Dynamic Behaviour of Materials

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Additional Considerations of Elastic Wave in Cylindrical Bar

Hello everyone, so we have already discussed about, the Wave Equation of a cylindrical bar, while a projectile bar, hits the cylindrical bar. So, though that equation we derived, and the analysis we did, it is not really very accurate. There are, some additional considerations, we need to take care of. So, in today's lecture, we will discuss about, the additional considerations, that we need to take care.

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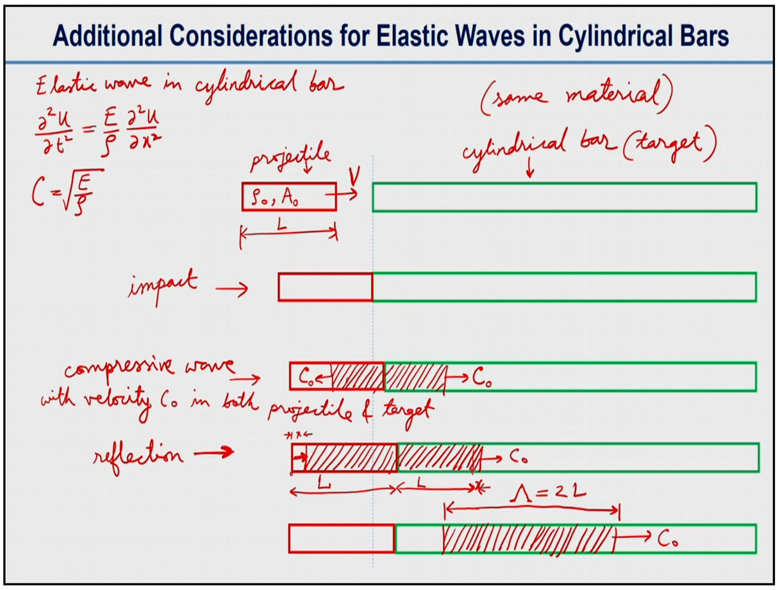

So, whatever we found in an earlier lecture, the Elastic Wave in a cylindrical bar, is expressed as, second partial of U, with respect to T, E by Rho, second partial of U, with respect to X, and where the velocity of the wave is, V is, E by Rho. So now, we will talk about the, the real situation, what happens, when a striking bar or projectile, hits the cylindrical bar. So, this is our projectile, and this is our cylindrical bar.

So now, this is moving at a velocity V, and it has a density, Rho-0 and A0. To remove any confusion, you can keep this, Wave Velocity as C. So, because here, we are showing the Projectile Velocity is V, and Mass Density is the Rho-0, and A0 is the Cross Sectional area, and we assume that, both the cylindrical bar and the projectile, the material is same, so that, the density is same.

And, in this case, we can take the cross section, it is also same. So, what happens, when a bar impact. Suppose, in this case, in this figure, it shows the impact, this projectile hits the cylindrical bar. Now, what will happen is, we will see in the third figure. So here, there will be, some stress pulses will be generated, the compressive stress. So, compressive stress will go forward, in this direction. Let us say, this is the stress pulse. This stress pulse is going, in this direction, with a velocity C. And then, similarly, there will be compressive wave, in the striking bar, as well. The striking bar also, because the materials, are also same.

So, with the same velocity, there will be wave on the other side, in the other direction. So, both, compressive wave. Compressive wave propagates in, or we can write here, C0 is the Wave Velocity. So, compressive wave, with velocity C0, in both, projectile and target. And, the cylindrical bar is the target. So now, after some time, what will happen is, it will propagate more.

And then, this in the projectile bar, the wave will actually reflect, in the free surface, this surface, and then, it will return back. So, it will return back. So, this will reflect back, with the same velocity, the reflection. So, at this end, a reflection at this, it will reflect. And, at that time, on the target also, whatever it will travel, in the projectile bar, and the same distance, it will travel in the target bar, as well, because, the velocity are same.

Now, in this case, the total length of the pulse, we will see. After some time, this wave will go into the target bar. And, there will be, it is a note, no wave in the projectile bar. So, this wave, the length, that is the pulse, is now, only in the target bar. And, this length is the wavelength, which is, can be depended with the capital Lambda. And, that is actually, equal to twice L. So, L is the length of the Projectile.

Let us try to understand, why it is twice L, when the wave reaches, the projectile back end, that is L. At the same time, the wave will travel, L distance here, L distance here. And because, this is the velocity, assuming both the medium, because they are the same material, both the bars, the velocity is same.

So, when it will reflect back, from this end, and at that time, it will also travel, that much. Let us say, this distance is X, after it travels back. And similarly, this distance will also be, X. So, this, till this point, it is L, and the other point, it is X. So, that means, after the reflection, the Stress Wave, that the pulse has a, actually total length of twice L, and that wave, it will go forward, in this direction.

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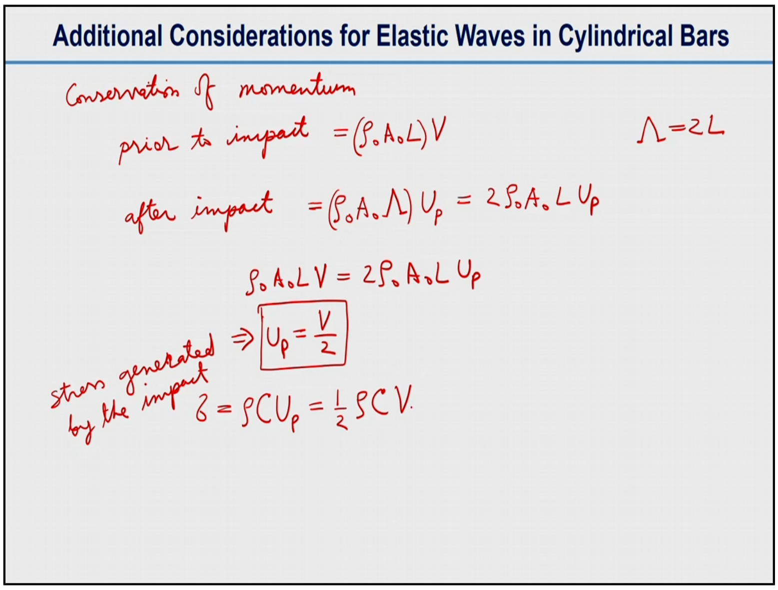

So now, if we use, the Conservation of Momentum. So, for the prior to the impact, and after the impact. So, parallel to impact, and after the impact. So, if you see, from the figure, so prior to impact, it has a velocity V. And, that can be written as, prior to impact, the momentum is equal to, Rho-0 A0 L V. So, this is the mass. And similarly, after impact, so this, Rho-0 A0, in the part of the bar, which has the stress pulse is, Rho-0 A0 into Lambda, which will be multiplied by, the Particle Velocity.

So, because that part of the bar only, have the compression wave, that is propagating towards the right direction. And, that will give us, because Lambda is equal to twice L. So, because the Lambda is equal to twice L, as we got, in the previous figure, so this will be, L U P. So now, if we use the Conservation of Momentum, so Rho-0 A0 L V is equal to twice Rho-0 A0 L U P.

So, this will give us U P, the Particle Velocity, is half of the Projectile Velocity. And again, from our earlier discussion, the stress generated, in our earlier classes, we have learned that, this Rho C UP, is equal to, half of Rho CV, now. This is, C the velocity wave. And so, UP is, V by 2, now. So, we can express the stress generated, by the impact.

So, stress generated by the impact, with a Projectile Velocity V, can be written, like this. So, we are discussing about, additional considerations, for Elastic Wave propagation in cylindrical bars. So, this is important for, Split Hopkinson Pressure Bar experiments, which is, the most widely used experiment for, Dynamic Material Testing.

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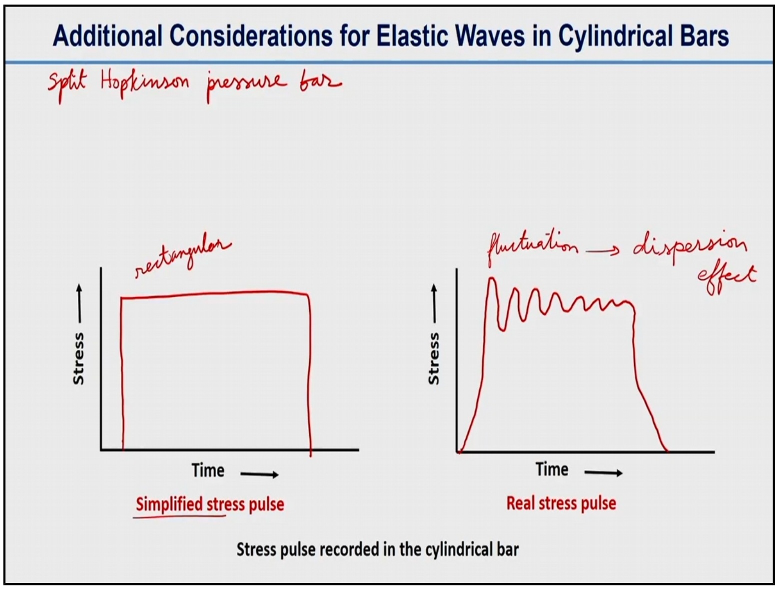

So, it is important for, Split Hopkinson Pressure Bar experiment. Now, the expected shape of the stress pulse, if it is in a simplified form. So, the expected pulse shape is, it is rectangular. However, in real case, so we will get, significant fluctuations. So, there in the real case, it looks like, something like this.

So, significant oscillation, in real case. So, these effects will influence, the interpretation of the Split Hopkinson Pressure Bar results. So, these fluctuation effects, that is actually also called, Dispersion effect. So, these fluctuations will influence, how we interpret, Split Hopkinson Pressure Bar results.

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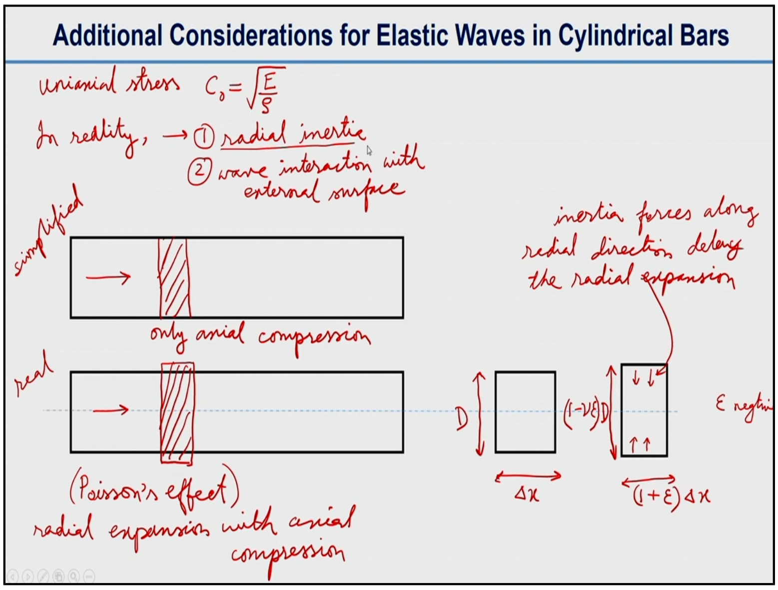

So, we will discuss, how there are deviation, from the simplified case. So, initially, what we got for uniaxial stress case. So, we found that, C0, the Wave Velocity can be expressed, as square root of the ratio of, Elastic Modulus by Mass Density, E by Rho. And however, in real case, in reality, so there are two effects. Number one is, Radial Inertia. And, number two is, Wave Interaction with external surface.

So, let us first check, what is Radial Inertia. So, in the simplified case, or simplified one, or idealized one, so what we see, that the stress waves, in this cylindrical bar, the longitudinal stress wave, so we think that this, compressive stress was coming from left to right, will compress the bar, above this small section, and that wave, the compressive wave travels, from left to right.

But, in real case or in reality, so what happened is, when the compressive wave travels from left to right, this section will be little, expanded in the vertical direction. So, that is, as you can understand, from Poisson's Effect. So, this is nothing but the, Poisson's Effect. So, radial expansion with, radial expansion or with axial compression. Now, in the first case, it is only assumed that, we have axial compression, and no radial direction, deformation.

So basically, if you see the cross section, the original cross section is, the length is Delta X, and the diameter is D. And then, the final shape will be, like this, which is, 1 plus Epsilon, which is the strain, multiplied by Delta X. And here, this will have, the Poisson Effect. So, new Poisson's Ratio, multiplied by a strain, the whole thing multiplied by D. So, and in this case, because this is now, elongated little bit.

So, in this case, we should understand, that this is, the strain is negative, here actually. Because, this is a compressive strain. So, Epsilon is negative, which will have an axial compression, and the radial expansion. And so, we will have some inertial forces, due to this radial expansion. That is, material flowing outward, due to the kinetic energy, for this compression wave. So, there will be some inertia forces. These are inertia forces, along radial direction. Radial direction, delay the, radial expansion. So, that is what, we mean with, by this Radial Inertia. Okay.

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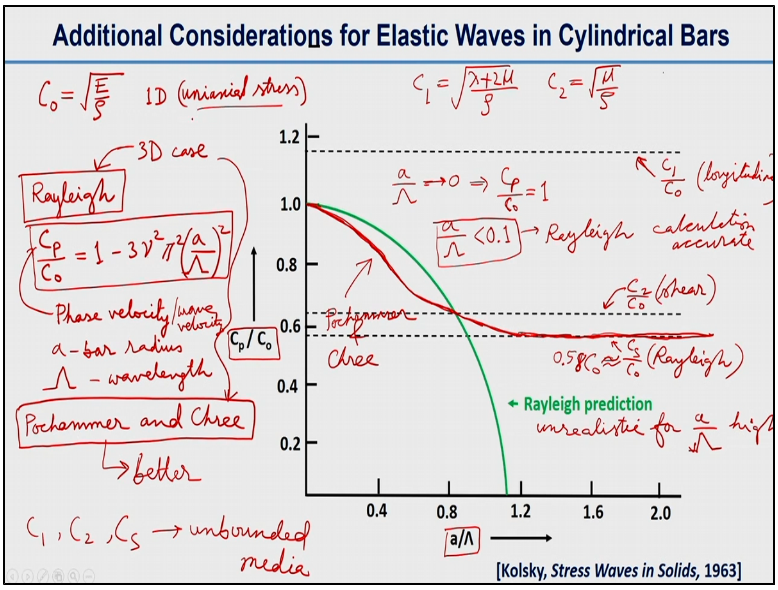

Now, we will talk about, the Wave Interaction, at the surface. So, initially, we found this for a, uniaxial stress case. So, C0 is, square root of the ratio of E by Rho, this is for 1-Dimensional case, that is uniaxial stress. But, if we assume, 3-Dimensional effects, 3-Dimensional case, so there are different calculations. First one is, we will talk about, Rayleigh's calculation. And, that gives us is, CP by C0, is 1 equal to 1 minus 3, Poisson's square, Pi square, E by capital Lambda, whole square.

So, CP here, is the Phase Velocity. The Phase Velocity, which is, nothing but the, Wave Velocity, in this 3-Dimensional case. So, when we allow that, multiaxial stress conditions, so this Phase Velocity is normalized by C0, which is for the uniaxial case. And, Mu is the, Poisson's Ratio, Pi. And then, A is the, bar radius or cylinder radius. And then, capital Lambda is, the Wavelength. So, this calculation proposed by, Rayleigh. And, the another one is proposed by, Rayleigh.

And then, another one, another calculation, was proposed by, Pochhammer and Chree. So, this is another one, we are not, will plot this, both of this. So, we do not include, the equations for the second one. So, what we can see, from here, so, in 3-Dimensional cases, so there are two. One is the, Rayleigh. And, the another one is, these two are, commonly used calculations. So, in this case, so what we have is, the Phase Velocity, normalized, by the Uniaxial Wave Velocity.

So, Phase Velocity is nothing but the, Wave Velocity in the 3-Dimensional case. And, that can be also, sometimes known as the, Group Velocity. But, the Group Velocity is can be different than, Phase Velocity. So, we are not going, into that details. But, we want to know, the simply the variation of the normalized, the Wave Velocity, for the 3D case, with the ratio of bar radius, by the wave length.

So, the first one, is the green, that curve shows for, Raleigh prediction. From this equation, whatever we found, we already mentioned this equation. And, this dotted lines are for, C C1 by C0, which is longitudinal Wave Velocity. And then, C2 by C0, we have already discussed about, C1, C2, and the CS, which is the, Rayleigh Wave Velocity. C1 is, we have found in the earlier lectures, that Lambda by twice Mu by Rho, for longitudinal case. And, C2 is, square root of Mu divided by Rho. And, CS, would be little less than, the Shear Velocity.

So, the Rayleigh Wave Velocity is, little less than, Shear Velocity. So now, here we are showing that, Rayleigh prediction, for the 3-Dimensional case, with the additional effects. And then, we can see that if it is the, A by Lambda, is very small. That means, the bar radius is very small. So, this implies that, CP by C0 equal to 1, or they are almost the same. And then, if this A by Lambda, smaller than equal to 0.1, the Raleigh calculations are, accurate.

Now, we will see, the second case, Pochhammer and Chree calculations, we have not any included in an equation, but we will plot it here. So, plot will look, something like this. So, I am just opening this curve, the same curve, so it will look, something like this. And, this curve is, Pochhammer and Chree calculations. As I already mentioned, that if, A by Lambda is, less than 0.1, these Raleigh predictions, are also accurate.

But, otherwise, if A by Lambda is very high, the Raleigh prediction are, unrealistic, for A by Lambda high values. And, in that case, we can say that, this Pochhammer and Chree predictions are, better. They are better than, the Raleigh predictions. So, you can see that, at higher value of A by Lambda, this PC, which we call in short form, Pochhammer and Chree predictions or calculations, actually approaches the Rayleigh Wave Velocity, which is like almost 0.5, 8 times of C0.

So, whatever we told about, C1, C2, and CS, so as we discussed, the dotted lines C1, for by C0, this is normalized longitudinal Wave Velocity. And then, C2 by C0. And, then again, CS by C0, the dotted lines. So, corresponding to C1, C2, CS, which are, I hope, you understood, from earlier lecture, they are for, Unbounded Media. And, our C0 is for, Finite Body. So, C0 is for, Finite Body with Uniaxial Stress.

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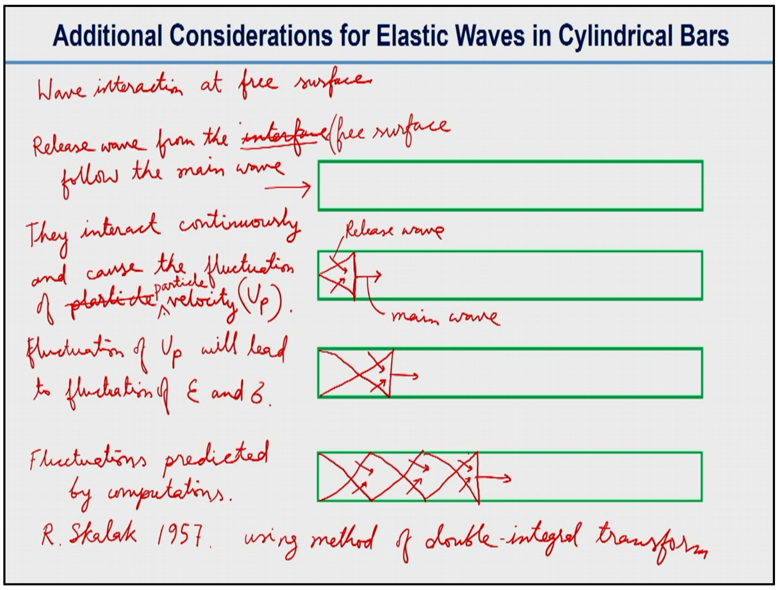

Then, we will discuss about, the Wave Interaction at free surface. So, let us see the first case. So, let us assume that, we are hitting this cylindrical, the rod, with a projectile or striker bar. And then, in the second case, what we get to know that, while this compression wave propagates, so there will be some waves, that the wave will interact, with the free surface.

And then, there will be some waves, will be reflected, from the interface. And, that is, we call the Release Wave, from the interface. So, similarly, if you go forward, so this wave will, so like this, the Release Wave. And then, similarly, we have this, the wave will propagate more and more. And, these Release Wave will, interact among themselves. So, these are the

Release Waves. We have learnt little about, the Wave Interaction earlier in, reflection and refraction.

So, the Release Waves, from the interface, will follow the Main Wave. So, this is the Release Wave, and this one is the Main Wave. They interact, continuously, and cause the fluctuation of Particle Velocity, which we denote it as, UP, earlier. Fluctuation of Particle Velocity, will lead to, fluctuation of strain and stress, as well. So, that means, in an earlier slide, we have shown that, this stress fluctuation.

So, that is due to, this Release Wave Interaction, release it from the, free surface. Okay. Actually, I wrote here, interface. But, better, we can write it as, free surface, which is interface between, the air and the cylindrical bar. But, we can write it as, free surface. So, these fluctuations can be predicted, by computation. For example, R Skalak has computed, Distress Wave Predictions, in 1957.

He used, Method of Double Integral Transform, to solve the problem of, this impact of a cylindrical projectile, on a cylindrical rod. I am sorry. This Particle Velocity, I have made a mistake in the spelling, so Particle Velocity, UP. So now, we will discuss about, another aspect of it. So, what will happen, when the cross section of the bar, changes?

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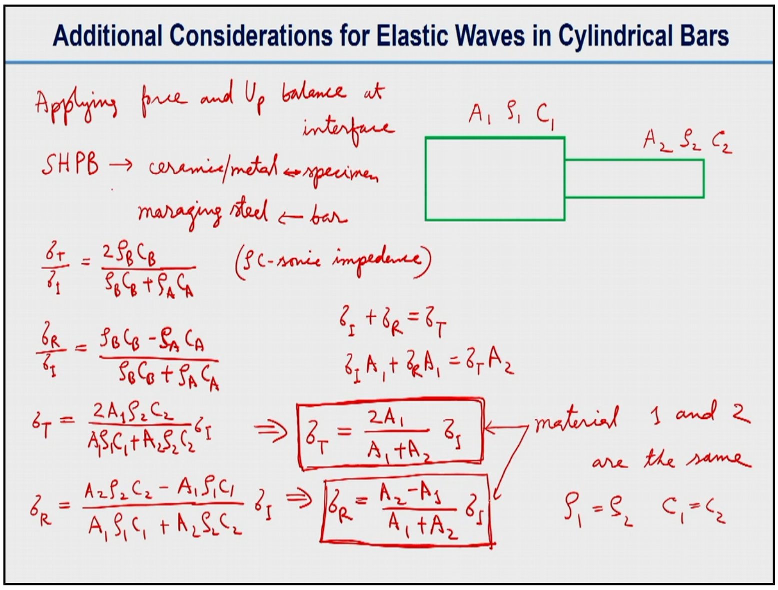

Suppose, we have a cross section, of the bar here is A1, and here, the cross section is A2. And, if we assume, the materials are also different, then that mass density will be Rho1 here, here the Mass Density will be Rho2, and similarly, the Wave Velocity, if we say, C1 here, and C2 here. This is, Longitudinal Wave Velocity. So, applying the force and particle balance, what we have used, even earlier, Particle Velocity balance.

So, we can find, the stress and the Particle Velocity, at the new cross section. So, we will apply the force, in Particle Velocity balance, at the interface. And then, we can find, the stresses and Particle Velocity, in the new cross section. So, this analysis is important in, Split Hopkinson Pressure Bar, which we commonly abbreviated as, SHPB. Generally, we take the specimen in a, smaller cross section or smaller diameter. Then the, bar diameter, and also the material, we test is, let us say, can be, any ceramic or metal.

And generally, this is the SHPB specimen. And, our bars are, used in the SHPB are, let us say, very widely used bars are, Maraging Steel, high strength steel. So then, the properties will be different, across the interface. So, as we can remember, from our earlier lectures, so Sigma transmitted by Sigma incident, is equal to, twice Rho-B CB, divided by Rho-B CB plus Rho-AC, where we know, Rho multiplied by C, is the Sonic Impedance. We call it as a, Sonic Impedance.

And similarly, Sigma reflected over Sigma I, is equal to, Rho-B CV, Rho-A CA, divided by Rho-B CV, Rho-A CA. So, if we remember, the earlier calculations we did, in a previous lecture that, the equilibrium at the interface, will require the inclusion of the area term. We did not do, at that time. But, now we can, check that. So, what we had earlier, the equilibrium is equal to Sigma I, plus Sigma R, equal to Sigma T. But now, we have the different areas. So, Sigma I A1 plus Sigma R A1, equal to Sigma t A2.

So, if you use these equilibrium conditions, so we will end up with our expression, Sigma t is equal to, twice of A1 Rho2 C2. Sigma I is equal to, A1 Rho1 C1 A2 Rho2 C2. Here, it is important, this is A1, not A2, which, Rho2 and C2, on the numerator. And, similarly, for Sigma R, Sigma subscript R, so this will be, A2 Rho2 C2, A1 Rho1 C1, divided by, you can write first, A1 Rho1 C1, plus A2 Rho2 C2, and multiplied by Sigma I.

So, which can be simplified as, Sigma transmitted, equal to, twice A1 plus A2 Sigma I, and Sigma R equal to, A2 minus A1, divided by A1 plus A2 Sigma I. So, these are important for, Elastic Wave propagation, in cylindrical bars, which will be important for our, Split Hopkinson Pressure Bar calculations. Sorry, I made a mistake here. Probably, I somehow, I forgot to mention this. So, we assumed in these two cases, that Material 1 and 2, are the same.

So, that means, Rho1 equal to Rho2, and C1 equal to C2. So, that will give these, simplified expression for Sigma T, and Sigma R. So, as we discussed, that is Split Hopkinson, the materials at the bar, and the specimen, will be different, and also, can be the cross section can be different. So, with this, so we are closing this, chapter of Elastic Wave propagation. So next, we will discuss, the Plastic Wave Propagation.