Fundamentals Of Artificial Intelligence
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Lecture – 13
First Order Logic -II

Welcome to fundamentals of artificial intelligence. We are looking at knowledge representation and reasoning. And have covered propositional logic. In the last class we introduce first order logic. Which is also called predicate logic or first order predicate calculus? We have looked at predicates and functions. Look very closely at quantifiers. For it is the quantifier that make first order logic more expressive than propositional logic.

Today we will continue our discussion of first order logic. In the previous lecture on first order logic we have discussed the syntax of a first order logic.

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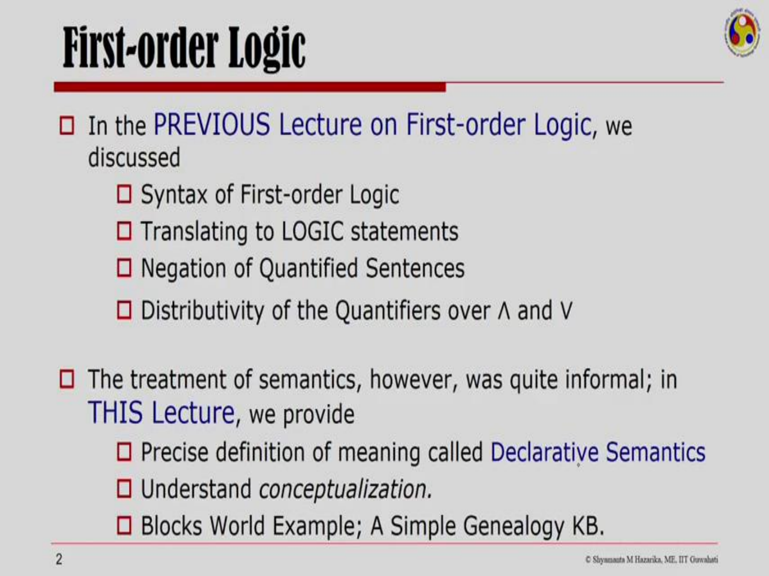

How to translate and English sentence into a logical statement, we have looked at negation of quantified sentences and we see how one could distribute the quantifiers over the conjunction and disjunction. The treatment of semantics however in our discussion in our last class was quite informal. In this lecture, we would provide precise definition of meaning what we refer to as declarative semantics.

We would understand the concept called conceptualization and as an example of knowledge representation. We would work through two example problems. One blocks wall example and another, a simple Genealogy knowledge base.

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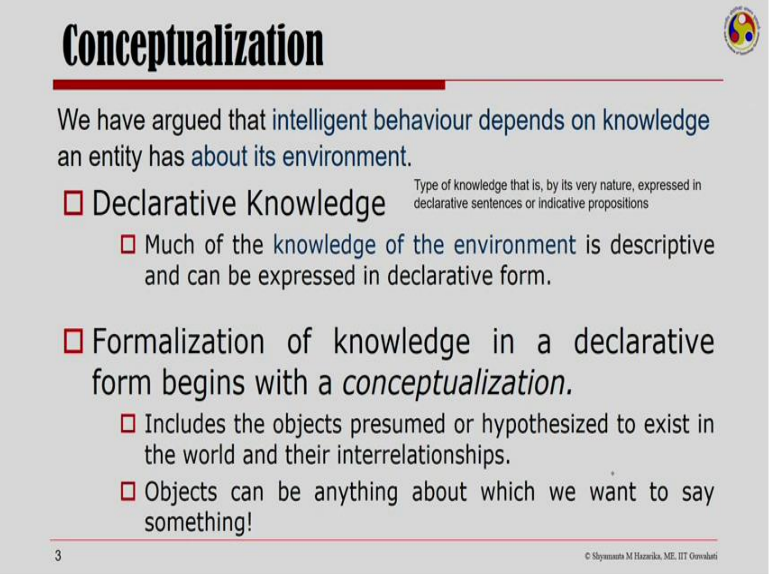

We have argued that intelligent behaviour depends on the knowledge. The knowledge that an entity has about its environment it is important to realise that this knowledge is declarative that is, it is by its very nature expressed in declarative sentences or indicated propositions. Much of the knowledge of the environment is descriptive and can be expressed in declarative form. Formalization of knowledge in a declarative form begins with a conceptualization.

Conceptualization is about identifying or presuming what are the objects that exist in the world and what are their inter-relationships? Now you the concept of object is very general. Objects can be anything about which we want to say something.

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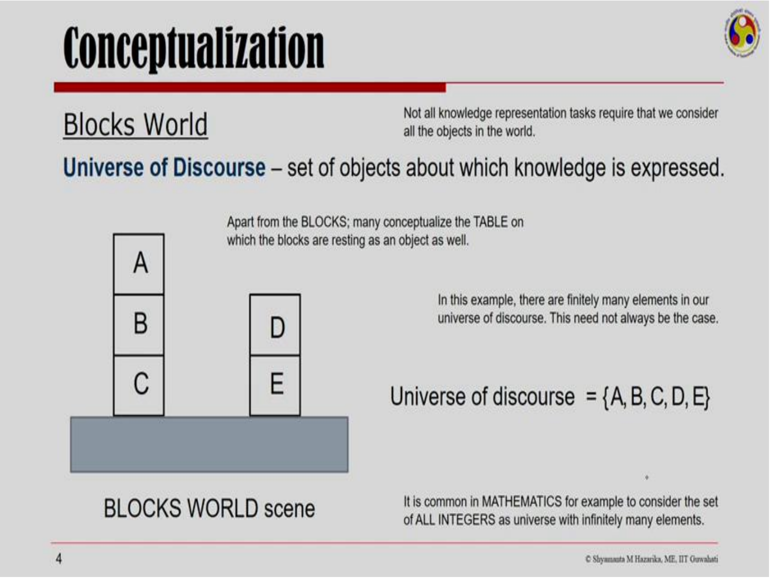

Let us look at a very simple toy conceptualization of the world which is referred to as the block world. We have 5 blocks on a table and here is the blocks wall seen. Now, one needs to realise that not all knowledge representation task require that we consider all the objects in the world in this example, we will considered objects. A, B, C, D and E which are the blocks? But we will not consider the table as an object for this illustration.

So we have a concept called the Universe of discourse. Which is the both the set of objects about which knowledge is expressed in this example hear the Universe of discourse is set containing the 5 blocks apart from the blocks as I was referring to many conceptualize the table on which the blocks are resting as an object as well. We live it out here. In this example there are finitely many elements in our Universe of discourse.

One needs to realise that this need not always be the case in many problems, which is common in mathematics, for example, we consider the universe to be of infinite many elements, like for example the set of integers as Universe with infinitely many elements.

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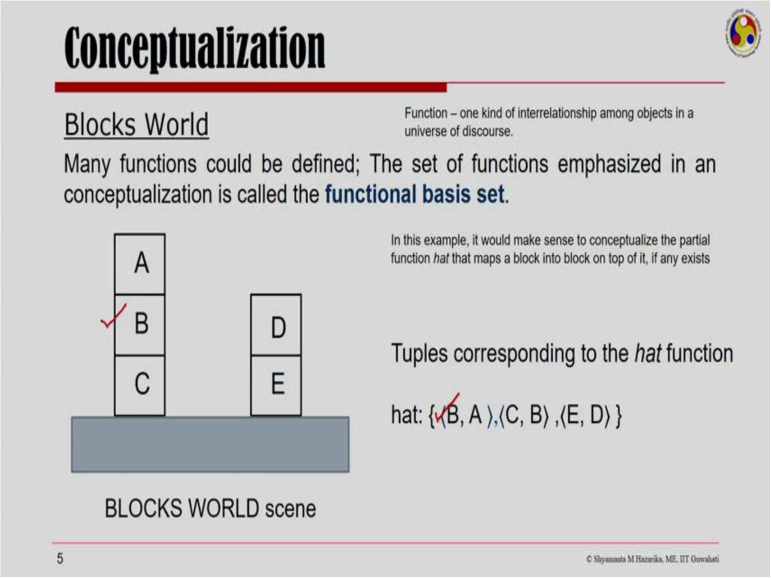

One important concept that we have looked yesterday is the concept of function. Function is one kind of inter relationship among objects in a Universe of discourse for this blocks wall scenario that we have here. We could have many functions define but then only a couple of them would be included in the conceptualization. The set of functions which are emphasize in a conceptualization is called the functional basis set.

In example it make sense, conceptualize the partial function have that maps block into block on top of it if any exist. Now suppose corresponding to the head function would be the pass BA, CB, ED let us locate one of them and try to understand what we mean by this. If you think of what is the function to take one object? And it would return another object and in this example the help maps a block into a block which is on top of it.

So, if you take B here the hat function returned to us and A which is a block on top of it similarly for CB and the pair ED.

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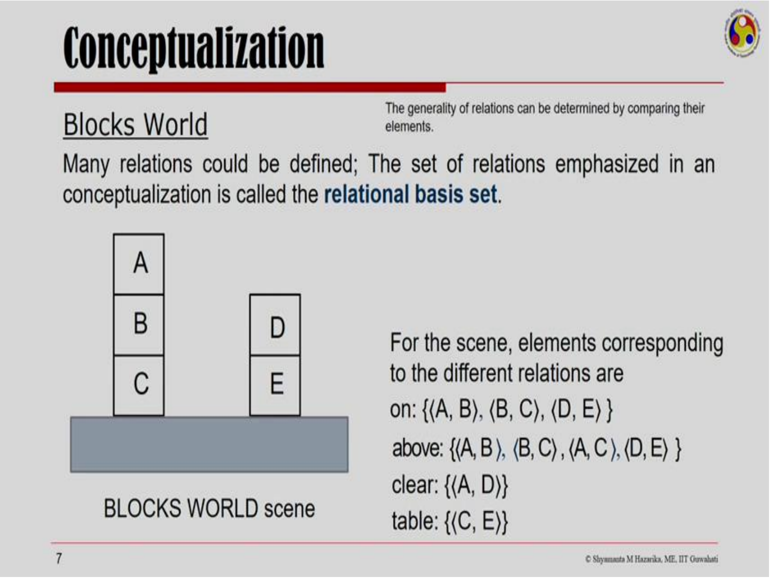

The other kind of relationship among objects in a Universe of discourse is the relation. In a special configuration of the walls block there are a number of meaningful relations. We could talk of blocks being on the table, we could talk of blocks being on top of one another so and so forth. Here we consider a relation called on which holds if and only if a block is immediately above the other. We talk of a relation above which is between two blocks if and only if one is above the other we then have a Unary relation clear to mean no block is on top of the other block.

Out of these numerous relations possible the set of relations emphasizing and conceptualization is called the relational bases set. For the same elements corresponding to the different relations are for the on relation I have couples for AB for A is on B, similarly BC and BA. And for the above relation I would have couples AB, BC, AC and DA thereafter I could have couples for clear as I can see it here nothing on top of A.

So I would have as an element of the clear relation and B. I have one more relation called the table to mean an object is on the table. And here I have the element size CE. Now if you look at the two relations on and above you could see that in terms of the on relation is a subset of the above relation and on relation is there for less general then the above relation.

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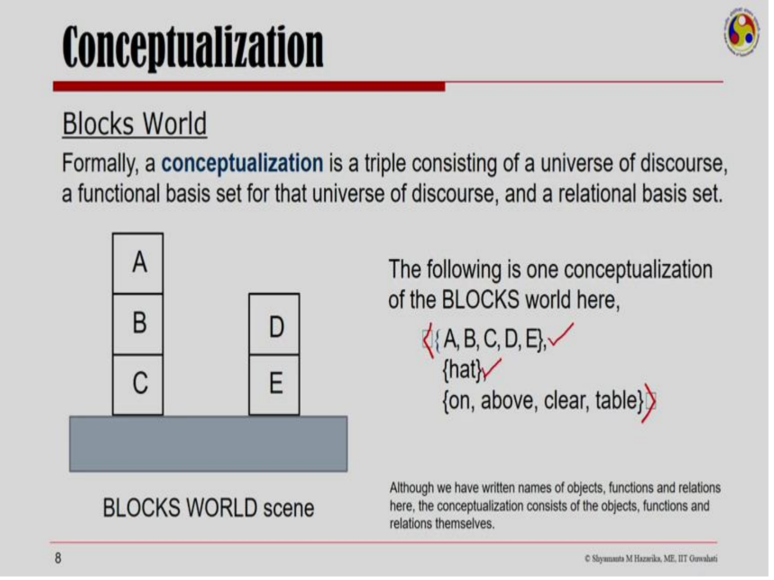

Given the concept of functional bases at a relational basis set and the Universe of discourse we are now in a position to formally define a conceptualization. A conceptualization is a triple consisting of a Universe of discourse, a functional basis set for the Universe of discourse and a relational bases set. Coming back to the blocks world seen that we are conceptualising. The following triple is one conceptualization of the blocks world.

I have the 5 blocks A, B, C, D, E as the objects in the Universe of discourse as for the functional bases set. I have a function hat and as for the relational bases I have on above clear and table therefore relations. So this triple is a conceptualization of the blocks world here. Although we have written names of objects functions and relations here, one needs to realise that the conceptualization consists of the objects, functions and relations themselves.

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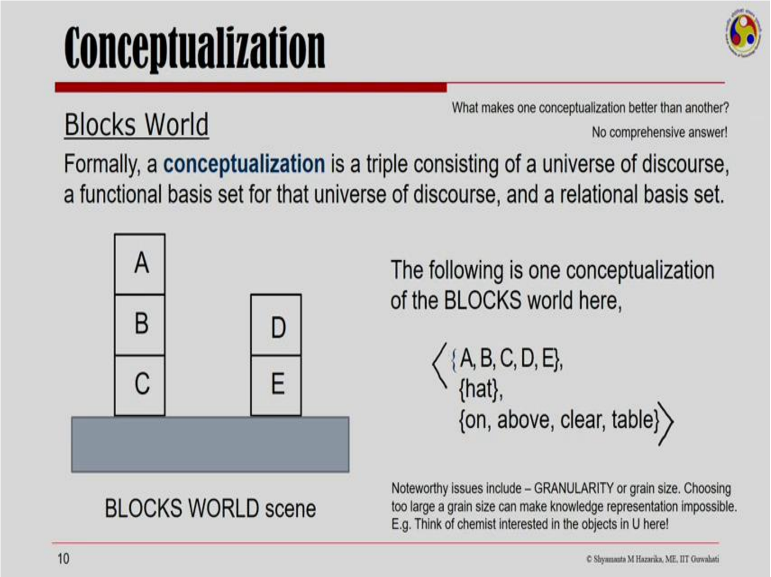

Now what makes a conceptualization better than another one? There is not a comprehensive answer for this question. However, one note worthy issue to look at it is the issue of granularity or grain size. Choosing to a smaller grain size can make knowledge representation tedious. Like here in the blocks world if you think of objects in the Universe of this course in terms of atoms you see that you have a huge number of elements to deal with when the only task is about stacking one on top of the other and therefore looking at them as blocks would suffice.

However, if you are thinking of looking at the chemical properties of these blocks then the grain size that we have considered of the objects as blocks would be to a large grain size and knowledge representation would be impossible.

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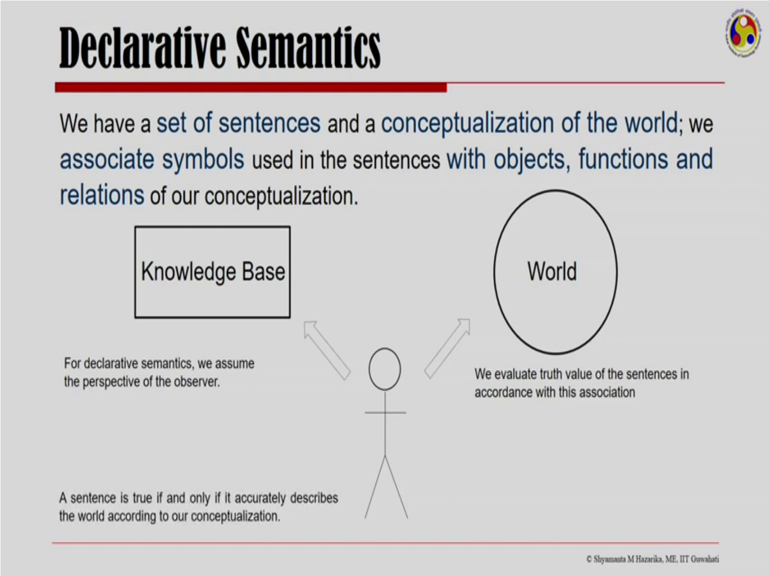

So for declarative semantics we assume the perspective of the observer as shown in the illustration here we have a set of sentences any conceptualization of the world. We associated symbols used in the sentence with objects functions and relations of our conceptualization. We evaluate the truth value of the sentences in accordance with this association and a sentence is true if and only if it accurately describes the wall according to our conceptualization.

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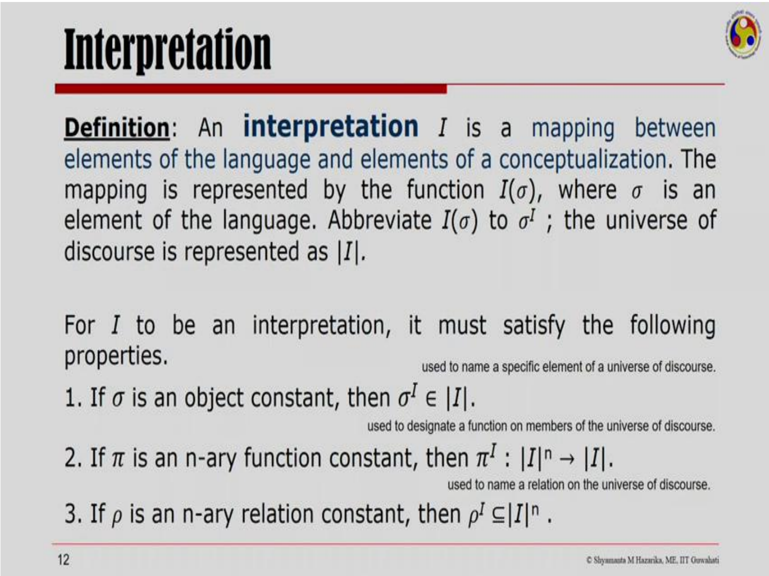

So let us now try and understand what we mean by and interpretation given a conceptualization. And elements of a language in which we want to do the representation. An interpretation is a mapping between the elements of the language and elements of a conceptualization. The mapping is represented by the function of Sigma where Sigma is the element of the language. The Universe of discourse for an interpretation I the following properties need to be satisfied.

If Sigma is an object constant then it needs to name a specific element of a Universe of discourse under the given interpretation. If Pi is an n-ary function constant then the mapping and given under the interpretation should map Pi as a function on members of the Universe of discourse. Finally if Rho is an n-ary relation constant then under that interpretation Rho need to be able to name a relation on the Universe of discourse.

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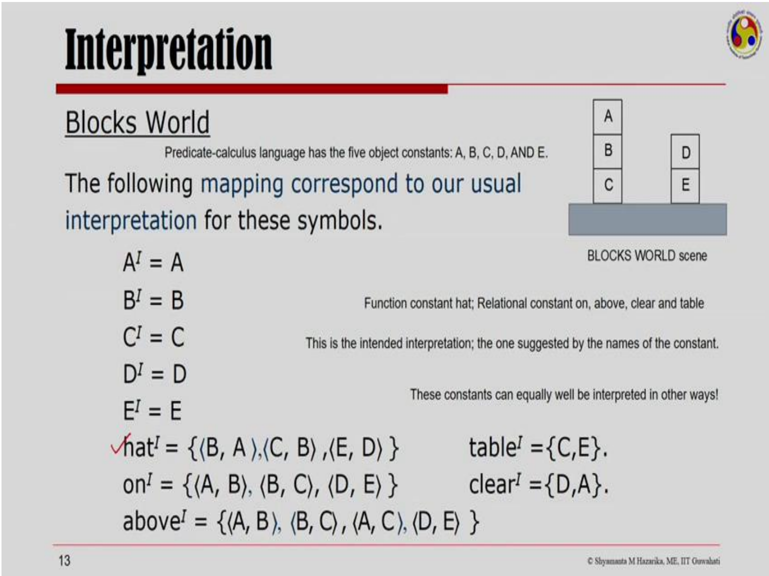

So, let us now come back to the blocks world problem that we were looking at and try to understand what we mean by interpretation. So here the predicate calculus language that we have has five object concepts A B C D and E which represent some of blocks in our world. We have a function constant hat and we have relational constants on above clear and table. The following mapping corresponds to our usual interpretation for the symbol.

So, here under interpretation I in the block A is mapped to the constant symbol A. Then we have B mapped to B so on and so forth and the function constant under the interpretation have the following tuple similarly for the other relation constant. This is the intended interpretation. The one suggested by the names of the constant one needs to realise that this consonants can equally well be interpreted in other ways and we will have a difference interpretation of the blocks world problem.

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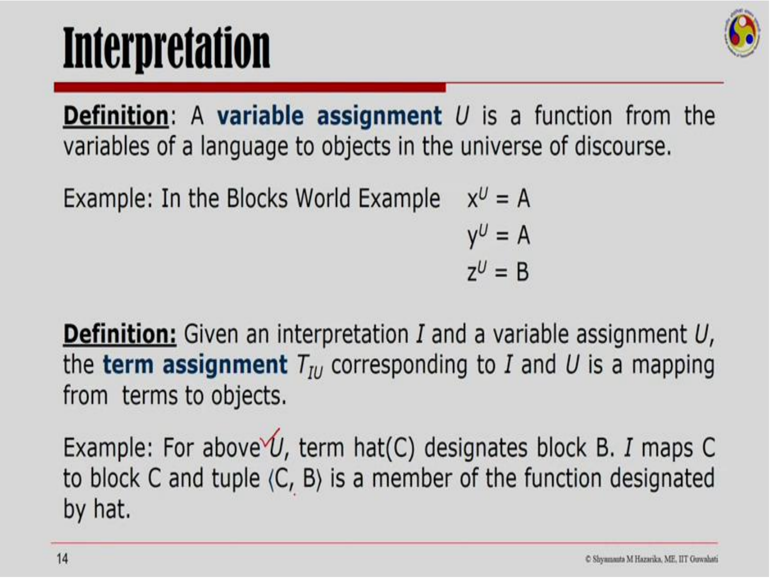

Now let us look at what we mean by a variable assignment. A variable assignment is a function for the variables of a language to objects in the Universe of discourse. So, in the box world example and variable assignment you the variable x could be assigned to A the variable y assign to A variable z could be assigned to B given an interpretation I and variable assignment U.

We can now talk of term assignment which is a mapping from terms to objects, for example we have the above variable assignment U. And we have a term hat of C. Now hat of C designates block B because the interpretation I maps C the constant symbol to block C. And tuple CB is a member of the function designated by hat and therefore the term assignment designates the term hat C as block B.

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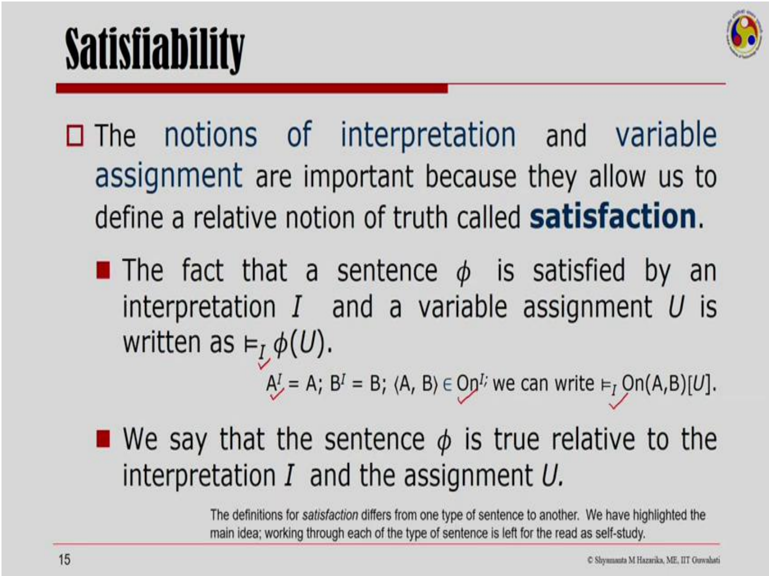

The notions of interpretation and variable assignment are important because they allow us to define a relative notion of truth which is called satisfaction. The fact that a sentence Phi is satisfied by interpretation I any variable assignment U is written as under interpretation I M tales Phi U, what this means is that the sentence Phi is satisfied by interpretation I when I have a variable assignment U.

Let us look at the blocks wall world and see what we mean. So here is an interpretation that takes me from the block A to A here and B Constant symbol B there. And the tuple AB is already within the tuples for on, so we can write that under the interpretation the on AB is entailed. We say that the sentence Phi is true relative to the interpretation I and the assignment U.

The definitions for satisfaction differs from one type of sentence to another. Here we have only highlighted the main idea. Working through each of the type of sentence is left for the reader as self study.

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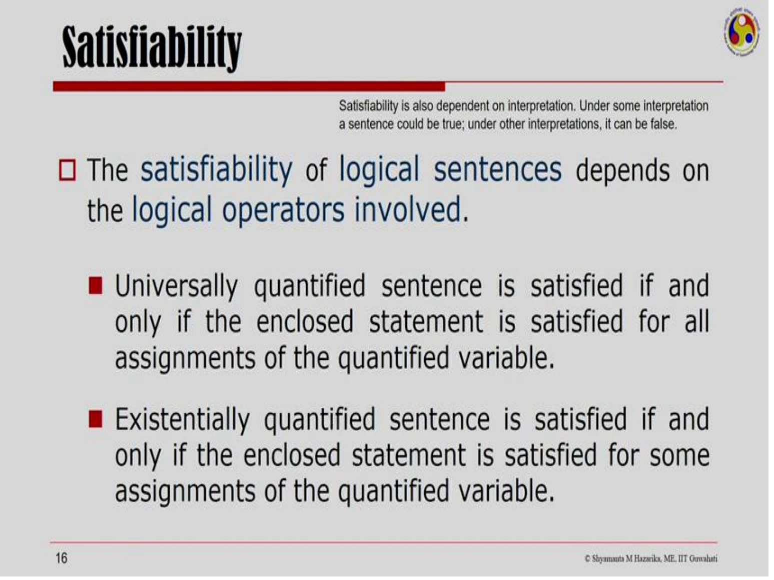

Satisfiability is also dependent on interpretation under some interpretation a sentence could be true under other interpretations. It can be false. The satisfiability of logical sentences depends on the logical operators involve, we will just note here that the universal quantified sentence is satisfied if and only if the enclosed statement is satisfied for all assignments of the quantified variable.

The existential quantifiers sentence is satisfied if and only if then close statement satisfied for some assignment of the qualified variable.

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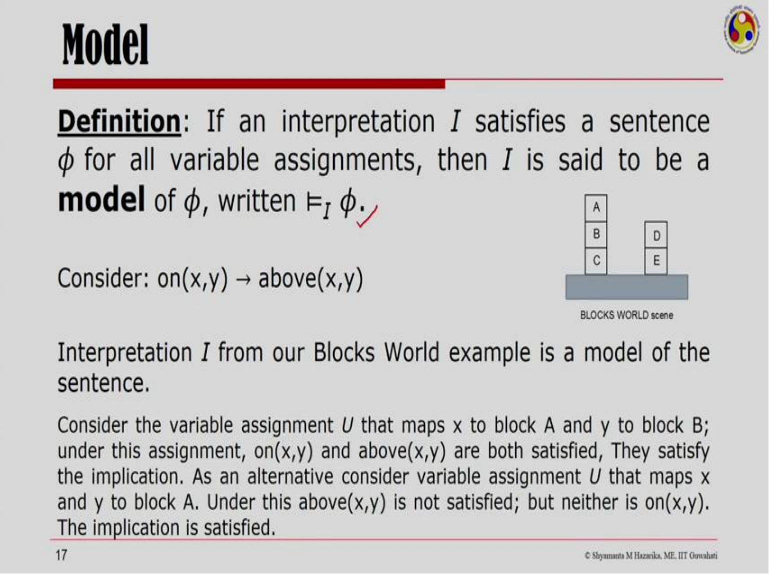

Now having understood assignment we define something called the model. If an assignment I satisfy is a sentence Phi for all variable assignments. Then I said to be a model of Phi written as under assignment I n tails Phi. Now let us look at the blocks for seen and consider the following expression on xy implies above xy. Now interpretation I from a block world example is a model of the sentence. Let us see how that is.

Consider a variable assignment U that maps x to block A and y to block B and under the assignment on xy and above xy are both satisfy. Therefore they satisfy the implication as an alternative consider a variable assignment U that maps x and y both to block A under these assignments above xy is not satisfied neither is on xy. And therefore the implication is satisfied.

So we have seen that for all variable assignments the interpretation I from our blocks world for example satisfies this above expression and therefore we say that interpretation I is a model of the sentence.

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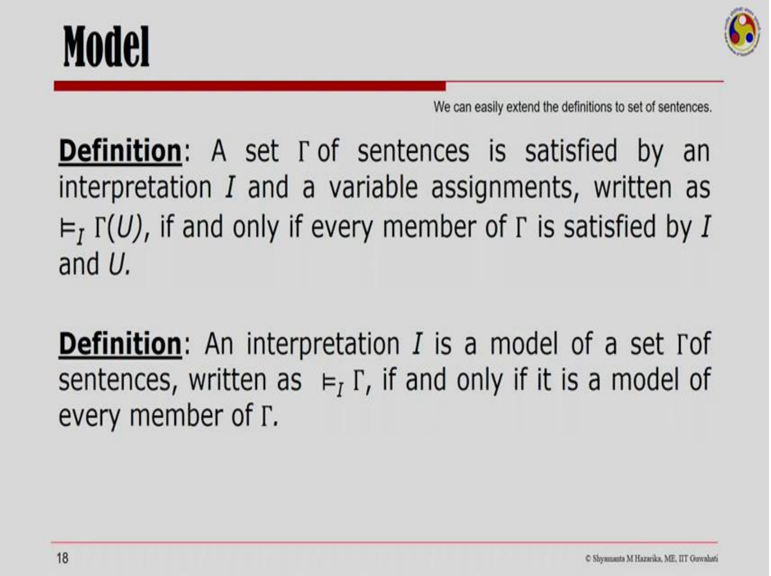

We can easily extend the definitions to set of sentences. A set gamma of sentences is satisfied by an interpretation I and a variable assignment written as entails gamma U if and only if every member of gamma is satisfied by the interpretation I and the variable assignment U and then we could think of a model for a set of sentences. So an interpretation I is a model of a set gamma of sentences written as entails gamma if and only if it is a model of every member of gamma.

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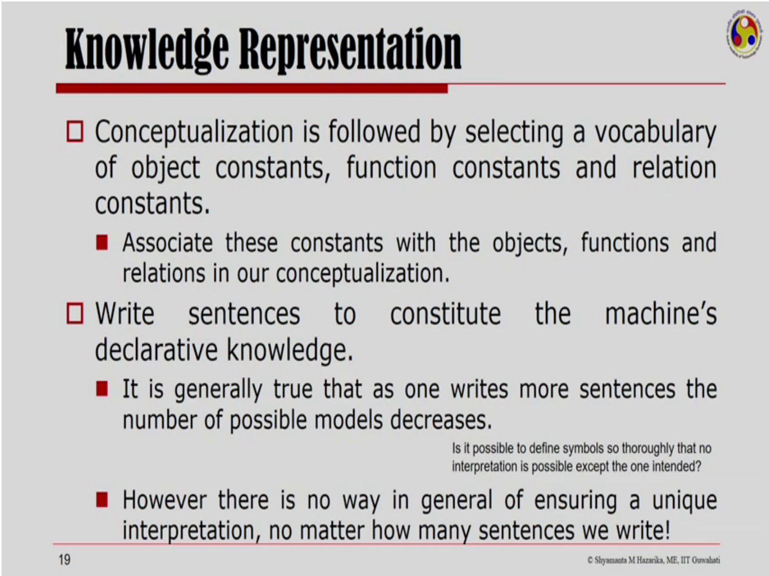

So, we have seen that conceptualization is followed by selecting a vocabulary of object constants, function constants and relation constants. We associate these constants with objects functions and relations in our conceptualization. We write sentences to constitute the machines declarative knowledge. It is generally true that as one writes more sentences. The number of possible models decreases.

Now the question is, is it possible to define set of symbol, so thoroughly that no interpretation is possible except the one that is intended. It turns out that this is not the case. There is no way in general of ensuring a unique interpretation. No matter how many sentences we write.

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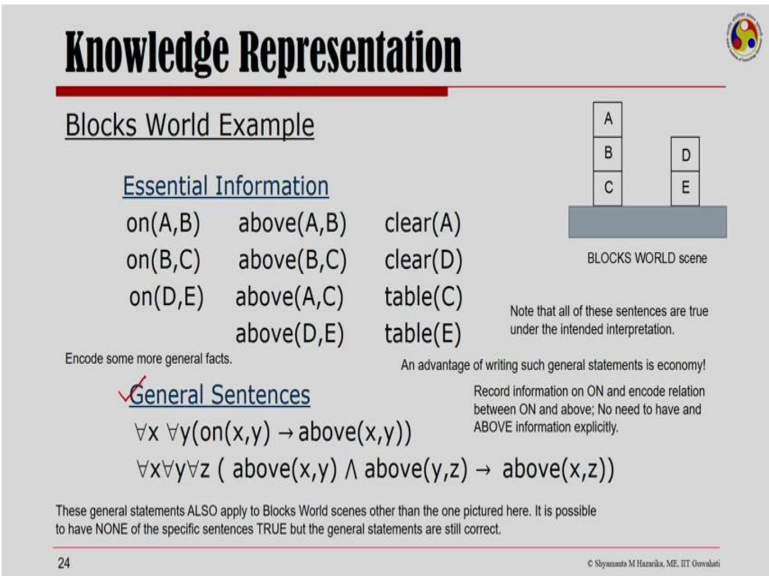

Under this scenario we would now look at an example of the blocks world and try to write down the essential and the general information for these blocks world seen. For the blocks for seen shown on your right we could write things as on AB above AB and clearing we could write similar relations for B and C. And D being clear near we can write clear D. We could write similar relationship of on between D and E and above between A and C.

Similarly we could write that C is on the table and E is on the table. Note that all of the sentences are true under the intended interpretation. Encode some general facts using general sentences is possible. So, here we have a general statement that relates on to above if one block is on another block then that block is above the other block. We could also write more interesting statements on above itself and show that the above relation is transitive.

In the second statement here, what we are saying is if one block is above a second and the second is above third. The first is also about the third so look at this scenario here. A is above B and B is above C? Therefore we could say A is above C and advantage of writing such general statement is of course economy. We need not write all the essential information explicitly things could be found out from the given set of facts and the general sentences that I have here.

We could record information on an encode relation between on and above. And we can avoid having above information explicitly stated. These general statements if you have this time realise is not restricted to the blocks world seem that I am taking here. It is possible to have none of the specific sentences that we have for the blocks world was seen true. But still the general statement that I hear it be correct.

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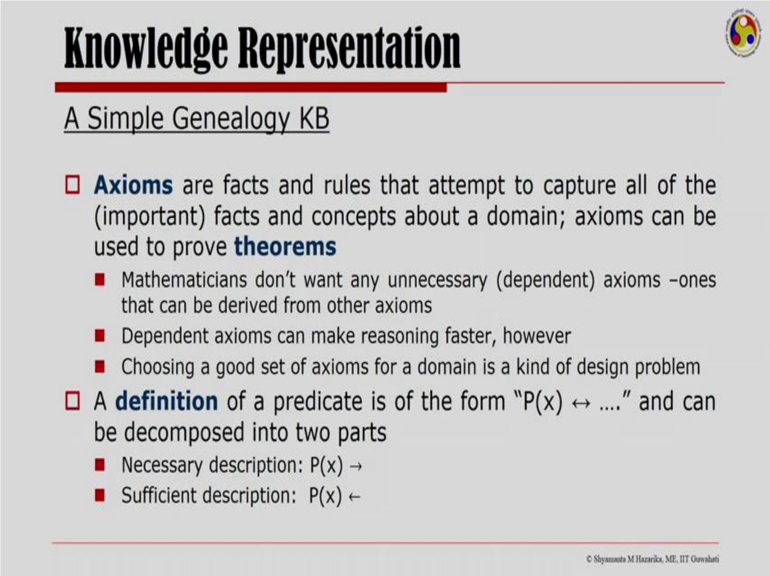

Now let us look at another knowledge representation example of a simple Genealogy knowledge base. So I have to have certain predicates and some facts, here I will only work with the predicates and leave the facts for you to work out through. Whenever we are creating such knowledge basis, we need to realise that we can write facts and rules. That attempt to capture all of the important facts and concepts about domain. So, these are referred to as axioms.

We can use these axioms to prove theorems. Now one point to know is that mathematician's do not want any unnecessary or dependent axioms one that can be derived from other axioms is something they would not love to call axioms at all. However one needs to realise that depended axioms can make reasoning fast. Therefore choosing a good set of axioms for a domain is kind of a design problem and as we go on working through more problems in knowledge representation, we would have a more better understanding of what to be taken as axioms and what to be left out to be derived.

So, a definition is an important concept whenever we are trying to write such a knowledge base. A definition of a predicate is of the following form it says Px if and only if that particular concept is satisfied. And definition can be decomposed into two parts. The necessary description at which point Px implies and the sufficient description at which point it would mean that the concept that has been define implies the predicate.

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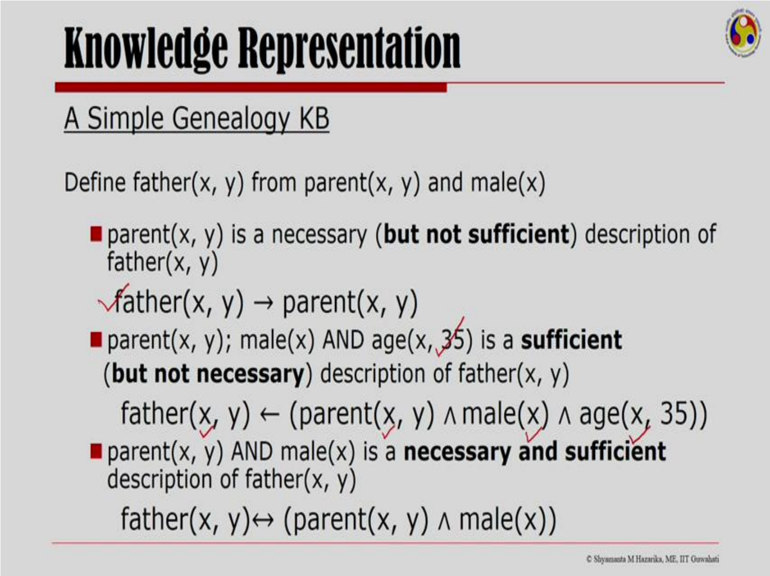

So, let us try and understand how one could define predicate father x, y that is x is the father of y from to primitives one call the parents x is the parent of y and male x. Now parent x, y is a necessary but not sufficient description of being a father? So someone who is a parent is a father. But parent is a necessary but not sufficient description of a father. If x is a father of y then definitely is the case that x is the parent of y.

If x is a parent of why it is not necessary he or she is the father, parent x, y male, if we put third condition that the age must be greater than 35 is a sufficient condition but not necessary description of the father predicates. So, somebody who is a parent and a male is above 35 years of age is a father. But this does not work on the other way. Further for any x which is the father of y it need not be the case that x should be greater than 35.

So when we are looking at the definition of father, parent and male is a necessary and sufficient description of father. So, x is a father of y if and only if x is a parent of y and x is male.

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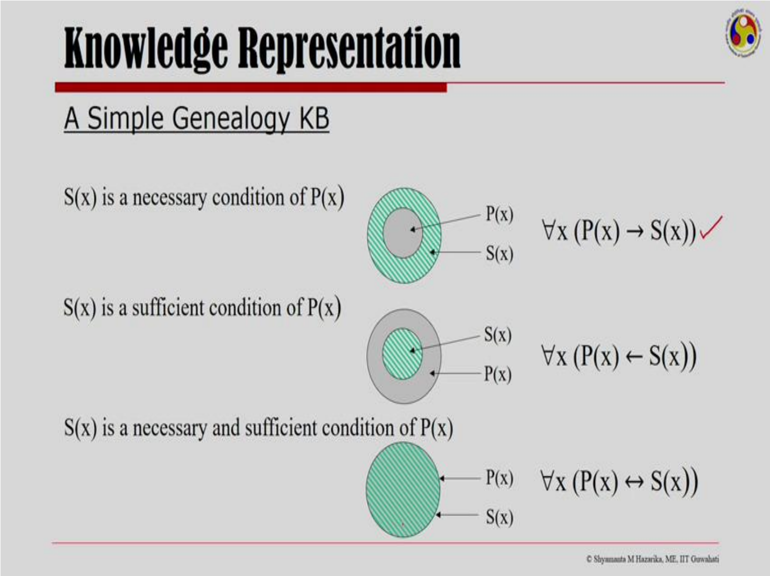

Now let us try to get this by understanding what we mean by the necessary condition and efficiency condition one more time. Sx is a necessary condition of Px what that mean? Is that for all x Px would imply Sx where is when Sx is a sufficient condition of Px would mean that for all x's Sx would imply Px and when we are talking of a definition for a predicate then these to must be equal that is here Sx is a necessary and sufficient condition of Px would mean that for all x Px if and only if Sx and that is what is a definition.

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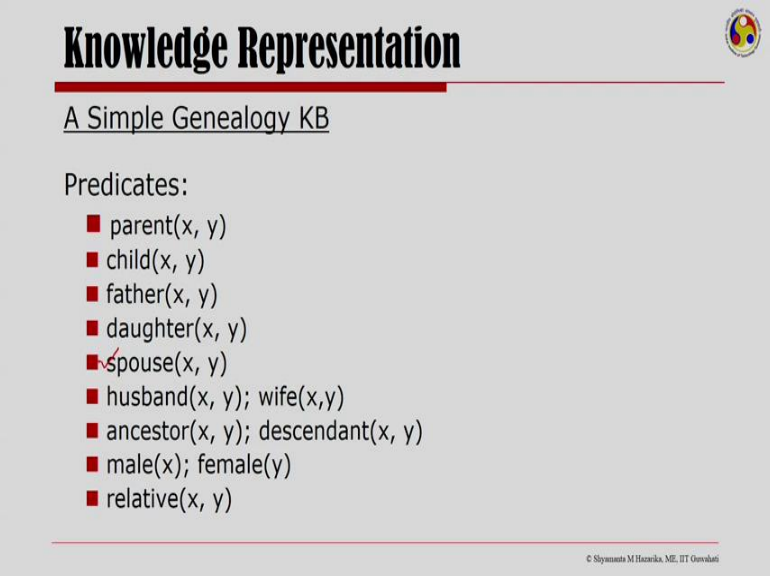

Try to get a more concrete knowledge representation of a simple genealogy base using a couple of predicates. So we have offers predicate parent x, y we says that x is a parent of y. We could have another predicate child x, y x is the child of y we have looked at father. Father x, y with mean x is a father of y, daughter x, y, x is a daughter of y, spouse x, y now this is an interesting predicate. For you could see if x is a spouse of y, y is the spouse of x.

So spouse x, y is asymmetric relation we could have husband x, y similarly wife x, y and then we could have ancestor descendant, male, female predicates defined and we could have a predicate cognitive. One can think of more predicates this is where the question of design. And question of how much of the facts to be taken into consideration to create a knowledge representation is to be explored.

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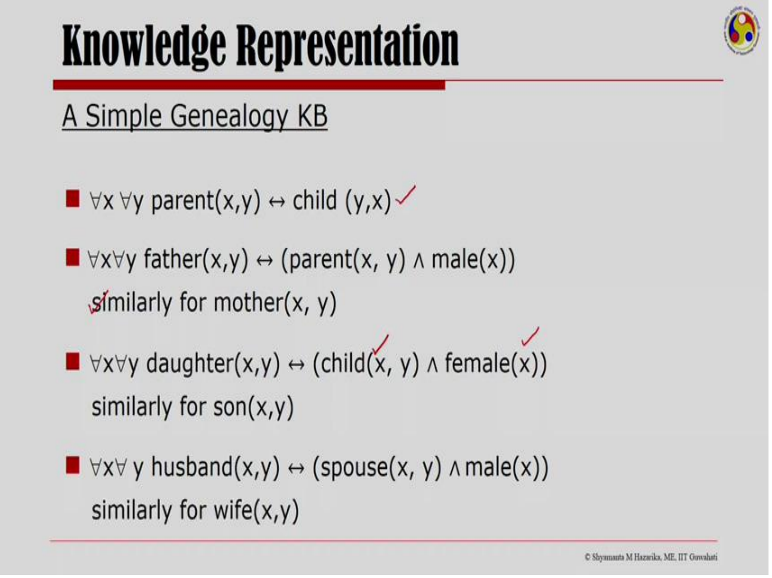

So let us try and define a couple of predicates before we include a couple of simple genealogy knowledge base. So, here is our first definition for all x for all y x is the parent of y if and only if y is the child of x. So, that is the most simplest of definitions that you could have for parents. We have already seen the definition for father. So, for all x and for all y x is the father of y if and only if x is a parent and male.

Similarly we could have a definition for mother in which case we will say that x is a parent of y and female x. We could define something like a daughter relation, so we have for all x for all y daughter x, y. If and only if x is a child of y and x is female. So here one needs to be a bit careful to see that we are talking of x where is y is one of the parent either father or mother and therefore for a daughter definition it is important that x is female.

We could similarly include a relation for son and then we have a relation here for husband. Now for all x for all y x is a husband of y if and only if x and y are spouse of each other and x is a male. Similarly we could include a definition for wife. One thing to note in these definitions are that when we are doing it? A couple of them we need to consider as primitives and then using them we need to look at which are the necessary and sufficient condition for the other predicate to be defined in terms of the primitives.

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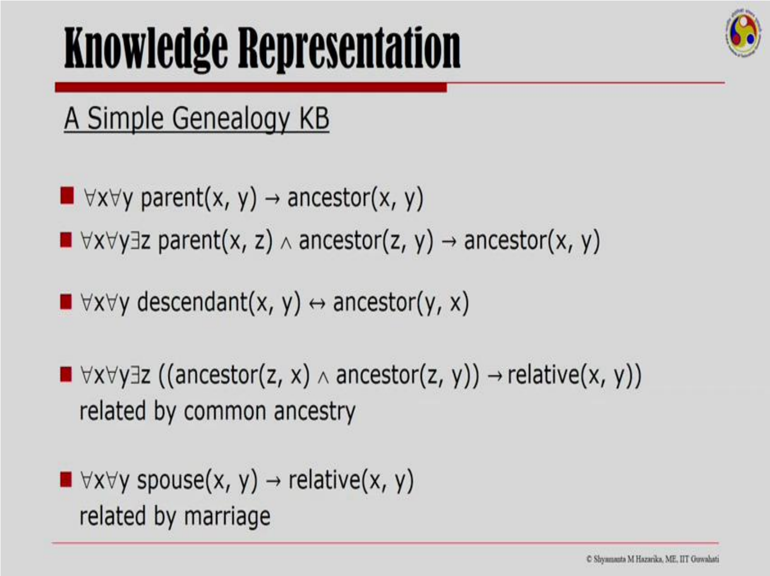

Now let us include a couple of rules. So, here is our first rule which says that if x is a parent of y x is the ancestor of y every parent is an ancestor. The next rule is saying that if x is a parent of z and z is an ancestor of y then we have x as an ancestor y. And similarly we could see that we could draw an equivalence between descendant and answers that by saying x is the descendant of y if and only if y is the ancestor of x.

Now here is an interesting rule on who is a relative. So, for any x and any y if there is a z and x and y share the common ancestor gel Then x and y are relative. So this is something like related by common ancestry. We could have another relative definition coming directly from the spouse relation which is related by marriage. So for all x for all y x is the spouse of why we could say x and y relative.

Now this is what we have done in this portion of the representation exercise. We have looked at a simple genealogy knowledge base and try to understand how we could define predicates how we could include rules into the system. So, we should take a couple of notes before we and our discussion today.

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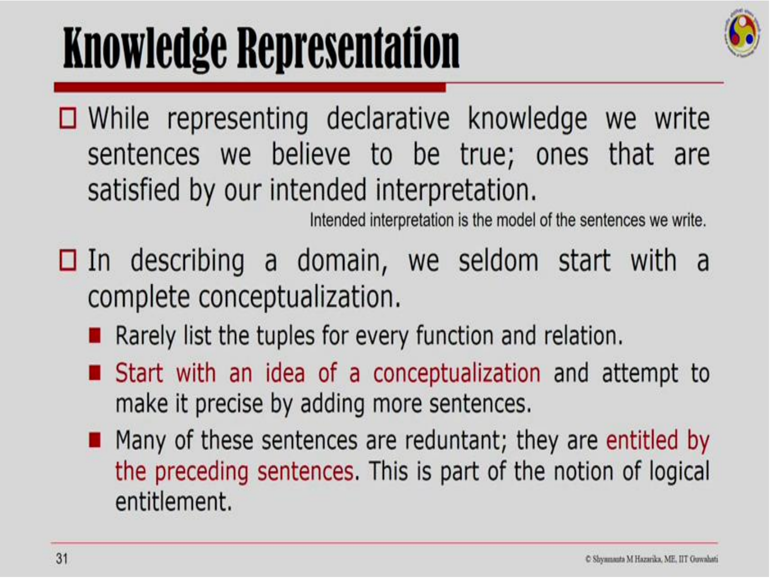

We have seen that while representing declarative knowledge. We write sentences we believe to be true and those are satisfied our interpretation. So, the intended interpretation is the model of the sentence we write. In describing a domain we need to realise that we seldom start with a complete conceptualization. It is neither feasible nor intended that we have a complete conceptualization before we start describing a domain. We really list the tuples for every function and relation.

For the blocks world, seen that I have used as an example here it was toy configuration and it was possible for me to leave the tuples for every function and really but we really do this in practice. We start with an idea of a conceptualization an attempt to make it precise by adding more and more sentences as we go about. Now many of the sentences that we had would be returned for they are entitled by the preceding sentences.

Now is a given sentence and entitled by the preceding sentences. This is a part of the notion of logical entitlement. So, what is more commonly call logical inference. We will take up logical inference in our next lecture. Thank you.