Steam Power Engineering
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Lecture - 28
Examples on Turbine 1

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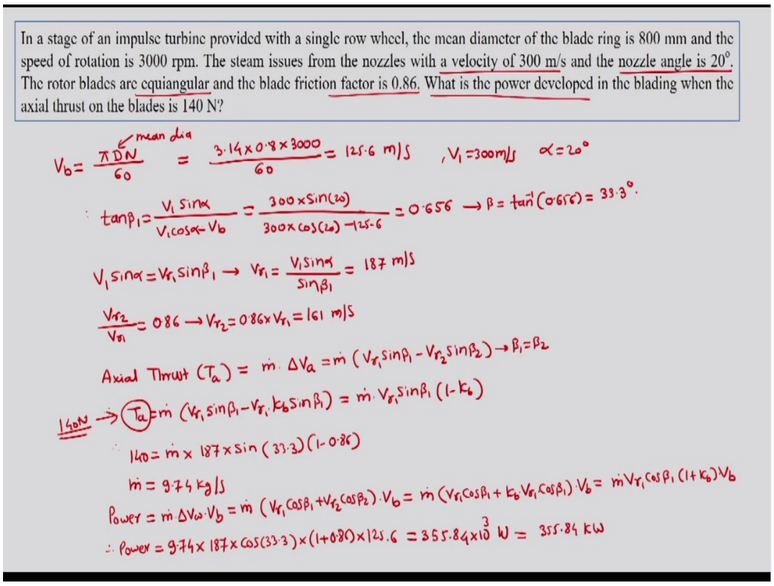

We will see the next example. It reads that a single stage, in a stage of an impulse turbine provided with a single row wheel, the mean diameter of the blade is 800 mm and the speed of rotation is 3000 rpm. The steam issues from the nozzle with a velocity of $300 m/s$and the nozzle angle is 20 degree. The rotor rotates, the rotor blades are equiangular and the blade friction factor is 0.86. What is the power developed in the blading when the axial thrust on the blade is 140 Newton?

So, in this example, we know we are given with the rpm, so we can find out V_b which is $\frac{\pi DN}{60}$. This is mean diameter and this is given, so mean diameter is given to us, so for that we can write down $3.14 \times mean\, diameter\, is 0.8 \times \frac{3000}{60}$. So, this gives you V_b as $125.6\, m/sec$. In the example, this is told us that velocity is $300\, m/sec$ and nozzle angle is $\alpha\, is\, 20\, degree$.

So, we can find out using our formula for β in terms of V and α. So, $tan\beta = \frac{V_1 \sin(\alpha)}{V_1 \cos(\alpha)} - V_b$.

So, $\frac{300 \sin(20)}{300 \cos(20)} - 125.6$ and then that is 0.656 and this gives us $\beta = \tan^{-1} 0.656$and we get $\beta \, is \, 33.3 \, degree$. Now, we can make use of this beta to find out V_r since we know $V_1 \sin(\alpha) = V_{r1} \sin(\beta_1)$.

So, this gives us $V_{r1} = \frac{V_1 \sin(\alpha)}{\sin(\beta_1)}$and this gives us $V_{r1} = 187 \, m/s$, it is told to us that friction factor is 0.86, so $\frac{V_{r2}}{Vr1} = 0.86$, so $V_{r2} = 0.86 V_{r1}$ and so we have $V_{r2} = 161 \, m/s$. Now, we know that power developed is, we are given with axial thrust T_a and this axial thrust for rest is $\dot{m} \Delta V_a$.

So, for ΔV_awe know we have to make $V_{r1} \sin(\beta_1) - V_{r2} \sin(\beta_2)$ but blades are equiangular, so $\beta_1 = \beta_2$. So, what we can write $T_a = \dot{m} V_{r1} \sin(\beta_1) - V_{r1} K_b \sin(\beta_2), \beta_2 = \beta_1$. So, we have $T_a \dot{=} m V_{r1} \sin(\beta_1)(1 - K_b)$. So, that we can make use of it but here left hand side is known to us and this is given as 140 Newton.

So, $140 \dot{m} V_{r1}$ which is $187 \sin(\beta_1)$ which is $33.3^{\circ}(1 - 0.86)$ and this gives us $\dot{m}$ which is $9.74 \, kg/sec$. Now, we can make use of it since our objective is to find out power developed and we need for power developed m dot and which we have found out. So, power developed is $\dot{m} \Delta V_w V_b$. So, we can make use of it.

So, this is $\dot{m}$ is $\dot{m} \Delta V_w V_{r1} \cos(\beta_1) + V_{r2} \cos(\beta_2) V_b$. So, we know again we can make in terms of K_b, so $V_{r1} \cos(\beta_1) + K_b V_{r1} \cos(\beta_1) V_b$, so it is $\dot{m} V_{r1} \cos(\beta_1) . 1 + K_b V_b$. So, power developed is equal to $Power = 9.74 \times 187 \cos(33.3)(1 + 0.86) \times 125.6$ and this gives us answer as 355.84×10^3 Watt and this is equal to $355.84 \, kilowatt$.

So, this is how we would have solved this example where we were given with the axial thrust and then we were making use of it to find out the mass flow rate.

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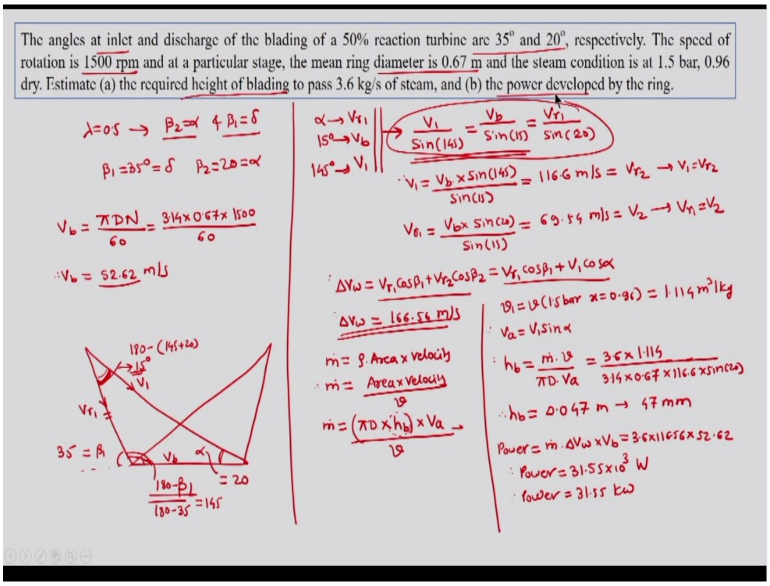

We will move ahead to the next example which states that the angles at inlet and discharge of the blading of a 50% reaction turbine are β are $35°$ and $20°$respectively. The speed of rotation is $1500\, rpm$and at a particular stage, mean ring diameter is given as 0.67 meter. The steam condition is at 1.5 bar, 0.96 dry. Estimate required height of the blading to pass 3.6 kg per second of steam and power developed by the ring.

Here, the very first thing which is given to us is degree of reaction which is $\lambda=0.5$and we know $\beta_2=\alpha \wedge \beta_1=\delta$or what we say if we name α_1 and α_2 then $\beta_2=\alpha_1$and $\beta_1=\alpha_2$and this is given to us, so we are told that $\beta_1=35°$ since it is at the inlet so this is equal to δ and $\beta_2=20=\alpha$.

Having said this, we should proceed, we are told that the mean diameter at the height is 0.6 meter and speed is 1500. So, $V_b=\frac{\pi DN}{60}, 3.14 \times 0.67 \times \frac{1500}{60}, so\, V_b$ for us is 52.62 m/s. Now, for this example to solve we can make use of the triangle law or sin law for the triangle where we are having this as V_1 and this is V_{r1} and this is α and then this is β_1.

This is β_1, so this will be $180-\beta_1$ and for us β_1 is 35, so $180-35$ is this $180-35$and then this α is given to us as $20°$. So, using this we can find out rest of the things. So, we have this angle as 145, so this angle is 145, this is 20. So, it tells us that this angle is $180-145+20$and this angle that is why it becomes $15°$. So, this is $15°$ angle, this is V_1 and this is V_b .

So, what we have is α is the angle across which we are having side $V_{r1}, \beta = 15°$ angle is this and for which across which side is V_b and this angle is $145°$and for which side is V_1. So, we can write down this, we can make use of this and write down $\frac{V_1}{\sin(145)} = \frac{V_b}{\sin(15)} = \frac{V_{r1}}{\sin(20)}$. So, having said this, we can make use of this first and find out V_1.

So, $V_1 = \frac{V_b \sin(145)}{\sin(15)}$. So, we can get V_1 from here as $116.6 m/sec$ but it is $\lambda 50\%$so this is equal to V_{r2} since we know $V_1 = V_{r2}$and then we can find out V_{r1} from here as $\frac{V_b \sin(20)}{\sin(15)}$ and this gives us V_{r1} as $69.54 m/sec$ but this is $equal ¿ V_2$ since for 50% reaction $V_{r1} = V_2$.

Knowing this, we can find out $\Delta V_w$and $\Delta V_w =$ $V_{r1} \cos(\beta_1) + V_{r2} \cos(\beta_2)$. So, V_{r1} we are knowing now which is $V_{r1} \cos(\beta_1) + V_{r2} is\, basically\, V_2$ and $\cos(\alpha) = sorry, V_{r2} = V_1 \cos(\alpha)$. So, we can either of them we can use and then we can find out $\Delta V_w \wedge \Delta V_w$ for us becomes $166.56 m/sec$. Now, we know that we are supposed to find out the area.

For area, we can use formula for m mass flow rate is basically density into area into velocity. So, we can make use of this by the fact that it is equal to area into velocity divided by specific volume where we can write down area as πD into which is mean diameter over here into h of the blade height of the blade, this is area into velocity, velocity is V_a axial velocity divided by specific volume.

So, for this formula to use where we can find out h_b we need V_a and specific volume. Specific volume can be found out from the state that it is at 1.5 bar and $x = 0.96$ and this gives us V_1 at that stage as $1.114 m^3/kg$ from the steam table. So, once this is known, we can make use from the fact that $V_a = V_1 \sin\alpha$. So, this formula can be rearranged and it can be written as $h_b = \dot{m} \times$ specific volume divided $by\, \pi D V a$ which is $\dot{m}$ since $\dot{m} is\, 3.6$.

So, we are knowing that 3.6 specific volume is 1.114, π is 3.14, D is given to us as $0.67 V_1$ and V_1 is $116.6 \sin(20)$ and this gives us h_b which is basically 0.047 meter which is 47 meter, so 47 mm. So, this is how we can find out first part which is height of the blade. Now, we can find out power developed, power developed is equal to $\dot{m}\,\Delta V_w V_b$.

And ΔV_w is found out 116.56, so 116.56 and $V_b$is also known 52.62, so we have this answer of power developed will be $31.55\times10^3\,Watt$. So, power is equal to 31.55 kilowatt. So, having said this we found out both power developed and the height of the blade. In this example, point to be noted that we were not given with any velocity like V_1 or V_{r1}. We were given with only one velocity but we were given with 2 angles then we made use of the sin law of triangle and found out required things.

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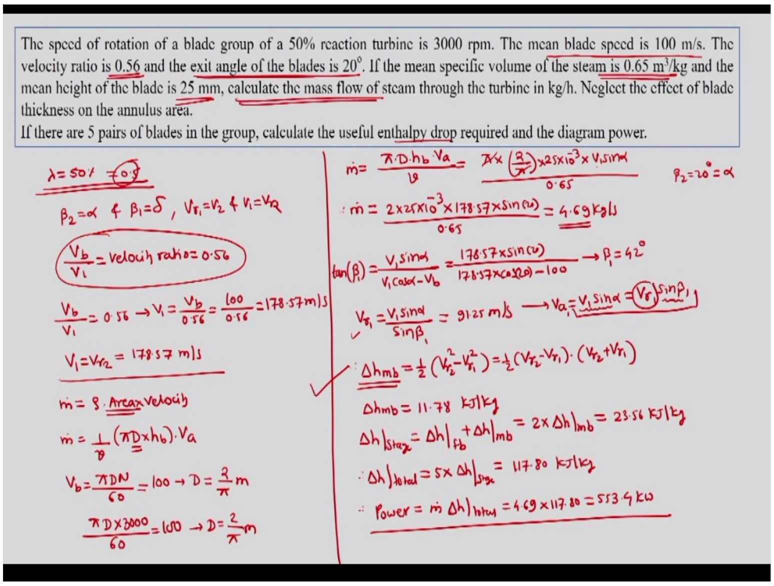

Then, we will go ahead with the next example which states that speed of rotation of blade group of a 50% reaction turbine is 3000 rpm. Mean blade speed is $100\,m/sec$, the velocity ratio is 0.56 and the exit angle of the blade is $20°$. If the mean specific volume of the steam is $0.65\,m^3/kg$ and the mean height of the blade is 25 mm, calculate mass flow of the steam through the turbine in kg/sec.

Neglect the effect of blade thickness on the annulus area. If there are 5 pairs of blades in the group, calculate useful enthalpy drop required and diagram power. In this example again what we have is $\lambda=50\%$ or 0.5 degree of reaction. So, we have β_2 is ¿$\alpha \wedge \beta_1=\delta$ or α_2 as per the sign convention and then we are also given with the fact that velocity ratio is 0.56 is the new thing which is $\frac{V_b}{V_1}$ is velocity ratio and that is given as 0.56.

Further for 50% reaction, we have $V_{r1}=V_2$ and $V_1=V_{r2}$. So, let us solve this example. We can basically make use of the fact that we are told that mean blade speed is $100\,m/sec$. So, $\frac{V_b}{V_1}=0.56$, so we can make use by saying $V_1=\frac{V_b}{0.56}=\frac{100}{0.56}$ which is $178.57\,m/sec$ but as the fact goes we have $V_1=V_{r2}$, so V_{r2} is also equal to $178.57\frac{m}{sec}$.

Now, we can find out V_r basically needing mass flow rate to be calculated. So, for that we will need as we know mass flow rate is equal to density into area into velocity. So, for this area to calculate, we need diameter since density can be replaced by 1 upon specific volume and then area is π diameter into h_b into velocity which is V_a. So, here we need diameter and that diameter we can find out from $V_b=\pi D\frac{N}{60}$ and that $V_b\,is\,100$.

So, we have D diameter is $\frac{2}{\pi}$ meter since we have πDN is given as $\frac{3000}{60} equal ¿ 100$. So, we have D is $2/\pi$ meter. Having said this, we should proceed and then find out $\dot{m}=¿\ \pi D h_b V_a$ divided by specific volume and that specific volume is given as 0.65, so we have π, $D=\frac{2}{\pi}h_b, h_b$ is given as 25 mm, so 25×10^{-3}

And then V_a is $V_1\sin(\alpha)$ divided by specific volume which is 0.65. So, $\dot{m}$ we can make use since we can make use of all these things and we have basically given that blade angle at the exit as $20°$. So, we are given as β_2 as $20°$ but that is equal to α. So, $\alpha\,is\,20°$, V_1 is calculated, so we can make, so $2/\pi$, π gets cancelled. So, we have $2\times25\times10^{-3}$, $V_1=\frac{178.57\sin(20)}{0.65}$ and this gives us $\dot{m}$ as $4.69\,kg/sec$.

And now we are going to find out other things enthalpy drop. So, for that first find out *beta* since we know $\tan(\beta_1)=\frac{V_1\sin(\alpha)}{V_1\cos(\alpha)}-V_b$. So, we know

$\frac{V_1 178.57 \sin(20)}{178.57 \cos(20)} - V_b \wedge V_b = 100\, m/sec$. So, this gives us $\beta_1 = 42°$. So, having this we can make use to find V_{r1} and $V_{r1} = \frac{V_1 \sin(\alpha)}{\sin(\beta_1)}$.

So, we have $V_{r1} = 91.25 \frac{m}{sec}$. Here, basically we are using $V_{a1} = V_1 \sin(\alpha) = V_{r1} \sin(\beta_1)$ and then in this process of equation we are getting $V_{r1} = \frac{V_1 \sin(\alpha)}{\sin(\beta_1)}$. So, we can write down Δh moving blade, there is enthalpy drop in the moving blade and we know in case of reaction turbine, there is enthalpy drop in the moving blade.

And in impulse turbine, there is no enthalpy drop in the moving blade since there is no enthalpy drop in the impulse turbine and in case of frictionless flow, we have $V_{r1} = V_{r2}$. Relative velocity does not change but in case of reaction turbine relative velocity changes, so there is increase in relative kinetic energy which is basically due to change in enthalpy in the moving blade so it is $\frac{1}{2}(V ¿¿ r2^2 - V_{r1}^2)¿$ which is equal to $\frac{1}{2}(V ¿¿ r2 - V_{r1})(V_{r2} + V_{r1})¿$.

So, we know V_{r1} and we know V_1 that is equal to V_{r2} since we have said here. So, Δh moving blade is equal to $11.78\, kj/kg$. So, we have Δh complete stage is equal to Δh nozzle or fixed blade plus Δh moving blade but we are told that it is 50% reaction, so it is $2\Delta h$ moving blade so it is $23.56\, kj/kg$. So, there is Δh total since it is told that there are such 5 rings.

So, $5\Delta h$ stage, so we have $\Delta h is 117.80\, kj/kg$. So, we have basically power developed is equal to $\dot{m}\, \Delta h$ total and we know $\dot{m}$ is 4.69. So, 4.69×117.8 and this give us 553.4 kilowatt. So, this is how we would have made use of the new things what we understood over here is this that the velocity ratio is $\frac{V_b}{V_1}$ and we also should remember that change in relative kinetic energy or increase in relative kinetic energy is enthalpy drop in the moving blade, this also we have made use of.

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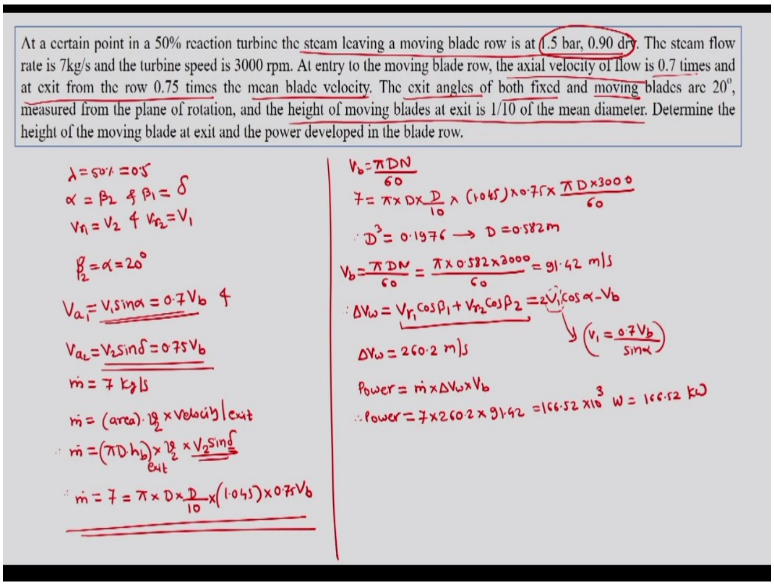

So, now we will go ahead with the next example which states that at a certain point in a 50% reaction turbine, the steam leaving a moving blade row is at 1.5 bar and 0.9 dry. The steam flow rate is 7 kg/sec and the turbine speed is 3000 rpm. At entry to the moving blade row, the axial velocity of the flow is 0.7 times at the exit from the row 0.75 times the mean blade velocity okay.

The exit angles of both fixed and moving blades are $20°$ measured from the plane of rotation and height of the moving blade at the exit is 1/10 of the mean diameter. Determine the height of the moving blade at the exit and power developed in the blade row. So, in this example we are again told that $\lambda = 50\% = 0.5$. So, $\alpha = \beta_2$ and $\beta_1 = \delta$. $V_{r1} = V_2$ and $V_{r2} = V_1$.

Further we are told that the angles are $20°$, exit angles of both fixed and moving blades are $20°$ that means $\beta_2 = \alpha = 20°$ and this is what it is given, so we can make use of the known fact of (()) (30:14) that axial velocity of the flow is 0.7 times the mean blade velocity. Axial velocity is $V_{a1} = V_1 \sin(\alpha)$ and that is equal to $0.7 V_b$ and it is also known that and axial velocity at the exit of the row is 0.75 the mean blade velocity.

So, we are knowing $V_{a2} = V_2 \sin(\delta) = 0.75 V_b$. Further, it is told that $\dot{m}$ is $7\,kg/sec$. So, we know $\dot{m}$ is equal to area into specific volume into velocity. So, we have $\dot{m}\,\pi D\,h_b$ in to specific volume into velocity which is $V_2 \sin(\delta)$ since we are told that properties are at the leaving state, so we are given with V_2 and this area is also at the exit, so that is where we are using axial velocity at the exit.

And this is axial velocity, so we have m dot is equal to basically $7 = \pi D h_b$ but for h_b it is told that height of the moving blade at the exit is 1/10 of the mean diameter, so $D/10$ into specific volume at exit we can find out from this state which is 1.5 bar and 90 dry. From the steam table if we find out, we will get $V\, is\, 1.045\, V_2 \sin(\delta) \wedge V_2 \sin(\delta)$ is also known as $0.75\, V_b$.

So, this is what it is told to us but further V_b we know it is equal to $\pi DN/60$. So, this also we can keep here and then we will have $7 = \pi D \times \frac{D}{10} \times 1.045 \times 0.75\, \pi DN$ and N is given as $\frac{3000}{60}$. So, here we know everything except D, so this becomes an expression of D^3, so we get basically D^3 which is 0.1976. So, we get D as 0.528 meter.

So, we got diameter or we basically got mean diameter at the exit. Knowing this mean diameter, we can find out V_b which is $\pi DN/60$ so which is $\pi\, 0.582 \times \frac{3000}{60}$ and this is $91.42\, m/sec$. So, we have $\Delta V_w = V_{r1} \cos(\beta_1) + V_{r2} \cos(\beta_2)$ but in general we can write this expression however in case of 50% reaction, the same expression can be written as $2 V_1 \cos(\alpha) - V_b$.

So, having said this we know now everything and we can find out ΔV_w from here and so we have $\Delta V_w = 260.2\, m/sec$ where we are going to make use of the fact that here we need to know V_1 and this is basically found out from this expression which says that $V_1 = 0.7 \frac{V_b}{\sin(\alpha)}$. So, this we can put over here and then find out ΔV_w.

And once ΔV_w is known, we can found out power since power is equal to $\dot{m}\, \Delta V_w V_b$. So, we get power which is also called as diagram power as $\dot{m}\, is\, 7, \Delta V_w\, is\, 260.2$ and $V_b\, is\, 91.42$, so we get 166.52×10^3. So, it is in Watt which is equal to 166.52 kilowatt. So, in this example, we saw that in a 50% reaction turbine, we can convert with the given constraints of the velocities at the exit or at the inlet.

And then find out first the expression for diameter and then make use of that to find out the necessary things.

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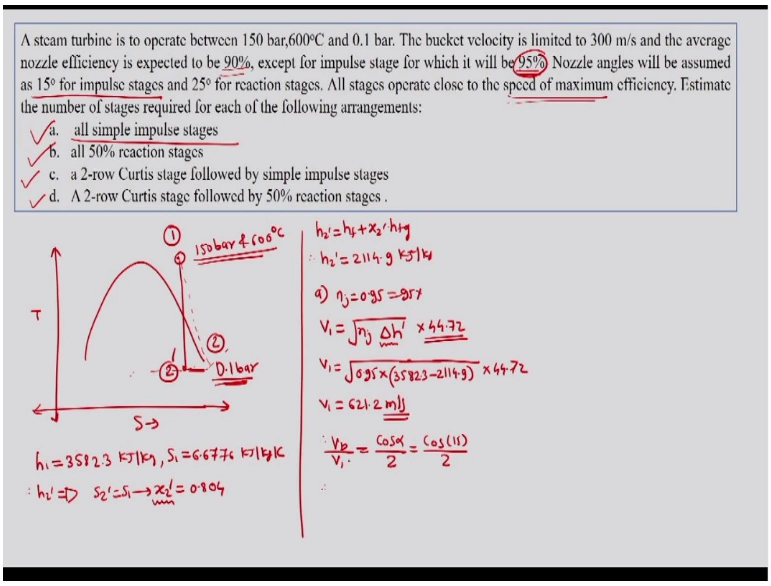

We will move ahead to the next example which states that a steam turbine is to operate between 150 bar, $600\,°C$ and 0.1 bar. The bucket velocity is limited to $300\frac{m}{sec}$. average nozzle efficiency is expected to be 90%, except for impulse stage for which it will be 95%. Nozzle angles will be assumed as $15°$ for impulse stage and $25°$ for reaction stages. All stages operate close to speed of maximum efficiency.

Estimate number of stages required for each case of the following arrangements. All simple, al 50% reaction, a 2-stage Curtis followed by a simple impulse stage, a 2-stage Curtis followed by 50% reaction. So, in this we will go ahead and see in this example, if we try to plot the T-s diagram, then we are told that this stage is 150 bar and $600\,°C$ and this state is given to us as 0.1 bar.

So, we can make use of these 2 states to find out the total enthalpy drop. So, for that let us find out this state as 1 and state as 2 okay but here we should remember that it will have 2 dash and 2 since the efficiencies are different and they are not 100%. So, in this let us find out h_1 which is corresponding to 150 bar and 600 degree Celsius, it turns out to be $3582.3\,kj/kg$.

Similarly, at the same state we will note down enthalpy which is $6.6776\,kj/kgK$ and then we can note down h_2' since for h_2' we are using the fact that $s_2' = s_1$ and this gives us the x_2' which is the dryness fraction at state $2\,dash$ which is inside the dome which has equal enthalpy as 1 and that gives us x_2' as 0.804 and this we can make use to find out h_2' which is $h_f + x_2' h_{fg}\, at\, 0.1$ ¿.

So, we get $h_2' = 2114.9\,kj/kg$. Knowing this, we can solve first example where all stages are impulse. Now, for impulse stages, the nozzle efficiency is given as 0.95 which is 95%, we can find out V_1 from here which is square root of nozzle efficiency into Δh isentropic or $\Delta h'$ and into 44.72. Now, $V_1 = 0.95, h_2' = 3582.3 - 2114.9 \times 44.72$.

So, for us $V_1 = 621.2\,m/sec$. We have to keep this point in mind; we are writing this Δh in kj/kgthat is why we have 44.72 as a number which is outside the square root. So, we get velocity in meters per second but we are told that all the stages were close to optimum, so we have $\frac{V_b}{V_1}$ in case of impulse turbine as $\cos(\alpha)/2$, so which is equal to $\cos(15)$since it is given $15°$for impulse stages $\cos(15)/2$.

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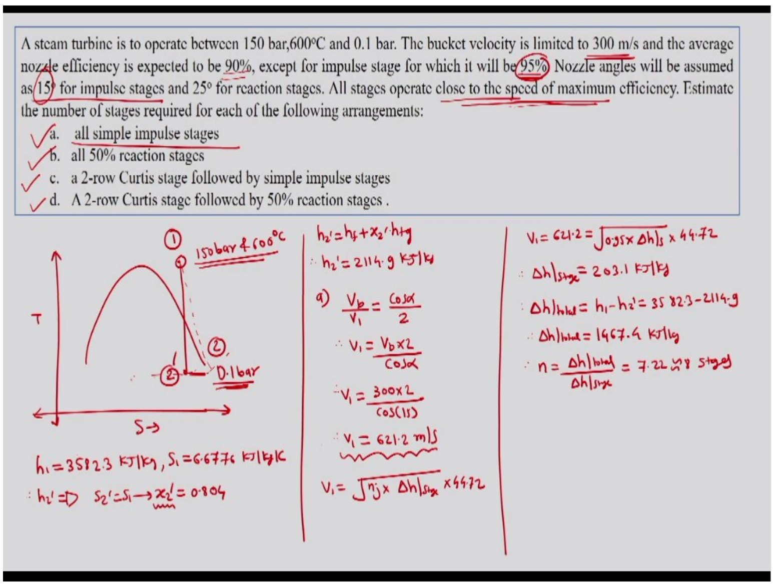

And then this gives us V_b so we will start the example with first part where all the stages are impulse and for all the impulse stages we can make use of the fact that we are having $\frac{V_b}{V_1} = \frac{\cos(\alpha)}{2}$ since it is told that this close to optimum condition. So, we have

$V_1 = V_b \times \frac{2}{\cos(\alpha)}$. So, V_1 and we are told that $V_b is 300\, m/sec$, so $300 \times \frac{2}{\cos(15)}$.

Angle is given as $15°$, so we have $V_1 = 621.2\, m/sec$. Now, we can make use of V_1 to find out stage enthalpy drop since V_1 is equal to nozzle efficiency into Δh stage into 44.72. So, we can say that V_1 which is $621.2 = 0.95 \Delta h$ stage into 44.72 and this gives us Δh stage as $203.1\, kj/kg$ but we have total $\Delta h is = h_1 - h_2'$ and this is equal to $h_1 is 3582.3$ minus we have h_2' which is 2114.9, so we have Δh total = $1467.4\, kj/kg$.

So, number of stages is equal to Δh total divided by Δh stage, so we have Δh total divided by Δh stage as 7.22. So, we have around 8 stages for impulse turbine. So, now we will see for 50% reaction.

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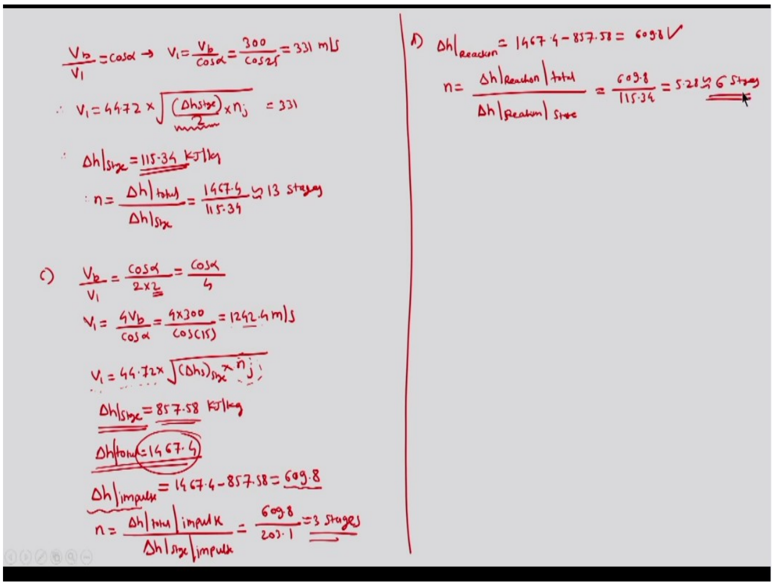

So, for 50% reaction we know we are having $\frac{V_b}{V_1} = \cos(\alpha)$, so we have $V_1 = \frac{V_b}{\cos(\alpha)}$ *upon*

which is equal to $\frac{300}{\cos(25)}$. So, this we can find out V_1 from here and which is 331 meters per second. Again, we know $V_1 = 44.72\sqrt{\Delta h}$ stage divided by 2 into nozzle efficiency. Here we are dividing it by 2 since half of the enthalpy drop of the stage takes place in the nozzle.

So, that will be giving us V_1 velocity and half will take place in the moving blades, so which was complete Δh in case of impulse that become $\frac{\Delta h}{2}$ for the 50% reaction and then this is equal to 331, so we can get Δh stage from the known data for 2-stage is $115.34\,kj/kg$. So, number of stages equal to Δh total divided by Δh stage and this gives us $\frac{1467.4}{115.34}$ which is equal to around 13 stages.

Now, we will see a 2-stage Curtis followed by impulse but for 2-stage Curtis in case of optimum we know $\frac{V_b}{V_1} = \frac{\cos(\alpha)}{2} \times 2$. So, this is for 2-stage, so this is basically $\frac{\cos(\alpha)}{4}$. So, we have $V_1 = 4\frac{V_b}{\cos(\alpha)}$ and then this is equal to $4 \times \frac{300}{\cos(15)}$ since it is $15°$ given in the example. So, this is 15, so for that we get V_1 as $1242.4\,m/sec$.

So, we have $V_1 = 44.72 \times \sqrt{\Delta h_s \times n_j}$ and so we have basically Δh stage from here we will get Δh_s $857.58\,kj/kg$ since we know n_j is 90% and then we have 44.72 known, V_1 is known 1242 and this gives us Δh stage. So, we have number of stages is equal to $\Delta h_{total}/\Delta h_s$.

So, it is equal to Δh total as we know basically Δh total is equal to 1467.4 and then we want to 1 stage of velocity compounded Curtis turbine, so what we will do, we will subtract from here 857, so we will have Δh only impulse simple impulse is equal to $1467.4 - 857.58$, so we have 609.8. So, this is the enthalpy drop if we use all the stages after the 2-stage Curtis as impulse.

So, we have to find out number of impulse stages. So, number of impulse stages is equal to Δh total for impulse divided by Δh stage for impulse. So, we have 609.8 and we have found out Δh stage for impulse is equal to 203.1. So, this is 203.1 so we have around 3 stages. So, once stage Curtis or rather 2-stage Curtis will be followed by 3 stages of simple impulse turbine where we would be needing 8 stages of simple impulse.

Now, last we will solve for row of 2-stage Curtis and 2-stage Curtis we know 1-row or 2-stage Curtis, so we have to find out Δh reaction is equal to this $1467.4 - 857.58$ and this gives us again 609.8 and we know 1-stage of reaction has 115.4 so number of stages of reaction is equal to Δh reaction total which is this divided by Δh stage reaction stage and this is equal to $\frac{609.8}{115.34}$.

So, we have 5.28 which is around 6 stages and we need as 50% reaction turbine. So, this is how we have seen that if we are having fixed boiler condition and fixed condenser condition, so for that how we would have different number of stage requirement for the different kind of turbines.