Steam Power Engineering
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Lecture - 21
Nozzle Flow

Welcome to the class. Last time, we have seen that the isentropic relations exist for the flows with inviscid and study case and then those isentropic relations for mass, momentum and energy equations we had seen. Using those relations, we had proved the speed of sound is equal $\sqrt{\gamma RT}$ for the gases and then we had also seen it for the other cases. Now, in today's class, we are going to study about the nozzle flows.

These studies whatever we have underwent in the last class and today's class will be helpful for the studies or for the analysis of the turbine so nozzle flow.

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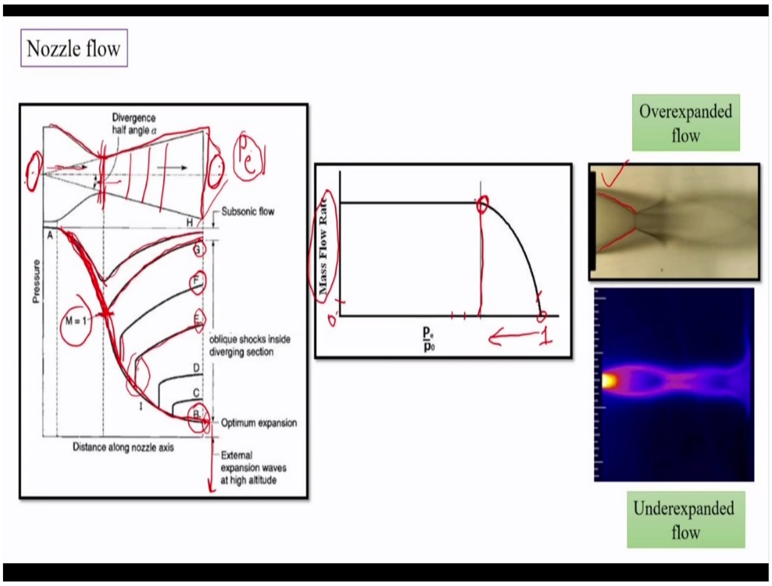

We know that let us consider that there is nozzle which is of convergent-divergent section where as per the last class, our information if velocity is 0, then such conditions are called as total conditions. So, let the nozzle inlet has total conditions. This is the nozzle inlet and this is the nozzle outlet and this is our nozzle okay.

So, having known this as nozzle and this is inlet and this is outlet condition, we are saying that now to start with if inlet and outlet pressure is same, then there will not be any flow taking place through the nozzle, but this nozzle exit pressure denoted by P_e is supposed decreased, then flow will start taking place in the nozzle. In our last class, we had seen that if the duct is convergent and flow is subsonic, then it will act as nozzle.

If duct is divergent and flow is supersonic, then it will act as nozzle. So, initially there will be subsonic flow and convergent section acts as a nozzle but it will not reach a critical condition to the minimum dimension which we call it as throat. So, it will still remain subsonic over here and then that is why pressure will initially decrease and then this divergent portion will act as a diffuser, so pressure will start to increase and it will try to reach to the exit pressure.

Further, in the next experiment, if we decrease this P_e to some value, which is corresponding to G, then we will have further deceleration of the flow and we will still follow the subsonic path in both the nozzles. So, this thing will continue till we will have a critical pressure in the exit where we will have the convergent portion acting as a nozzle such that the Mach number at this minimum cross-section will become 1.

And then if we decrease the pressure below slightly to that critical pressure, then we will have this convergent portion acting still as nozzle and no property will change in this portion and then we will have certain part of the divergent section will continue to act as nozzle but if it continues to act as nozzle, then we will reach a state which will be B but the exit pressure for us is corresponding to F, so it is not advisable for the flow that it should continue to get expanded in the nozzle.

That is why intermediate shock will appear and then that shock will give rise to the pressure of the flow and it will make the flow to be in subsonic range and since the flow is subsonic and the section is divergent, it acts as diffuser such that pressure will rise and reach the state F. If we still decrease the exit pressure to E, then the shock will appear little late but till that portion the convergent. and divergent part of divergent nozzle would continue to act as nozzle, then there will be shock and then flow will be subsonic and then it will try continuing as diffuser. This process continues till a point where we will have exit pressure matching with the isentropic expansion case and then in that case flow will be continuously expanded in the nozzle and we will have a nice expansion of the flow in the nozzle.

And then we will have flow expanding on decreasing its pressure from state A which is a stagnation condition to state B. So, this is how flow would take place in a nozzle and in this state, as we were doing the experiment where nozzle exit pressure was decreased. So, in this direction, we are decreasing the nozzle exit pressure. So, as in initial case, we had nozzle exit pressure and the stagnation pressure at the inlet are same.

So, state is 1, so at the state 1, there is zero mass flow rate going through the nozzle but if we decrease the exit pressure slightly, then that means we have increased the mass flow rate of the nozzle and then as we continuously decreasing the exit pressure, then we will continuously increasing the mass flow rate, but this increment will take place till only a critical value when we have minimum cross-section reaching Mach 1.

And then once minimum cross-section is reached with Mach 1 and we have the subsonic part acting as nozzle, then in that case there onwards further any decrement in exit pressure will not change the pressure in the, will not change the mass flow rate to the nozzle. In that case, we are having in some of these cases we have the shock in the divergent portion but this is not going to alter the mass flow rate.

So, mass flow rate is frozen for the flow. So, this mass flow rate which is fixed at a state where Mach 1 is reached into this minimum cross-section is called as choked mass flow rate for the nozzle. So, if we have choked mass flow rate, then there are 2 types of conditions, which are possible and one condition is called as over expanded flow and other condition is called as under expanded flow.

These conditions correspond to the fact that in case of over expanded flow, we are seen over here where we are having shock in the nozzle. If there is shock in the nozzle, shock in the nozzle means flow is expanded but if flow would continuously expand till the exit, it will go to a pressure very lower than the exit pressure which is not expected. So, this is called as over expanded condition.

So, for that nature will provide a shock in the nozzle or may be outside the nozzle such that the shock would give rise to the pressure and such rise in pressure would match with the exit pressure, such condition is called as over expanded where we can see that there is a shock in the exit of the nozzle or there is shock inside the nozzle. Further, if we have pressure in the exit of the nozzle, which is below the expected exit pressure for the isentropic expansion.

Then, flow will continuously expand in the nozzle and will reach a pressure which is PB but exit pressure is lower than PB such case is called as under expanded case which are the precious lower than this and in this case we will have an expansion plan appearing in the nozzle exit and then it would lead to expansion, continuous expansion outside the nozzle as well till the flow matches its pressure with the ambient pressure. So, these are the some details of flow through the nozzle.

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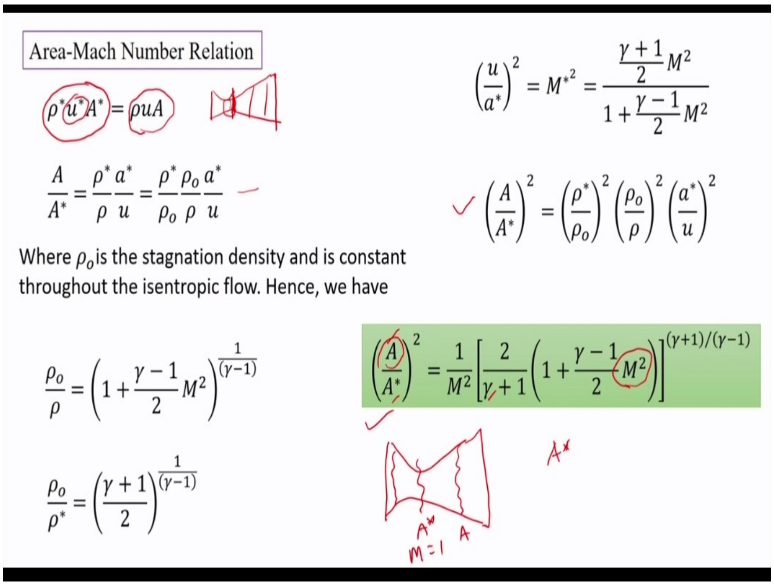

Now, we have seen that there is a choking condition and for that we have to use this understanding of choking condition to find out mass flow rate but before that we will try to find out what is the area Mach number relation. We know that $\rho^{\cdot} u^{\cdot} A^{\cdot} = \rho u A$. Here, we means star by the velocity area and density corresponding to Mach 1 case. In our earlier case, we had a C-D nozzle and then we will assume that now our nozzle is choked or our nozzle has a condition which is Mach 1 in the throat.

So, density, velocity and area in the throat which gives Mach 1 for the nozzle gives its mass flow rate through this section and this mass flow rate is equal to mass flow rate in any section, so we are considering this ρ, u, A in any section and this $\rho^{\cdot}, u^{\cdot}, A^{\cdot}$ is mass flow rate only through the sonic section. Then $\frac{A}{A^{¿}}=\frac{\rho^{¿}}{\rho}\frac{a^{¿}}{u}$ So, this is sonic velocity since we have Mach number 1 over here.

So, this $u^{¿}$ is basically $A^{\cdot}$. So, this $\frac{\rho^{¿}}{\rho}$ can be replaced as $\frac{\rho^{¿}}{\rho_0}\frac{\rho_0}{\rho}\frac{a^{\cdot}}{u}$. Here, $\rho^{¿}$ is the stagnation density at the inlet and it is constant throughout the isentropic flow. Hence, we have $\frac{\rho_0}{\rho}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}$ We have derived this relation in our last class. Further, if we keep Mach number is equal to 1, then we can as well obtain the relation which is $\frac{\rho_0}{\rho^{¿}}=\left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}$

Further, we will take one relation known from the gas dynamics that $\frac{u}{a^{¿}}$ which is velocity at any point divided by corresponding velocity at Mach 1 state is called as $M^{¿}$ and that $M^{¿}$ is related with M. M is Mach number which is velocity divided by local speed of sound but that local speed of sound is a. Here $a^{¿}$ is the speed of sound when Mach number is 1.

Then, putting these in this relation but for this equation if we make a square of it, then we will get this relation and in this will keep all the equations whatever we have obtained then we will get relation which is standard relation which is called as $A/A^{\cdot}$ which is area and Mach number relation. Here, area Mach number relation says us that if we know area at a particular location.

And if we know $A^{\cdot}$ which is a location at which Mach number is 1, then that area ratio if it is known to us for a given fluid, then we know what is the Mach number and the area of cross-section which is A. So, if $A^{\cdot}$ is known, $A^{\cdot}$ is if this is a nozzle, which is a convergent-divergent nozzle and then in this nozzle this is called as $A^{\cdot}$ which is minimum area at which Mach number is 1.

Then, if this area is known to us, then we can find out Mach number in any area, suppose that this is A. This $\frac{A}{A^{\cdot}}$ is known from left side since we know these areas, we know which gas is flowing, so we know γ . So, we can iteratively work out and find out what is Mach number M in this cross-section. Similarly, we can as well find out Mach number M any other cross-section. So, this is called as area-Mach number relation.

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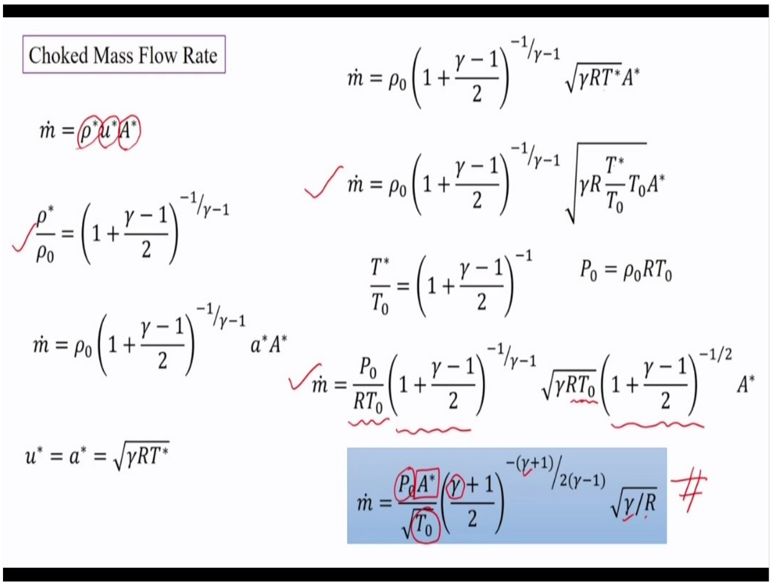

Then, we will use this to find out choked mass flow rate. So, mass flow rate at any section could have been taken by us but we will take a particular cross-section since we are interested in choking phenomena and as we have seen choking is said to be happened when Mach number reaches 1 at the minimum cross-section.

Since Mach number has reached 1, mass flow rate formula which$\rho u A$ would get replaced by or can be particularly written for the minimum cross-section area where $A^{\cdot}$ is throat area, $u^{¿}$ is corresponding velocity and $\rho^{¿}$is corresponding density but we know$\frac{\rho^{¿}}{\rho_0}=\left(1+\frac{\gamma-1}{2}\right)^{\frac{-1}{\gamma-1}}$

Further, this rho star will be replaced using this formula and we can get $\dot{m}=\rho_0\left(1+\frac{\gamma-1}{2}\right)^{\frac{-1}{\gamma-1}} a^{¿} A^{¿}$ but $a^{\cdot}=\sqrt{\gamma} R T^{¿}$ and then this will be replaced and our mass flow rate formula will be updated as shown in the screen. So, we will alter this $T^{¿}$ by saying it as equal to $\frac{T^{\cdot}}{T_0} T_0$ and then we know relation of $\frac{T^{¿}}{T_0}=\left(1+\frac{\gamma-1}{2}\right)^{-1}$.

This relation is obtained for a very particular case where Mach number is equal to 1 and further we will say $P_0 = \rho_0 R T_0$. Keeping all these relations in this mass flow rate formula, we will get a big relation for mass flow rate as shown in the screen. This can be simplified since this term and this term is same. We have T_0 over here and R over here; we also have R and T_0 over here.

So, thus the mass flow rate formula in a concise form will be$m = \frac{P_0 A^{¿}}{\sqrt{T_0}} \left(\frac{\gamma+1}{2} \right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \sqrt{\frac{\gamma}{R}}$. It is evident from this formula that mass flow rate depends upon gamma and total conditions for a given $A^{\cdot}$ and also on a particular gas. So, if gas is changed, then choked mass low rate will change since changing gas will change gamma and also it will change R specific gas constant which is $C_p - C_v = R$.

Further it is also dependent upon P_0 and T_0 and it also depends upon $A^{¿}$. Major take away from this formula is that once mass flow rate is choked, we whatsoever alter the pressure at the exit of the nozzle is not going to change the mass flow rate since in this formula there is no term which corresponds to the exit condition okay.

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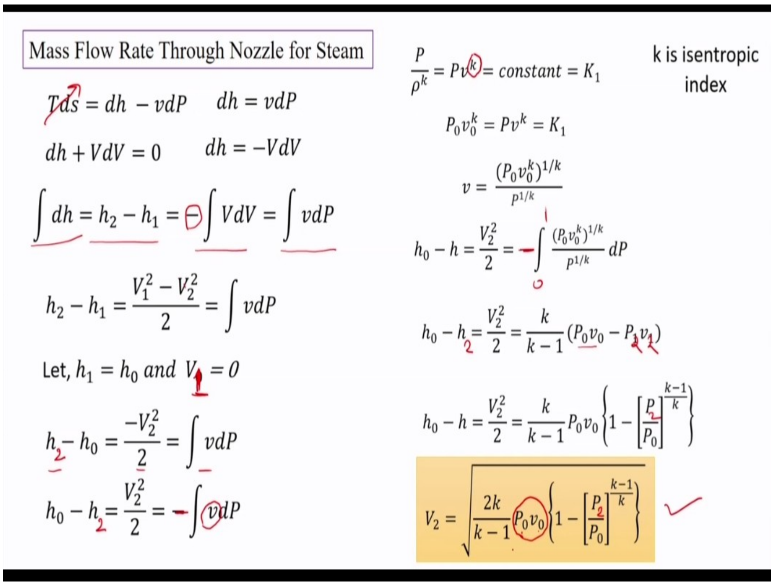

Now, this formula we could as well use for the steam but it is difficult for us to find out what is the gamma and R for steam and since it is a two phase flow situation becomes more complicated, but this mass flow rate formula which is very particular for gas is going to give us an idea about the nozzle flow and how nozzle will be choked and what will happen if nozzle is over expanded and nozzle is under expanded.

This understanding we can carry forward for steam, although these conditions will be different at different means this over and under expanded conditions for steam will be at different pressures and temperature as that of air but philosophically the flow pattern would be similar. So, in this case, we will try to find out mass flow rate through nozzle for steam. We know from the first law and second law combined thermodynamics expression $Tds = dh - vdP$, so $dh = vdP$ since we are considering the nozzle flow to be isentropic.

So, Tds will be 0, so $dh = vdP$but from the energy equation we also know that $dh = VdV$ since we have derived an expression which says that $h + v^2/2 = constant$ and the differential form of that expression was $dh + VdV = 0$, so knowing this we can have other expression for change in enthalpy which is $-VdV$. So, this gives us $dh = -VdV = vdP$ and now if we integrate all the 3 sides, we get $\int dh = -\int VdV = \int vdP$

But this is $\int dh = h_2 - h_1$which is enthalpy at the exit minus enthalpy at the inlet and this VdV integration will be $\frac{V_1^2 - V_2^2}{2}$since it has minus sign before it and this V_1 is the velocity at the inlet and V_2 is the velocity at the outlet but if we say that at the inlet we have stagnation condition, so $h_1 = h_0$ and since at the inlet condition is stagnation we have $V_1 = 0$.

So the same expression can be rewritten as $h - h_0 = \frac{-V_2^2}{2} = \int vdP$okay and this is $h_2 - h_0 = \frac{-V_2^2}{2} = \int vdP$ integral from inlet 1 to outlet 2. Further, we will also write down this expression in positive line that $h_0 - h_2 = \frac{V_2^2}{2} = -\int vdP$ but we know $Pv^\gamma = constanta$ or $\frac{P}{\rho^\gamma} = constant$ and let that constant be k_1 which is called as isentropic.

Here this k is an isentropic index for steam, so we have $P_0 v_0^k = P v^k = k_1$, so we have v over here and this v can be replaced by $\frac{\left(P_0 v_0^k\right)^{\frac{1}{k}}}{P^{\frac{1}{k}}}$. So, this can be put over here and then we have $h_0 - h = \frac{V_2^2}{2} = \int \frac{P_0 v_0^k}{P^{\frac{1}{k}}} dP$. That is why we will have an expression which says that we will have $h_0 - h = \frac{V_2^2}{2}$ is equal to okay sorry.

We have this expression written as negative $h_2 - h_0$ was said as $h_0 - h_2$, $\frac{-V_2^2}{2}$ said as V_2, so we multiplied all the 3 sides by minus 1, so we have minus here and so we also have minus over here. So, now we are integrating this expression from state 0 to state 1 or practically state 1 to state 2, so we have $h_0 - h_2 = \frac{V_2^2}{2} = \frac{k}{k-1}\left(P_0 v_0 - P_2 v_2\right)$.

So, we have $h_0 - h = \frac{V_2^2}{2} = \frac{k}{k-1} P_0 v_0 \left\{1 - \left[\frac{P_2}{P_0}\right]^{\frac{k-1}{k}}\right\}$. Since we have taken P_0, v_0 common from this bracket, so we have 1 minus this. So, this gives us the velocity at the exit of the nozzle. So, now we can know what is the velocity at the exit of the nozzle since this formula will be used to find out velocity at the exit of the nozzle.

So, we should know what is the condition for the boiler, for our steam power plant since boiler corresponds to maximum pressure and maximum temperature and it is the steam turbine inlet and the steam turbine inlet means practically for us a nozzle inlet. So, for the nozzle inlet, we have stagnation condition. So, if we know stagnation condition and if we know nozzle exit pressure which is P_2, then we can find out velocity at the exit of the nozzle.

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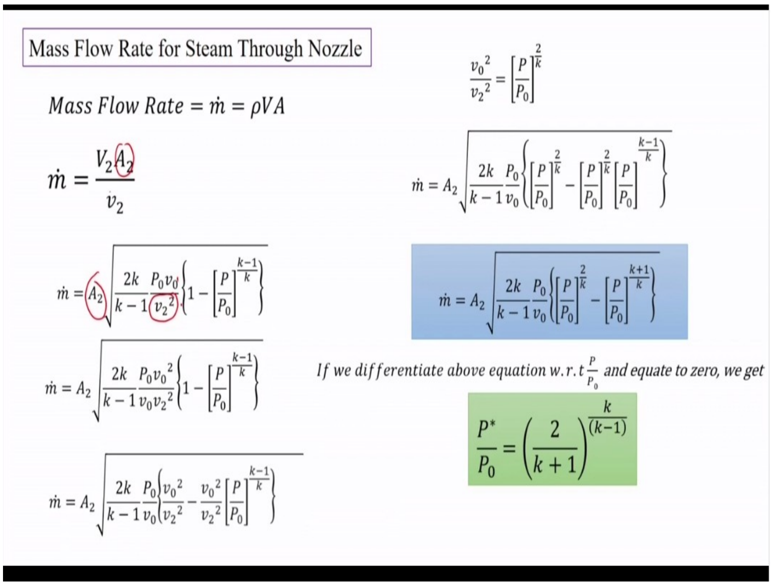

So, the mass flow rate formula we will continue $\dot{m}=\rho AV$. So, $\dot{m}=\rho_2 V_2 A_2$ So, we have just now derived a formula for V_2 and we can say it is equal to A_2 and this A_2 appears over here, we wrote V_2 as what we wrote in the last slide but we also have V_2 and this V_2 is taken inside the square root by $V_2^2$and this gives us with readjustment we are multiplying numerator and denominator over here by V_0.

So, it is $\frac{V_0^2}{V_0}$and then we are also saying 1 as $\frac{V_0^2}{V_2^2}$ and $\frac{V_0^2}{V_2^2}$, so we are multiplying it since $\frac{V_0^2}{V_2^2}=\left[\frac{P}{P_0}\right]^{\frac{2}{k}}$ so we can use this. By this formula, we say that

$$\dot{m}=A_2\sqrt{\frac{2k}{k-1}\frac{P_0}{v_0}\left\{\left[\frac{P}{P_0}\right]^{\frac{2}{k}}-\left[\frac{P}{P_0}\right]^{\frac{2}{k}}\left[\frac{P}{P_0}\right]^{\frac{k-1}{k}}\right\}}$$

This leads to the formula

$$\dot{m}=A_2\sqrt{\frac{2k}{k-1}\frac{P_0}{v_0}\left\{\left[\frac{P}{P_0}\right]^{\frac{2}{k}}-\left[\frac{P}{P_0}\right]^{\frac{k+1}{k}}\right\}}$$

If we differentiate this expression by $\frac{P}{P_0}$, we can equate it to 0, then we will get a state which is the choked mass flow rate condition which corresponds $\frac{P^{¿}}{P_0}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$

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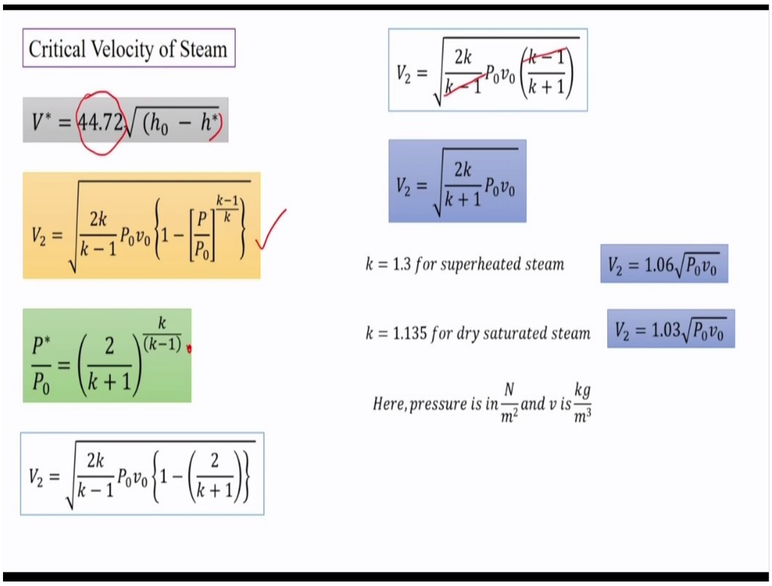

So, we can find out critical velocity in the nozzle which corresponds to be star that means when the steam has reached Mach number 1 and that corresponds to $44.72\sqrt{h_0 - h^{¿}}$ Basically, we had seen that $dh = VdV$ so kinetic energy change is equal to change in enthalpy. So, change in velocity is equal to square root of change in enthalpy, but here as the steam table gives enthalpies in kilojoule per kg.

So, enthalpy in kilojoule will be multiplied by 1000, make it in joule so this 1000 into kinetic energy will have $\frac{V^2}{2}$ So, that 2 will be multiplied, so square root of 2000 is 44.72 assumption or here is inlet velocity to the nozzle is negligible. Under that fact, we can find out the critical velocity or the velocity which gives us Mach number 1 in the nozzle, that velocity can be find out using this formula.

So, otherwise also we can find out we can use the formula for V_2 which is equal to $\sqrt{\frac{2k}{k-1} P_0 v_0 \left\{1 - \left[\frac{P}{P_0}\right]^{\frac{k-1}{k}}\right\}}$ but when the velocity is to be found out for the critical case, we mean that P is basically $P^{¿}$ and so $\frac{P^{¿}}{P_0}$ relation is known to us which $\frac{P^{¿}}{P_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$.

So, putting this in this equation, putting this formula in this equation, we can find out $V_2 = \sqrt{\frac{2k}{k-1} P_0 v_0 \left\{1 - \left(\frac{2}{k+1}\right)\right\}}$. So, V_2 which is a critical velocity is $V_2 = \sqrt{\frac{2k}{k-1} P_0 v_0 \left(\frac{k-1}{k+1}\right)}$

Further this k – 1 gets cancelled, so we have $2V_2 = \sqrt{\frac{2k}{k+1} P_0 v_0}$ is the critical velocity formula for the steam corresponding to the Mach 1 condition.

Now, from the experiments people have found out that k which is an isentropic index for superheated steam is around 1.3. So, this formula will become $V_2 = 1.06\sqrt{P_0 v_0}$ and for dry saturated steam the k value is 1.135. So, for this the formula becomes $V_2 = 1.03\sqrt{P_0 v_0}$. Here, P_0 is in Newton per meter square and specific volume is in kg per meter cube. So, these formulas can be used to find out critical velocity in the nozzle.

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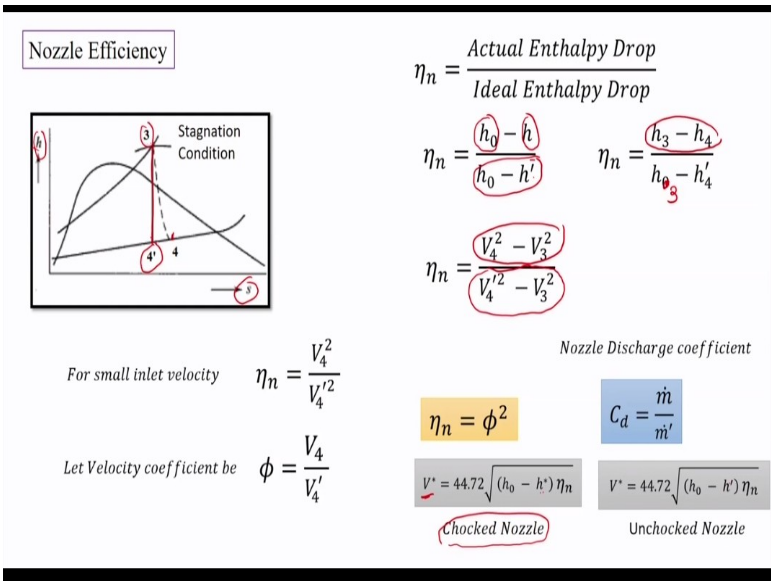

Now, we are going to see what is the term which is called as nozzle efficiency. Here, actually we expect if there is a nozzle, then it will have certain inlet and certain outlet and nozzle flow is said to be isentropic. So, on h-s diagram where h y-axis, s is x-axis we expect the line to be vertical. Let us say that state is 3 at the inlet and state is 4 dash at the outlet and this vertical line is isentropic. So, this is the ideal expansion of the flow in the nozzle.

But in reality we get point 4 at the outlet where we deviate from the reality. So, nozzle efficiency is said to be the actual enthalpy drop divided by ideal enthalpy drop. So, actual enthalpy drop is h_0 which is the stagnation enthalpy at the entry to the nozzle minus h which is the enthalpy at the outlet of the nozzle divided by $h_0-h^{'}$ where $h^{'}$ is ideal enthalpy at the exit of the nozzle.

So, this formula in the current context as 3, 4 dash and 4 becomes $\frac{h_3-h_4}{h_0-h_4^{'}}$ where $h_0=h_3$, so $h_3-h_4^{'}$ we can as well write down this formula in terms of velocity since we know change in enthalpy is equal to change in kinetic energy for the flow. So, $V_4^2-V_3^2$ is change in kinetic energy of the flow in reality divided by $V_4^{'2}-V_3^{'2}$ which is change in kinetic energy in ideal sense.

For small velocity at the inlet means V_3 is to be almost 0. We can write down this formula as $\eta_n=\frac{V_4^2}{V_4^{'}2}$. So, let the velocity coefficient be ϕ which is $\phi=\frac{V_4}{V_4^{'}}$. Then, in that case, nozzle efficiency (η_n) is said to be $\phi^2.\eta_n=\phi^2$. We have to know one new term which is called as velocity coefficient and that term is said to be ϕ.

Now, further we are adding one more terminology which is nozzle discharge coefficient which is equal to basically is coefficient of discharge which says that actual mass flow rate through the nozzle divided by ideal mass flow rate through the nozzle. Here, if we are having nozzle which is non-ideal, then for non-ideal nozzle if I want to find out choked critical velocity, then the velocity formula which was just this earlier for ideal case would have nozzle efficiency multiplied since now these enthalpies were ideal enthalpies.

So, the reality will have the velocity at the exit of the nozzle or critical velocity at the throat is $V^{¿}=44.72\sqrt{\left(h_0-h^{¿}\right)\eta_n}$ where these enthalpies are in kilojoule per kg. This is for choked nozzle condition but if nozzle is unchocked, then we will not have $h^{¿}$, we will have any h as $h^{'}$ which is the isentropic exit enthalpy to the nozzle. Now, we are going to see what are the types of nozzle.

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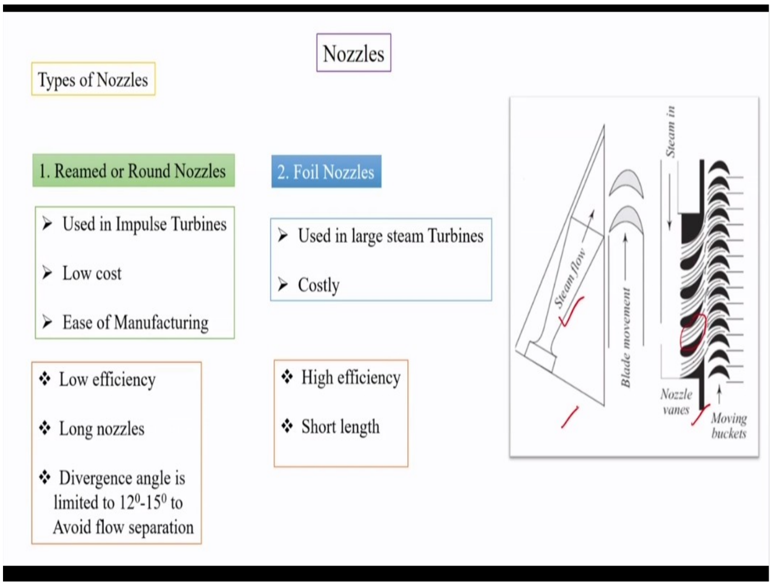

There are basically 2 types of nozzle; one nozzle is called as round nozzle or reamed nozzle and other nozzle is called as foil nozzle. So, in case of round nozzle, we have basically them mostly used in impulse turbines, these nozzles are very conventional, they are low cost nozzles, easy to fabricate, so easy to manufacture. Foil nozzles are costly and they are used for large steam turbine applications.

Further problem with reamed or round nozzle is they are having low efficiency, they are large length nozzles or long length nozzles and to avoid flow separation, the divergence angle in the nozzle is limited to be around 12 to 15 degree but in case of foil nozzle, they are high efficiency and then they are short nozzles. So, these two are these nozzles. So, we can see this is a convergent-divergent type or maybe only convergent or only divergent type.

Nozzle is this which is reamed or round nozzle and foil nozzles are basically the passages between two aerofoils, the passage between two aerofoils will act as nozzle. So, here we have to fabricate the aerofoils and so aerofoil fabrication is not that simple as the round nozzle. So, this is the foil nozzle and this is round nozzle.

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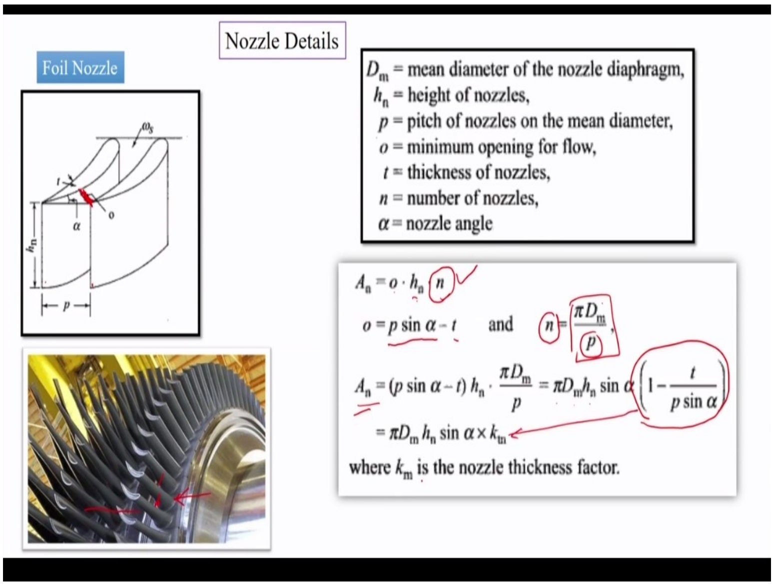

There are some more details we can see over here. In case of steam turbine, flow will come axially. In case of axial turbine, flow will go axially. So, this passage between two aerofoils will act as nozzle and this nozzle is basically a stator, then once the steam passes through the stator, steam would go from the rotor. So, there will be stator and rotor combination and that stator and rotor combination would act as the complete stage of a steam turbine.

So, this is the passage between two aerofoils where p is the distance between the edge, trailing edge of the aerofoil, α is the angle which is the flow deflection angle at the exit and then t is a local thickness of the aerofoil, o is the distance perpendicular to the flow at the exit and h_n is the height of the nozzle in the exit. So, this terminology is given over here.

And knowing this terminology, we can find out different quantities which are required which is area which is exposed for the flow which is $o \times h_n \times n$ number of such nozzles around the perimeter of the turbine. So, $o = p\sin(\alpha) - t$, o over here is $p\sin(\alpha)$ minus thickness but n is number of such nozzles which is $\pi D_m / p$. This gives us total number of nozzles present.

So, keeping all these terminologies in this formula, we can get $\pi D_m h_n sin\alpha \left(1 - \frac{t}{psin\alpha}\right)$ and this term can be said to be equal to k_m which is called as nozzle thickness factor. Now, we can use these relations and this understanding for further analysis of the nozzle flow in the steam turbine and those details we will see in the next class. Thank you.