Steam Power Engineering
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Lecture - 26
Reaction Turbine

Welcome to the class. Till time, we have considered that there can be an impulse turbine and then there is a necessity of compounding and this necessity has led to 2 types of impulse turbines. One is the impulse turbine of pressure compounding type and other is impulse turbine of velocity compounding type. So, we have seen that there is Rateau or Curtis staging of the impulse turbines.

Now, we will come back, come out from the understanding of impulse turbines and we will move towards the reaction turbines. So, today's discussion is majorly towards reaction turbines.

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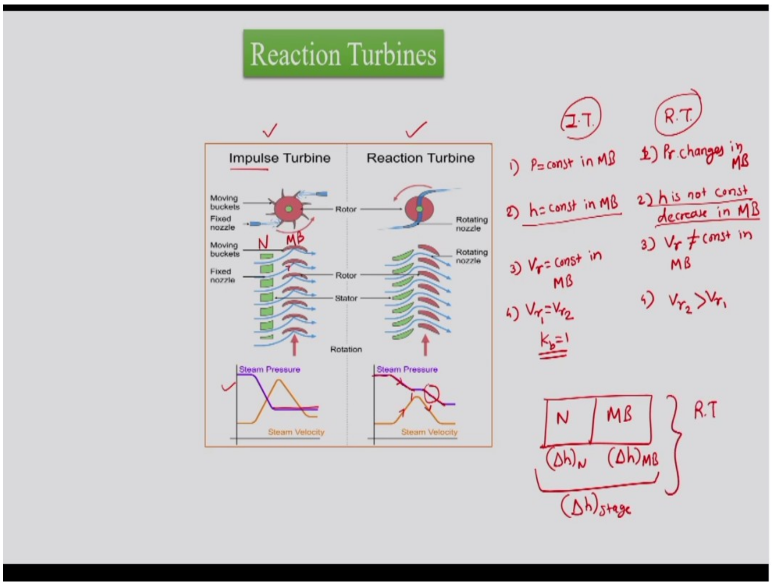

So, in case of reaction turbine let us first understand what is the difference between reaction and impulse turbine. So, in an impulse turbine, we have nozzles, this is impulse turbine and this is reaction turbine. In case of impulse turbine, this figure is a demonstration from the perspective of the action or the working fundamentals for impulse and reaction turbine and this is not exactly related with one has to one related with the steam turbine.

But in general we will have nozzle and we will have moving blades. So, there will be enthalpy drop on pressure decrease in the nozzle and then pressure does not change in the moving blades. Velocity will increase in the nozzle, then velocity will decrease in the moving blades. So, this is what the fundamental in which impulse turbine works. The flow in the impulse turbine is such that the velocity is directly going to heat.

Velocity vector is arranged such that there will be a stagnation point here and which will lead to action or which will lead to an impulse to the moving blades and the moving blades will rotate. However, in case of reaction turbine, we will have smooth flow into the moving blades. Here, there is partial pressure drop partially or some pressure will drop in the nozzle and rest of the pressure will drop in the moving blades.

So, moving blades unlike in case of impulse turbine will face the pressure decrease. So, this pressure decrement in the moving blade will actually help to raise the relative velocity into the moving blades and then we will have that relative velocity further consumed to the work generation or energy generation in the turbine. So, there is absolute velocity which will be rising in the nozzle and then that absolute velocity will decrease in the moving blades.

So, very first point what we should understand, in case of moving blade, in case of impulse turbine, if we have impulse turbine and then we have reaction turbine. In case of impulse turbine, we have pressure, constant in moving blade in case of reaction turbine pressure changes in moving blades. So, enthalpy is constant in moving blades and then second in case of reaction turbine, enthalpy actually is not constant rather it decrease in moving blades.

As an outcome of this, relative velocity is constant in moving blades of impulse turbine but relative velocity is not constant in moving blades of reaction turbine. So, what is the maximum constraint we have seen, $V_{r1}=V_{r2}$ if there is no friction or friction factor is equal to 1. So, maximum value of $V_{r2}=V_{r1}$but here it would happen that V_{r2} is always greater than V_{r1}. This is what the fourth point in case of impulse and reaction turbine difference. So, we will move ahead and see how the impulse and reaction turbines will work.

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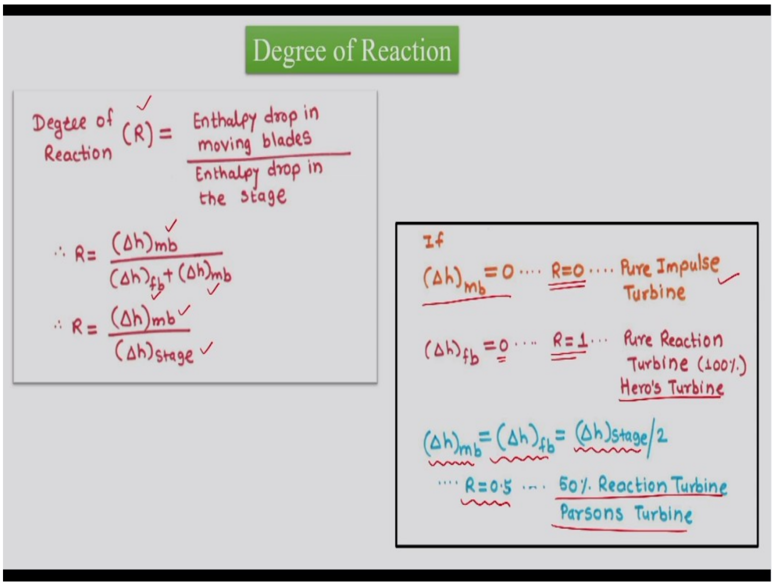

Very first point to be understood here is the degree of reaction. As we have said here that there is enthalpy constant in the moving blades of impulse turbine, but enthalpy decreases so there is an enthalpy drop in case of reaction turbine. So, in case of reaction turbine, we have nozzle and moving blades so there is Δh in the nozzle and then there is Δh in the moving blades, so there is total Δh in a stage.

So, this is what it will happen in case of a reaction turbine. So, in view of this there is a relation which states that there is degree of reaction. So, degree of reaction R is defined as the enthalpy drop in moving blades divided by enthalpy drop in a stage. So, there is total enthalpy drop in a stage, total absolute enthalpy drop in a stage and how much percent of it is having dropped in the moving blades that is degree of reaction.

So, degree of reaction will be Δh in moving blades divided by Δh in fixed blades plus Δh in moving blades or Δh in moving blades divided by Δh in stage. So, this is the degree of reaction. So, what we mean here is if moving blades had zero enthalpy drop so if moving blades had zero enthalpy drop, then it becomes degree of reaction zero, so it is a pure impulse turbine.

If fixed blade had zero enthalpy drop, then degree of reaction is 1 and it becomes pure reaction turbine or also called as Hero's reaction turbine or Hero's turbine. Now, there is a very particular case which is of interest for us mainly for solving the examples where we will have equal enthalpy drop in moving blade and fixed blade. So, they have enthalpy drop which is $\frac{\Delta h}{2}$

So, there is degree of reaction 0.5 it is called as 50% reaction turbine or it is also called as Parsons Turbine. So, we have one parameter, one governing parameter or one performance parameter, one parameter of the impulse turbine which should be known to us and that is degree of reaction which basically tells us the percentage enthalpy drop in the moving blade out of the total enthalpy drop in a stage.

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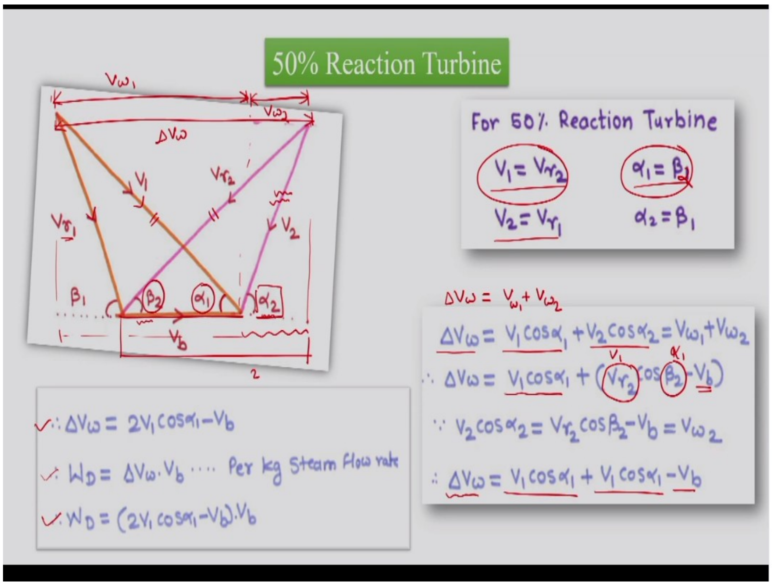

So, now we will move for a particular case of 50% reaction turbine. Now, in case of 50% reaction turbine, we will draw the velocity triangle. So, the velocity triangle is as usual we will have the blade and then there will be velocity which is appearing which is coming into the blade which is relative velocity and then we have V_b which is blade velocity and then we have V_1, so as we have drawn the velocity triangles in the impulse turbine.

So, this is the V_1 velocity at the entry which is absolute velocity, this is V_{r1} which is the relative velocity at the inlet as we had defined earlier alpha 1 is the nozzle angle at the inlet, β_1 is the blade angle at the inlet. Then, since we have enthalpy drop in the moving blades, so relative velocity is about to change due to change in enthalpy, so V_{r2} will have certain value, so we have V_{r2} and we have V_2.

So, absolute velocity angle at the outlet is α_2 and relative velocity angle at the outlet is β_2. So, we will take it for granted few things. For 50% reaction turbine and those things, we would state that in case of 50% reaction turbines, we have $V_1=V_{r2}, so V_1=V_{r2}$. We have $V_2=V_{r1}$, so we have $V_2=V_{r1}$ and then it leads to $\alpha_1=\beta_2$ and $\beta_1=\alpha_2$. So, this is a take away for a special case of 50% reaction turbine.

Having said this, we can draw the velocity triangles for the 50% reaction turbine and write down the formulas as what we did in earlier case. Here, ΔV_w is the change in velocity which is in tangential direction. Now, we know here that in case of reaction turbine as well or in case of impulse turbine, the tangential velocity V_{w1} is this and tangential velocity V_{w2} is this. So, we have this complete distance as ΔV_w .

So, $V_{w1}=V_1\cos(\alpha_1)$, so $V_1\cos(\alpha_1)$ is this distance. So, this is $V_1\cos(\alpha_1)$ and $V_{w2}=V_2\cos(\alpha_2)$ so this distance is $V_2\cos(\alpha_2)$. Practically, we are writing it as $\Delta V_w=V_1+sorry V_{w1}+V_{w2}$. This is what we are writing. So, here $V_2\cos(\alpha_2)$ can be expressed as $V_{r2}\cos(\beta_2)$, so this distance is $V_{r2}\cos(\beta_2)$ minus this distance is V_b. So, this distance is equal to $V_{r2}\cos(\beta_2)-V_b$.

And then we know this which is equal to V_{w2}, so ΔV_{w1} becomes $V_1\cos(\alpha_1)+V_{r2}\cos(\beta_2)-V_b$ but we know that $V_{r2}=V_{r1}$. So, since $V_{r2}=V_1$ sorry $V_{r2}=V_1$ so this V_{r2} will be replaced by V_1. Similarly, we also know that β_2 sorry $\alpha_1=\beta_2$ and $\alpha_2=\beta_1$. We also know that α_1 ¿ β_2. So, this β_2 will be replaced by α_1 and this will be replaced by V_1.

So, we will have $V_1\cos(\alpha_1)$ as it is and $V_{r2}\cos(\beta_2)$ will replaced by $V_1\cos(\alpha_1)-V_b$. So, this is the formula for ΔV_w and then this leads to the next step which states that there is $2V_1\cos(\alpha_1)-V_b$ that is the ΔV_w. So, we can use $\Delta V_w$to find out what is the work done in a stage per kg of steam flow rate. Otherwise, we have to multiply by steam flow rate.

So, this is $\Delta V_w V_b$, so we have work done in a stage is equal to $2V_1\cos(\alpha_1)-V_b V_b$. So, this is the work done by the 50% reaction turbine.

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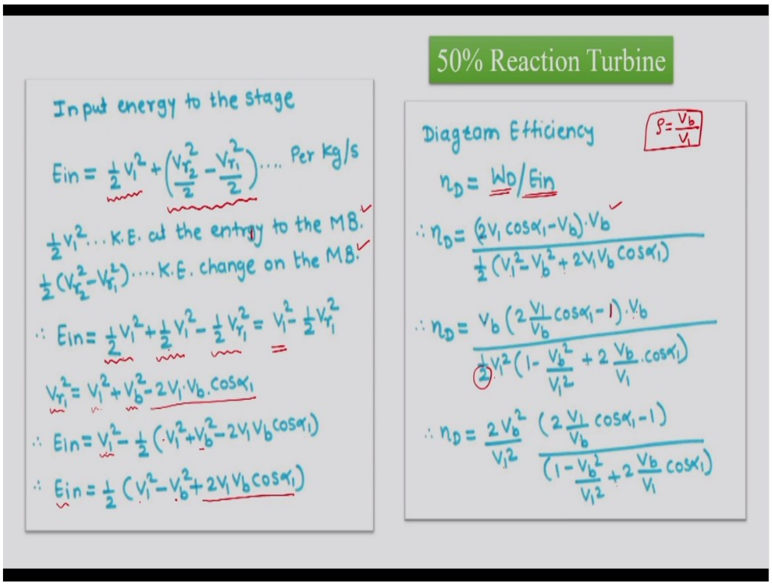

Now, let us find out what is the input energy to a stage. This is very important for us since in case of the impulse turbine, we know that the kinetic energy at the exit of the nozzle is given as an input to the moving blades. So, input kinetic energy was used to divide the work done to get the formula for diagram efficiency. So, diagram efficiency in case of impulse turbine was work done divided by $1/2\,\dot{m}_s V_1^2$ and that $\frac{\dot{m}_s V_1^2}{2}$ was the kinetic energy at the outlet of the nozzle or at the entry to the moving blades.

But their whole enthalpy drop was taking place in the nozzle only. However, in the present case it is not true in the reaction turbine. We have enthalpy drop in the moving blades and in the nozzle or in the guide vanes also. So, since in the stationary blades also we have enthalpy drop and we have enthalpy drop in the moving blades. We have to account the enthalpy drop in the moving blades.

So, this is the kinetic energy at the entry to the moving blades and that is trivial since that was also available in the impulse turbine but this is the kinetic energy which we got generated, we got changed increase in kinetic energy on the moving blades at the expense of changing or decrease in enthalpy of the moving blades. So, there are 2 kinetic energies which should be summed to get the total energy which was received by the moving blade to do the work.

So, we have this $\frac{1}{2}(V ¿¿ 2+V_{r2}^2-V_{r1}^2) ¿$. Actually, here we are assuming that entry kinetic energy to the nozzle is negligible in comparison with the exit kinetic energy. So, $\frac{1}{2}V_1^2$is the kinetic energy entry to the moving blades and then we have $\frac{1}{2}V_{r2}^2-Vr_1$ which change in kinetic energy on the moving blades. So, we have $E_¿=\frac{1}{2}V_1^2$ but $V_{r2}=V_1$ so we have again $\frac{1}{2}V_1^2$ and we have $\frac{V_{r1}^2}{2}$ which is minus.

So, these two will sum up then we have $V_1^2-\frac{1}{2}V_{r1}^2$but if we go back and see what is V_{r1} then this is V_{r1}, so this is V_{r1} and then from the velocity triangle, we can write down $V_{r1}^2=V_1^2+V_b^2-2V_1V_b\cos(\alpha_1)$. So, $E_¿=V_1^2-\frac{1}{2}V_1^2+V_b^2-2V_1V_b\cos(\alpha_1)$. So, we have this V_1^2 will go in, so we will have $\frac{1}{2}V_1^2-V_b^2+2V_1V_b\cos(\alpha_1)$.

Having known the input kinetic energy, we can now find out what is the diagram efficiency for 50% reaction turbine. So, it is equal to blading efficiency or diagram efficiencies work done divided by $E_¿$ and we knew earlier that work done is $2V_1\cos(\alpha_1)-\frac{V_bV_b}{2V_1^2}-V_b^2+2V_1V_b\cos(\alpha_1)$. We know that we are interested in finding out the optimum efficiency condition.

And optimum efficiency's condition is always corresponding to certain velocity ratio and we had earlier also defined ρ as a velocity ratio which is $\frac{V_b}{V_1}$. So, we want a ratio of ρ which is $\frac{V_b}{V_1}$, so we will take V_b common from this bracket, so we have 1 over here and we will have

$\frac{V_1}{V_b}$ over here and V_b will come out and then this will be as it is. Similarly, we will take V_1^2 common from the denominator.

So, we have $\frac{1}{2} V_1^2$¿ will get cancelled into cos (α_1). So, this V_b and this V_b will constitute V_b^2

and this 2 will go into the numerator. So, $\frac{\frac{2V_b^2}{V_1^2}\left(2\frac{V_1}{V_b}\cos(\alpha_1)-1\right)}{(1-\frac{V_b^2}{V_1^2}+2V_bV_1\cos(\alpha_1))}=\eta_D$. So, this is the formula for diagram efficiency.

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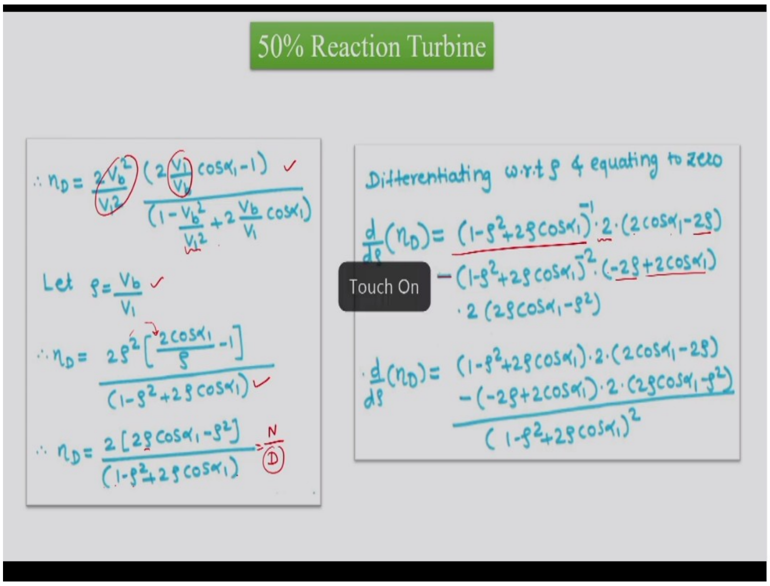

So, this formula will be placed by $\rho=\frac{V_b}{V_1}$ so we will have $\frac{2\rho^2\left[\frac{2\cos(\alpha_1)}{\rho}-1\right]}{1-\rho^2+2\rho\cos(\alpha_1)}$. We can put this ρ^2 into the bracket and then this leads to $\frac{2[2\rho\cos(\alpha_1)-\rho^2]}{1-\rho^2+2\rho\cos(\alpha_1)}$. We can differentiate this formula with respect to ρ and equate it to 0.

This differentiation is seen as what we use a product or division rule of differentiation. So, this term is numerator divided by denominator, so denominator raise to – 1 as it is so we have first differentiation of numerator. So, differentiation of numerator 2 is a constant and then we are differentiating with respect to ρ, so $2\cos(\alpha)-2\rho$. Since it is a differentiation of denominator, we will have raise to minus 2 into the differentiation of denominator 1 is a constant.

So, it will have differentiation $0-2\rho+2\cos(\alpha_1)\left[2\rho cos(\alpha_1)-\rho^2\right]$. So, we will equalize the denominator of 2 terms, so we will have

$$\frac{\left(1-\rho^2 2\rho\cos(\alpha_1)\right)2\left(2\cos\alpha_1-2\rho\right)-\left(-2\rho+2\cos(\alpha_1)\right)2\left(2\rho\cos(\alpha_1)-\rho^2\right)}{\left(1-\rho^2+2\rho\cos(\alpha_1)\right)^2}.$$

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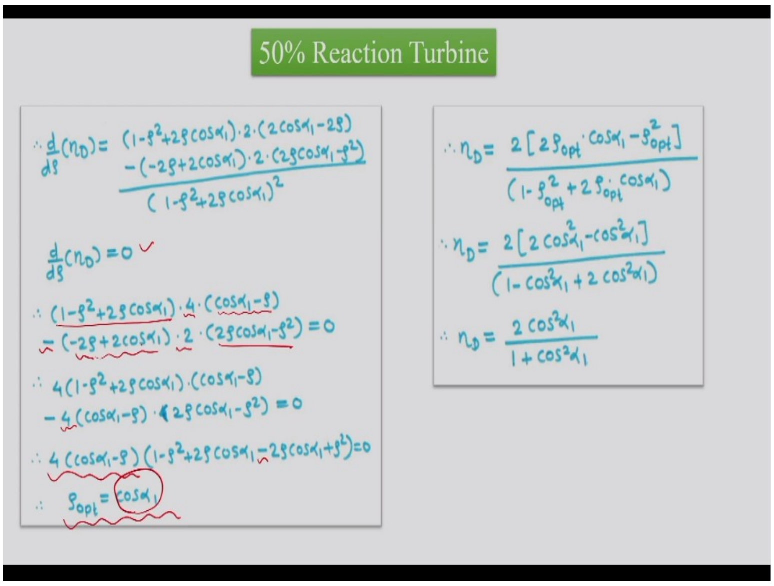

Having said this, we will equate this with respect to 0, denominator is not definitely 0. So, we will divide it, multiply both the sides by this, so this will get cancelled and then we will have a term which is $1-\rho^2+2\rho\cos(\alpha_1)\times 2$ and then we will have 2 as common so it will be $4\cos(\alpha)-\rho-2\rho+2\cos(\alpha_1)\times 2\times 2\cos(\alpha_1)-\rho^2$.

So, here as well we have to so here as well it will become 4, so this 4 and this 4 are there and then further we have this term as $\cos(\alpha)-\rho$ and here also we have $\cos(\alpha)-\rho$. So, these 2 terms and the 4 term will be common, so we have $4\cos(\alpha)-\rho(1-\rho^2)+2\rho\cos(\alpha_1)-2\rho\cos(\alpha_1)+\rho^2$ and this if we equate it to 0, we ultimately will get ρ_{opt} or $\rho=\cos(\alpha_1)$.

So, maximum efficiency of a reaction turbine will be obtained if we have rho or $V_b = V_1 \cos(\alpha_1)$. Then, we can put it into the formula of diagram efficiency and as per that formula we have $\frac{2\left[2\rho_{opt}\cos(\alpha_1) - \rho_{opt}^2\right]}{1-\rho_{opt}^2 + 2\rho_{opt}\cos(\alpha_1)}$. We will replace $\frac{\rho_{opt}}{\cos(\alpha_1)}$, so we will have $\frac{2(2\cos^2\alpha_1 - \cos^2\alpha_1)}{1-\cos^2\alpha_1 + 2\cos^2\alpha_1}$.

So, that leads to $\frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$. This is the maximum efficiency of a reaction turbine. Having said this, we have thus derived the formula for work done. We have first drawn the velocity triangle for 50% reaction turbine rather we have drawn in general the velocity diagram for 50% reaction turbine. From that, we found out the work done, from that we found out the first initial input energy to the reaction turbine.

Then, we derived the expression for diagram efficiency and then using that expression we found out what is the optimum condition for maximum diagram efficiency and hence what is the maximum diagram efficiency.

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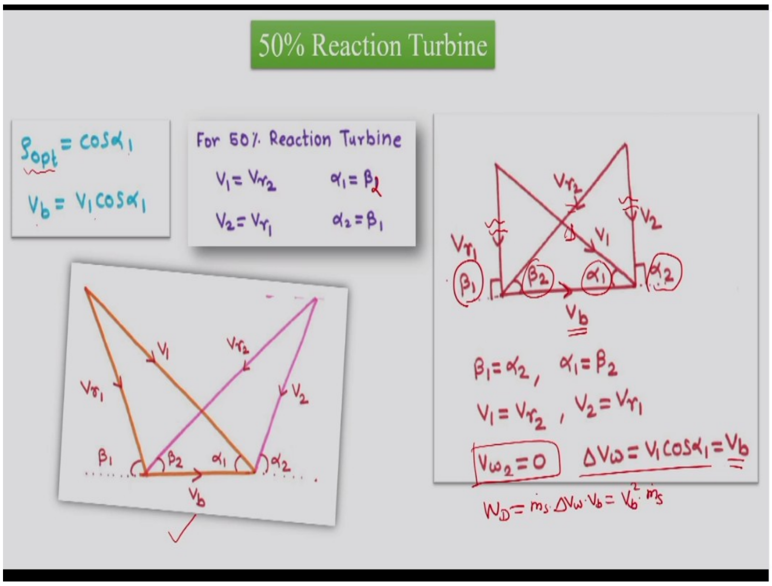

Having said this, we know now that $\rho_{opt}=\cos(\alpha_1)$ and $V_b=V_1$. So, what is ρ_{opt} ρ_{opt} is $\frac{V_b}{V_1}$. So,

$\frac{V_b}{V_1}=\cos(\alpha_1). So, V_b=V_1\cos(\alpha_1)$ but further for 50% reaction turbine, we know that $V_1=V_{r2}$ and $V_2=V_{r1}\wedge\alpha_1=\beta_2\wedge\alpha_2=\beta_1$. So, if we try to reconstitute the velocity triangle which was earlier like this, now this velocity triangle which was general for 50% reaction turbine becomes this in case of optimum efficiency or maximum efficiency case.

Here, this is V_1, so this V_1 and this V_b will have relation which is ρ_{opt} which is V_1 $\cos(\alpha)=V_b$ and this is V_2, V_2 is having α_2 which is 90 which is equal to β_1 and α_1 and β_2 are equal, it leads again to V_{r2} and V_1 to be equal and V_{r1} ¿ V_2 to be equal. Here, we have V_{w2} to be 0, so $\Delta V_{w1}=V_1\cos(\alpha_1)$ which is V_b . So, we have work done is equal to $\dot{m}_s\Delta V_w V_b$ which is $V_b^2\dot{m}_s$.

This is a particular case for 50% reaction turbine with maximum efficiency or optimum working of 50% reaction turbine.

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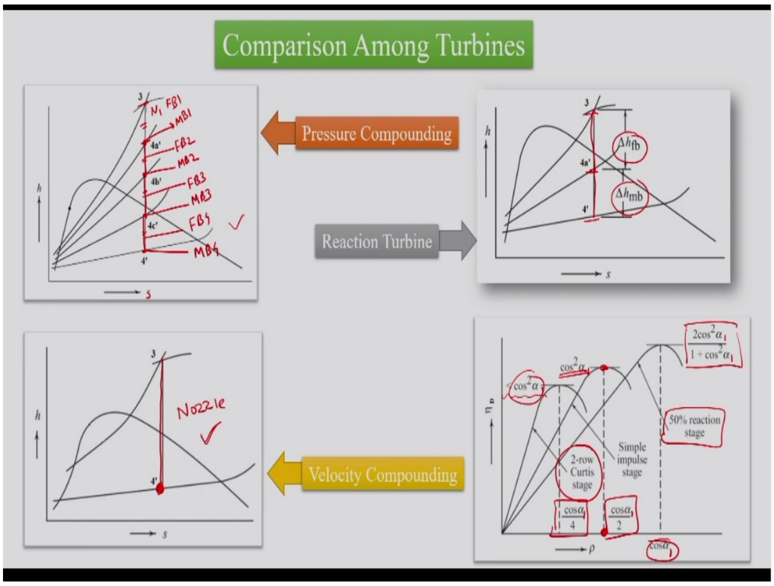

Now, we will compare different turbines. So, first we said that we will have pressure compounding. So, if we have pressure compounding, so in case of pressure compounding this was the $h-s$ diagram which we should be expecting. If we have 1, 2, 3 and 4 stages of pressure compounding, then first nozzle will have this enthalpy drop and then this point corresponds to this is first fixed blade.

Then, this point corresponding to moving blade 1, then this line corresponds to fixed blade 2, this point corresponds to moving blade 2, this line corresponds to fixed blade 3, this point corresponds to moving blade 3, this line corresponds to fixed blade 4 and this point corresponds to moving blade 4. So, this is the complete expansion in the turbine which is the 4-stage turbine in case of pressure compounding.

Now, the same h-s diagram we will change like this in case of velocity compounding. Here, suppose we are working for 2-stage velocity compounding turbine but as we know in case of velocity compounding we have only 1 nozzle and rest of the stationary blades are acting as guide vanes. So, whole enthalpy drop is taking place in only one nozzle and all the stages are sitting here at a point.

If it is a 2-stage, then one point corresponds to all the 2-stages moving blades and corresponding stationary blades as well. If this is for 3-stage, then this point corresponds to all the 3-stage moving blades and also the stationary blades. In case of reaction turbine, we have one stage reaction turbine, we have 3 to $4a'$ as the enthalpy drop in the fixed blade and then this is the enthalpy drop in the moving blade.

So, this diagram is for exclusively for single stage reaction turbine. This is for multiple stage or single stage velocity compounding and this is for the multistage impulse turbine which is pressure compounding and then this diagram is same for the single stage impulse turbine which is pressure compounding. Having said this, now we can compare the efficiency and rho formula.

So, as we see that in case of simple impulse turbine, we have derived rho optimum is $\frac{\cos(\alpha)}{2}$ and corresponding efficiency is $\cos^2\alpha$. In case of velocity or 2-stage Curtis turbine, ρ_{opt} *is* $\frac{\cos(\alpha)}{4}$ corresponding efficiency is $\cos^2\alpha$ but it will be always lower than this due to the frictional loss. Then, the reaction turbine has α_{opt}, ρ_{opt} as cos (α) or all these are 1 for us.

And then this leads to $\frac{\cos^2\alpha \times \left(2\cos^2\alpha\right)}{1+\cos^2\alpha_1}$ as the maximum efficiency for 50% reaction turbine. So, here we discussed what do we mean by reaction turbine and how to work out with the velocity triangle of a reaction turbine, how we can compare the performance of various reaction turbines based on the optimum condition.

So, for that we evaluated what is the optimum condition for 50% reaction turbine. So, here we end the discussion for reaction turbine and next time we will see the rest of the part of the steam power plant. Thank you.