Steam Power Engineering
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Lecture - 30
Gas Mixture

Welcome to the class. Our topic of discussion is gas mixture. This topic has concern for our present study of steam power plant since in the steam power plant in the condenser actually we will have cooling tower as well. So, in the cooling tower, we will have air and water vapour as their mixture. So, their air will be also treated as a gas and water vapour will also be treated as gas, so the mixture will be present over there.

So, the mixture properties we would have to calculate. So, this class or this topic of gas mixtures will let us know how to find out different mixture properties from the known properties of individual gases.

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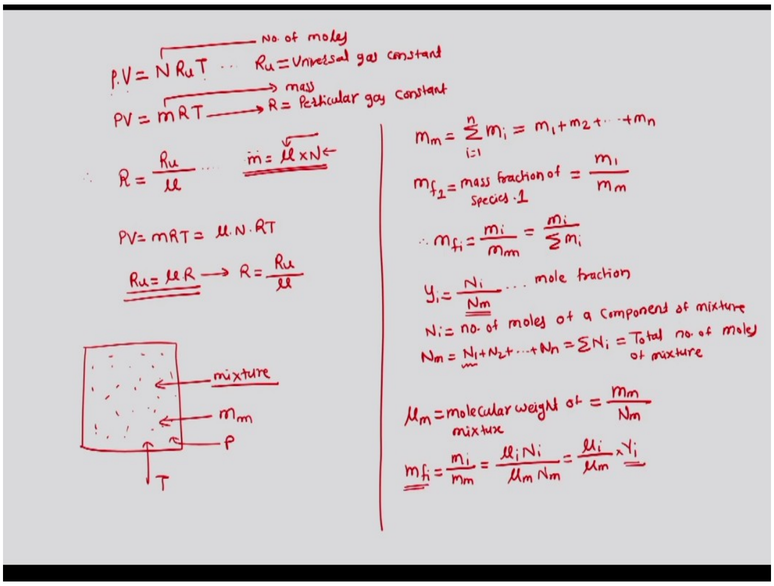

So, let us start. We know that for any ideal gas, we have $PV = N R_u T$ where we have R_u as universal gas constant and this N is number of moles. Then, P is pressure, V is total volume and T is temperature. So, we have one more relation which says that $PV = mRT$ where R is particular gas constant and here we have m as mass. So, if we divide both the equations, we get r or particular gas constant is equal to universal gas constant divided by molecular weight since m is equal to molecular weight into number of moles.

This is weight of 1 mole of a gas into number of moles, this gives us mass. So, practically when we write $PV=mRT$ we are writing $PV=\mu NRT$ and then that is why we get universal gas constant is equal to molecular weight into particular gas constant. So, we say that particular gas constant is equal to universal gas constant upon molecular weight. So, then if we have a gas, if we have a gas which is mixture of many gases.

Let us consider this is a container which has a mixture of many gases. So, what will happen? We can say here that this mixture has mass m_m, this mixture has pressure P, this mixture has temperature T. So, mixture mass is equal to mass of all species which are present, so it is equal to $i=1$to n, so $m_1+m_2+\ldots+m_n$. So, if there are n components in the mixture, then there will be summation of all m masses to get the mixture mass.

So, we can write down mass fraction of any one species say 1 m_{f1} which is called as mass fraction of species 1 is equal to $\frac{m_1}{m_m}$. So, in general $m_{fi}=\frac{m_i}{m_m}$ or ¿$\frac{m_i}{\Sigma m_i}$. Similarly, we can also find out mole fraction where we have $y_i=\frac{N_i}{N_m}$where this is called as mole fraction where N_i is equal to number of moles of a component of mixture.

And $N_m=N_1+N_2+..+N_n=\Sigma N_i$ that means total number of moles of mixture. So, this N_1 is number of moles of first component, so that divided by number of moles of mixture will give us mole fraction of one component or first component. So, we can find out molecular weight of mixture is equal to mixtures mass divided by number of moles of mixture. This is the molecular weight of the mixture.

Then, we can write down here mass fraction of any species $i=m_i$ divided by mixture mass where mass m_i can be written as molecular weight of i into number of moles of i divided by molecular weight of mixture into number of moles of mixture. So, we can write down this as molecular weight of i divided by molecular weight of mixture into y_i. So, this is the relation between mass fraction and mole fraction.

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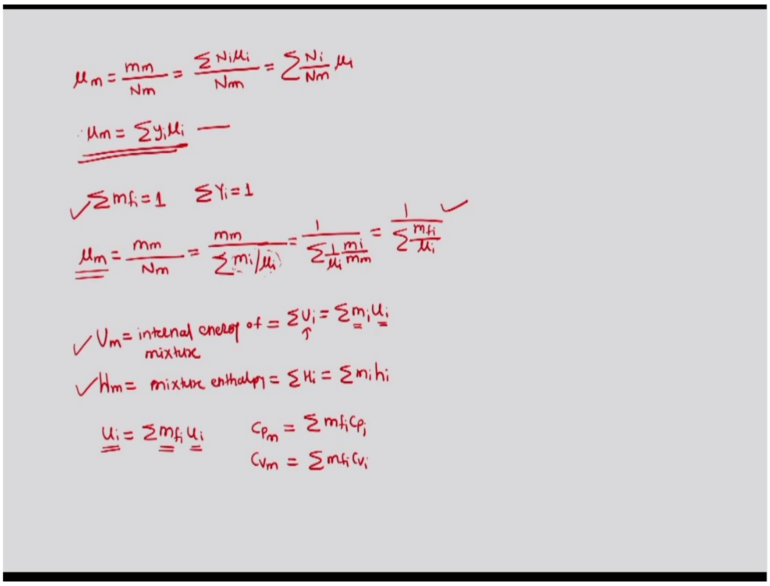

Further, we can find out other relation where we can write down molecular weight of mixture is equal to mass of mixture divided by number of moles of mixture. So, mass of mixture is $\Sigma N_i \mu_i$number of moles is N_m. So, this is $\frac{\Sigma N_i}{N_m}\mu_i$ so molecular weight of mixture is $\Sigma y_i \mu_i$. So, this is what molecular weight of mixture. We should remember one thing that summation of mass fraction of all the species is equal to 1 and summation of mole fraction is also 1.

We can also write mixture molecular weight is equal to this is one relation, there can be other relation as well divided by number of moles. So, this is mixture mass, number of moles of mixture can be written as molecular weight of mass i divided by molecular weight of i so this is basically m_i and this is summed over all i's so this is N_m, $\Sigma m_i is N_m$. So, then we can have

$\frac{1}{\Sigma \frac{1}{\mu_i}\frac{m_i}{m_m}}$ so we can write it down as $\frac{1}{\Sigma \frac{m_{fi}}{\mu_i}}$.

So, this is the formula for molecular weight of mixture. Now, we know that if we know all the masses then we can find out mass fraction. If we know all the moles, then we can find out mole fraction. Then, we should also going to find out the properties from the mole mass and mole fraction. The properties like mixture's internal energy, mixture's entropy, mixture's enthalpy and mixture's internal energy.

These things can be calculated from the individual mole mass fractions and also from the individual mole basically properties like entropy, internal energy, enthalpy. So, that way we should know suppose U_m is internal energy of the mixture, then it is equal to U_i which is summation of all internal energies and then into then what we can do, this U_i is extensive, we can make it as intensive, then we can basically write it as $m_i U_i$.

So, specific internal energy of ith component into mass of ith component summed over all components will give us internal energy of mixture. Similarly, mixture's enthalpy is equal to $\Sigma H_i = \Sigma m_i h_i$. Now, we are not needing the extensive internal energy or extensive enthalpy, we need intensive properties. Then, in that case, we should write down as $m_{fi} U_i$.

So, mass fraction into internal energy of one species and then summed over all species will give us specific internal energy of the mixture. Similarly, we should also find out C_p which is specific heat at constant pressure, C_v specific heat at constant pressure for the mixture using this formula $m_{fi} C_{pi}$ and then $m_{fi} C_{vi}$. So, this is how we should find out all the mixture properties. Now, this we can make use of to solve some examples. So, let us see some examples over here.

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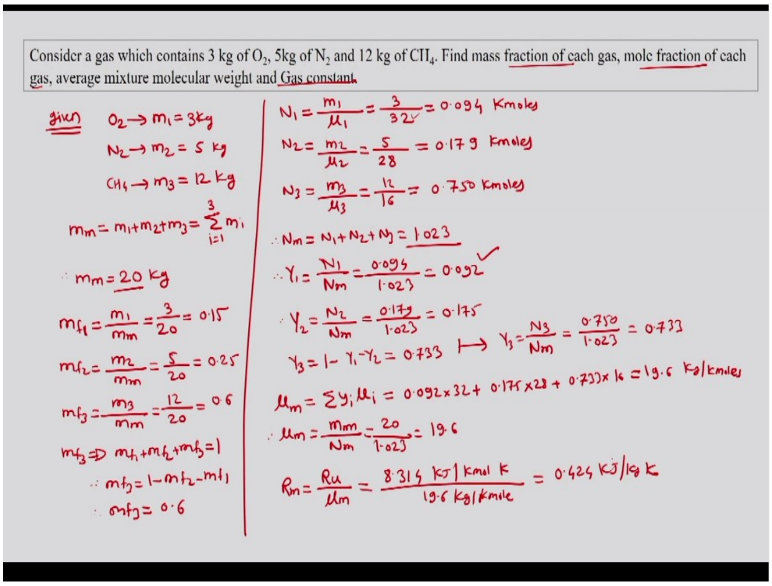

So, first example says that consider a gas which contains 3 kg of oxygen, 5 kg of nitrogen and 12 kg of CH_4 methane. Find mass fraction of each gas, mole fraction of each gas, average mixture molecular weight and gas constant. So, in this we are given that O_2 has suppose m_1 as 3 kg, we have N_2 then which is m_2 as 12 kg, we have CH_4 which is m_3 as sorry m_2 is 5 kg and CH_4 is 12 kg. Then, we know we can find out the mixture mass first which is $m_1 + m_2 + m_3 \vee \Sigma m_i, i = 1$ to 3.

So, we get mixture mass is equal to 20 kg. Now, we should find out basically mass fraction, which is simple to find out. We can find out $m_{f1} = \frac{m_1}{m_m}$, so it is 3 divided by 20, so it is 0.15.

Then, $m_{f2} = \frac{m_2}{m_m}$, so it is 5 divided by 20, so it is 0.25. Then, m_{f3} is also can be found out by 2 ways, first way is we are saying it as $\frac{m_3}{m_m}$which is 12 divided by 20 which is 0.6 or otherwise m_{f3} can also be found out from the fact that which is $m_{f1} + m_{f2} + m_{f3} = 1$.

So, $m_{f3} = 1 - m_{f2} - m_{f1}$, so also $m_{f3} = 0.6$. Now, having said this we can find out number of moles of oxygen by knowing mass of oxygen which is mass of 1 divided by molecular rate of 1. Mass of 1 is given as 3, we know molecular weight of diatomic oxygen is 32, so we have 3 divided by 32 which is 0.094 which is number of moles, number of kilo moles. Since it is in kg, we have kilo moles of oxygen.

And then $N_2 = m_2$ divided by molecular weight of 2 which is 5 divided by 28 is the molecular weight of nitrogen, so 5 divided by 28 so we have this sorry so it is 0.179 kilo moles. Similarly, $N_3 = \frac{m_3}{\mu_3}$. So, it is 12 divided by 16 so CH_4 has molecular weight as 16 which is 12 for carbon and 4 for hydrogen. So, 12 + 4 = 16 so 12/16 is equal to 0.750 kilo moles.

So, we can find out $N_m = N_1 + N_2 + N_3$ which gives us 1.023. So, we can find out mole fraction of first species which is Y_1 which is $\frac{N_1}{N_m}$, N_1 is 0.094 divided by 1.023 so it is equal to 0.092.

$Y_2 = \frac{N_2}{N_m}$, N_2 is 0.179, N_m is 1.023 so this is equal to 0.175. So, $Y_3 = 1 - Y_1 - Y_2 = 0.733$. We could have as value it $Y_3 = \frac{N_3}{N_m}$ and N_3 *is* 0.750 *divided by* N_m 1.023 and then also we get 0.733.

So, this is the way we just found out mass fraction and mole fraction of each species. Now, we have to find out average molecular weight. So, we know we can find out mu mixture is equal to Σ mole fraction of species which is basically $y_i\mu_i$, this is one way. We know y_i $Y_1\mu_1$, so it is 0.092 into 32 + 0.175 into 28 + 0.733 into 16 and this gives us mixture's molecular weight as 19.6 kg per kilo moles.

So, we can as values μ_m is equal to mass of mixture divided by number of mole of mixture, so mass of mixture is 20, number of moles of mixture is 1.023 and this will also give us close to 19.6. Now, our aim is to find out gas constant and we know gas constant for the mixture would be universal gas constant divided by mixture's molecular weight. So, universal gas constant is 8.314 kilojoule per kilo mole Kelvin divided by 19.6 kg per kilo mole and this gives us 0.424 kilojoule per kg per Kelvin.

So, this is how we would have solved the example based on the mass fractions of or masses of different components for a given mixture.

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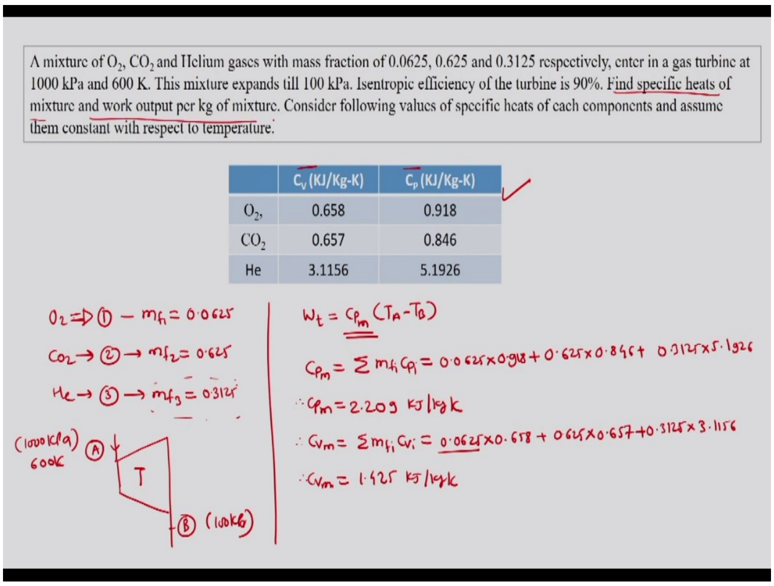

So, let us solve the other example which says that a mixture of O_2, CO_2 and Helium gases with mass fractions of 0.0625, 0.625 and 0.3125 respectively, enter in a gas turbine at 1000 kilopascal and 600 Kelvin. This mixture expands till 100 kilopascal. Isentropic efficiency of turbine is 90%. Find specific heats of mixture and work output per kg of mixture. Consider the following values of specific heats basically they are given in the table for each component and assume them constant with respect to temperature.

So, here C_p and C_v both are constant, so gas is ideal for us and we will again say $O_2=1$ so we are given with m_{f1}0.0625, we will say CO_2 as 2 and this is equal to m_{f2} and is 0.625, then we have Helium which is 3 and we have m_{f3} and that is 0.3125. We can cross verify by making Σm_{fi} it will give 1. Actually this data of m_{f3} or any mf is redundant since we could have as well calculated that.

Then, we will sketch for turbine and then we can notice that this is what it is happening we know state A and we have to find out stage B, this is turbine. So, first for that we know that turbine output is difference in enthalpies and enthalpy of a gas is $C_p into T$. So, $C_p T_A$ so work done in the turbine is C_p of mixture into $T_A - T_B$. So, we need to find out *Cp* of mixture. So, for that we can say *Cp* of mixture is ¿ $\Sigma m_{fi} C_{pi}$.

So, we know it is $C_{pm} = \Sigma m_{fi} C_{pi} = 0.0625 \times 0.918 + 0.625 \times 0.846 + 0.3125 \times 5.1926$. So, this gives us C_{pm} which is the mixture specific heat at constant pressure as 2.209 kJ/kgK. We also know that the pressure ratio and temperature would have a gas constant also associated with that, so for that let us calculate mixture specific heat at constant volume which is again $C_{vm} = \Sigma m_{fi} C_{vi} = 0.0625 \times 0.658 + 0.625 \times 0.657 + 0.3125 \times 3.1156$. So, we have C_{vm} as 1.425 kJ/kgK.

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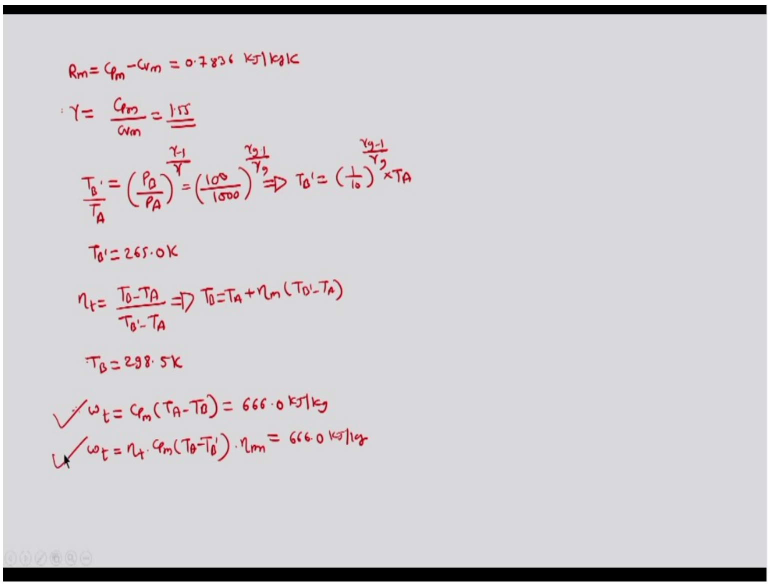

So, we can know two things, we can know $R_m = C_{pm} - C_{vm}$ and this R value is 0.7836 kJ/kgK.

Similarly, $\gamma = \frac{C_{pm}}{C_{vm}}$ and so we have $\gamma = 1.55$. Knowing that, we can find out the temperature basically, so we have T_2' which is the ideal temperature at the exhaust on the turbine so we call it as $\frac{T_B^{'}}{T_A} = \left(\frac{P_B}{P_A}\right)^{\frac{\gamma-1}{\gamma}}$.

So, which is equal to as we know turbine entrance condition is 1000 kPa and 600 Kelvin and bottom or end of the turbine is 100 kPa. So, we can say it has 1000 kPa divided by basically $\left(\frac{100\,kPa}{1000}\right)^{\frac{\gamma_g - 1}{\gamma_g}}$. So, this gives us $T_B^{'} = \left(\frac{1}{10}\right)^{\frac{\gamma_{g-1}}{\gamma_g}} T_A$. So, we can get $T_B^{'} = ¿$ 265.0 Kelvin.

So, we know efficiency of turbine is$\eta_T = \frac{(T ¿¿ B - T_A)}{T_B^{'} - T_A} ¿$, turbine efficiency is given to us, so we can write down $T_B = T_A + \eta_T (T_B' - T_A)$. So, we have T_B is equal to real temperature at the exit of the turbine is this 298.5 Kelvin. So, now we can find out turbine work is equal to C_p mixture into $T_A - T_B$. So, we know now T_A is 600 degree Celsius or 600 degree Kelvin, 600 Kelvin.

And then we know now T_B as 298 Kelvin, so using that we can find out $C_{pm}(T_A - T_B)$ and we get work output is equal to 666 kilojoule per kg. So, this is how we can solve the example for the mixture. We were asked in the example to find out specific heat of the mixture and work output per kg of the mixture. So, we found out work output. Work output could also have been found out as $W_T = C_{pm}(T_A - T_B^{'})$.

So, that would also have given into efficiency that would also have given 666.0 kilojoule per kg. So, this is how we end the discussion on the gas mixtures. Now, we have understood in the individual properties are known and fraction of each component are known we can fetch the properties of the mixture. Thank you.