use for two cases, one we can plot since efficiency and net work now we have expressed in terms of t. We can plot it for different t's or since we want to express them as pressure ratios so we can plot them as C since we said that.

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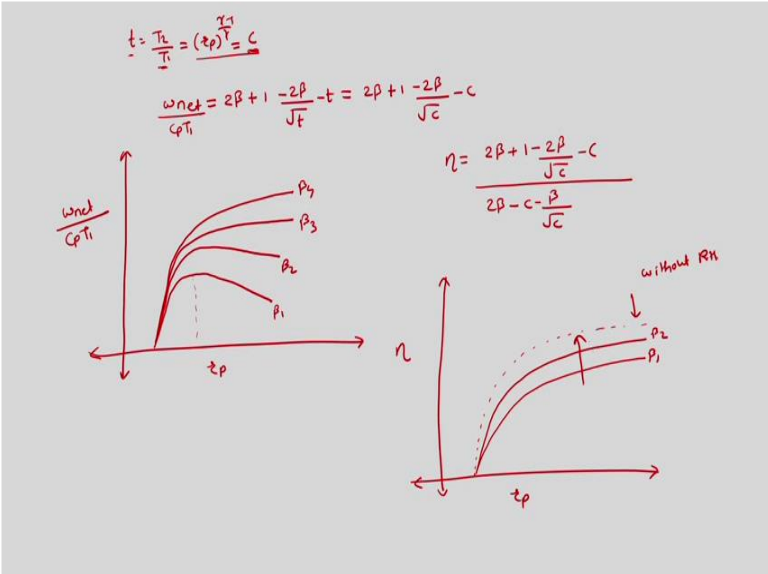

$$t = \frac{T_2}{T_1} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} = C$$

$$\frac{W_{net}}{C_pT_1} = 2\beta + 1 - \frac{2\beta}{\sqrt{t}} - t = 2\beta + 1 - \frac{2\beta}{\sqrt{C}} - C$$

$$\eta = \frac{2\beta + 1 - \frac{2\beta}{\sqrt{C}} - C}{2\beta - C - \frac{\beta}{\sqrt{C}}}$$

Here we can use this to plot $\frac{W_{net}}{C_pT_1}$ then versus r_p which is pressure ratio. So, we will plot it for different betas this is β_1 *and this is* β_2. So this is for β_3 and this is for β_4 so here as well at higher pressure ratios we can see that the net work decreases. So, this is $\frac{W_{net}}{C_pT_1}$. Now we can go back and write down the formula for efficiency in terms of C.

Since we have represented earlier that formula in terms of t so efficiency for us if we write down in terms of C then efficiency

$$\eta = \frac{2\beta + 1 - \frac{2\beta}{\sqrt{C}} - C}{2\beta - C - \frac{\beta}{\sqrt{C}}}$$

So these things will make the new formula for efficiency which corresponds to maximum work. And we can plot the efficiency versus r_p for Brayton cycle with re-heater so efficiency versus rp. Here again will plot efficiency for the Brayton cycle without re-heater and as what we said with re-heater since we are adding more heat.

So, this is efficiency for β_1, this is the efficiency for β_2, this is how efficiency increases which increase in beta but at lower betas efficiency is lower than without reheat cycle this is the understanding for Brayton cycle with re-heater and Brayton cycle with heat exchanger. Rest of the attachment and associated performance alteration we will see in the next class. Thank you