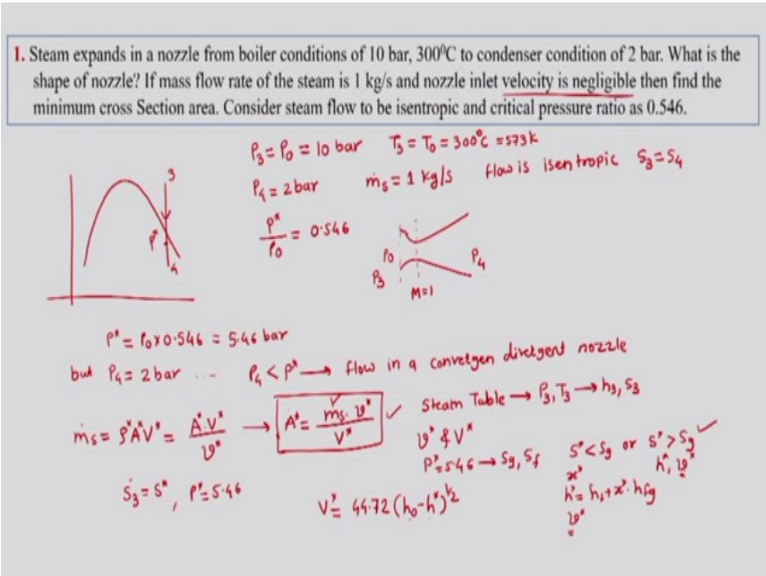
Steam Power Engineering
Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology - Guwahati

Lecture – 22
Nozzle Examples

Welcome to the class, till time we have seen about various aspects of the turbine, one of the parts of the turbine is basically nozzle and today's class is devoted for solving examples about nozzle. So, here we will see how to solve examples which are given for nozzle flow basically, especially for the steam flow through the nozzle.

(Refer Slide Time: 00:57)



So, we have first example which states that steam expands in a nozzle from boiler condition of 10 bar and 300 °C to condenser conditions of 2 bar. What is the shape of nozzle, if mass flow rate of the steam is 1 kg per second and nozzle inlet velocity is negligible, then find out minimum cross section area, consider steam flow to be isentropic and critical pressure ratio as 0.546.

So, here now, the given thing for us, first let us plot T-S diagram, so as we know we will be here 3 to 4, we are in the state of nozzle 3 to 4 as per our conventions of the cycle, so here things given to us are P_3 is equal to total condition since, we are told that velocity is negligible, so $P_3 = P_0 = 10$ bar and $T_3 = T_0 = 300°C$, so 300 + 273 is 573 K, so it is given to us.

Now, we are also told that condenser; P_4 = 2 bar. Now, we are also knowing that $\dot{m}_s$; mass flow rate of steam is 1 kg per second, we are told that flow is isentropic, so we are told that s_3 is equal to s_4, we have also been told that $\frac{P^¿}{P_0}=0.546$, so this is the critical condition. We know that if a nozzle is a CD nozzle then, in case of CD nozzle minimum cross section area will attain Mach number for the expansion from subsonic to supersonic and $P^¿$ is the pressure at the sonic condition.

And then, we will have the flow expanding from P_0 to $P^¿$ star but here flow is expanding from P_3 to P_4 and now, we are going to see if $P^¿$ is less than P_4 , then we will not have expansion in CD nozzle, if $P^¿$ more than P_4, then the expansion is as the convergent divergent nozzle. We have seen that the choking condition leads to a particular pressure in the exit, if pressure is below than that, then the flow is in a CD nozzle.

So, we can find out $P^¿=P_0\times 546$, so since P_0 is 10, it will be 5.46 bar but we are told that P_4 = 2 bar but so since, P_4 is less than $P^¿$, we have flow in a convergent divergent nozzle, since flow has already been choked by the pressure difference and then P_4 is so low, then $P^¿$ such that flow has to be in the convergent divergent nozzle. So, having said this we will move to the next point where it is asked to us to find a minimum cross section.

We know that $\dot{m}_s=\rho AV$, where we will take V as $V^¿$, A as $A^¿$, ρ as $\rho^¿$ but instead of that we can say $\frac{A^¿V^¿}{\upsilon^¿}$, so $A^¿=\frac{\dot{m}_s\upsilon^¿}{V^¿}$. So, here we will do this, we need to find out what is the area for $\dot{m}_s$ which is given to us, $V^¿$ and $\upsilon^¿$ and $V^¿$.

So, we will do one thing, we can do; we can find out what is the entropy; we will go to the steam table and from the steam table, we know P_3 and we know T_3 , we can find out h_3 and also we can find out s_3, so s_3 is known to us. So, knowing the s_3, we can do to way this calculation either we can go to the condition which is corresponding to 4 and find out at what the; what is the state and which we will have this entropy s_3.

But instead of that we can do one more thing, we can go since by that means, we will find out exit area, here we are asked for minimum cross section area which is for the throat and this is the expression for minimum cross section area for the throat, so now our target is to find out $\upsilon^{¿}$and $V^{¿}$specific volume and velocity and the throat.

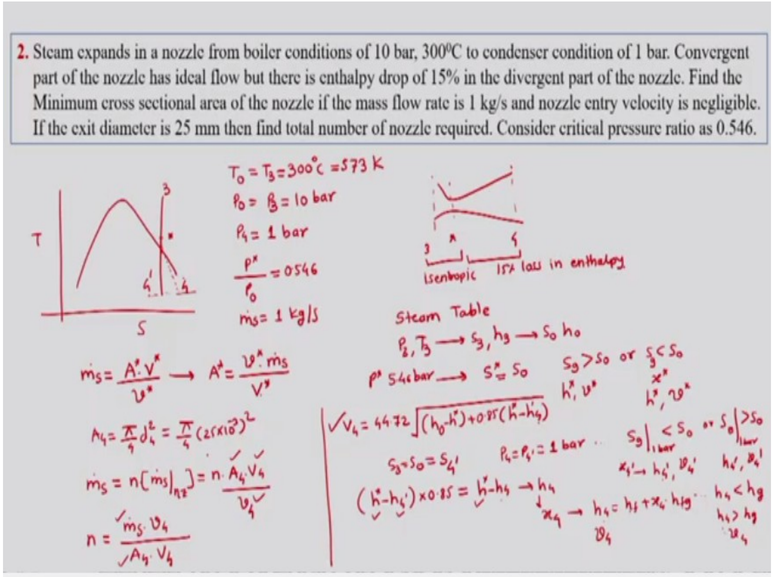
So, let us see how we can find out, now we know the $P^{¿}$; so here let us assume that we are somewhere here in the star, so we know that s star is equal to s_3, so basically we know s_3 is equal to $s^{¿}$, so knowing this and further we know $P^{¿}=¿$ 5.46, so we will go to the steam table and see for $P^{¿}=5.46$, what is the s_g and what is the s_f.

Now, we will see if $s^{¿}<s_g$ or $s^{\cdot}>s_g$, if $s^{¿}<s_g$, then we will have to find out $x^{\cdot}$ otherwise for this condition we can directly get $h^{¿}$, we can directly get $\upsilon^{¿}$, so these two points it is we can directly get otherwise, in the case where $s^{¿}<s_g$, we will first find out $x^{\cdot}$, from $x^{\cdot}$ we know $h^{\cdot}=h_f+x^{\cdot}.h_{fg}$.

And then similarly, we will also find out v start, so by this means of calculation, we have found out $\upsilon^{¿}$. Now, our target is to find out velocity $V^{¿}$, we can find out using the formula, $V^{¿}=44.72\left(h_0-h^{\cdot}\right)^{1/2}$, so we know now $h^{\cdot}$ by any means that we will put over here we know h_0 basically, h_0= h_3 that is also known to us and then we can know $V^{¿}$.

So, knowing the $V^{¿}$and knowing the specific volume, we can put everything in this formula and we can find out what is the minimum cross section in this particular example, $s^{\cdot}$ happens to be more than s_g so we do not have to find out dryness fraction, we just have to find out $h^{\cdot}$ from the steam table and we have to find out specific volume. So, I hope this is the way to solve this example.

(Refer Slide Time: 10:06)



I have elaborated the procedure how to solve the example, so we will solve the next example. Next example says that steam expands in a nozzle from boiler condition of 10 bar 300 degrees Celsius to condenser condition of 1 bar. Convergent part of the nozzle has ideal flow but there is enthalpy drop of 15% in the divergent part of the nozzle, find the minimum cross section area of the nozzle, if mass flow rate is 1 kg per second.

And nozzle entry velocity is negligible, if diameter; exit diameter is 25 meter mm, then find out total number of nozzles required, consider critical pressure ratio as 0.546. So, here as well we will first find out the T-S diagram, here as well we will have 3 and 4, so given conditions are $T_0 = T_3 = 300^o C = 573\,K$ And $P_0 = P_3 = 10\bar{¿}$.

And then, we are also told that $P_4 = 1\bar{¿}$, we have been given that $\frac{P^{\cdot}}{P_0} = 0.546$, so we are explicitly told that we have a CD nozzle, so we will say somewhere we have star, so this is star, this is 3 and this is 4. So in the CD nozzle we have this part as isentropic and this part has 15% loss in enthalpy. So, we are supposed to find out minimum cross section for $\dot{m}_s = 1\,kg/s$.

Let us first find out minimum cross section again, I will write down the formula $\dot{m}_s = A^{\cdot} \times V^{\cdot} / \upsilon^{¿}$, so from this formula again, $A^{\cdot} = \upsilon^{¿} \times \dot{m}_s / V^{\cdot}$ so, given thing to us is $\dot{m}_s$, so we have to find out the specific volume and velocity at the sonic state but it is very simple we should follow the same procedure what we did follow in the last example, we will go into the steam table.

And in the steam table, we will put P_3 and T_3 state for the P_3 , T_3 state, I will find out $s_3$and h_3, which is basically s_0 and h_0, this thing we will find out. Now, we are told that P_0; $P^{\cdot}/P_0=0.546$, this leads to $P^{\cdot}=5.46¿$, so I will go to 5.46 bar which is $P^{\cdot}$ and for that I will see where $s^{\cdot}=s$. Now, I will again see if $s_g>s_0 \vee s_g<s_0$.

Accordingly, I will put star is above the dome, inside the dome or outside the dome, so if $s_g>s_0$, then we will find out $x^{\cdot}$ and if this is not there, then we do not have to worry, knowing the $x^{\cdot}$, we can find out $h^{\cdot}$ and we can also find out specific volume. Otherwise if s is; if $s_g>s_0$, then we do not have to bother, if $s_g<s_0$., then we have to find out $x^{\cdot}$.

And knowing the $x^{\cdot}$, we will find out $h^{\cdot}$and specific volume but if $s_g>s_0$, we will directly go to superheated part of the steam table and there we will find out $h^{\cdot}$and specific volume. Knowing these specific volume and $h^{\cdot}$, we can find out $V^{¿}=44.72\left(h_0-h^{\cdot}\right)^{1/2}$, so $V^{¿}=44.72\left(h_3-h^{\cdot}\right)^{1/2}$, So, this is what we can do and then we can find out what is the velocity at the star state.

So, knowing this condition we can put that velocity here, we can put that specific volume over there and then we can put mass flow rate and find out $A^{\cdot}$, where a $A^{\cdot}$ is minimum cross section, so this is how the procedure to find out minimum cross section is. Now, we are told that what is the number of; what are the number of nozzles if we have exit diameter as 25 mm.

So, we know rather A exit; so $A_4=\pi/4$, we will rub this, so for this now, we have to find out mass flow rate through one nozzle and then we will divide this for the total mass flow rate and then we can find out number of nozzles. So, first thing is A_4; $A_4=\frac{\pi}{4}d_4^2$ and d_4 is given as 25 mm, so we can find out A_4. Once A_4 is found out again, $\dot{m}_s=n\times\dot{m}_{snz}$.So this is equal to

$\frac{n \times A_4 \times V_4}{\upsilon_4}$ so $n = \frac{\dot{m}_s \times \upsilon_4}{A_4 \times V_4}$ Now, here we will be in trouble since $\dot{m}_s$ is known to us, A_4 is known to us but we do not know again V_4 and υ_4, so that we need to find out.

But for that, we can see that how to find out, first V_4; I can think how to find out V_4, the formula is $44.72 \times ¿¿$, since we are told that after the nozzle there is loss and we will put it here, so isentropic point is $4'$, real point is 4.

So, we have to first find out what is the isentropic 4 point, same procedure we will follow, we can do one thing this 0.85 is the loss of 15% accounting this, this is the enthalpy drop in the divergent portion, this is the enthalpy drop in the divergent portion, this is the enthalpy drop in the convergent portion, so this summation is the total enthalpy drop in the nozzle and this is the velocity.

Now, we can find out h_4', for that we will again see $s_3 = s_0 = s_4'$, so we will go to the steam table and then and the steam table will go to the $P_4 = P_4' = 1\bar{¿}$, we will go in that and see where we can get $s_4' = s_3$, again we will have few things s_g at 1 bar can be less than s_4'can be less than s_0 or s_f or s_g at 1 bar can be no more than s_0.

Now, if s_g is less than s_0, then we can we need to find out x_4' and if x_4' is known to us, I can find out $h_4'$and V_4' but if s_g is greater than s_0, then we can directly get from the superheated past of the steam table, h_4' and V_4' that is how we can find out, so these are the specific volumes and enthalpy, so this is how I know now h_4', I will put that as h_4'here.

And then, I know what is the velocity at the exit of the nozzle once I know velocity, I can put that velocity over here, I knew area, I should know now V_4, so there is a way what we can find out V_4. We know now that there is 15% of loss, we know now there is 15% loss in the diverging section. So, for that we can do this way$(h¿¿{}^{*} - h_4') \times 0.85 = h^{*} - h_4¿$.

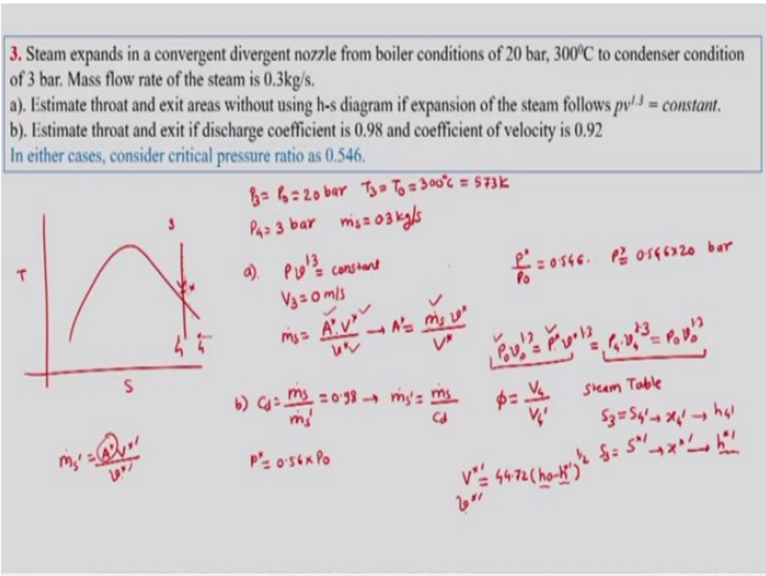
Here from this earlier method, we have found out h_4', we have found out h^{*}, we know h^{*}then we can find out h_4. So, once h_4 is known, we can find out x_4, since we know that

$h_4 = h_f + x_4 h_{fg}$ is $h_4 < h_g$, otherwise this is x_4 is 1, then we do not have to bother, then in that case if $h_4 > h_g$, then directly we can get the specific volume υ_4 from the superheated part of the steam table which is not the present case.

This is a wet steam, so we need to find out x_4, so $h_4 = h_f + x_4 h_{fg}$, now we know h_4, we know h_f, we know h_{fg}, corresponding to 1 bar and then we can find out x_4 and then using the specific volume for the liquid and fluid at the saturation state, we can find out V_4. So, now we know V_4 and then this is how we can find out the total mass flow rate of the steam is given and then we can find out total number of nozzles.

Here, we should remember that this number will always be an integer, this cannot be a float, so for that whatever be the nearest integer, we have to consider that as an answer, this is how we have to solve the example which is given to us in this form which will account the losses.

(Refer Slide Time: 23:42)



So, now let us see the next example; next example states that steam expands in a convergent divergent nozzle from boiler conditions of 20 bar $300°C$ to the steam condenser which is at 3 bar, mass flow rate is 0.3 kg per second. Estimate throat and exit areas without using h-s diagram, if expansion of the steam follows $pv^{1.3} = constant$.

Also, estimate throat and exit areas, if the discharge coefficient is 0.98 and the velocity coefficient is 0.92, in either cases consider critical pressure ratio as 0.546. Again, we will plot the T-s diagram and as per the T-s diagram, we are in 3 to 4 where we are given $P_3 = P_0 = 20$

bar, $T_3 = T_0 = 300°C$ = 573 Kelvin. We are told that $P_4 = 3¿$ $\dot{m}_s$steam flow rate is 0.3 kg per second.

Let us consider part a; here in the part a, it is told that $pv^{1.3} = constant$, we are told that $pv^{1.3} = constant$ and here we are specifically told that we cannot see the steam table, so there are ways by which we can solve the example. We can here as well; it is not told but still we can consider V_3, velocity at 3 is equal to 0 meter per second, so expansion is told to have an isentropic index of 1.3.

So, in this case we know that basically, change in kinetic energy which is $\frac{V_4^2}{2} - \frac{V_3^2}{2} = \frac{K}{K-1}(P_3 \upsilon_3 - P_4 \upsilon_4)$, so this is what we can use but before that we are solve it for a and then for that we will first solve for the throat, so we will; we knew that $\frac{P^*}{P_0} = 0.546$, so for us $P^* = 0.546 \times 20¿$, this is P^*value for us. Now, our objective is to find out area; minimum area?

And so for that minimum area again, we know the formula; mass flow rate is given to us, $\dot{m}_s = A^* \times V^* / \upsilon^¿$, so $A^* = \dot{m}_s \times \upsilon^¿ / V^*$, so here A^* is to be found out, mass flow rate of the steam is given and we need to found out, find out specific volume and velocity in the throat basically, we had seen till time that we can use the formula directly of velocity is equal to 44.72 into square root of enthalpy drop.

But for that we have to find out enthalpy at the star condition suppose, star is here, for that we have to use this pressure go to the steam table find out compare this is what we can do but here we cannot do that, since it is told that we cannot see the h-s diagram or steam table, h-s diagram; we cannot see the h-s diagram, so mollier chart, we cannot go to the mollier chart, otherwise it is very simple.

So, before that what we can do is without solving, without taking the enthalpy directly from mollier chart, we can find out the velocity using this formula, where we can use the formula for velocity as this V_3 sorry; $V^{\cdot 2}$ here, we can find out $\frac{V^{\cdot 2}}{2}=\frac{K}{K-1}(P_0\upsilon_0-P^{\cdot}\upsilon^{\cdot})$. We are told that $P_0\upsilon_0^{1.3}=P^{\cdot}\upsilon^{\cdot 1.3}=P_4\upsilon_4^{1.3}$.

So, using first relation we know now, $\frac{P^{\cdot}}{P_0}=0.546$, so we know $P^{\cdot}$ we know P_0 and then from the steam table, we can find out υ_0 corresponding to 20 bar and 300 °C, so this is what we can find out from the steam table. So, knowing this we can knowing this P_0, knowing this υ_0, we can and $P^{\cdot}$we can find out, $\upsilon^{\cdot}$, so specific volume as the sonic state is found out.

So, $P^{\cdot}$ is known, υ_0; $\upsilon^{\cdot}$ is known, P_0 is known, υ_0 is known, K is equal to 1.3, we found out $V^{\cdot}$, so this formula is basically, telling that the change in kinetic energy is leading to the change in specific volume due to the expansion as pv raised to 1.3 is equal to constant. So, knowing this we can put the specific volume and $\upsilon^{\cdot}$ and $A^{\cdot}$can be found out for a given mass flow rate.

So, this is how we can find out the throats area, similar procedure we can execute for the finding out exit area, where now $\dot{m}_s=\frac{A_4V_4}{\upsilon_4}$, so by this formula where we have $P_0\upsilon_0^{1.3}$. Using this, we will find out υ_4, so υ_4 is known, A_4 is to be found out, so $A_4=\frac{\dot{m}_s\upsilon_4}{V_4}$, so these 2 things are known, so we have to find out V_4.

Same procedure we can follow; $\frac{V_4^2}{2}=\frac{K}{K-1}(P_0\upsilon_0-P_4\upsilon_4)$, υ is specific volume, so knowing these things we would find out what is the velocity at the exit and then we can put it in and then find out what is the area, so this is what the procedure we would have adopted for finding out the or finding out the exit and the areas at the throat. Now, in the part b; so we will rub this and then in the part b, we are not told that there is $p\upsilon^{1.3}=constant$.

But we are told something else where we are told that there is a discharge coefficient of 0.98, so there is a change in mass flow rate, so for that part b; for part b now, we have discharge coefficient $C_d = \frac{\dot{m}_s}{\dot{m}_{s'}} = 0.98$, then we can find out $m_{s'} = \frac{\dot{m}_s}{C_d}$, actual $\dot{m}_s$ is known, C_d is known, we can find out.

Further this is the condition, this is the $4'$ and this is 4, then we are told that there is a discharge coefficient, so for discharge coefficient sorry; this is $4'$ and this is 4, so for discharge coefficient ϕ as what we know, it is $\frac{V_4}{V_4'}$, so these are the facts and now, we are told to find out throat area suppose to start with. So, again $\dot{m}_{s'} = \frac{A^{\cdot} V^{\cdot}}{\upsilon^{\cdot}}$

So, this is the formula which we know, so knowing this we can what are the things which are known to us, those things we will put, so we know that P in either case $P^{\cdot} = 0.546 \times P_0$ and this is known to us so, we know what is the value of $P^{\cdot}$, then we can find out the rest of the things. So, here we are told with velocity coefficient $\frac{V_4}{V'}$ and $P^{\cdot} = 0.546 \times P_0$

Now, we know that we can go to this steam table and then from the steam table, we know we will say that in the steam table, we know $s_3$and then $s_3 = s_4'$. s_4' is known to us, knowing this s_4' , we can if required we can find out x_4'. Similarly, we can do it for the throat, so we can know $s^{\cdot}$ basically, $s_3 = s^{\cdot'}$.

And for that we will find out $x^{\cdot'}$, so we will know the dryness fraction at the throat and also the dryness fraction at exit in the ideal condition and knowing this, we can find out h_4', and we can find out $h^{\cdot'}$. Then knowing this, we can find out the formula which is $V^{\cdot'} = 44.72 \left(h_0 - h^{\cdot'} \right)^{\frac{1}{2}}$, so this $h^{\cdot'}$is known to us, we will put here h_0 is known to us we will put here.

And then, we know now $V^{\cdot'}$, so this $V^{\cdot'}$is known to us, so once $V^{\cdot'}$is known to us, we will use the formula for the mass flow rate and then this $V^{\cdot'}$and similarly, using this we can also find out specific volume, so specific volume dash is also known to us, so velocity is known to us, this specific volume is known to us, we know ideal mass flow rate knowing the discharge coefficient, we can find out area at throat.

If we execute same procedure instead of star, we will have at 4 then, we can know exit area also, okay. So, there are 2 ways however, we can make use here itself of ϕ and then knowing the velocity as dash we can find out the velocity in reality, in reality velocity is known to us, then we can make use of that velocity to find out enthalpy at that state knowing that enthalpy, we can find out the specific volume.

And then, we can use that everything to find out the area in reality where we would have actual the actual mass flow rate in that condition. So, this is how we will solve the example which is given for us for the nozzle, we will see about rest of the things in the next class, thank you.