Fundamentals Of Artificial Intelligence
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Lecture – 12
First Order Logic -I

Welcome to fundamentals of artificial intelligence. We are looking at knowledge representation and reasoning. And, in the last class we have discussed propositional logic. Propositional logic deals with preposition, complete statements used as a whole. The truth values and logical reasoning involving these propositions, propositional logic commits only to the existence of facts.

The wall being represented may not be so; we have objects their properties, relationship between objects and generalization those are to be dealt with. Therefore language which is more expressive than propositional logic is required. Today we will look at first order logic also called predicate logic for first order predicate calculus. A language co-opted by the artificial intelligence community for knowledge representation purposes.

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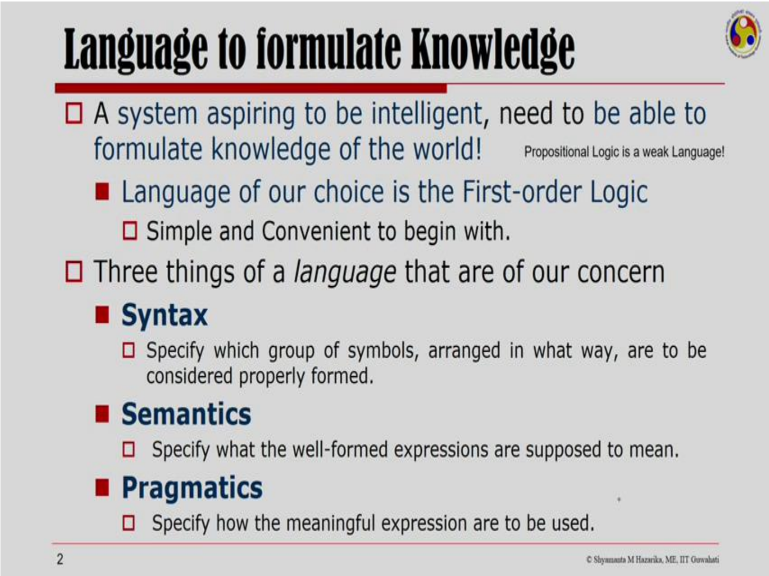

A machine aspires to be intelligent to somehow formalize the knowledge of the world in which it in habits language for this is the first order logic. The first order logic is our language of choice because it is simple and convenient to begin with and we have seen that propositional logic is a weak language. Any, language that needs to formulate knowledge of the world need to deal with three things that are of our concern here.

Number one, syntax; we need to specify which group of symbols arrange in what way are to be considered legal or well-formed expressions within the language. The next is sementics which is about associating meaning to these well formed expressions and then we have pragmatics, which is specifying how the meaningful expressions are to be used. Now Syntax and semantics is easy to understand.

Let us take an example to understand pragmatics. In English the statement there is someone behind you would be a warning or a request. And depending on the scenario one would become cautious or one would make way. Similarly in the knowledge representation and reasoning when we say pragmatics, We are interested in figuring out how to use the meaningful sentences as part of a knowledge base from which inferences are to be drawn?

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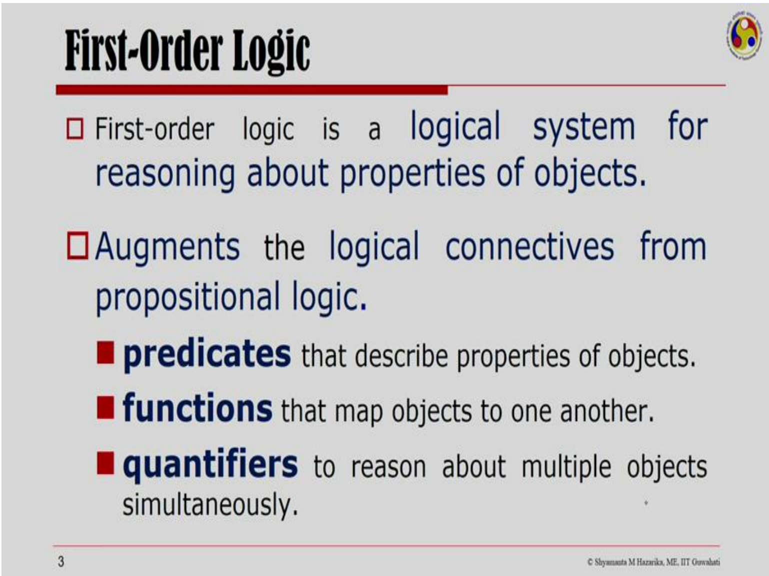

First order logic is a logical system for reasoning about properties of objects. First order logic actually arguments propositional logic. And it has a predicate that describes properties of objects, functions that maps object to another object, quantifiers to rigid about multiple objects simultaneously. It is this quantifier that makes first order logic more expressive than propositional logic.

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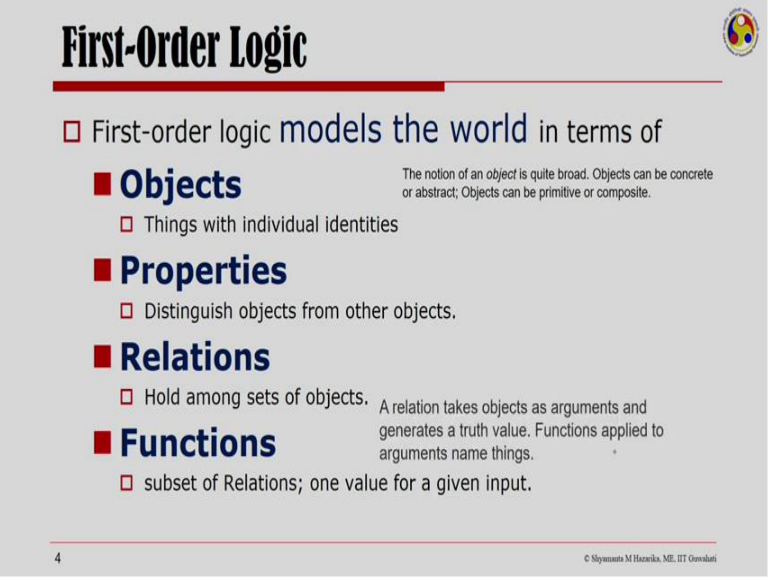

First order logic models the wall in terms of objects which are things with individual identities? It is important to realise that concept of object is very broad objects can be as concrete as a person or as abstract as the notion of justice. Objects can be primitive or composite such as an electric circuit being made up of smaller circuits. Then we have properties which distinguish objects from other objects relations define between set of objects and function which is a subset of relation is returning a value for a given input.

Now a relation takes objects as arguments and generated truth value whereas functions are applied to arguments and then the name things. We will come back to this when we are talking of converting English to first order languages. Let us now look at propositional logic and try to understand, what is it that makes first order logic difference?.

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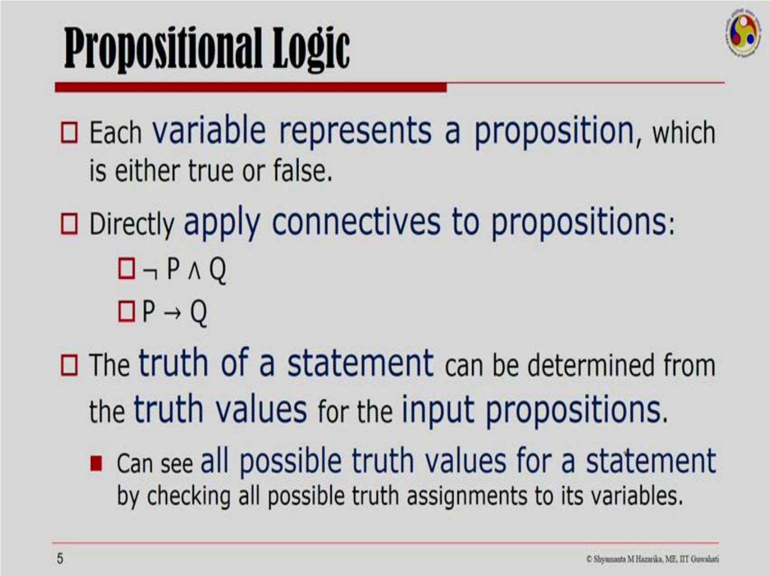

In propositional logic each variable represents a proposition which is either true or false. And we apply connectives to prepositions directly. The truth of a statement in propositional logic can be determined from the truth values for the input propositions. That is, one can see all possible truth values for a statement by checking all possible truth assignment to its variables.

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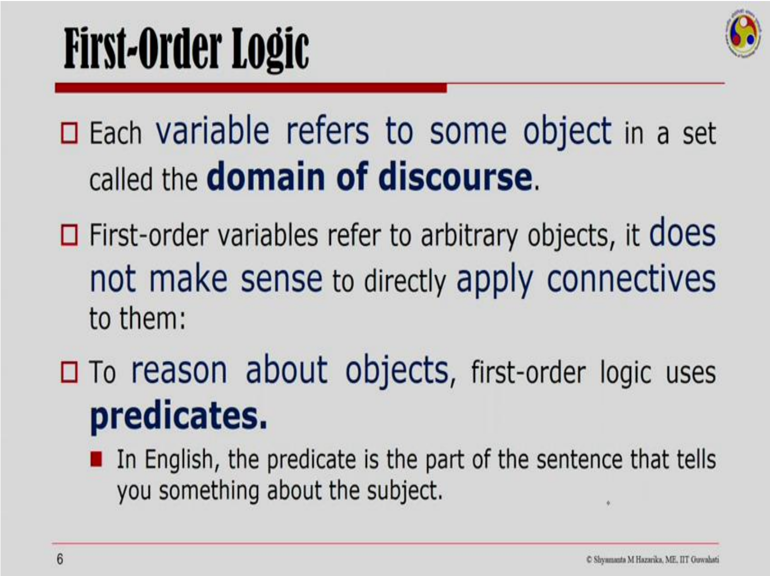

On the contrary each variable in first order logic refers to some object in a set called the domain of discourse. The first order variable actually refers to arbitrary objects. And it makes no sense to apply connectives directly to them. Infact first order logic uses predicates to region about objects like in English the predicate is part of the sentence that tells you something about the subject.

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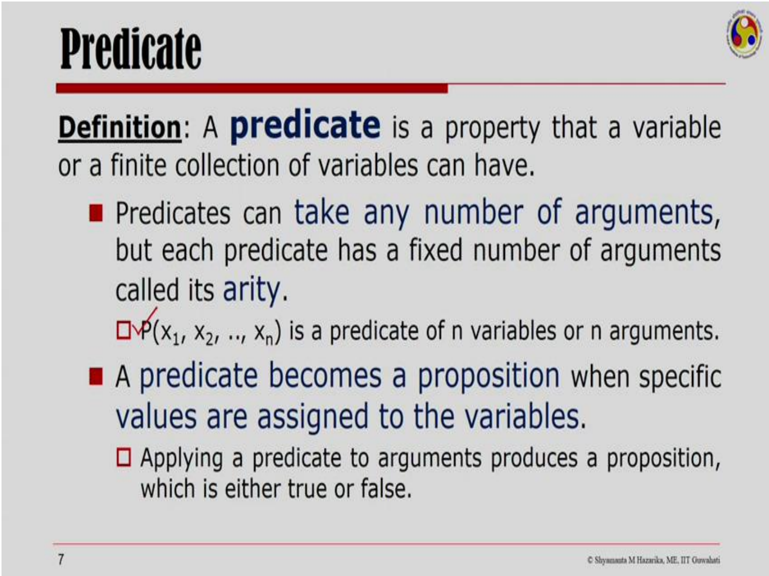

Here, in first order logic a predicate is a property that a variable or a finite collection of variables can have. Predicates can take any number of arguments and each predicate but must have a fixed number of arguments called its arity. Here the predicate p x 1 to x n is a predicate of n variables or n arguments. A predicate becomes a proposition when specific values are assigned to the variables.

That is when I apply a predicate two arguments it produces a preposition. Which is either true or false?

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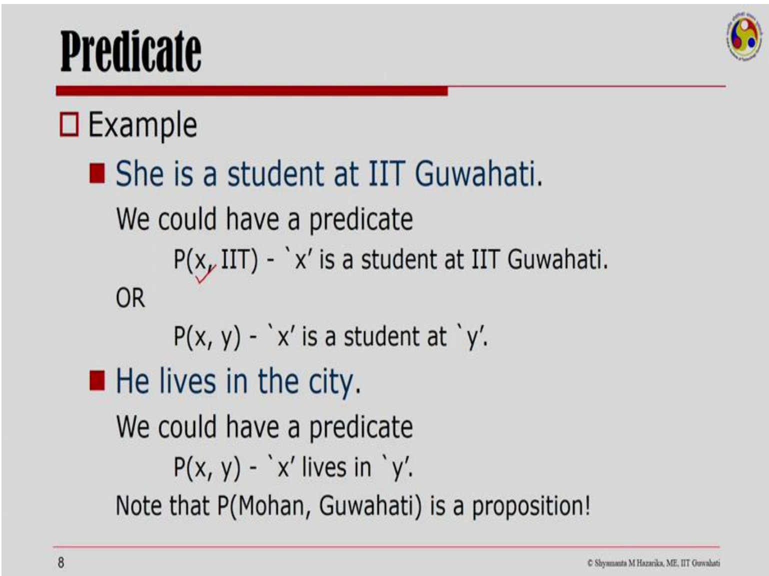

Let us look at a couple of predicate to understand the same here is an English statement saying she is a student at IIT Guwahati, we could have a pretty P x, IIT to mean x is a student at IIT Guwahati in which case this x is the only variable that specifies it is a student at IIT Guwahati. We could also have a predicate p x, y to say that x is a student at y. Here is another example, he lives in the city we could have a predicate p x, y to state x lives in y.

Now if I instantiate x to Mohan and y to Guwahati then here I have p Mohan, Guwahati which is a preposition to mean Mohan lives in Guwahati.

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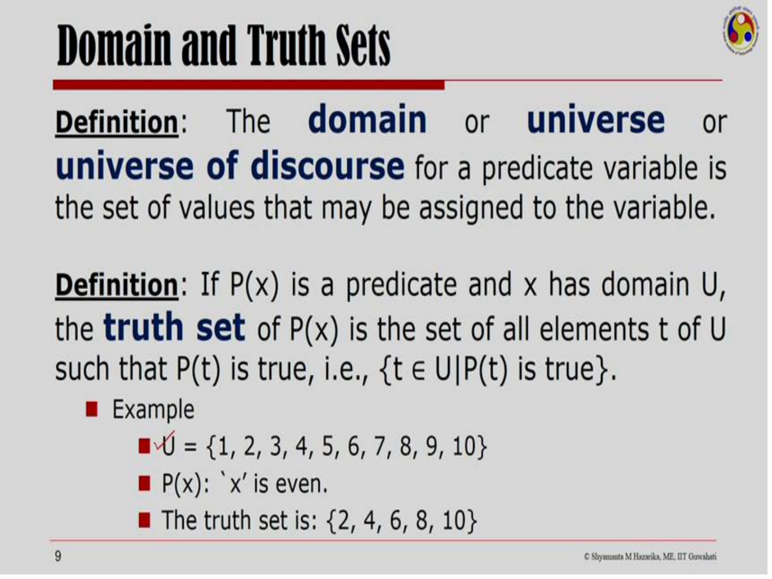

The domain or universe or universe of discourse for a predicate variable is the set of values that may be assigned to the variable. And the truth set of a predicate given a domain U is the set of all elements of you such that the predicate Pt is true. Let us try to understand this with an example here. Here, I have a domain U which is the first 10 integers 1 to 10, I have a predicate P of x which is saying x is given. Given the predicate Px the truth set is there for the set of even integers which is 2 4 6 8 and 10.

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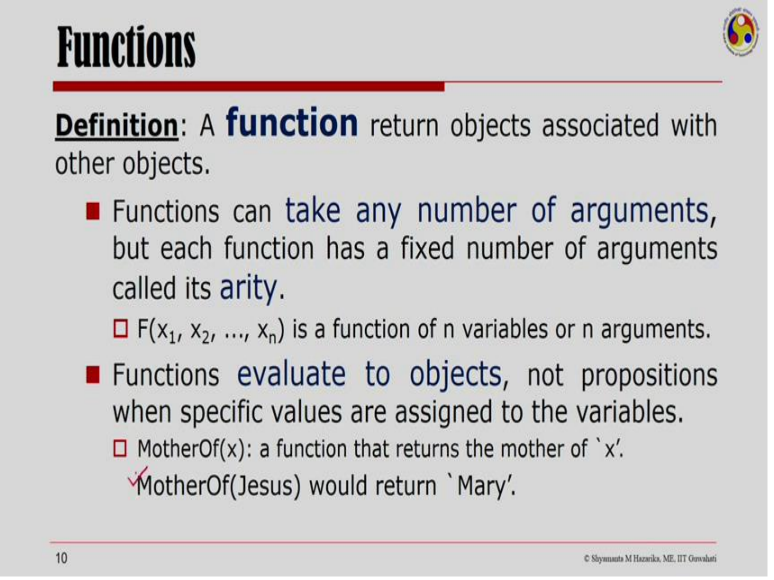

Now let us look at what we mean by function in first order logic. A function return objects associated with other objects. Functions can take any number of arguments but each function would have a fixed number of arguments called its airty. Here I have a function of n variables of n arguments. Unlike predicates that evaluate into prepositions functions evaluate to objects when specific values are assigned to the variable.

Let us say I have a function mother of x with the meaning that it returns the mother of the variable x. Under a common interpretation we should not be very difficult to see if it is a mother of Jesus that would return only Mary.

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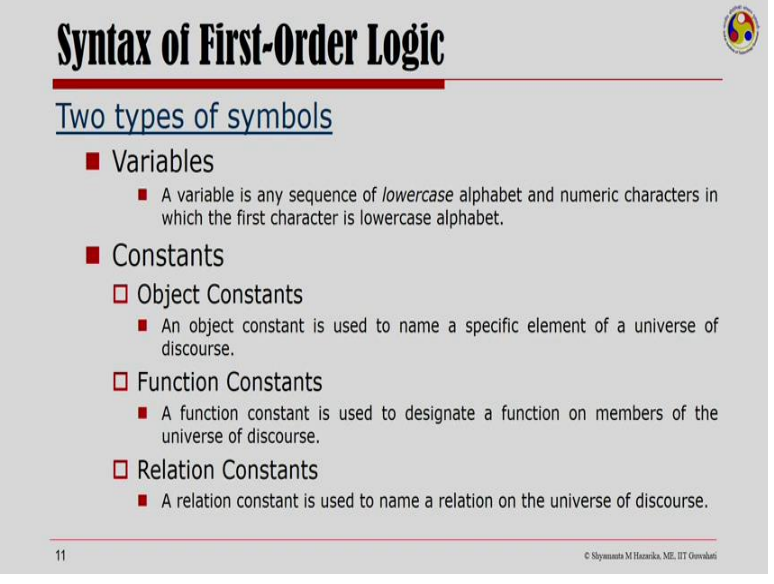

Let us quickly look at what are the types of symbols we have in first order logic. First order logic has two types of symbols as you may have made from our previous discussion. One, it is variable, a variable is any sequence of lowercase alphabet and numeric character in which the first character is lowercase alphabet. Then we have constants. We are three types of constant symbols in first order logic.

One the object constant; the object constant is used to name a specific element of a Universe of discourse. We have the function constant, which is used to designate a function on members of the Universe of discourse. And we have a relation constant which is used to name a relation on the Universe of discourse. Having introduced a couple of concept from first order logic, let us now look at what the first order logic provides and what the user provides in order to understand a first order logic representation.

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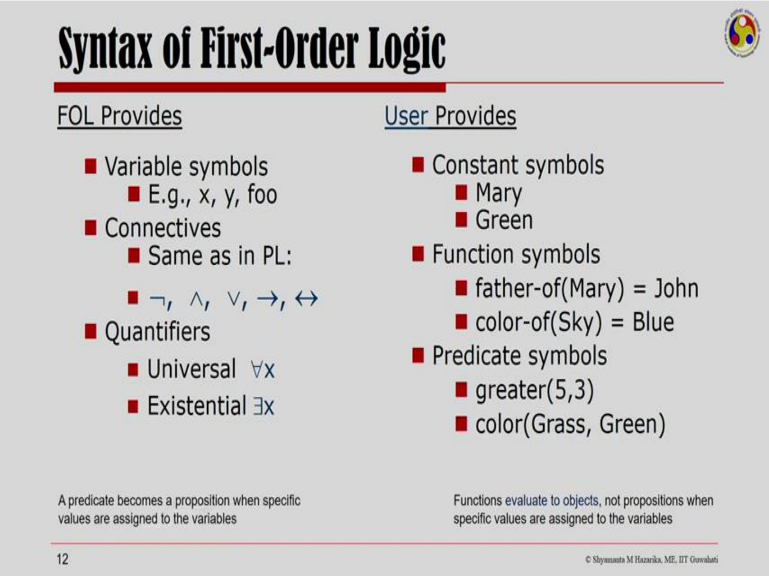

The first order logic provides variable symbols it provides connectives. Same as in propositional logic it provides the quantifiers the universal and existential quantifier. The user provides constant symbols meaning of which is clear under certain interpretation. The user provides functions symbols and these functions if you remember evaluate to objects when the specific values are assigned to the variable.

Like here, I could say father of Mary and it could evaluate to John or I could have a function symbol like colour of the variable being sky and it should return an object blue. The user also provides predicate symbols the predicate symbols when instantiated evaluates to a proposition. So, we can have predicates like greater x, y and then if I have 5 and 3, then this becomes a proposition greater 5, 3. Here I have a predicate called colour x, y, x is y in colour that I could write things like grass is green because grass is green in colour.

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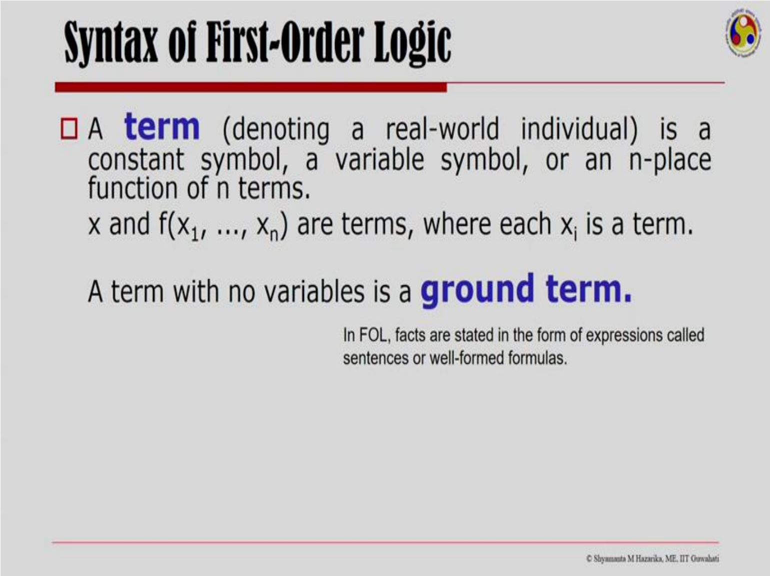

So a term in first order logic is a constant symbol, a variable symbol or an n-place function of n terms. A term with no variables is a ground term. In first order logic facts are stated in form of expressions call sentences for well formed formulas. Of this, the atomic sentence is an n-place predicate of n terms.

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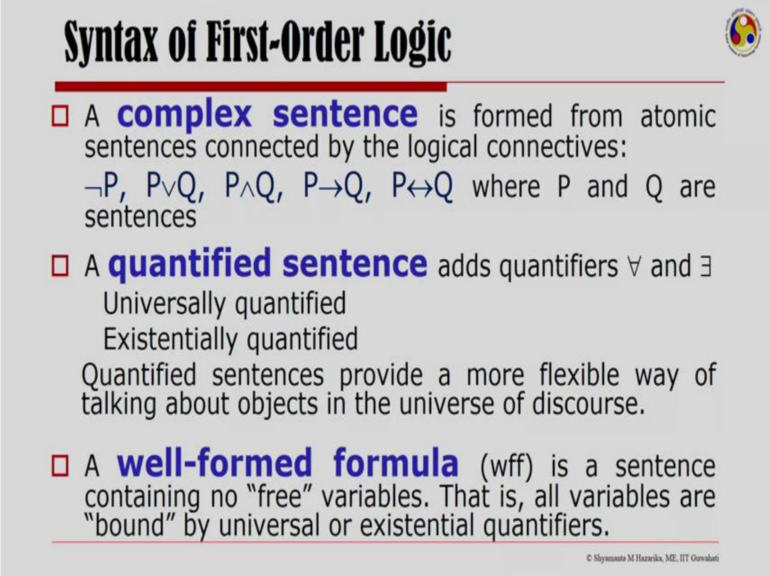

A complex sentence is formed from atomic sentences when they are connected by the logical connectives. So, we have sentences p and q, and we could generate not of p, p or q, p and q, p implies q, if and only if q. Quantified sentence as quantifiers for all and there exist the universally quantified sentence and the existential quantifier sentence, quantified sentences providing more flexible way of talking about objects in the Universe of discourse.

Find me a well formed formula in first order logic is a sentence containing no free variable that is all variables are bound by universal or existential quantifiers.

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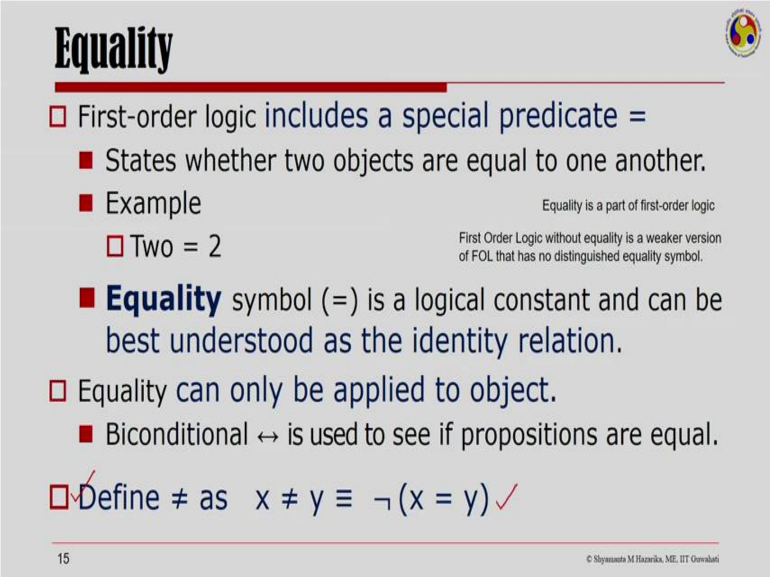

Let us now quickly look at a very special predicate in first order logic that is the predicate of equality. The predicate of equality state whether two objects are equal to one another. Example here could be I could have an object two and object 2. And I could say two is equal to 2. Equality symbol is a logical constant and can be best understood as the identity relation. First order logic without equality is also possible but its weaker version of first order logic that has no distinguished equality symbol.

Equality can only be applied to objects. If I am talking of propositions then I need to use by conditional. So, not equal to given the equality symbol could be defined as negation of x equal to y. Let us the quantifiers more closely.

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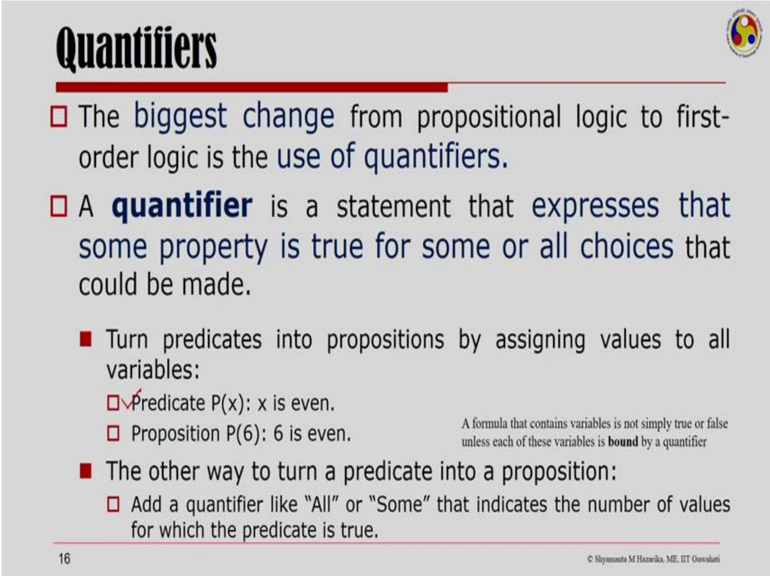

The biggest change from propositional logic to first order logic is the use of quantifiers. It is the quantifiers that make first order logic more expressive then propositional logic. And a quantifier is a statement that expresses that some property is true for some or all of the choices that could be made. So, it turns predicates into propositions by assigning values to all the variables. So here is a predicate P of x to mean x is even any five instantiate x to 6 then I have a preposition P of 6, 6 is a even number.

The other way to turn a predicate into a preposition is to add a quantifier like all or some, that indicates the number of values for which the predicate is true. A formula that contains variables is not simply true or false unless each of these variables is bound by a quantifier. And that is what these quantifiers are used for.

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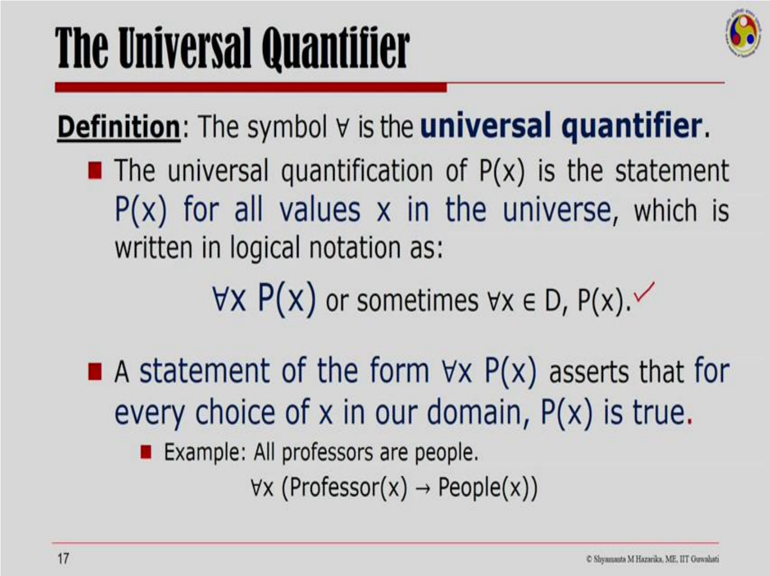

We will first look at the universal quantifier. the symbol for all is the universal quantifier. The universal quantification of Px means x for all values of x in the universe which in the logical notation could be written as for all x Px or sometimes for all x in the D is the domain of this course P of x. A statement of the form for all x Px asserts that for every choice of x in our domain system Px is true.

For example, we could say all professors are people by saying for all x if x is a professor then ht is a people.

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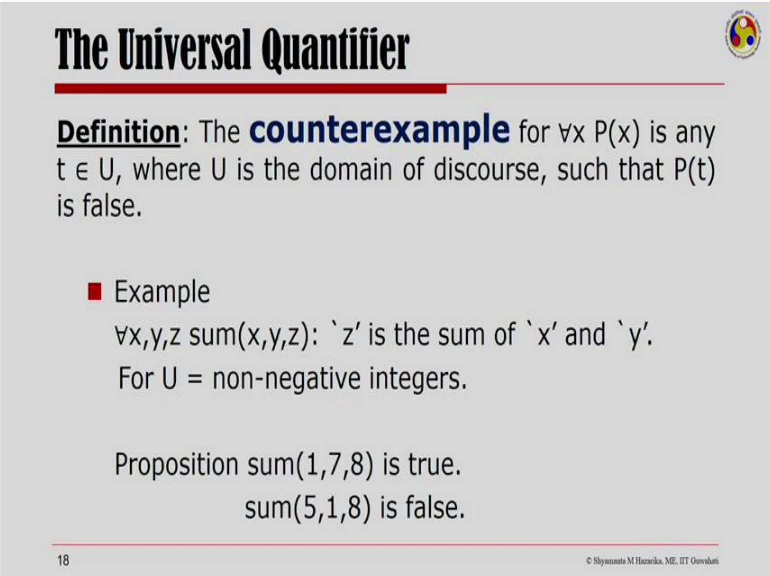

The counter example for all x Px is any t in the domain of discourse? Such that P of t, is false. Let us look at this example to understand what we mean by the counter example for all x Px.

For all x, y, z let some x, y, z refer to z sum of x and y. Now if I think the Universe of discourse as the non-negative integers preposition 1, 7, 8 is true, where is 5, 1, 8 is false.

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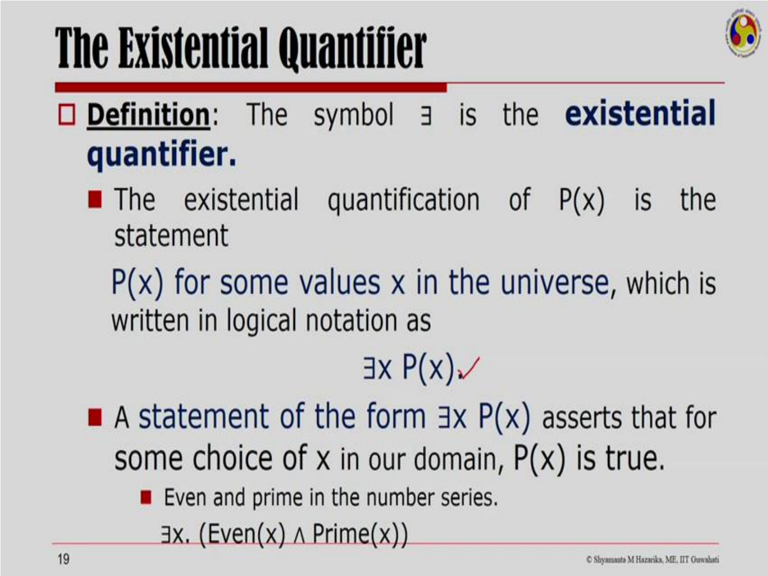

So, let us now look at the existential quantifier the symbol there exist is existential quantifier. The existential quantification of P of x is the statement that P of x for some value of x in the universe which is written as there exist x P of x. A statement of the form there exist x P of x asserts that for some choice of x in our domain P of x is true. For example, we could s**a**y even and prime in the numbers that the number 2 because there exist even x and prime x. There exist a number which is even and prime.

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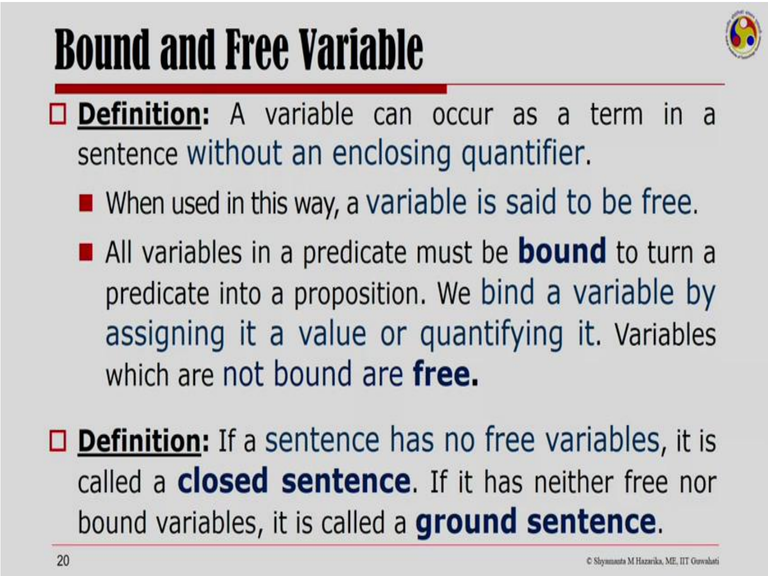

A variable can occur as a term in a sentence without an enclosing quantifier. When use in this way, the variable is said to be free. All variables in a predicate must be bound to turn a predicate into a preposition. We bind a variable by assigning it value for quantifying it. Variable which are not bound are called free variables. If sentences has no free variable it is called the close sentence if it is neither free not bound variables is called the ground sentence.

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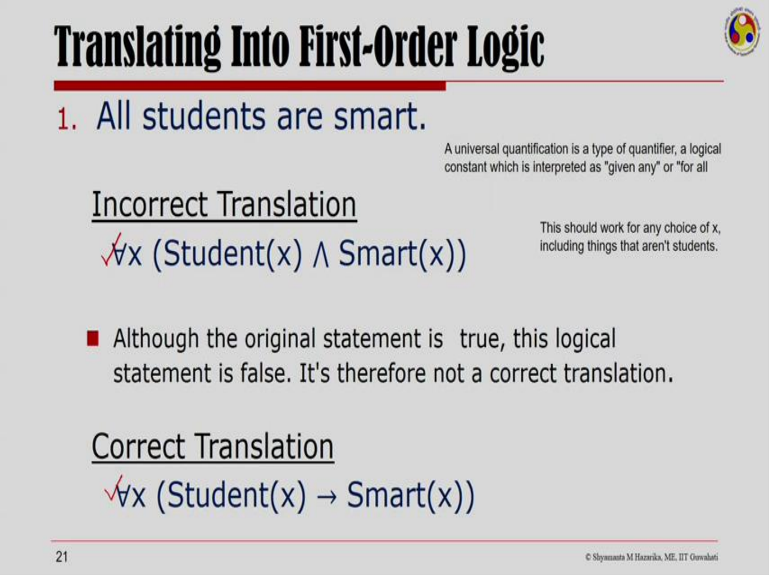

So let us now look at translating a couple of English statements to first order logic. Here is our first example? All students are smart a Universal quantifier needs to be used here because we are talking of all the students. But then if we write for all x student x and smart x, this is an incorrect translation. This is because our translation should work for any choice of x including things that are not student.

If we take certain x which is not a student this statement fails. Although the original statement is true this logical statement will be false is definitely not a correct translation. A correct translation would be for all x if x is a student then he is smart. One needs to remember that a Universal quantification is a type of quantifier logical constant which is to be interpreted as given any or for all and this statement will work for any x.

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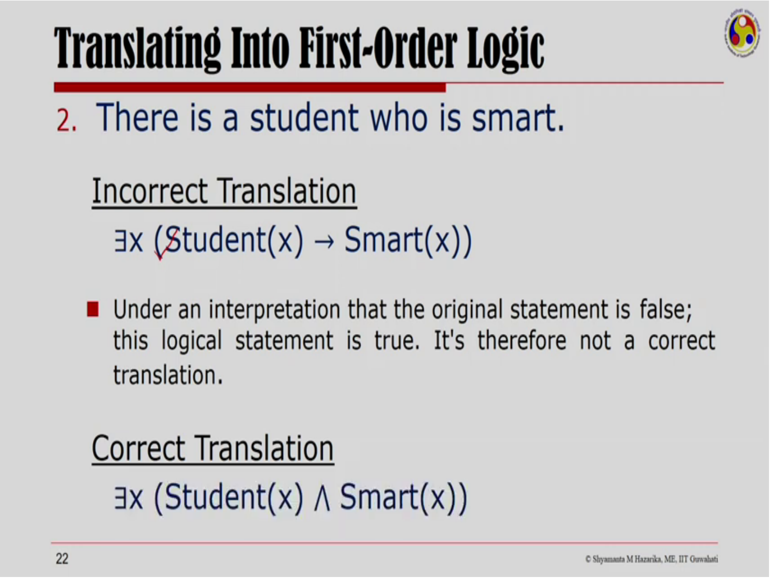

Let us take another example and here we are saying there is a student who is smart. Now we are talking of existence of and therefore. One thing is clear that we would be using and existential quantifier. But if you say there is a student x that would mean that he smart this should be an incorrect translation. The original statement is false whereas is this logical statement for any interpretation that I take where is not tested would become true therefore it not a correct translation. A correct translation would be there exists an x student x and smart x.

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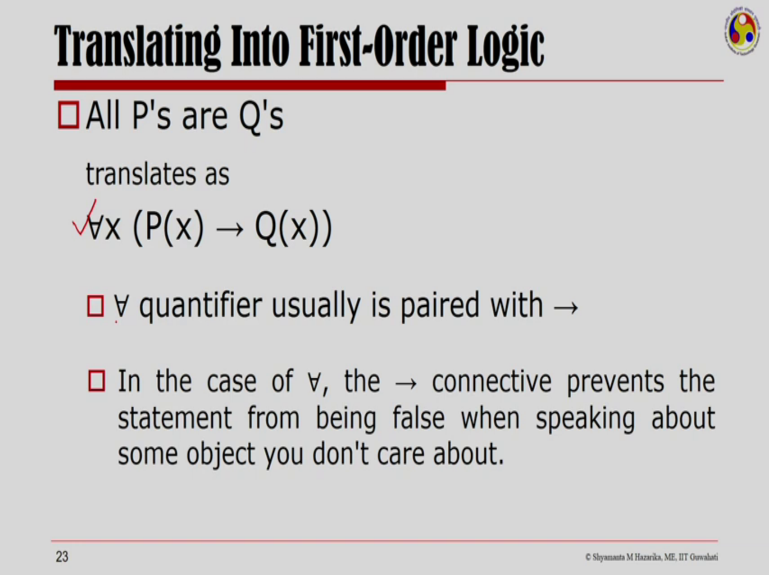

Two things that have come to focus in this examples is number one if statement that says all P's are Q's . Then the translation should be for all x if P x then Q x. If I have statement that says some P's are Q's then I need to translate it as there exist x, Px and Qx. So, when I look at this statement all P's are Q's this for all quantifier. I need to remember is usually to be paired with an implies. In case of the fall quantifier the implication connective prevent the statement from being false when speaking about some object you do not care about.

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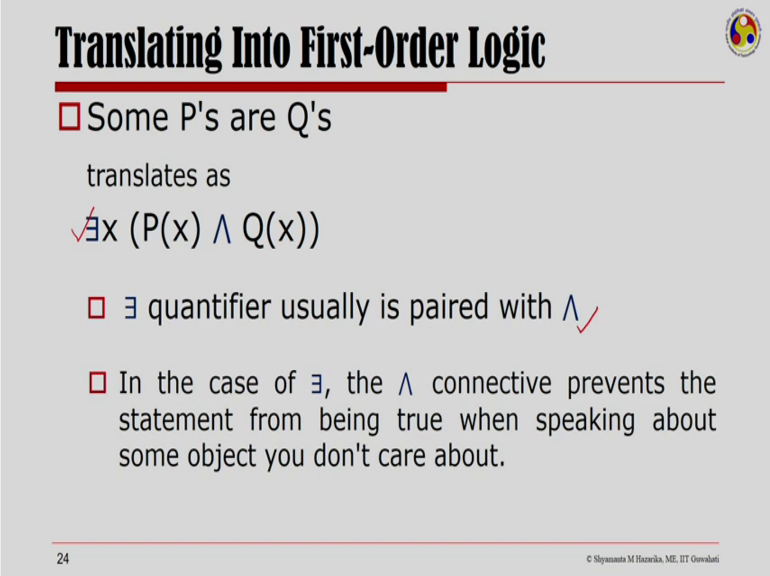

Whereas when you are talking about the existential quantifier, the existential quantifier is to be paired with a conjunction. In case of the existential quantifier, the And connective prevents the statement from being true when speaking about some object you do not care about. And this is something that is a Pitfall on conversion of English to first order logic that needs to be avoided.

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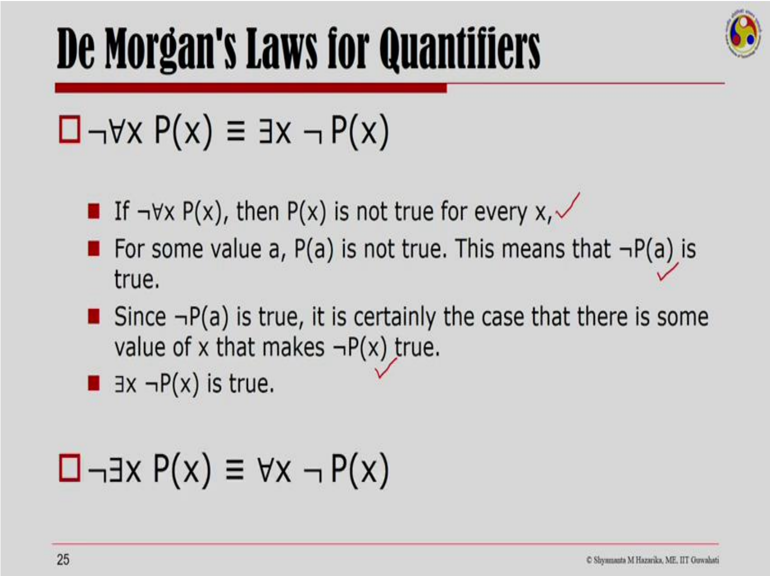

Now let us quickly look at De Morgan's laws for quantifiers. The first one states that not for all x Px is equivalent to there exist not of Px. Now, how do you go about checking this let us argue it out. If not of all x Px, then we could see that Px is not true for every x. When Px is not true for every x I will have some value a where Pa is not true. What this would mean is that not Pa is true. If not Pa is true.

It is certainly the case that there is some value of x that makes not Px true. And that is what has been written that there is an x where not of Px is true. We can proceed long the similar lines and argue that not there exist x Px is equivalent to for all x not of Px.

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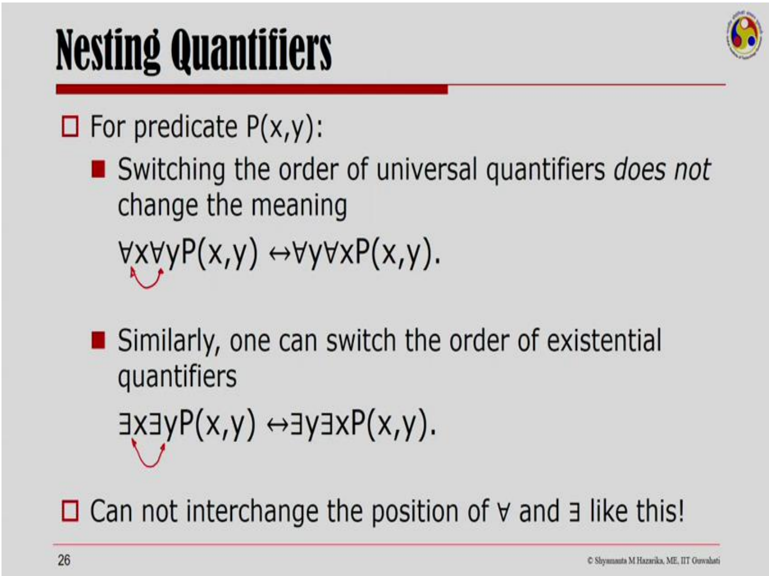

Now let us look at the vital point in first order logic that is about nesting quantifiers. For predicate Px switching the order of universal quantifier does not change the meaning. If I have a statement like for all x for x all y p of x y I could very well write it as for all y for all x P of x y. Similarly one can switch the order of existential quantifiers like there exist x there exist y P x, y this would be the same as there exist y there exists x P of x, y so I could switch this for all x for all y.

And I would still have the same meaning. Similarly I could switch the existential quantifier and I could have the same meaning. But one needs to remember that one cannot interchange the position of for all and there exist like this.

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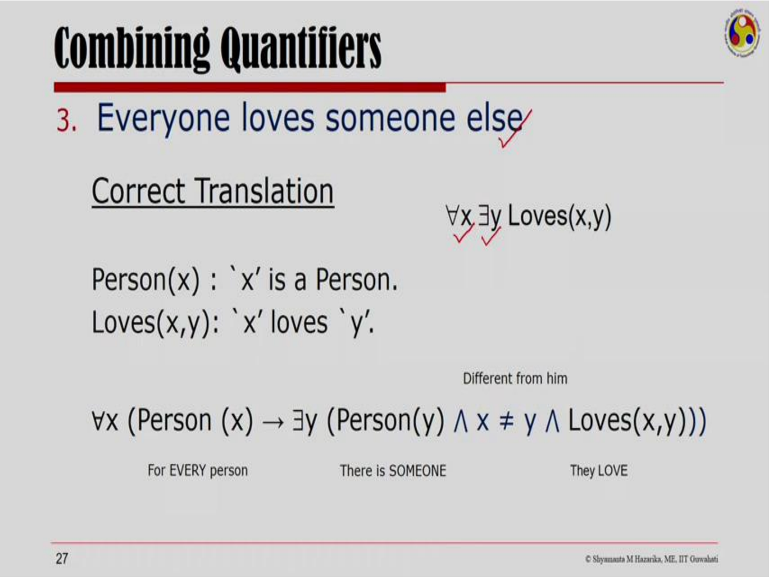

So, let us look at an example and try to understand them. Here is a 3rd English statement which says everyone loves someone else. In the statement we can make out that the correct translation would be if I was just saying everyone loves someone it will be that I would for all x to mean everyone and there exist a y to mean there is someone and x loves y here in order to take the fact into consideration that we are saying love someone else I will have to include a statement to somewhere. We says that x and y are not the same ones.

So I take person x to mean x is a person. I take a predicate loves x, y to mean x loves y and then I could write a statement here like this that for all x if x is a person. It imply that there is a y. Where y is a person and x is not equal to y and x loves y so here in this statement I would have for every person I would have someone who is different from him and they love each other, now here the order of the quantifiers is Universal first followed by existential.

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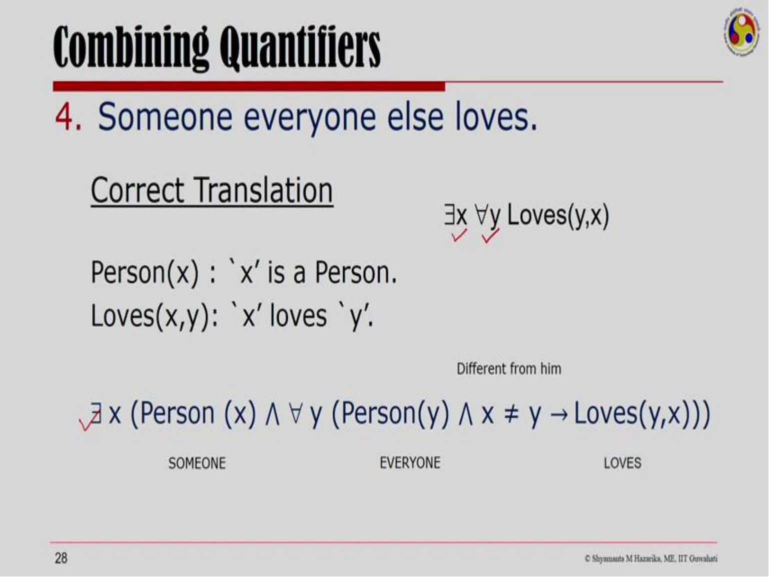

Let us look at very close statement but usually different in first order logic. Someone everyone else loves so correct translation would be there is someone. Everyone else loves so this I would again use the same predicate x is a person and x loves y I would there after write that there is a person x, so first they would be someone and then for all why there will be everyone. Someone is different from everyone and everyone loves that is someone.

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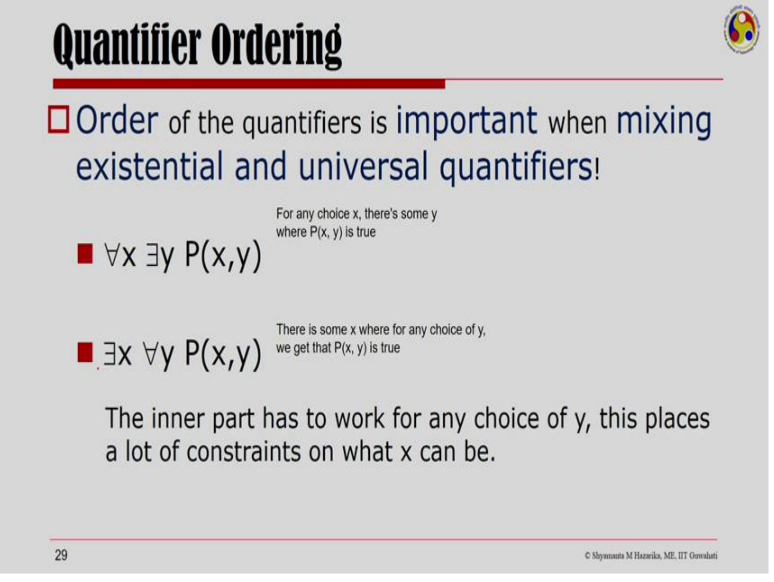

Order of quantifier is important when mixing existential and universal quantifiers for all x there exist y P of x y would mean for any choice x the some y, where P of x y is true, where is there exist x for all y P of x y would mean there is some x where for any choice of y we get the P of x y is true. Now in here in the second statement the inner part has to work for any choice of y and this place is a lot of constraints on what x can be.

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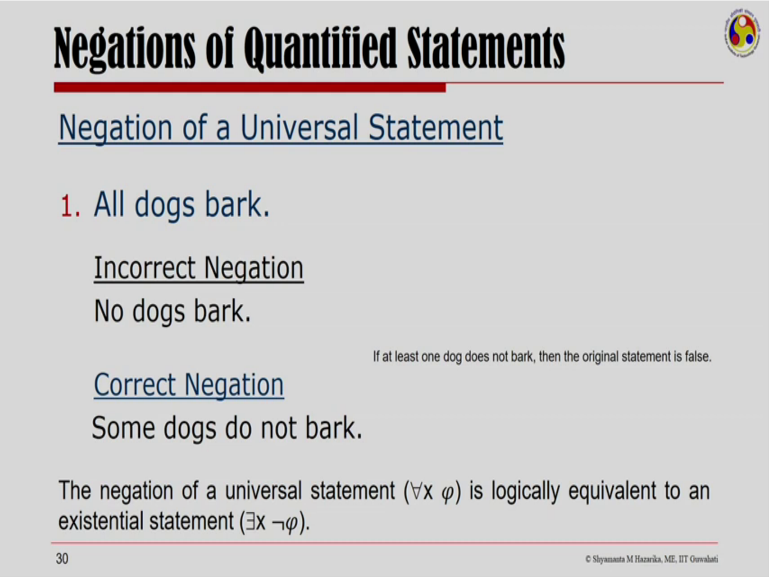

Now let us look at how we negate quantify sentences. So, first we look at negation of universal statement. If I give you a statement like all dogs bark, and I want you to negative it. The immediate statement that comes to mind is no dog bark but that is an incorrect negation. The correct navigation for this statement would be that there is some dogs that do not bark. All dogs bark if at least one dog does not bark then the original statement is false.

And therefore the negation would be that some dogs do not bark. We do not all the dogs not to bark. The negation of a Universal statement is logically equivalent to an existential statement with only the Phi being replaced by not Phi. So, here we have all dogs bark we replaced by saying some dogs do not bark.

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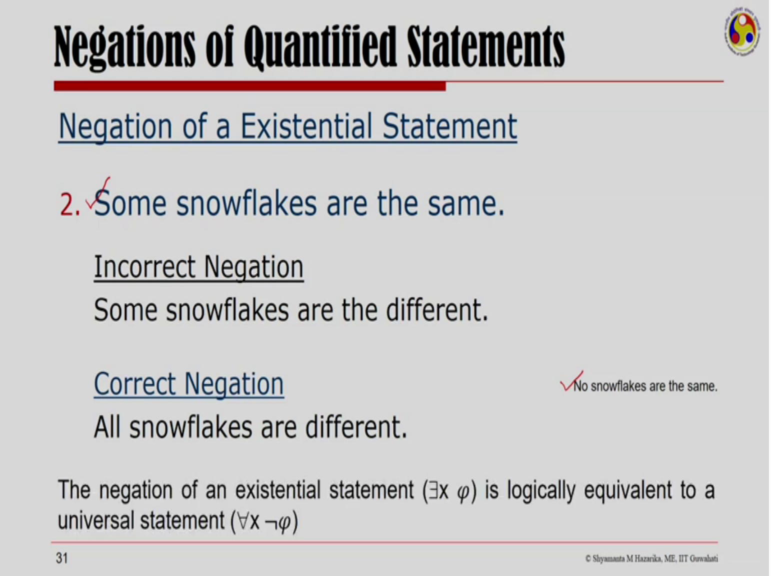

Now let us look at negation of an existential statement. Some snowflakes are the same immediately and incorrect negation comes to mind some snowflakes are different. But the correct navigation would be no snowflakes are the same because here it is saying some of them are same another for them to be false all of them need to be actually different. So, we will say all snow facts are different.

So the negation of an existential statement there exist x is logically equivalent to Universal statement with the phi being replaced by not of phi. So here we have some snowflakes. We replace it by for all snowflakes and say the same not of same we are usually returns different. So, we could write there all snowflakes are not same.

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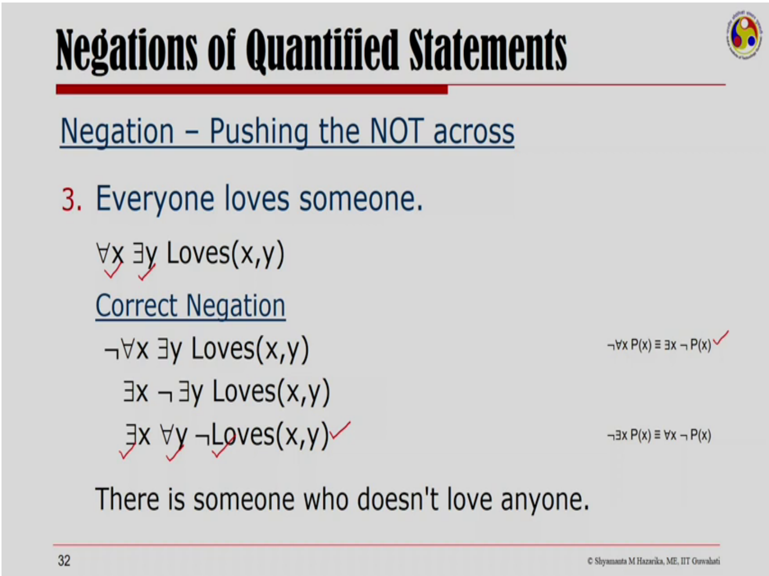

Let us look at how we could do it for complex statements. The negation for Complex statement would mean pushing the not across. So, let us look at this statement. Everyone loves someone. So here is a statement which says everyone loves someone because I have everyone and there is someone who everyone loves. I want to know this, what is the negation of the statement.

The correct negation of this statement would be that not for all x there exist y loves x, y and then I could push the not forward by looking at the de-Morgan's law that we have little while ago try to argue that for all x Px is same as there exist x not Px. So, we could write this and push this not inside or I could now replace this statement as there exist x not of there exist y loves x, y.

And then I could take this one step forward to replace it with the second de Morgan's law that we had seen that not of their exists x P of x is same as for all x not P of x and replace this here to write for all y not loves x,y. So, the negation of this statement that for would be that there is someone who does not love anyone. Even if everyone loves someone you want to negate and immediate treatment that comes to my mind is everyone does not love someone. Actually the correct negation is there is someone who does not love anyone.

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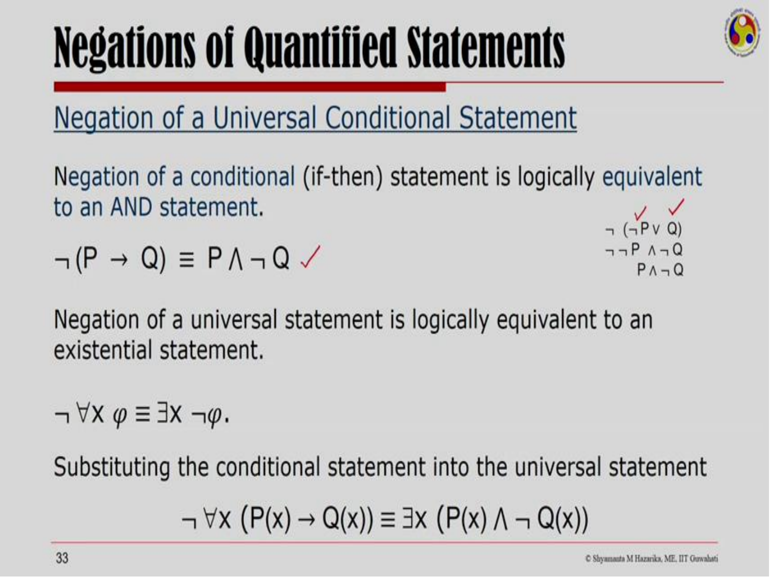

Now let us look at the negation of a Universal conditional statement that is a statement that is conditional that has a Universal quantifier in its location of a conditional statement is logically equivalent to an end statement. That is what we have looked in propositional logic. Not of P implies Q is same as P and not Q we could look at this way, not of P implies Q, P implies Q, I could write as not P or Q. And then when I have a not of P junction, it is not of the first and not of the second so I have not of not of P and not of Q.

So, double negation I am the having P and not a Q so negation of a if then statement is logically equivalent an end statement and then we have seen that negation of a Universal statement is logically equivalent an existential statement that is Not of for all x Phi is same as there exist x not Phi. Now, substituting the conditional statement into the universal statement here we would have not for all x Px implies Qx that is the conditional statement which is universal conditional statement.

The negation of the universal conditional statement is same as saying there exist x Px and not Qx.

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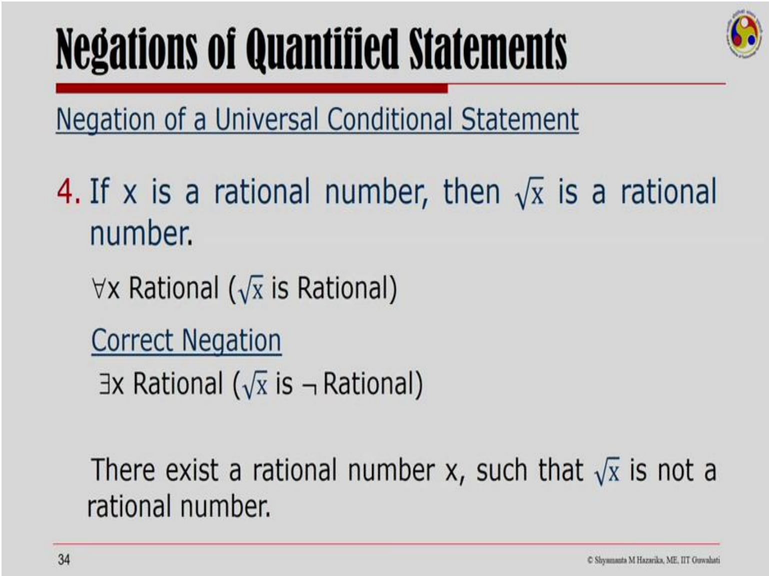

Let us take an example and try to understand this. If x is a rational number then root of x is a rational number, how do you get this statement? Now first you need to realise. This is sound form of a Universal statement because it is saying that for all x which are rational then it is saying root x is rational. So what is the correct negation? For the correct navigation I need to realise that I can introduce the existential there exist x which is rational and root x is not rational.

So, the correct negation of the statement if x is a rational number then root x is a rational number is the statement saying there exist a rational number x such that root of x is not a rational number.

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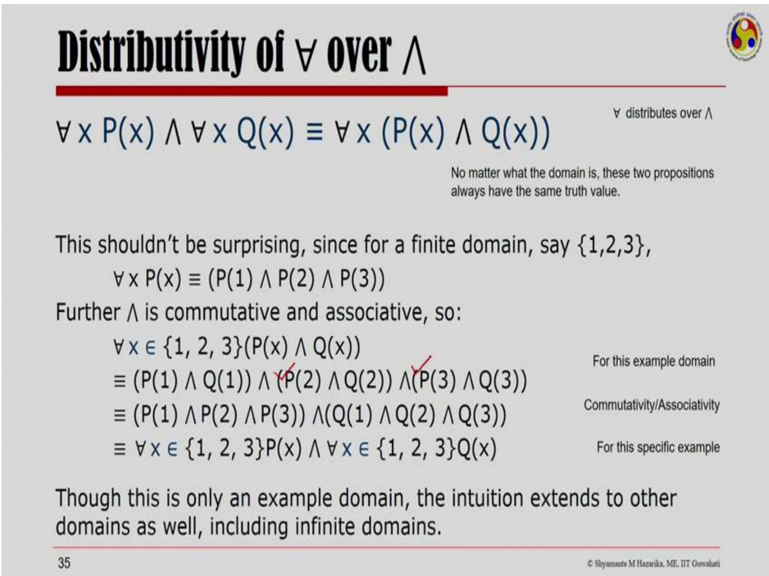

Now let is look at the distributivity of our Universal quantifier over a conjunction. So, for all x Px and for x Qx this is same as for all x Px and Qx that is that for all distribute over n and no matter what the domain is these two propositions always have the same truth value. Now this should not be surprising, we can I give it for a finite domain. Say I have my domain of this course being the digits 1, 2 and 3 then when I say for all x Px, I could write it as Pi and P2 and P3.

And for all x Qx I could similar thing further n is commutative and associative. So, I could write for all x in the domain 1, 2, 3 Px and Qx could be written as P1 and Q1 and P2 and Q2 and P3 and Q3 for this example domain. I could not bring the P's together and the Q's together because of and being commutative and associative. And end of having for all x in the domain 1, 2, 3 Px and for all x in the domain 1, 2, 3 Qx for this specific example.

Now this is example domain the intuitive extends to other domains as well including infinite domains. And we can say that the existential quantifier distributes over the And connective.

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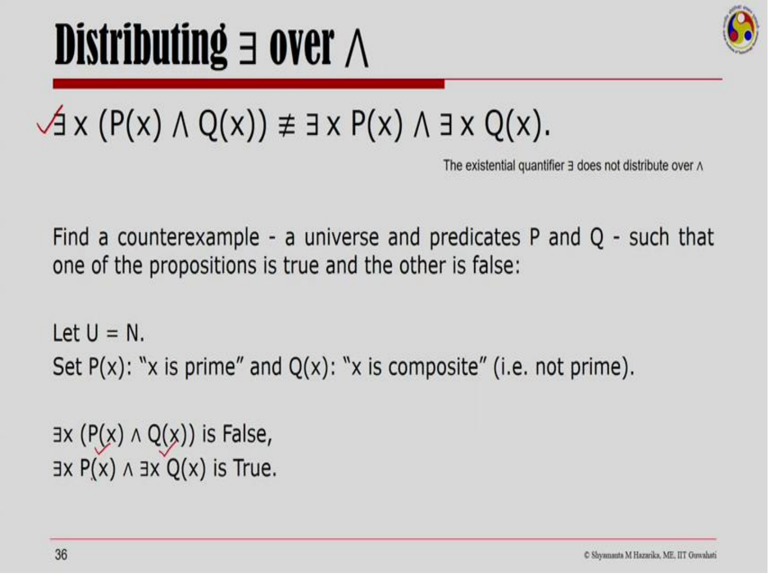

Now let us distributing the existential quantifier over and there exist x Px and Qx not equivalent to there exist x Px and there exist x Qx what it means is that the existential quantifier does not distribute over the AND. We need to find the counter example our universe and predicates P and Q such that one of the propositions is true and the other are false then literally we would have shown this.

So, let us take U to be the set of numbers x is prime and Qx composite that is not prime then there exist a number which is prime and composite is false. Where as I could definitely have a prime number and composite number and this is true.

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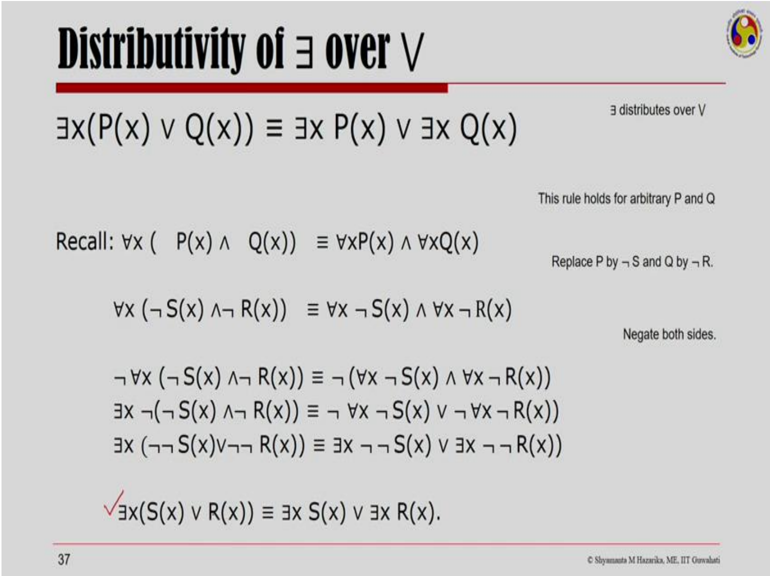

So, if you look at the distributivity of the existential quantifier over the disjunction. We see that on the contrary the existential quantifier distribution over the distinction. So, recall that for all x P of x and Qx we have shown that for all x is Px and for all x for Qx. Now this statement here holds for arbitrary P’s and Q’s therefore let us take P to be not s and Q to be not R and replace them in this formula.

So we have for all x not of x and not of Rx and this side also be replaced by not of S and Q by not of R and now we negate both sides. So, here one we negate for all x when we negate we will introduce and existential quantifier. Which is there exist not of whole of this expression here and this side. The negation being applied to a conjunction we will have the navigation of the first part or the negation of the second part.

So, we will then take the negation inside here and we will have the negation of the first part or negation of the second part. So, these two negations with give me a S, here I will end up with a R and therefore finally I will have this statement which is saying that there exist x Sx for all Rx there exist x Sx or there exist x Rx that is there exist distributes over the disjunction. So, we have seen some fundamental concepts in first order logic in our lecture today, we will continue this discussion in our next lecture. Thank you very much.