Dynamic Behaviour of Materials
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Lecture – 3
Introduction to Waves

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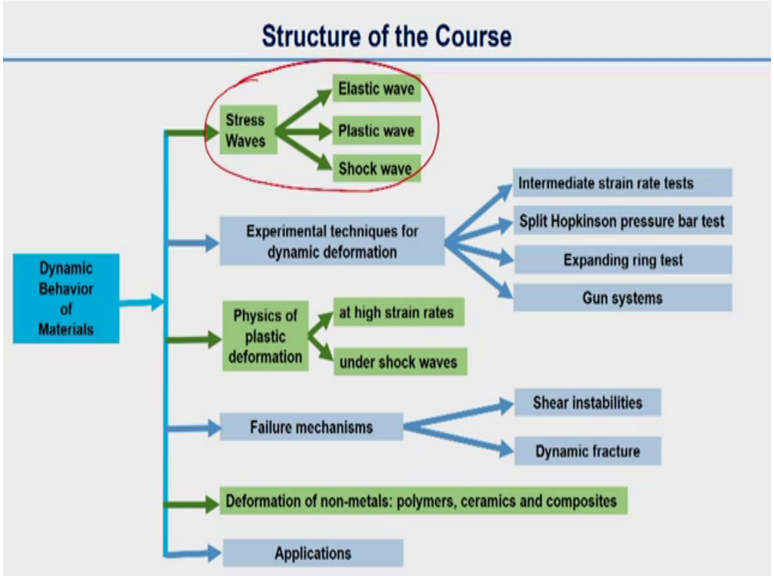

Hello everyone, in the last lecture we discussed about applications where we need dynamic behavior of materials knowledge and we also discussed about the structure of the course, dynamic behavior of materials. So here, in the structure of the course, we have the stress waves. So, we will first start with the stress waves. So today, we will discuss some fundamentals of waves.

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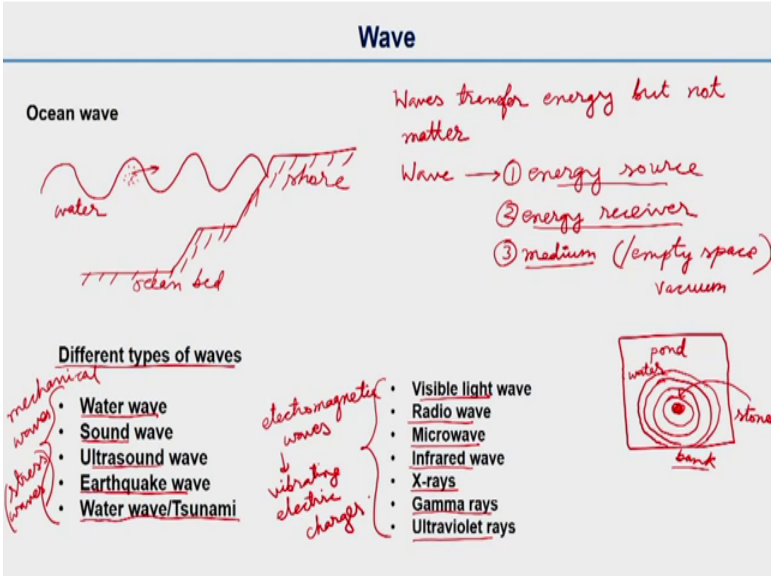

The basic of the waves I am sure all of you have learnt in your school days. So basically, if we talk about the ocean wave, let us say, this is ocean bed and this is the shore, so we call wave is like this, that is created in a medium that is water. So, this is ocean wave. So, this is the first picture that comes to our mind when you talk about wave. But there are different types of waves.

We know light is also a wave. We know sound is also a wave. So, there are different types of waves. We will talk about those waves. So, waves basically transfer energy, but not matter. It can transfer energy or motion without the transfer of matter. So, as you know, in this case, the wave transfers energy without much displacement of these water particles. So, like any other energy transfer processes, this wave transfer, the wave that is energy transfer, needs 1 energy source, 2 energy receiver, and 3 a medium through which the energy will be transferred.

But sometimes we know that some waves can go without medium as well, even empty spaces, that is, you can call vacuum, or I can write vacuum. For example, if we throw a stone in a small pond, let us say, we have a small pond, I am drawing the top view of the pond, if you throw a stone, as we know wave will be generated and it will travel like this.

So here, the stone is the energy source, energy receiver we can assume that is the bank of the pond. This is the pond, a small pond, and we are throwing a stone here. So, the bank of the pond is the energy receiver and the medium is water. So, if we talk about the different types of waves, we know whatever we discussed is water wave, then, there can be sound wave or ultrasound wave.

Then, we know about earthquake wave. So, water waves also can be called, so these water waves can call it as tsunami which is produced from earthquakes. And then, other kinds of waves are visible light, radio wave, microwave, infrared wave, x-rays, gamma rays, ultraviolet rays. So, this group of waves we call them mechanical waves.

So, this is also sometimes known as stress waves because the resulting waves in the medium are due to mechanical stress effects. This can be called as stress waves as well and these waves are called as electromagnetic waves. For electromagnetic waves, they are created by vibrating electric charges.

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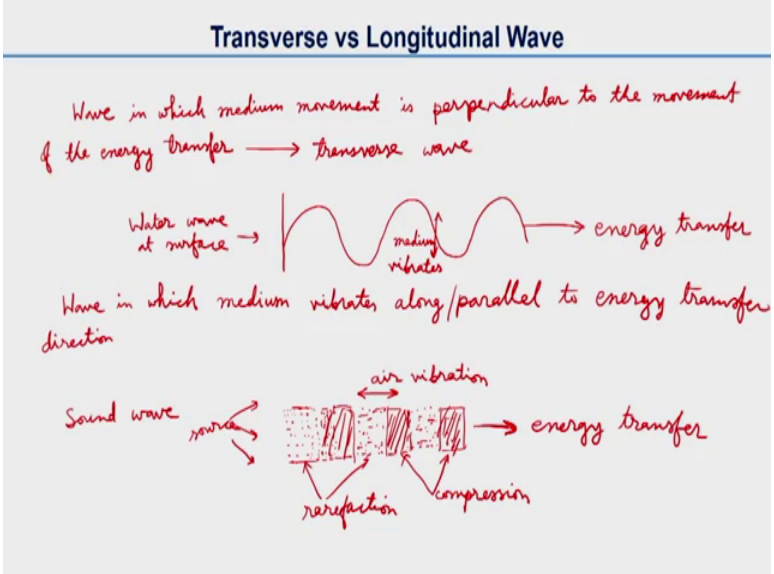

So, before going to details about the mechanical wave and electromagnetic wave, we will now discuss about another classification of waves which is transverse wave and longitudinal wave. So, the direction of movement of the medium is not always towards the direction of the energy transfer. So, a wave in which the medium movement is perpendicular to the movement of the energy transfer is called transverse wave.

So, for example, as we talked about a wave like this. The energy transfer is in this direction, and this is like, we are talking, let us say, about a water wave, whatever we showed in the last slide. So, the water particles, the medium movement is in the vertical direction. The water particle will vibrate in this direction. So, this medium vibrates in a perpendicular direction to the energy transfer.

On the other hand, the wave in which the medium vibrates along or you can say parallel to the energy transfer direction, for example, sound wave. The earlier one was water wave at surface of the water, this is the example. Here, we will talk about sound wave. So, whenever some source, this is the sound source, is emitting sound, suppose we are talking about the medium air, air particle will be compressed.

Let us say, at this part, air particle will be compressed and then again the density will be less. Then again, it will be compressed again, density will be high. And again here density will be low. Let us say density will be high here. So, this is called rarefaction and these are called compression. Here, this is the direction of energy transfer and the air molecules vibrate in this direction. Thus, the medium vibration is parallel to the energy transfer direction. Later we will demonstrate these transverse and longitudinal waves with the help of a spring, that we will discuss soon.

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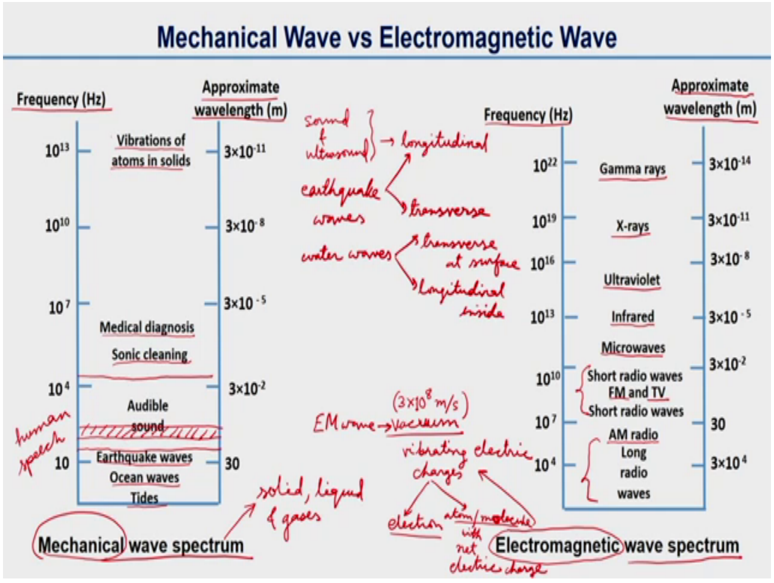

Okay, so, let us discuss this. Let us discuss about the different mechanical waves and electromechanical waves. In this slide, we will discuss about different mechanical wave spectrum and electromagnetic wave spectrum. So, we have frequency in hertz and approximate wavelength in meters. So, you can see that the range of frequencies and wavelength for mechanical wave spectrum and electromagnetic wave spectrum is very different.

So, if you see the mechanical wave spectrum, vibration of atoms in solids has a very high frequency. It is around 10 to the power 13 hertz. And then, we have medical diagnosis, which has also high frequency and then sonic cleaning. And then, we have audible sound range, we know that is from 20 to 20,000 hertz, this is the audible sound range. In this range we have human speech. It is a very narrow range, probably in this region, that is human speech. And then, the low frequency waves, mechanical waves are earthquake waves, and then ocean waves, and also these tides which are bigger ocean waves.

Similarly, for electromagnetic wave spectrum, we have very high frequency waves of gamma rays, then x-rays, then ultraviolet rays, infrared, then comes the microwaves. Short radio waves has this range. In this range we have FM and TV and then lower frequencies are AM radio. This is actually a long radio wave range.

So, let us discuss how these waves travel. The mechanical waves can travel through solid, liquid and gases. Generally, it does not go through vacuum. But electromagnetic waves are created by vibrating electric charges. So, that can be electron or that can be atoms or molecules which has a net electric charge, so it can be either electron or atom or molecule which needs electric charge.

So, one electric charge can affect another electric charge at a distance even when there is no matter in between. That is why the electromagnetic waves can travel through vacuum as well. So, I will write electromagnetic wave, EM, can travel through vacuum, like you know that sunlight travels through millions of kilometres to reach earth. So, that is millions of kilometres of vacuum it crosses. In vacuum electromagnetic waves have a speed of 3 into 10 to the power 8 meter per second. In other mediums, this wave velocity will be different.

So, if you see the nature of the waves whether it is transverse or longitudinal, the mechanical wave, sound and ultrasound, they are longitudinal. Earthquake waves can be both longitudinal and transverse. Then water weaves are generally transverse at the surface and longitudinal inside.

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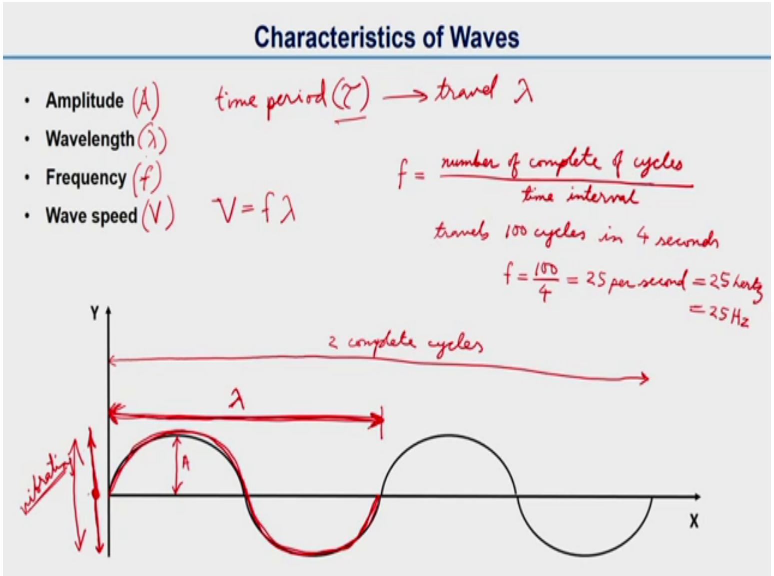

So, we will talk about amplitude, wavelength, and frequency.

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So, let us demonstrate a simple wave propagation. Here I have a string. So, if I want to produce a transpose wave, I need to vibrate, but my left hand is fixed. And then, my with my right hand I am trying to produce some vibration. So, this is kind of transverse wave. So, how much my right hand going up and down will give us, I mean, basically, the measure of amplitude, and how fast I am going up and down that will give us the frequency, or in other way, it will give us the wavelength. **(Video Ends: 18:50).**

So, we will see this. So, basically, to describe the differences among waves, we have these parameters amplitude, wavelength, or frequency. So, if we vary the distance, let us say, we have the string here and we are vibrating like this, and the amplitude, the maximum distance, the parts of the medium move from their natural position. So, this is our amplitude. Let us write it as A, amplitude is A. So, varying the rate of vibration, if we change the rate, if we vibrate faster or slower, that will give us the frequency f.

If you are vibrating faster, than the frequency is higher. If we are vibrating with less speed, than our frequency is lower. Then, the wavelength, the lambda, is the distance between one complete cycle of this. So, one complete cycle means when the string will complete one cycle. That means when you are vibrating from this point to this position and then we are going back, then this position, and then again coming back to the original position, that will give you one complete periodic motion. And that distance we call it as lambda.

We have another parameter which is called time period. Time Period tau, which is the time required for the wave to travel a distance lambda. So, the time required to this distance, that is lambda, we call it as the time period, tau, and the wave speed capital V is equal to f multiplied by lambda. That is actually the total distance divided by the time required to travel that distance.

The frequency can be defined as the number of complete cycles divided by time interval. So, in this case, let us say, x equal to 0 to this portion. So, we have two numbers of cycles, two complete cycles. So, if this wave, assume that this waves travels 100 cycles in, let us say, four seconds, then the frequency of the wave will be 100 divided by 4, it will be 25 per second and we write it as 25 hertz or hertz can be written as Hz in short.

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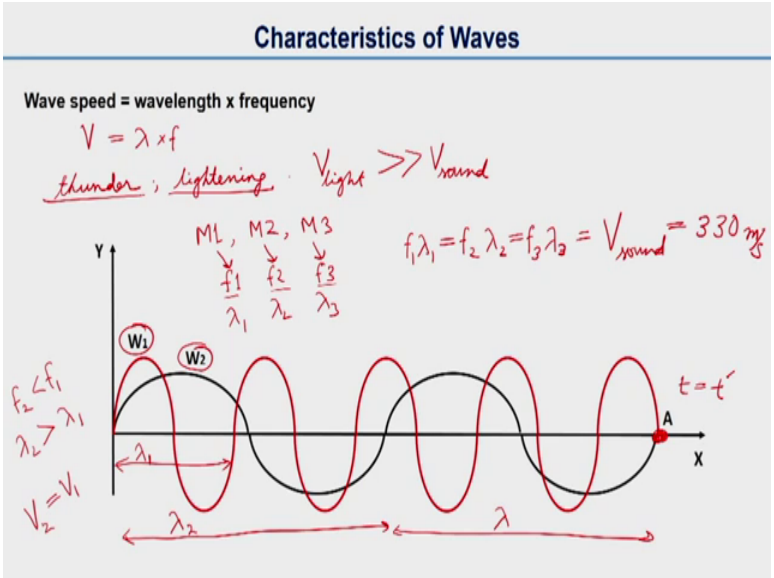

So, as I already mentioned that wave speed capital V is equal to wavelength lambda into frequency f. So, now, let us discuss about this. So, we hear sound of thunder much after we see lightning. So, first we see the lightning and after some time we hear this sound of thunder. That is because the wave speed, V, for light is much faster than V for sound.

Now, let us imagine there is some music function going on a few hundred meters away and there are different types of music instrument. Let us say, you have guitar, you have tabla, or, you know, you have piano. They are all instrument playing together. So, basically, you will get the sound of all the instruments at the same time because sound speed, the wave speed, in air is same for all the instruments.

However, these instruments, musical instruments, M 1, that is, let us say, guitar, and then M 2 and M 3, they will have different frequencies. Even for guitar you have different strings and the frequencies are different and even for other instruments the frequencies will be different. So, frequencies f 1, f 2, and f 3 will be different. That is why their wavelengths should be different. So, that means your wavelength should be, sorry, the frequency for this is lambda 1, lambda 2, and lambda 3.

So, ultimately, you should get your frequency of the first instrument multiplied by the wavelength of the first instrument will be equal to the other instruments f 2 lambda 2, f 3, that means f subscript 3 lambda subscript 3, that should be equal, because we know that sound speed in air is around 330 meter per second.

And here, we are demonstrating with the help of a transverse wave. We know that sound is basically a longitudinal wave but here in this case what we are showing is that black wave, that is we are showing with the black line, it has low lambda actually, this is lambda; there are only two cycles that is completed till point A and in the red one, so this lambda is smaller.

Let us assume that both the waves, we call it W 1 and W 2, both the waves reach the A at the same time, let us say, t equal to t prime. So, at the same time both the waves reach the point A. Here we can see that the lambda for W 1 is lambda 1, let us assume it as lambda 1, and for W 2 wavelength is lambda 2, that means lambda 2 is, here we will write, lambda 2 is bigger than lambda 1, but as both the waves reach the point A at the same time that means our velocities are equal, V 1 equal to V2, and that means our frequencies are different.

So, frequencies are different, that is, the frequency of wave 2 will be less than the frequency of the first wave.

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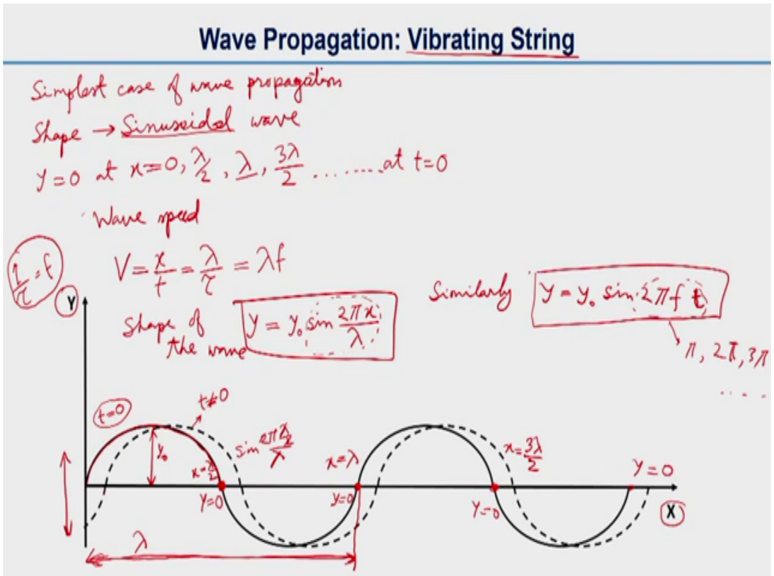

So, let us discuss about the vibrating string. Now we will do some derivation to obtain the wave equation. This is the string and we have X and Y coordinate here at time t equal to 0. We are representing the wave with a solid line and then time t is not equal to 0. That means after some time the wave reaches here, that is shown by dotted line. So, the vibrating string example is the simplest case of wave propagation. So, basically, the shape of the wave is, we are taking the simplest one, the sinusoidal wave.

As you can see, the y equal to 0, we are talking about now the wave position at t equal to 0, y equal to 0 at these positions, y equal to 0, y equal to 0, y equal to 0. That means y equal to 0 at X equal to 0 or X equal to lambda by 2, we already know lambda means the wavelength, which is lambda or 3 lambda by 2. Similarly, we can go on. So, we are talking about this condition at time t equal to 0.

Amplitude is y 0 and wavelength is lambda, we already told that. It is understood that we are vibrating this end and that is why at time t is not equal to 0, that means, let us say, after some seconds, it is not actually second, it may be after some, let us say, millisecond, we got this dash line. So, we will now see the wave speed expression. Wave speed V will be equal to the total distance travelled X divided by total time of travel which will be equal to lambda by tau.

As we discussed in a previous slide that tau is the time required to travel one wavelength, that is lambda. So, that is why we can express V is equal to lambda by tau and this 1 by tau is nothing but the frequency because tau is the time to required to travel one wavelength. So, 1 by tau is equal to frequency. So, that is why we can write as V equal to lambda multiplied by small f, f is the frequency.

So, as we say that this is sinusoidal wave, it can be expressed as Y equal to Y 0, which is the amplitude multiplied by sin of twice pi X divided by lambda. So, that is basically, you can see from here, even with this, we mentioned it earlier also, if X equal to 0, let us see what happens, if X is equal to 0, the entire expression will be 0. If X is equal to lambda by 2, at this point, this term will be sin twice pi lambda by 2 by lambda. So, that means sin pi which is zero.

And then similarly, if you talk about X equal to lambda, then again Y will be 0. Similarly, X equal to 3 lambda by 2, and here X equal to lambda, in this case, X equal to 3 lambda by 2. So, at these positions, the Y will be equal to 0. That is why we can represent the shape of the wave with this expression.

Similarly, we can also have Y equal to Y 0 sin twice pi f t, that is another way to express it. We can check that, if we multiply suppose at different time intervals, we will get that this expression will give you the values pi, twice pi, 3 pi, and so on. So, Y will be 0 at those positions.

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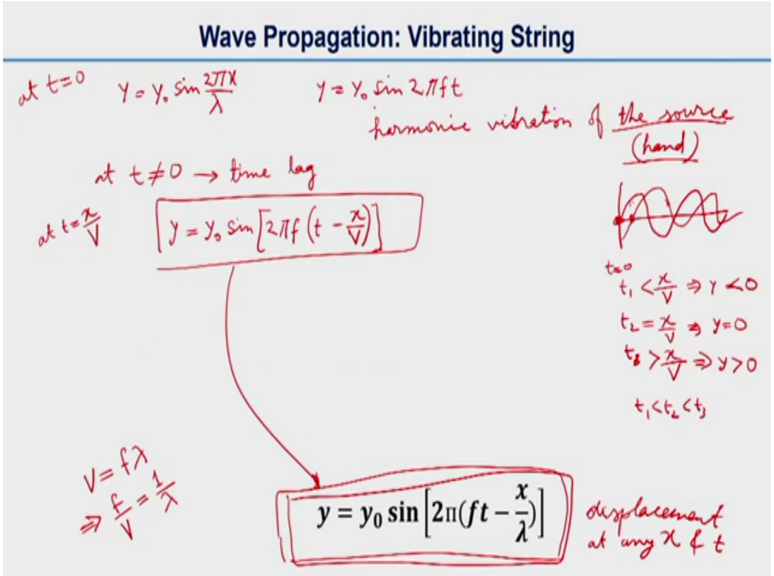

So, in the previous slide we have derived these equations, Y equal to Y 0 sin twice pi X divided by lambda, Y equal to Y 0 sin twice pi f t, frequency and time, and these two equations are different version of the same one equation of harmonic vibration of the source. That means, in the case of string, my hand is the source, so I was vibrating like this, my hand was the source.

So, the equation of the source, that means at t equal to 0, we have this equation, but if you want to know equation at any X and t, that means when t is not equal to 0, we need to consider a time lag. So, how we represent this equation, it is written as, Y equal to Y 0 sin twice pi f t which is we found it here. Now, what we will do is for the time lag we can write it like this. So, this is the equation for at any time X and t.

So, the time lag is like this. At Y equal to 0 earlier at the source, let us say after time t prime, let us say this is an X actually, this let us say X prime and let us say it takes time t prime which is X prime by V. Okay, let us assume this is distance X and let us assume it takes time t prime to cover this distance. So, X by V is the time. That means that time lag, from t we subtract the t prime so that we know now this is a new position of this wave. So, why we are subtracting this minus sign, because the wave is traveling towards right direction that is why this negative sign comes.

Now, this equation will give the displacement at any time t and any position X. So, this equation can be written in another form, this form. How we can do it, we know that V is equal to f lambda, and then if you rearrange it, f by V, from here if you see, here we have f and V, so f by V it will be equal to 1 lambda and that will give you this equation, this relation will give you this equation, and this is the equation of displacement at any position X and time t.

So, now, we will take a segment from that string. Let us say we are taking a small segment out of the string. Let us say this is the small segment d S. We will take this segment and we will go to the next slide.

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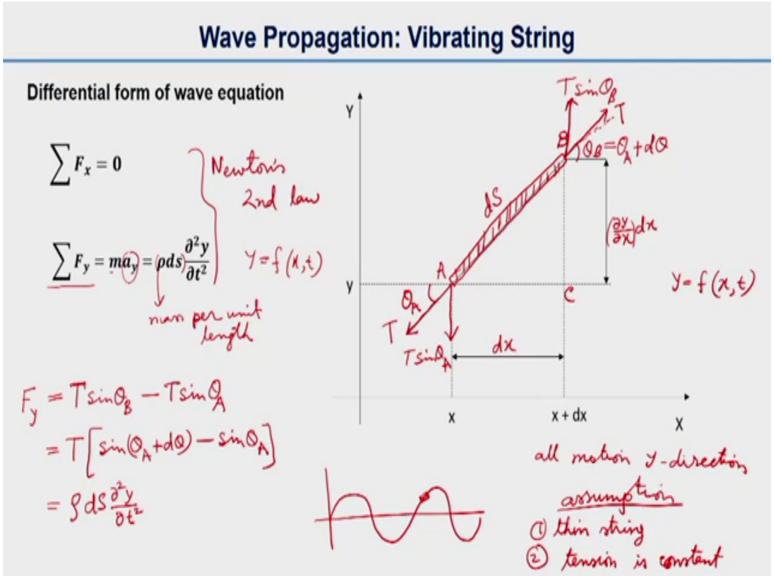

So, this entire portion is the segment d S of that string. Let us say, this is point A, this is B, and for our friends we are putting this as C. So, the X direction length of that segment is dx and the Y direction length we can get it as the partial of Y with respect to X multiplied by dx.

So, as you know Y is a multivariable function, as a function of X and t, both position and time. So, we can express this vertical distance from B to C this way, and this angle, first we should understand that the string is in tension and this is the line tension acting along the string. There are presumptions here. The assumptions are the string is thin and also the tension along from point A to point B will not be very different, that is, tension is constant.

Then, we will see that there is an angle. This angle is theta A and this angle will be theta B. We should understand that the string is not exactly a straight line if you see from the earlier string vibration figure. So, it is like this. So, if you take a string from this portion, this will not be exactly a straight line. That is why theta A and theta B will not be same. So, probably theta A will be something like that, and this theta B will be theta subscript A plus differential of theta.

So now, first, as this segment is in motion, we will apply the Newton's second law of motion. So, all the motion takes place in the Y direction we assume that, and so there will be some component of tension in this direction. So, basically, the vertical forces, mass multiplied by the acceleration in Y direction, which should be rho is the line density or mass per unit length, and as we know Y is a function of X and t, so rho d S will give us the mass and the acceleration is double partial of Y with respect to t.

So here, if we consider all the vertical forces, we see that, this is the t sin theta subscript B and this is t sin theta subscript A. So, the resulting vertical force is equal to line tension multiplied by sin theta subscript B minus line tension multiplied by sin of theta subscript A which is then again equal to, we can take T common out of this expression and then sin theta A plus d theta and then sin theta A. This expression will be equal to rho ds second partial of Y with respect to t.

So, we are assuming that all the motion takes place in the Y direction. So, there is another assumption that all motion is in Y direction. Why all the motion in Y direction, because we are giving only Y direction motion of the string.

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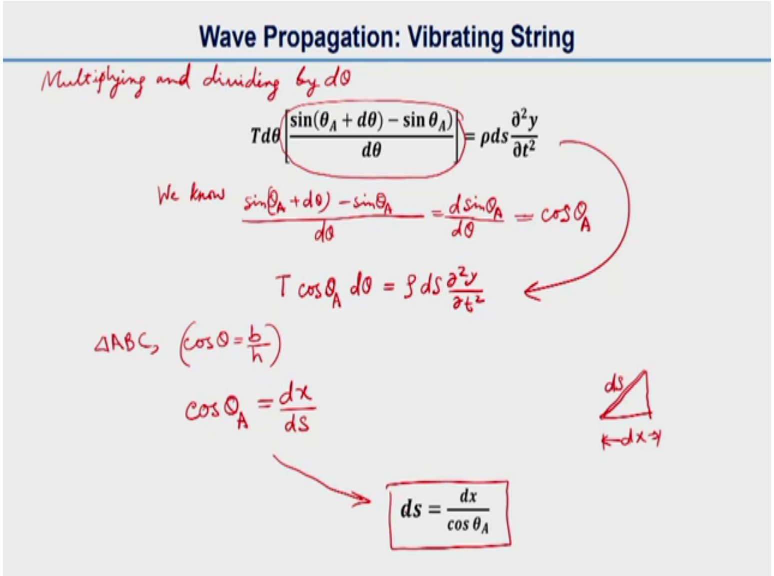

So, moving on from this, the expression what we got in the previous slide, we will multiply the previous expression and at the same time multiplying and dividing the expression by d theta, so we got this, but we know that this expression sin theta A d theta minus sin theta d theta is a kind of differential of sin theta A, the expression at the top, and d theta. So, basically, it is a derivative of sin theta A with respect to theta, that will give you cosine of theta A.

So, from the previous expression what we got is t cosine of theta A d theta is equal to rho ds second partial of Y with respect to t. So, this comes from this equation. And also we know that from the right angle triangle ABC, cosine of theta that is base by hypotenuse of the right angle triangle ABC, we can get that cosine of theta A will be equal to dx by ds. So, basically, if we have the line segment ds and so we have dx, so this is cos theta A. You can get the expression finally like this. ds will be equal to dx divided by cos theta A.

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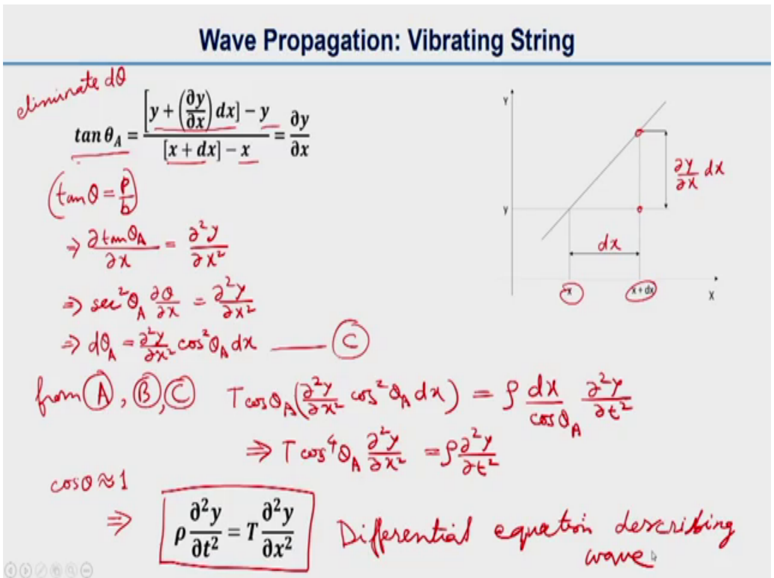

So, we will now try to eliminate d theta. What we can do is we know that tangent of theta for a right angle triangle is perpendicular by base, here in this case, so we have X plus dx and X, this one is partial of Y with respect to X and multiplied by dx. So, ultimately from the tan theta A we have X plus dx minus X, that means this dx, and we have this minus this.

So, basically, this minus this, and finally it ends up with partial of Y with respect to X and from that we can get partial of tan theta A with respect to X. I mean, taking partial derivative on both the sides, that will give you second partial of Y with respect to X and this will give you sec squared theta A partial of theta with respect to X and this will give you differential of d theta A will be equal to dx 2 cos square theta A dx.

So now, from the earlier equation, we got this, ds equal to dx divided by cos theta A and another we got that this one, so let us assume, we call this as A and this one is B. So, from A and B and C, what we can get is T cos theta A, cos squared theta A, dx is equal to rho dx cos theta A second partial of Y with respect to dt 2 and this will give you T cos to the power 4 theta A second partial of Y with respect to T… equal to rho second partial of Y with respect to t.

So, for very small oscillations, if we consider oscillations are very small, so cos theta will be equal to 1 and that will give you this equation. So, this is the differential equation for wave or you can call wave equation or you can call it as a wave equation.

We will show some demonstration on the basic aspects of web propagation. Two of the teaching assistants, my Ph.D. students Akshay and Vikram will show some demonstration with the help of a helical spring. We will show longitudinal and transverse wave propagation. You will see what happens when the spring is tighter or it has higher stiffness.

So far we have discussed about the wave speed of vibrating string and we saw that that depends on the line tension and that means if the line tension is higher the wave speed is higher and the square root of the ratio of line tension and mass density. Similarly, in a solid medium, the elastic wave speed depends on the Young's modulus. So, if we have a higher stiffness spring, that means if we stretch the spring, then we will see what will happen to its wave speed.

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So, we will show the longitudinal wave, that is actually longitudinal compression. Now we will show the transverse wave. Now we will show that if we stretch the spring more, we are stretching it more, then we can see that the wave will propagate faster. So, we can see that the this is a faster wave propagation than the previous case where the spring was not stretched much in the previous case, then, we will see what will happen if we increase the frequency. So, you can show two cases; first one is slower and then it can be higher. First we did lower frequency that means slower one, okay, and then we will go to higher frequency.

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So, in this lecture, we have discussed about the basics of wave propagation and then we also discussed about the wave equation for a vibrating string, and then in the next lectures we will discuss about the difference between quasi-static and dynamic deformation from atomic perspective and then elastic wave and its classification. Thank you.