Steam Power Engineering
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Lecture -24
Impulse Turbine 2

Welcome to the class. in the last class we had seen how to find out the basic performance parameter like efficiency in case of that turbine stage where we had considered 1 nozzle and 1 row of the blades there we found out that of the nozzle plus blade combination we will get the optimum efficiency maximum efficiency as cos square alpha. And that efficiency would be at a particular velocity ratio which is cos alpha by 4. So now we will see the rest of the things.

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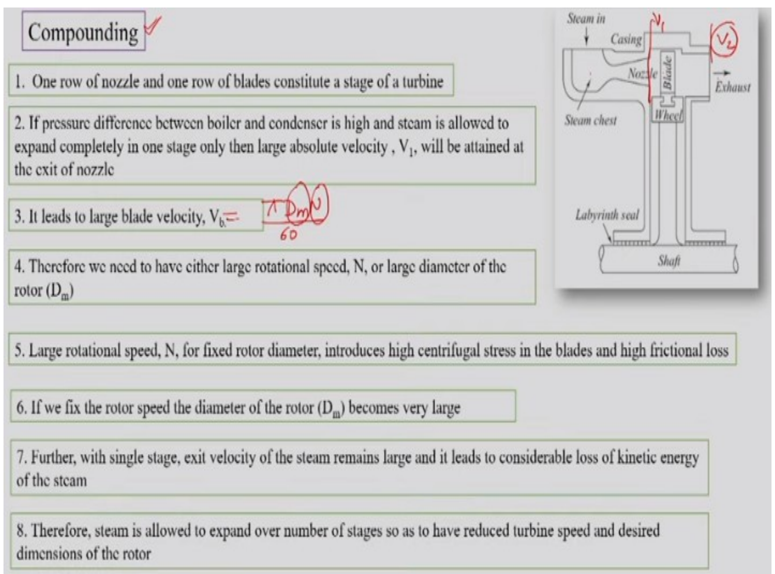

In the turbine as we know that we have nozzle and a blade. This is a stage of the turbine. now if we have very big very large pressure in the nozzle at the entry to nozzle in the boiler and very low pressure at the exit of the turbine that is in the condenser then we need to have complete expansion in the nozzle and then complete work extraction has to be done by the blades. So with this fact we have to work however it is not logical.

We will see that if we have only 1 row of nozzle and only 1 row of blades then it constitutes a stage of a turbine as shown in the figure. But if we have complete pressure drop then in the nozzle whatever the pressure drop which should take place between the boiler and the condenser then we will get very large velocity at the exit of the nozzle. So here we will have very large velocity absolute velocity V_1 then as what we have seen there is a particular relation between absolute velocity at the exit of the nozzle and the blade for getting the maximum efficiency.

It turns out that there is very large velocity of the blades as well. So the large velocity of blade we know that $V_b = \pi D_m N/60$ so we get very large velocity of V_b if we are going to obtain the complete extraction through only 1 stage then this V_b will be large. Then this large V_b can be accommodated by 2 facts. Either we should have a large N or which we should have with odd we should have really large D_m which is the mean diameter of the blade.

But if we have very large N to accommodate the complete V_b large V_b then we should have fixed diameter D_m and large value of N introduces large centrifugal stresses and very large friction in the bearing. So we will have a large frictional loss and we will have large centrifugal stress. So further if we fix the speed and if we change the D_m then it turns out that we need a very large diameter mean diameter of the blade.

So that also creates a problem so further there would be a problem that we will have further very large velocity V_2 at the exit of the complete stage. So we would not able to completely extract the affected energy from the stage. So this tells us the fact that it is not good idea to have only 1 stage of a nozzle and blade combination. We should have multiple stages and that combination is called as compounding. So we should not have complete enthalpy drop in only 1 stage.

We should have enthalpy drop in the multiple stages or we should have the not complete velocity extraction in 1 stage. We should have velocity extraction or kinetic energy extraction into multiple stages and such facts are called as compounding of steam turbine. So we will see so steam is allowed to expand over large number of stages so as to have reduced turbine speed or desired dimension of the rotor.

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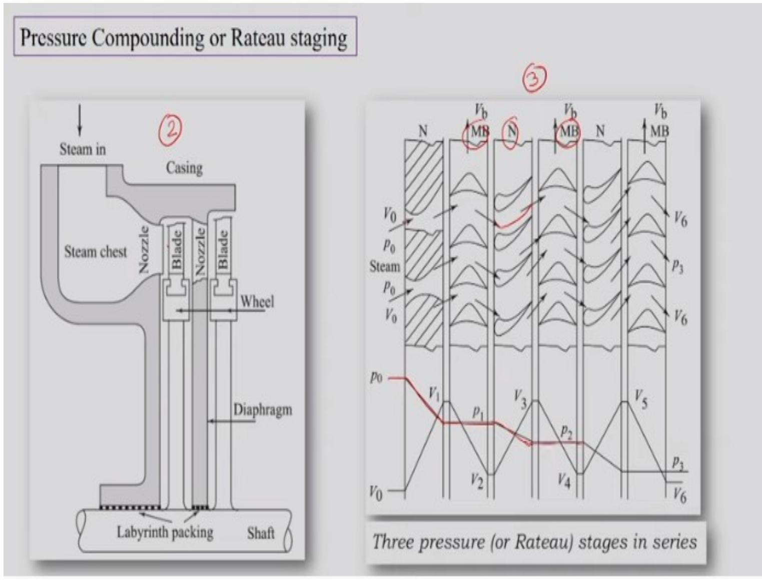

So very 1st method of compounding is pressure compounding are called as Rateau staging. So here as what we see that there will be multiple nozzles and multiple blades. So 1 nozzle would lead to some enthalpy drop in it and create corresponding kinetic energy. That kinetic energy will be extracted by the blades. and then we will have certain else remaining kinetic energy change or remaining enthalpy drop and corresponding rise in kinetic energy in the next nozzle and then corresponding blade will extract that kinetic energy into the work done.

So we have multiple nozzle and blade combinations or we have multiple stages in case of pressure or Rateau staging. Here we are calling it as pressure compounding since not complete enthalpy drop or pressure right drop will take place in that 1 nozzle but we will have multiple nozzles in the stack. Further each blade will not have any static enthalpy drop or each blade will not have a new relative velocity change here.

This is what we will have staging called last pressure compounding. So as what expected we said that 1 nozzle will rise the velocity and drop the pressure and drop the enthalpy and then it will go over the moving layer. Then next it will go to the next set of nozzles. Next set of nozzles will further rise the velocity dropped the pressure then it will go over the next or 2nd set of nozzles and it will continue.

So this figure is for 3 staging. This figure is for 2 staging and as what we have said we have some pressure drop in the 1st nozzle. Then we will not have any pressure drop in the moving blades then in the nozzle we will have a pressure drop and then again in the 2nd set of moving blades there is no pressure drop and this will continue.

Similarly, velocity rise will take place in the nozzle across the pressure drop or enthalpy drop and then this will a kinetic energy will be extracted by the nozzle here. Remember that we are talking about the extraction of absolute kinetic energy but when we are telling that the impulse factor binds are there then there is no change in relative velocity or what we have seen that if at all that is change in the velocity.

That is mainly due to friction and we are saying that there is a blade friction factor and this will be continuing to the rest of the stages. So such staging is called as pressure compounding.

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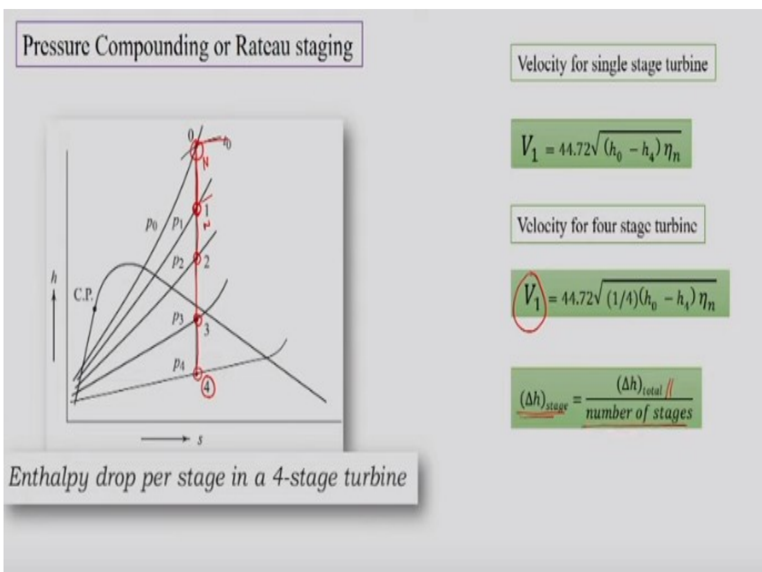

So we can see the TS diagram or HS diagram for the pressure compounding. We know this is the boiler and then this is the condenser. So in the 1st stage we will have 1st enthalpy drop in the nozzle. So this is the nozzle enthalpy drop and then we have at this point only we have the rotor. Since the rotor will not have the enthalpy drop then 2nd nozzle we will have 2nd enthalpy drop then this is the 2nd rotor.

Then we have third enthalpy drop this is the 3rd rotor and 4th enthalpy drop and 4th blade. So this is the complete expansion from 0 to 4 that in the 4 stage pressure compounding turbine. Now we can see that velocity for single stage as what we know can be found out from this formula which is $V_1 = 44.72\sqrt{(h_0 - h_4)\eta_n}$ into nozzle. So this is if we have only 1 stage then we would have this as the absolute velocity. But now we have 4 stages so we should have this enthalpy drop is divided into 4 parts.

So this $V_1 = 44.72\sqrt{\Delta h \times \frac{1}{4} \times \eta_n}$ is the 1st velocity or velocity at the exit of 1st nozzle. If we would our 3 stages, then this 4 would be replaced by 3. So enthalpy drop in 1 stage = total enthalpy drop/number of stages. So we basically have in our formula $V_1 = 44.72\sqrt{\Delta h_{stage}\eta_n}$ So this is ideal enthalpy drop in a stage. So if we can find out enthalpy drop in 1 stage then we can find out velocity of the exit of the nozzle or corresponding stage nozzle.

So if we know enthalpy drop in 1 stage and if we know total enthalpy drop required then we can also find out number of stages. So number of stages is equal to enthalpy drop total divided by stage enthalpy drop. So this is all about the pressure compounding or Rateau staging.

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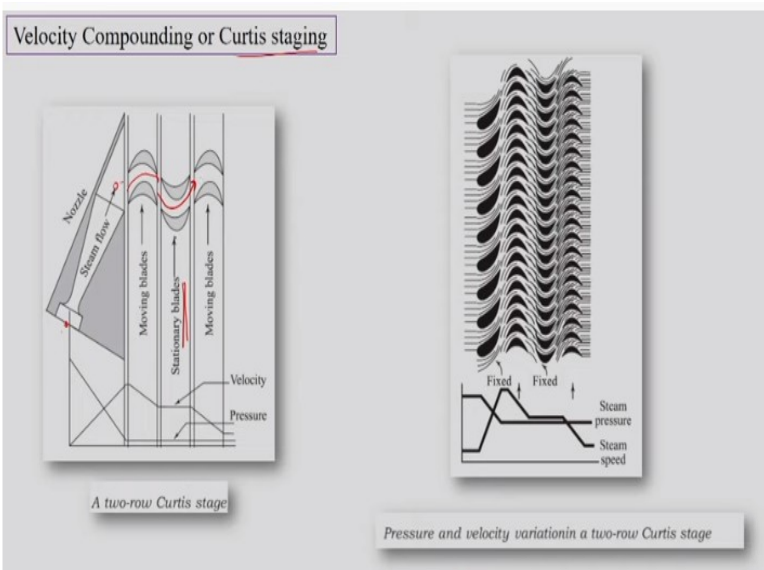

Now we will see next staging is called last Curtis staging or velocity compounding. This is a 2nd type by which we can reduce the dimension of the construction and dimensions of the nozzle or we can have a basically lower speed. In case of velocity compounding we will have 1st nozzle which will actually completely expand the steam from boiler pressure to condenser pressure. It is very important in case of pressure compounding.

We were not completely expanding the flow of the steam but we are partially expanding in multiple nozzles. We were having partial pressure drop in multiple nozzle but in case of Curtis staging or velocity compounding we will have complete enthalpy drop into the nozzle and then we will have the moving blades were partially we will have the work done or partial extraction of kinetic energy.

Then the next set of blades are less stationary blades or they are also guide vanes blades where it will guide vanes the flow to the next move in place. In these guiding blades or just stationary blade there will not be any enthalpy drop or there is no in ideal case there is no velocity change. Then further moving blades will change the kinetic energy of the flow. So here we will have velocity compounding we will have complete enthalpy drop in of 1 nozzle and partial kinetic energy change will take place into that multiple rotors.

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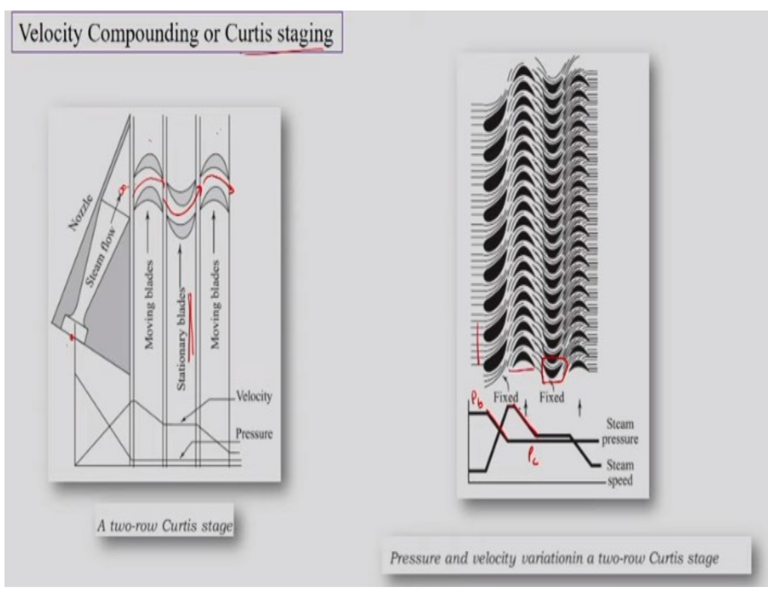

So such turbine it is called Curtis staging as what we can see into this 2-stage Curtis turbine. There is a nozzle which this is a nozzle where we will have complete enthalpy drop or complete pressure drop and then in the moving blades then we go to the moving blade in the same nozzle rather 1st velocity rise and this velocity rise will be decreased in 1 set of nozzle sorry 1 set of moving blade and then into the fixed blades or when there is no change in velocity and the next set of moving players we have the rest of the change in kinetic energy.

So we will have change in speed. So pressure drop has taken place completely from the boiler pressure to the condenser pressure in the 1 nozzle. But the fixed blades are just the guide vanes vents. So, they are change in the direction of absolute velocity. The fixed or fixed blades or the guide vanes vents they are just change in the velocity. But we will have kinetic energy change into the moving blades partially.

So 1 set of moving blade it change in the real absolute velocity and then 2nd set of moving blade is change in the rest of the kinetic energy. So it is only velocity change is taking place into the moving blades.

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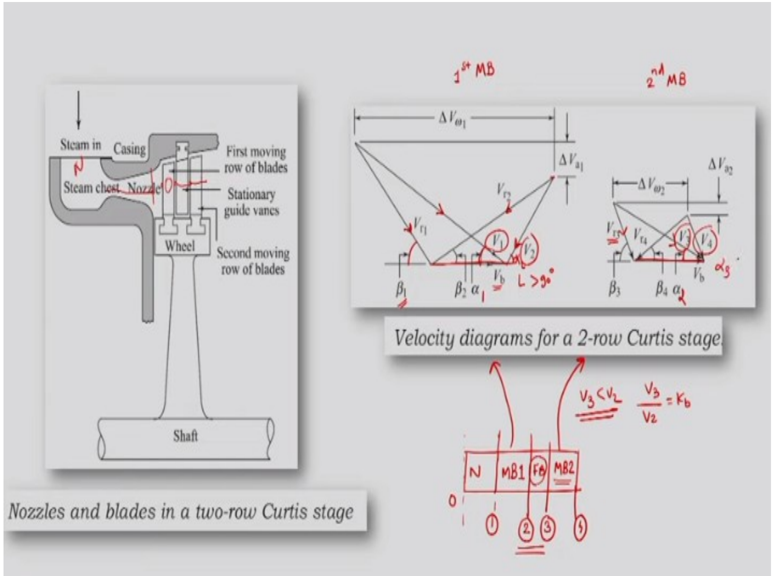

And we have these fixed blades as only the stationary. We have fixed blades as did guide vanes. So let us see how would it proceed so as what we have seen this is a 2 stage Curtis turbine. We have nozzle here and then in the nozzle we will have complete enthalpy drop and maximum velocity at the exit of the nozzle. Then there will be blades and these moving blades will change the absolute velocity to some value at the exit.

Then there will be stationary blades or guide vanes they will guide change in the direction of velocity such that it will be smoother than the 2nd moving blades. And then in the 2nd moving blades we will have a rest of the velocity change taking place. So if we see the velocity diagram then for the 1st set of moving blade this is 1st set up the moving blades and this is 2nd set of moving blades.

For the 1st set of moving blades this is absolute velocity very larger magnitude at the entry and this absolute velocity has angle α_1 and then we know that the moving blades are rotating with velocity V_b tangential speed of V_b and they are rotating with speed RPM and then there is an angle α_1 between them. So we have a relative velocity at the entry as V_{r1} and corresponding angle as a β_1.

But while going out from the 1st set of moving blades the blade will absolute velocity V_2 and it will have relative velocity V_{r2}. Here we are drawing this diagram for the fact that the α_2 or Δ is greater than 90 this angle is greater than 90 degree and here we are saying that blades are not symmetric or blades maybe symmetric where β_1 and β_2 can be accordingly adjusted. But at this stage we are saying that $V_{r1} \neq V_{r2}$.

And so there is mismatching magnitude due to friction factor. Now the 2nd stage of moving blade the same V_2 will be here as V_3. Now if there is frictional loss between the while the flow is taking place on the stationary blades are the guide vanes then V_3 will be less than V_2. And we can again see that $V_3/V_2 = V_b$ which is a frictional factor. So V_3 is there and then we have 2nd angle as $alpha2$ for the moving blades and then we have same V_b as the tangential speed for the 2nd stage or for the 2nd set of moving blades and then we so have V_{r3}.

So here what we are trying to say is this way we are having nozzle so nozzle at 0 at the starting point nomenclature and then at the exit of the nozzle we are saying 1 then we are having moving blades at the exit of the 1st set of moving blades. We are saying 2 then we have fixed blades at the exit of the fixed blades we are saying 3 and we are in the 2nd set of moving blades we are saying 4 and present velocity diagram for 1st set of moving blade is here and 2nd set of moving blade is here.

So for the 1st set of moving blades and 3 velocity is V_1 exit velocity is V_2. For 2nd set of moving blades and 3 velocities V_3 and exit velocity is V_4. Since 2 to 3 change is happening in the fixed set of blades so we are drawing the same velocity diagram. But here my rank velocity diagram we have drawn this special velocity diagram for the fact that we have all α_3 is basically less than basically we have here α_3 and α_4 which is the absolute velocity angle the exit which is less than $90°$ for which we are drawn these diagram.

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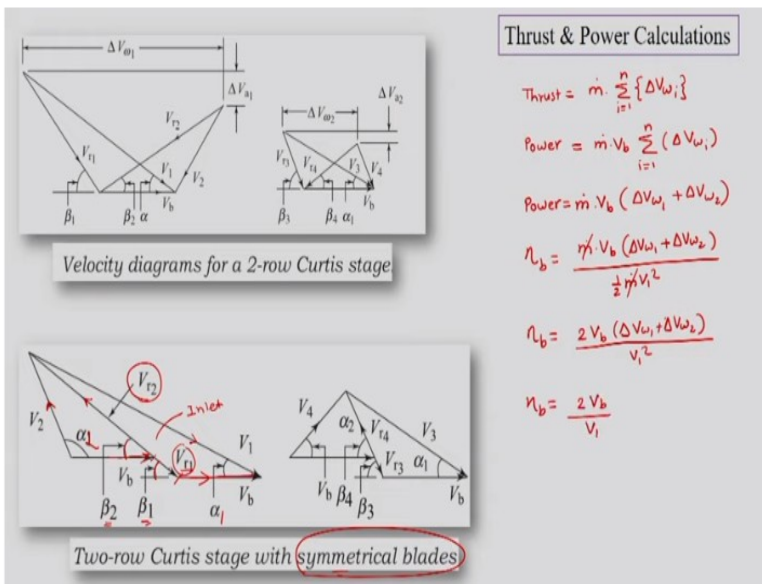

Now let us go and see the same velocity diagram we can draw and we can as well draw this diagram by a new method where we have this V_b inlet velocity triangle V_1 where angle α_1, then we have V_{r1} the velocity relative velocity at the entry. So this is inlet velocity triangle now we know that there is some fraction by which V_{r1} will be reduced to V_{r2}. So, we will reduce that magnitude and then now this becomes V_{r2} or V_{r2} for a.

Now this V_{r2} we have suppose same angle for a symmetrical blading case. So we have β_2 and β_1 same and then we have so V_2 with the absolute velocity with angle α_2. But here while drawing diagram we have to keep one thing in mind that V_1 has direction downward V_{b1} has direction towards the right. So V_{r1} has direction towards downward but here we have to keep in mind that V_b cannot change its direction.

It is still in positive x direction or right hand side but V_{r2} is in opposite direction and V_2 is also in opposite direction. this diagram helps us to such diagram helps us to simplify the complicacy in drawing the diagram which is conventionally otherwise would be drawn for the fact that symmetrical blade and for the further derivations. So now we can make use of this diagram for finding out thrust and power calculations for doing thrust and power calculations.

So we know that tangential $Thrust=\dot{m}\sum_{i=1}^{n} V_b \Delta V_{wi}$and this is for number of stages. So this is V_b into ΔV_{wi}. So this is sorry $Thrust=\dot{m}\sum_{i=1}^{n} \{\Delta V_{wi}\}$ that is thrust total thrust or total axial force or torque. Now we know that $Power=\dot{m}V_b\sum_{i=1}^{n} (\Delta V_{wi}¿)¿$ okay so we will have here for 2 stage $Power=\dot{m}V_b(\Delta V_{w1}+\Delta V_{w2})$ we have defined V_w as the tangential velocity and ΔV_w means change in tangential velocity.

So that thrust is force which is change in angular momentum or change in tangential momentum. okay. So this is the formula for us so we can define efficiency or diagramming efficiency or blade efficiency is so far 2 stage $\dot{m}V_b(\Delta V_{w1}+\Delta V_{w2})$ for this particular case of 2 stage we have $1/2\dot{m}V_1^2$ so $\dot{m},\dot{m}$ will cancel. So blade efficiency or diagram efficiencies $\eta_b=\frac{2V_b(\Delta V_{w1}+\Delta V_{w2})}{V_1^2}$ is taken up so we will have in general blade efficient okay.

We will have divided by V_1^2 so we will have efficiency = $\eta_b=2\frac{V_b}{V_1^2}\sum_{i=1}^{n} (\Delta V_{wi}¿)¿$ and so this is what our formula for efficiency.

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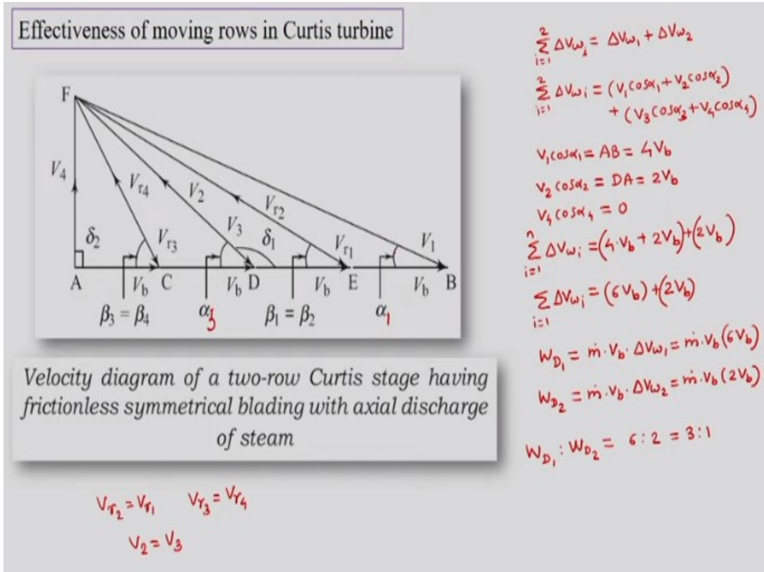

Now we can find out same what we did in earlier case and we can find out the extraction of the work in each stage in proportion with each other. So now let us draw the velocity diagram for 2 stage Curtis turbine with frictionless blade and axial discharge. There are 2 terms over here 1 term says that we are frictionless blade. So the blades are frictionless that means we are considering that $V_{r2} = V_{r1} \vee V_{r3} = V_{r4}$ and similarly $V_2 = V_3$ this is also we are seeing.

So velocity at the exit of the 1st moving blades is equal to velocity exit of the 1st stationary blade. So, stationary blade is going to change in velocity. So for this particular case this is the velocity diagram where we will have the $V_{r1} = V_{r2}$. Now as per notation we will have this angle as $\alpha_1$and symmetrical blades $\beta_1 = \beta_2$ and then this angle = α_3 so we can find out the frictionless of the moving blade.

So as per the formula or work done we have we need basically ΔV_{w1} or 2 stage it is sorry we need $\sum_{i=1}^{n} (\Delta V_{wi}¿) = \Delta V_{w1} + \Delta V_{w2}¿$ and for us we know that this $\sum_{i=1}^{n} (\Delta V_{wi}¿)¿$, ΔV_{w1} is tangential velocity at the entry and that is at the inlet we have $V_1 \cos(\alpha_1)$ and at the exit we have $V_2 \cos(\alpha_2)$ but $V_2 \cos(\alpha_2)$ but $V_2 \cos(\alpha_2)$ is in the since we are considering δ or this angle α_2 as more than 90 so it will be $V_2 \cos(\alpha_2)$ okay + $V_2 \cos(\alpha_3) + V_4 \cos(\alpha_4)$.

Now we know $V_1 \cos(\alpha_1)$ from this diagram is equal to $4V_b$ each this line corresponds to V_b. So $V_1 \cos(\alpha_1)$ is a complete AB basically this is equal to AB. Since $V_1 \cos(\alpha_1)$ is AB we know $AB = 4V_b$. So $V_1 \cos(\alpha_1) = AB$ and $AB = 4V_b$ then we know $V_2 \cos(\alpha_2) = V_2 \cos(\alpha_2)$ is here and $\alpha_2 \wedge \alpha_3$ are the same. So it is DA and $DA = 2V_b$.

Similarly, we knew that $V_4 \cos(\alpha_4) = 0$. Since we are considering axial discharge so there is the angle between the absolute velocity at the outlet and the tangential speed are 0. So we will have this $\sum_{i=1}^{n} (\Delta V_{wi} ¿) = 4V_b + 2V_b + 2V_b ¿$. So we will have $\sum_{i=1}^{n} (\Delta V_{wi} ¿) ¿$ this is 1st stage and this is for the 2nd stage $\sum_{i=1}^{n} (\Delta V_{wi} ¿) = 6V_b + 2V_b ¿$ here now we can see that this is corresponding to the 1st stage and this is corresponding to the 2nd stage.

Now if I want to find out isolated the work done for the 1st stage then work done or power for the 1st stage the I will write down suppose WD were done for the 1st stage is $W_{D1} = \dot{m} V_b \Delta V_{w1}$ and this turns out to be $\dot{m} V_b 6V_b$. So for the 2nd stage or 2nd rotor blade we are not saying 2nd stage we are saying that for the 2nd moving blade or practically the 2nd stage of Curtis turbine will have $\dot{m} V_b \Delta.\ Vw2 = \dot{m} V_b 2V_b$.

Now if we take a ratio then $W_{D1} : W_{D2} = 6:2$ or we will write it as 3 : 1. So 1st stage of the Curtis turbine does basically 3/4th of the work and 2nd stage does 1/4th of the work. So total work is 4 like 3 + 1 but out of 3 + 1, 3 units of the work is done by the 1st moving blades and 1 unit of the work is done by the 2nd unit of the blade. Now if we would have instead of 2 stages if we would have 3 stages then we can prove that $W_{D1} : W_{D2} : W_{D3} = 5:3:1$.

So there is total 9 unit of the work out of 9 unit of the work 5 units of the work will be done by the 1st stage. 3 units will be done by the 2nd stage and only 1 unit of the work will be done by the 3rd stage. So, only 1/9th of the work will be done in that third stage. So we do not have to have too many stages in the Curtis turbine.

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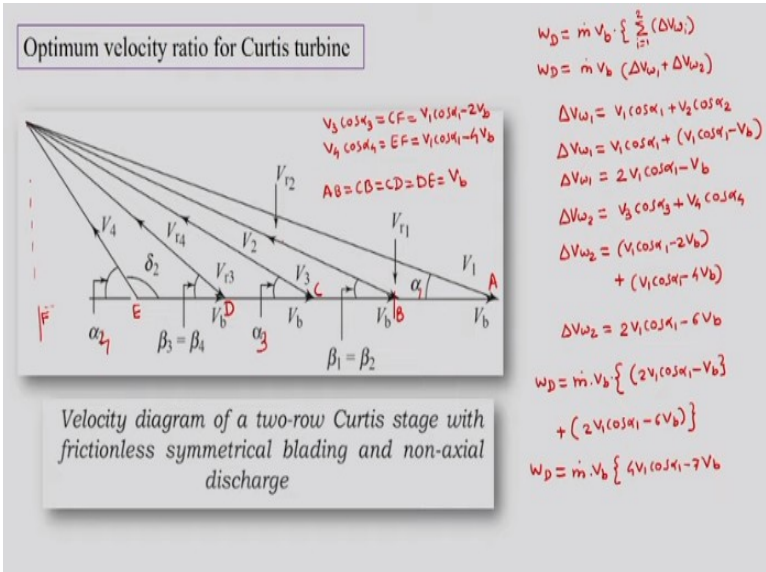

Velocity diagram of a two-row Curtis stage with frictionless symmetrical blading and non-axial discharge

Now we will go ahead and see that what is the formula for optimum velocity ratio for Curtis turbine? So let us draw velocity diagram for Curtis turbine again and in this case we are not drawing the diagram for radial exit but we are saying that we have not said that drawing diagram for axial outlet. We are saying that it is non-axial discharge but here still the diagram corresponds to the fact that the blades are symmetric and there is no friction.

So we have ideal blades so this is α_3 and this is α_4. So for this fact we know now how to write down the formula for total work done which is $W_D = \dot{m} V_b \sum_{i=1}^{2} (\Delta V_{wi} ¿) ¿$ so we have $W_D = \dot{m} V_b (\Delta V_{w1} + \Delta V_{w2})$ but here we can know that ΔV_{w1} and ΔV_{w2} can be found out in this case where we have you can write down the formula for $\Delta V_{w1} = V_1 \cos\alpha_1 + V_2 \cos\alpha_2 \wedge V_2 \cos(\alpha_2)$ basically $V_2 \cos(\alpha_2)$is this part for as this is V_2.

$V_2 \cos(\alpha_2)$is practically this big length from here to here, so, if we name A, B, C, D, E, and F the $V_1 \cos(\alpha)$ is practically for as $V_1 \cos(\alpha_1)$we will write down here $V_1 \cos(\alpha_1) = AF$ but

$V_2 \cos(\alpha_2) = BF, AF + BF$. So the distance $AB = BC = CD = DE = V_b$. So looking at that we know $\Delta V_{w1} = V_1 \cos(\alpha_1) + (V_1 \cos(\alpha_1) - V_b)$ $so\, \Delta V_{w1} = 2V_1 \cos(\alpha_1) - V_b$.

Similarly, we can write down $\Delta V_{w2} = V_3 \cos(\alpha_3) + V_4 \cos(\alpha_4) \wedge V_3 \cos(\alpha_3) = ¿$ we can write down. Now here $V_3 \cos(\alpha_3) = V_1 \cos(\alpha_1) -$ but basically $V_3 \cos(\alpha_3) = CF = V_1 \cos(\alpha_1) - 2V_b$ and $V_4 \cos(\alpha_4) = V_1$ basically EF and $EF = V_1 \cos(\alpha_1) - 4V_b$ okay. So, we can write down $\Delta V_{w2} = V_1 \cos(\alpha_1) - 2V_b + V_1 \cos(\alpha_1) - 4V_b$. $So, \Delta V_{w2} = 2V_1 \cos\alpha_1 - 6V_b$.

So we can use this in and write down the formula for W_D and it becomes $\dot{m} V_b$ into for the 1st stage we have $2V_1 \cos(\alpha_1) - V_b$ and for the 2nd stage we have $2V_1 \cos(\alpha_1) - 6V_b$ and then bracket complete. So we have $W_D = \dot{m}.V_b(4V_1 \cos(\alpha_1) - 7V_b)$.

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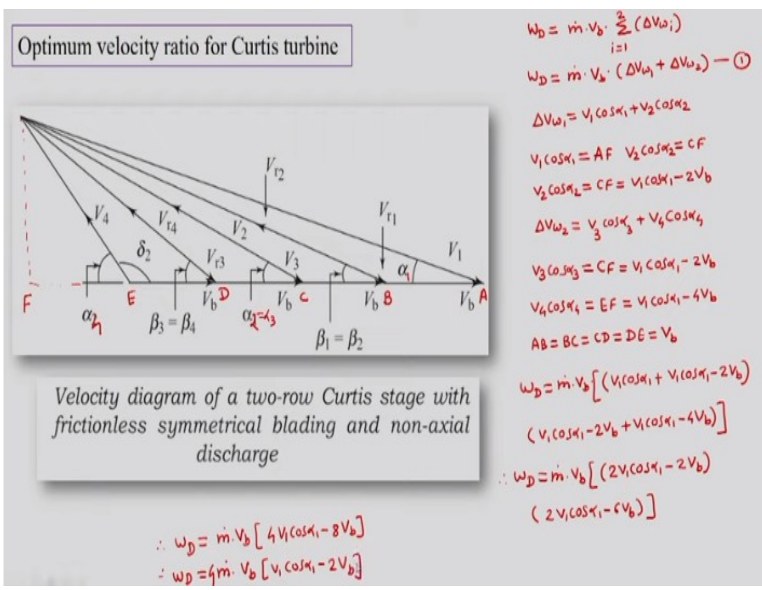

Now let us see the optimum condition of two stage of the turbine, so here we are again during the same velocity diagram as we have done earlier. But here while drawing this we are not considering the axial discharge we are considering non-axial discharge but we are still considering the blades to be symmetric. Here we have α_1 here we have α_2 here basically here we have α_3 and then we have α_4. Let us name them as A, B, C, D, E and then I will name this part as F.

Now let us find out various quantities for this 1st and our objective is to work out for optimum velocity ratio for Curtis turbine. The reason is as follows we know that total work done = $\dot{m}V_b\sum_{i=1}^{2}(\Delta V_{wi}¿)¿$ So work done = $\dot{m}V(\Delta V_{w1}+\Delta V_{w2})$. Now let us find out ΔV_{w1} and ΔV_{w2}.

So we are interested in finding out ΔV_{w1} we know that, let us name this as equation number 1. We know $\Delta V_{w1}=V_1\cos(\alpha_1)+V_2\cos(\alpha_2)$ as per this diagram $V_1\cos(\alpha_1)=A\times F, AF$ and this is equal to further we know that $V_2\cos(\alpha_2)=CF$. So $V_2\cos(\alpha_2)=CF=V_1\cos(\alpha_1)-2V_b$ and this would be ΔV_w. Further $\Delta V_{w2}=V_2\cos(\alpha_2)$ sorry we will have $V_3\cos(\alpha_3)+V_4\cos(\alpha_4)$.

Here $V_3\cos(\alpha_3)$ equal to as per this diagram we will have again = CF and that is $V_1\cos(\alpha_1)-2V_b$ and $V_4\cos(\alpha_4)=EF$ and then that is $V_1\cos(\alpha_1)-4V_b$. Since as per this diagram we will have $AB=BC=CD=DE=V_b$. We can put everything into equation number 1 now we can put all these equations into equation number 1 and then we can write $\dot{m}V_b\Delta V_{w1}$ and then this is $V_1\cos(\alpha_1)-+V_1\cos(\alpha_1)-2V_b$ this is for 1st stage and for 2nd stage we have $V_1\cos(\alpha_1)-2V_b+V_1\cos(\alpha_1)-4V_b$ bracket complete.

So we have W_D = $\dot{m}V_b$ and then we will have for the 1st stage $2V_1\cos(\alpha_1)-2V_b$ and then for 2nd stage we have $2V_1\cos(\alpha_1)-6V_b$. So we will have $W_D=\dot{m}V_b[4V_1\cos(\alpha_1)-8V_b]$. So we have $W_D=\dot{m}V_b[4(V_1\cos\alpha_1-2V_b)]$.

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$$\eta_D = \frac{W_D}{\frac{1}{2}\dot{m}V_1^2} \qquad \eta_D = \cos^2\alpha$$

$$\therefore \eta_D = \frac{2V_b \dot{m} \cdot 4[V_1\cos\alpha_1 - 2V_b]}{\dot{m}V_1^2}$$

$$\therefore \eta_D = \frac{2V_b[4(V_1\cos\alpha_1 - 2V_b)]}{V_1^2}$$

$$\rho = \frac{V_b}{V_1}$$

$$\therefore \eta_D = 2\left(\frac{V_b}{V_1}\right)\left[4\left(\frac{V_1\cos\alpha_1}{V_1} - 2\frac{V_b}{V_1}\right)\right]$$

$$\therefore \eta_D = 2\rho[4(\cos\alpha_1 - 2\rho)]$$

$$\therefore \frac{d}{d\rho}(\eta_D) = 0 \longrightarrow 8\cos\alpha_1 - 32\rho = 0$$

$$\rho = \frac{\cos\alpha}{4}$$

Then we can use this for our derivation of efficiency. So let us find out the formula for efficiency and then we know formula for efficiency is W_D efficiency diagram. Efficiency is $\eta_D = \frac{W_D}{\frac{1}{2}\dot{m}V_1^2}$ so we have diagrammed efficiency as $2V_b\dot{m}$ into these 2 will go up so you will have $\eta_D = \frac{2V_b\dot{m}4(V_1\cos\alpha_1 - 2V_b)}{\dot{m}V_1^2}$ and $\dot{m}$and $\dot{m}$will get cancelled.

So we have diagram efficiency as $2V_b$into we basically have $8V_b$we will write down as $2V_b$and then this will help us later on for $\eta_D = \frac{2V_b[4(V_1\cos\alpha_1 - 2V_b)]}{V_1^2}$. Now let us say that $\rho = V_b/V_1$so we have diagram efficiency = 2 we will make it a point here $\frac{V}{V_1}$, $V_1$is given here and other V_1we will keep it for inside. So, $\eta_D = 2\left(\frac{V_b}{V_1}\right)\left[4\left(\frac{V_1\cos(\alpha_1)}{V_1} - 2\frac{V_b}{V_1}\right)\right]$.

So we have diagram efficiency as $\eta_D = 2\rho[4\cos(\alpha_1) - 2\rho]$. Now we know for given α_1 diagram efficiency is just function of ρ and then if we differentiate it with ρ and equate it to 0 we will get a formula as $8\cos of\,\alpha_1 - 32\rho = 0$ and then this leads to the fact that $\rho = \frac{\cos(\alpha)}{4}$ and then this formula, if we put into the efficiency, which is this then we get diagram efficiency as $\cos^2\alpha$.

So we have seen that in earlier case when we were having pressure compounding or only one stage of nozzle and moving blades then we got optimum efficiency or maximum efficiency at optimum ρ at $\frac{\cos(\alpha)}{2}$. Now we are getting it at $\frac{\cos(\alpha)}{4}$ but the maximum efficiency same which is $\cos^2\alpha$. These are our discussion was about impulse turbine. The next set of turbine which is reaction turbine we will see in that next class. Thank you.