Steam Power Engineering

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Lecture - 20
Stagnation Conditions and Nozzle Flow

Welcome to the class. We had seen in last class about the when it is concerned with the theoretical part, we had seen the one of the components of the steam power plant as steam generator, we had also seen its different parts and mechanism of heat transfer. Now, we are going to see the second component as steam turbine.

But before going directly to the steam turbine, we are going to understand some basic concepts of compressible flows which are required in the phase of understanding of steam turbine. So, let us start with that. So, today's class is dedicated for stagnation conditions and nozzle flow.

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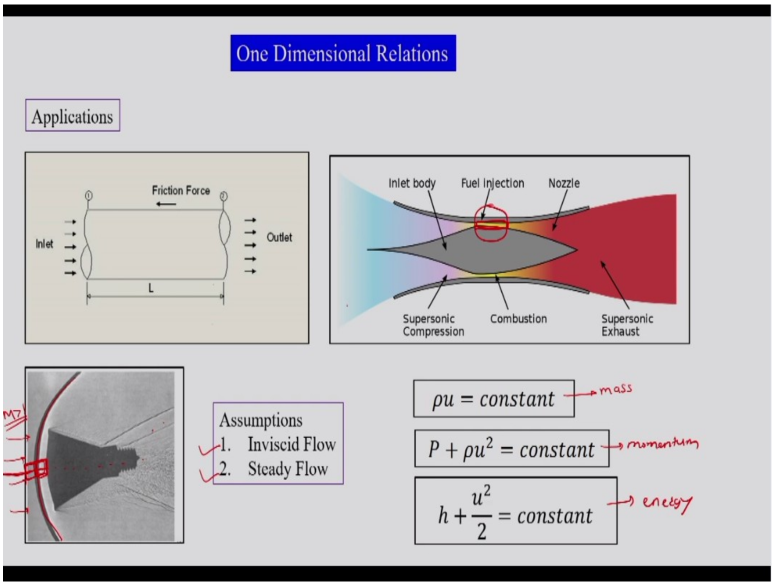

So, in this case, first we have to make use of some relations, which are called as standard one-dimensional relations but where are the applications of standard one-dimensional relations. See, the fact is we are assuming that flow is having major variation in only one direction and such flows are called as one-dimensional flows. So, what are the applications? One such application is flows with friction.

So, here we will have certain inlet in a duct and then we will have certain outlet in the duct for the duct and then due to the walls of the duct we will have friction. So, there are certain losses due to friction and then if we try to apply one-dimensional relations for compressible fluids, then we can try to estimate the properties at the outlet from the known conditions of the duct at the inlet.

Then, we have, in some engines; we have the flow through the engine where we have compressed gas or air in the combustion chamber or where heat addition takes place. So, here variation of the area in this fuel injection around would be not dominant and further there is chemical reaction which is taking place and that leads to the heat addition to the compressible flows. So, we practically are going to consider the effect of heat addition on to the compressible one-dimensional flows.

That is one more application where we will try to get what is the outlet conditions in such a domain where heat addition is going to take place. Then, we have one more relation or one more application where we have a blunt body and a supersonic flow flows over this blunt body and as what we know supersonic flow means flow having Mach number more than 1. In such cases, there is a strong shock which is going to appear in front of the body.

But very special is a streamline which is called as the stagnation streamline or the streamline which is going to go from the centre of the object when velocity is parallel to the centre line. Then, the variation for such specific streamline across the shock will be one-dimensional only. So, there are various specific applications where we can make use of the one-dimensional relations.

However, we in our case of steam turbines, we would be making use of these relations later on for few things that we will tell in detail in the following classes. What are those? Assumptions which involve in knowing the one-dimensional flow relations and those assumptions include flow to be inviscid and flow with steady and in some cases we will also assume that flow to be having no frictional work in these 2 cases.

And here will incorporate friction as not the term as what we are going to consider in some cases but in general these are the 2 assumptions involved in deriving one-dimensional relations and one-dimensional inviscid flow relations with steady-state particularly are these

3 where we have this as mass conservation equation, this as momentum conservation equation and this as energy conservation equation.

And these 3 relations do not get directly applied to these 3 philosophies or 3 applications but yes these equations with some specific extra atoms would be eligible to get applied for these 3 situations. Hence, we know that there are certain specific applications in which approximated one-dimensional flow can be helpful to predict the flow properties.

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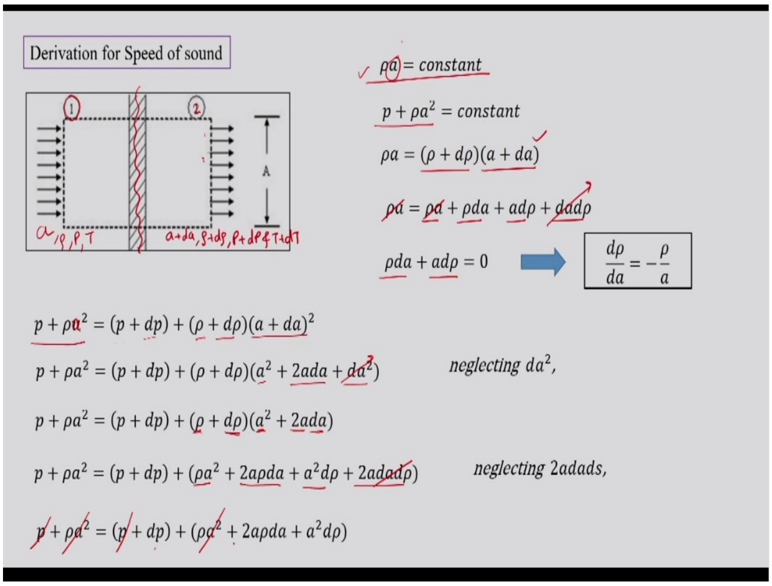

So, now derivation for speed of sound; we always know that sound is a medium or sound is a wave which is passing in a medium and then that wave is having certain speed and that wave speed which is having the infinity symbol changes in the medium due to the pressure variation and those minute pressure variations are said to be the propagation of the sound and then we are trying to derive the relation for speed of sound.

So, this is suppose the wave which is a sound wave, actually it is traversing with certain velocity in a medium which is still or which is having zero velocity but we assume that the wave is standing still and the flow fluid which is given the velocity of the wave. So, fluid is not stagnant and now wave is stagnant. So, there is wave which is stagnant and then there will be fluid at station 1 coming into our control volume shown by dotted lines.

And then there is the same fluid which would be leaving from the same control volume. So, now we are going to understand how would be the speed of sound okay. So, let us say that *a* is the velocity by which fluid is entering into the domain and this is practically our speed of sound and this speed of sound is appearing here in the mass conservation equation where we are saying that $\rho a = constant$ since we have area of the computational or area of our control volume is same.

Otherwise rho a would have been constant but momentum equation says that $p+\rho a^2 = constant$ and then this equation which is mass conservation equation if we put 1 as inlet and 2 as outlet but since we know that the speed of sound or sound wave leads to infinitesimal changes and if there is a as speed, ρas density, p as pressure and T as temperature at the inlet, then those quantities at the outlet would become $a+da$, $\rho+d\rho$, $p+dp$ and $T+dT$.

So, this is the variation of properties across the sound wave. Then, using these conditions we can put it in the mass conservation equation. Practically, we mean over here that ρaat the inlet is equal to ρaat the outlet but ρa at the outlet is $\rho+d\rho \wedge a+da$. So, the same mass conversation equation as what we have said as constant between inlet and outlet is getting represented as $\rho a = (\rho + d\rho)(a+da)$.

Then, we can expand the bracket and then say that $\rho a + \rho\, da + a d\rho + da d\rho$ but $da d\rho$ would be already da is a small quantity, $d\rho$is also a small quantity, so multiplication of 2 infinitesimal quantities would make their order of magnitude much lower than the other terms and hence let us neglect this term to be out from the above expression and then further we know that ρa ρa can be cancelled.

So, we can get an expression which say that $\rho da + a\, d\rho = 0,$ so we get $d\rho / da = -\rho / a$. So, this is what we could achieve from the mass conservation equation across the sound wave. Further, let us apply momentum conservation equation across the sound wave. Momentum conservation equation says that $p+\rho a^2$ before the wave and after the wave are same, so $p+\rho a^2$ is before the wave but p after the wave is $p+dp$ and ρ is $\rho+d\rho$ and a u and here we have basically $a$and $(a+da)^2$

Then, we can get here the same term, this bracket can be expanded and said that $a^2 + 2\, a da + d\, a^2$ and rest of the terms can be kept as same. Now, we can as neglected in earlier case da itself is a small quantity, so $d\, a^2$ will be further small, so let us neglect that and then neglecting that we can write down the same expression without just $d a^2$. Further, let us multiply these 2 brackets and say that ρa^2.

Then, we have $2 a\rho da$ and then we have ρa^2sorry then we have $d\rho a^2$ and then we have $d\rho da 2a$. So, these are the 4 terms. Further, higher order terms $da\, d\rho$ will be neglected here and then we can get the same expression without the term of $dad\rho$.

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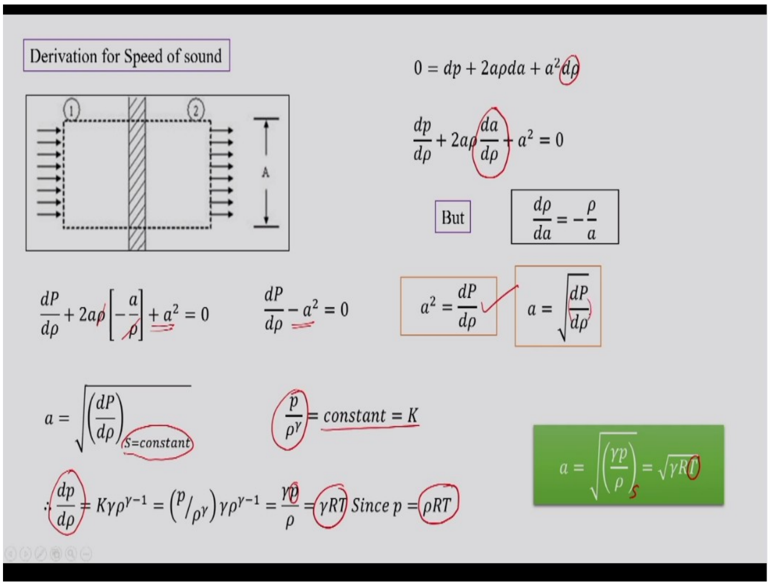

Now, the same thing, we can further if we go then we can neglect, we can cancel dp, we can cancel $d a^2$ and then we can have only, we have only 3 terms in the equation and equal to zero. So, we get 3 terms in the equation and equal to zero. Let us divide the terms by $d\rho$ and we have $\frac{dp}{d\rho}+2a\rho\frac{da}{d\rho}+a^2=0$ but we know that from mass conservation equation $d\rho/da=-\rho/a$.

So, we can put this over here which will be becoming $\frac{-a}{\rho}$ so we can further have ρ,ρ cancelled, then this will be becoming $-2a^2$ but we have$+a^2$, so basically it leads to $\frac{dp}{d\rho}-a^2=0$, so this is the speed of sound expression which says that $a^2=\frac{dp}{d\rho}$, so $a=\sqrt{\frac{dp}{d\rho}}$ but here the changes are so small such that we can treat this as an isentropic condition where practically $\frac{dp}{d\rho}$ is having variation of pressure with respect to density changes at constant entropy.

But we know for the isentropic case, we have $\frac{p}{\rho^{\gamma}} = constant$, so we can differentiate this and get $\frac{dp}{d\rho}$ and that would give you as γ ,$\sqrt{\gamma RT}$, $\sqrt{\gamma RT}$ since we have $p = \rho RT$. So, this p can be replaced as ρRT and then we can get the speed of sound expression as $a = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma RT}$

So, this tells that speed of sound in any medium depends upon the temperature. If temperature is high, then speed of sound will be higher. Obviously, we are using these relations for gases; however, this relation is valid in general.

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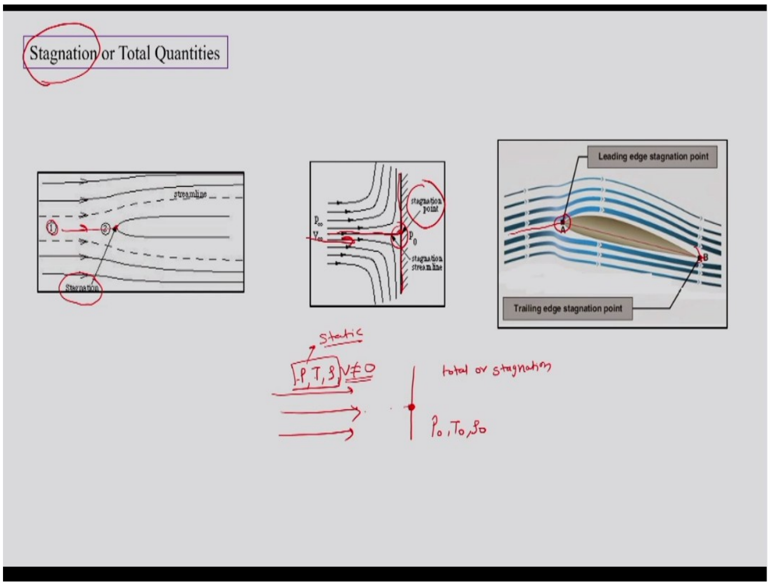

Now, let us talk about stagnation quantities. So, what do we mean by stagnation quantities? First of all, let us understand what do we mean by stagnation. Let us see there are 3 situations in which first is there is a flow over ogive shaped body where we can see it is a low speed flow flowing over ogive shaped body and then this flow at point 1 has certain amount of velocity when it comes closer to the body and when it hits the body at this point very particular point it gets zero velocity.

So, such point is termed here stagnation point. Here, in the second example, we have a vertical plate and flow is coming perpendicular direction to the plate and then or flow is coming in the direction parallel to the area vector of the plate such that when velocity vector hits or here velocity, stream line hits or here we get a flow with zero velocity and here as well the same point is called as stagnation point.

Similarly, there will be an aerofoil and flow over the aerofoil where initially streamline would become stagnant at this point or get terminated at this point. So, this is the leading edge stagnation point and when the 2 streamlines meet over here, we have a second stagnation point, which the trailing is stagnation point. So, practically stagnant means we are meaning over here that velocity is zero for the flow.

So, when the flow is adiabatically stopped to zero velocity, then such changes in the flow which are basically related to reversibility of such stopping leads to isentropic stopping of the flow to zero velocity from its initial state. Hence, if the flow is stopped isentropically from its initial state to zero velocity, then the corresponding conditions are called as stagnation or total condition.

So, if we have a streamline of the flow where flow has P pressure, T temperature and ρ density, then it has v velocity which is not 0. So, if velocity is not 0, then corresponding quantities are called as static quantities. So, we have static pressure, static temperature and static density but now if this flow is isentropically stopped, then at this point pressure, temperature and density are called as total or stagnation.

And generally they are represented as $P_0, T_0 \wedge \rho_0$. So, these are the stagnation quantities of the flow. So, we have static pressure, stagnation pressure or total pressure; static temperature, stagnation or total temperature. Now, we are supposed to find out the values of stagnation quantities if we feel that we know the static quantities or other way we are trying to find out what is the relation between static and stagnation quantities such a relation is called as, such relations are called as isentropic relations.

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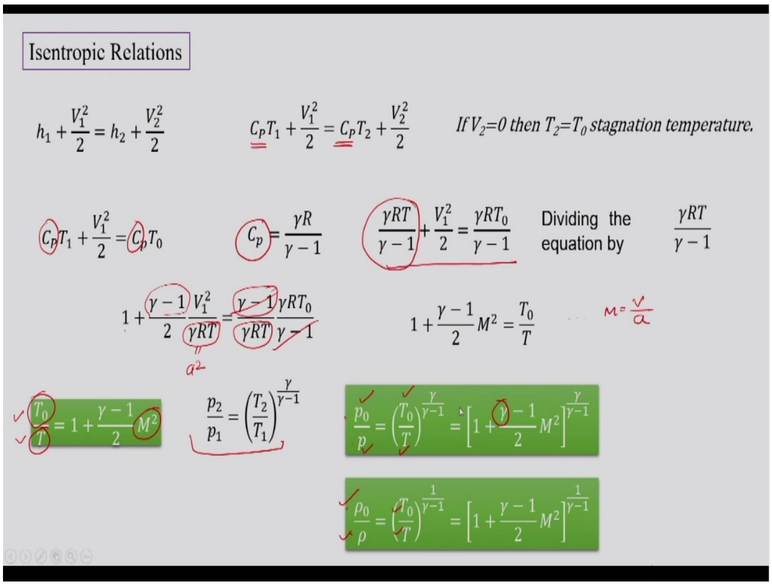

So, let us take the third equation as energy conservation equation for one-dimensional case which says that $h_1+\frac{V_1^2}{2}=h_2+\frac{V_2^2}{2}$where flow is flowing between station 1 and station 2. We know that $h=C_pT$, so $C_pT_1+\frac{V_1^2}{2}=C_pT_2+\frac{V_2^2}{2}$. Our argument at this moment is C_p is constant for the fluid flow and since it is a perfect gas with constant C_p it is called as calorically perfect gas.

So, this relation if we feel that we have brought the flow to the zero velocity isentropically at station 2, then we will say that $V_2=0$ and corresponding temperature T_2 is T_0 and which is a stagnation temperature. Then, the expression become $C_pT_1+\frac{V_1^2}{2}=C_pT_0$. but we know that $C_p=\frac{\gamma R}{\gamma-1}$. Therefore, we get $\frac{\gamma RT_1}{\gamma-1}+\frac{V_1^2}{2}=\frac{\gamma RT_0}{\gamma-1}$after putting the expression for C_1 in this energy equation form.

Now, let us divide this equation by $\frac{\gamma RT}{\gamma-1}$. Practically, we are dividing by this term to the complete equation. Therefore, this term will be 1 and we will have $\gamma-1$in the numerator and γRT in the denominator, same thing would happen to the right hand side as well. So, practically, this $\gamma-1$ and this $\gamma-1$ will cancel and we have $\gamma RT=a^2$ as what we have derived.

So, $\frac{V_1^2}{a^2} = M^2$ which is using the concept Mach number is equal to velocity of the fluid divided by local speed of sound. So, this is the definition of Mach number. Therefore, we get an expression which says that $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$. So, here basically if we know static temperature and if we know Mach number of the flow, then we can know total temperature.

Or otherwise if we know total and static temperature, then we can find out what is the Mach number of the flow. Further here let us use isentropic relation which says that $\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$. Hence, as what we have said that in case of stagnation conditions, we are feeling that flow is brought to the rest isentropically. So, we can use this relation to correlate the temperature ratio and pressure ratio between the static and stagnation quantities.

So, this is stagnation pressure, static pressure, stagnation temperature, static temperature and these can be correlated using this expression which again we can correlate for the density, the stagnation point divided by static density using its relation with the total temperature divided by static temperature. Thus, we can see here that the ratio of total temperature to the static temperature, ratio of total pressure to the static pressure or ratio of total density to the stagnation density, this ratio depends upon only Mach number and ratio of specific heats.

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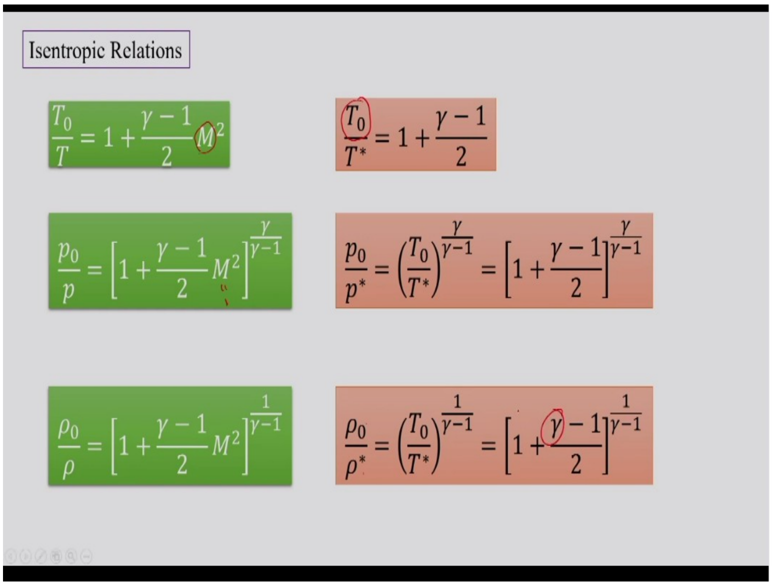

So, if we put Mach number to be 1, if this Mach number is 1, then we get a special relation which is the relation between total temperature and the sonic temperature. Here, sonic condition is specifically denoted by star. Further, total pressure and static pressure, they can also be found out from Mach number but if we put Mach number is equal to 1, then this relation also become a specific relation between total pressure and the sonic pressure.

Similarly, density ratio between the total and the static at the sonic condition can be found out in the same way and the star quantities can be seen to have only dependence upon the specific heat ratio gamma since we are putting a specific value of Mach number which is 1.

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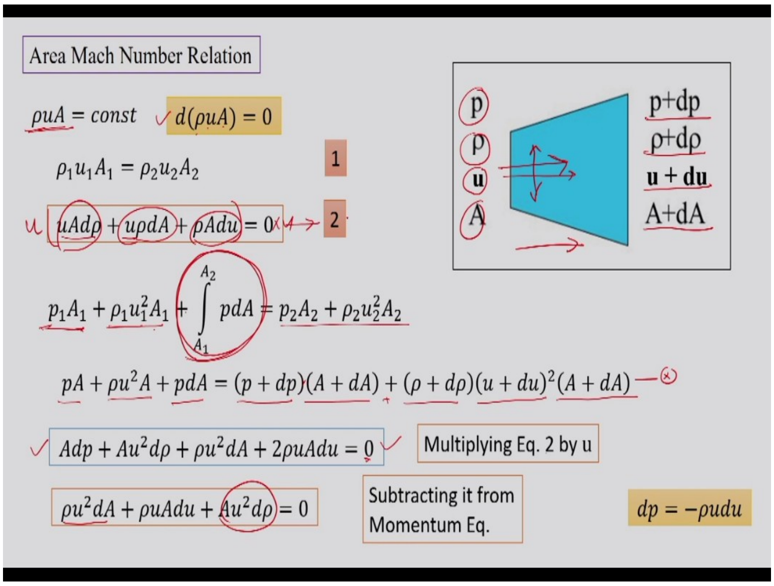

Now, let us try to design an expression or let us try to find out an expression which is expression for area and Mach number relation. Let us consider a duct which is having area A at the inlet, it is getting fluid flow with velocity u, density ρ and pressure p; however, in the direction of the flow duct is having varying area such that at outlet we are $u+du$, $\rho+d\rho$, $p+dp$ and $A+dA$.

Such condition is no more one-dimensional since here area is varying in 2 directions okay, area is varying in the direction of flow that is why there are changes in the fluid properties in the direction of flow and there are certain changes away from the or normal to the direction of flow. If the changes which are happening normal to the direction of flow, if they are small, then such flows are called as quasi one-dimensional flows.

So, let us try to derive the relation which is area Mach number from the perspective of quasi one-dimensional relation. Now, earlier in one-dimensional relation we had considered $\rho u = constant$, but now for quasi one-dimensional or in general, we will have $\rho Au = constant$. This relation tells to us that $d(\rho Au) = 0$. So, practically, $\rho_1 A_1 u_1 = \rho_2 A_2 u_2$

Also, we can differentiate this expression which is $d(\rho Au) = 0$ and then we can get $uAd\rho + \rho udA + \rho adu = 0$, so this is the differentiation of expression corresponding to mass conservation equation. For quasi one-dimensional equations, quasi one-dimensional equation for momentum is this where this term was not present when we were considering one-dimensional relation and it was $pA = \rho$, it was $p + \rho u^2 = const$.

But now we have area variation, so we have $p_1 A_1 + \rho_1 u_1^2 A_1$ plus area variation in the direction of the flow is equal to outlet pressure into area plus outlet $\rho\ u^2$ into area. So, this expression in differential form can be written between inlet and outlet as $pA + \rho u^2 A + p\,dA$. So, this is a dA variation of area in the direction of flow is equal to $(p + dp)(A + d\,A) + (\rho + d\rho)(u + du)^2 (A + dA)$.

After expanding this term and again as what we did in earlier case, neglecting all terms which are second order terms, we can get an expression which will be $Adp + Au^2 d\rho + \rho u^2 dA + 2\rho uAdu = 0$. After multiplying by, this expression is basically obtained after multiplying equation 2 by u. We have got this expression, let us star this. Let us take this expression which is equation number 2 and multiply this equation completely both sides by u.

So, 0 into u is 0 but these 3 terms would get multiplied by u and then we have an expression which is $\rho u^2 dA$, r$\rho u^2 dA$ this term plus $\rho uAdu + Au^2 d\rho$. So, these 3 terms would belong to equation number 2 after multiplying equation 2 by u. Further, this equation belongs to the modified form of this starred equation.

Now, we can subtract these two equations from each other and in we can get an equation which says that $dp = -\rho u du$. So, this is what we will get from this expression when we subtract the u multiplied equation from the momentum equation.

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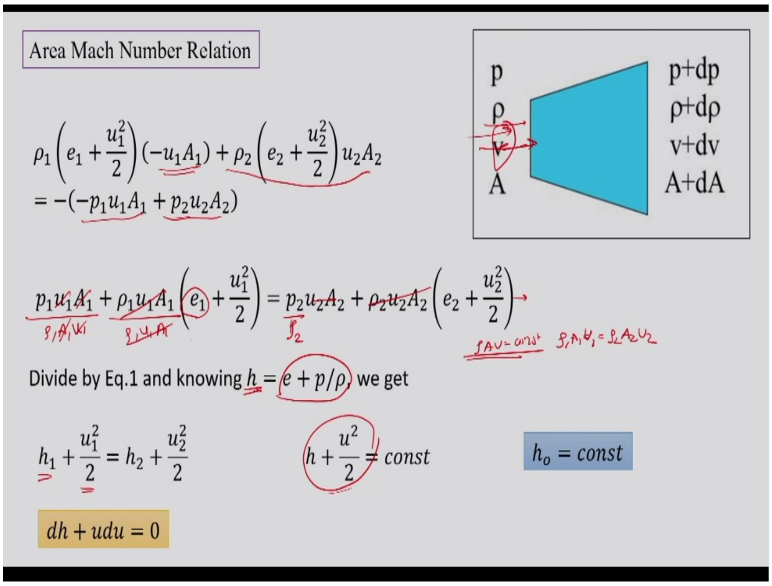

Now, we know that the energy equation for one-dimensional relation is ρ is $h_1+\frac{u_1^2}{2}=h_2+\frac{V_2^2}{2}$

Here, let us consider the similar expression where we are considering energy flux at the inlet plus energy flux at the outlet is equal to the reason for changing flux and here we are only considering pressure forces. So, $\rho_1\left(e_1+\frac{u_1^2}{2}\right)(-u_1A_1)$ that is for the inlet of this domain where minus sign neglects, minus sign tells that velocity vector and the area vector at the inlet they are opposite to each other.

Similarly, this term belong to same energy flux at the outlet, this is the pressure into area into velocity at the inlet. This is pressure into area into velocity at the outlet. We will rearrange the terms in this expression, we will get $p_1u_1A_1+\rho_1u_1A_1\left(e_1+\frac{u_1^2}{2}\right)$ is equal to same two terms at the outlet. Dividing equation 1 by h, dividing by equation 1, this equation let us divide by equation number 1 we say that$\rho Au=const$.

So, $\rho_1A_1u_1=\rho_2A_2u_2$and now we can divide this expression completely, we can get here as $\rho_1A_1u_1$, so u_1 u_1 will get cancelled, A_1, A_1 will get cancelled and $\rho_1A_1u_1$ so this term will be cancelled. Similarly, we will have this to be is equal to $p_2\rho_2$ and this will be cancelled. So, we have $\frac{p_1}{\rho_1}$ plus e_1.

So, this $p/\rho+e$ is basically h and hence we will have an expression which states that $h_1+u_1^2/2=h_2+u_2^2/2$ which otherwise says that $h+u^2/2=constant$ which also says that h_0 is equal to constant. So, if we differentiate this expression, then we get $dh+u\,du=0$.

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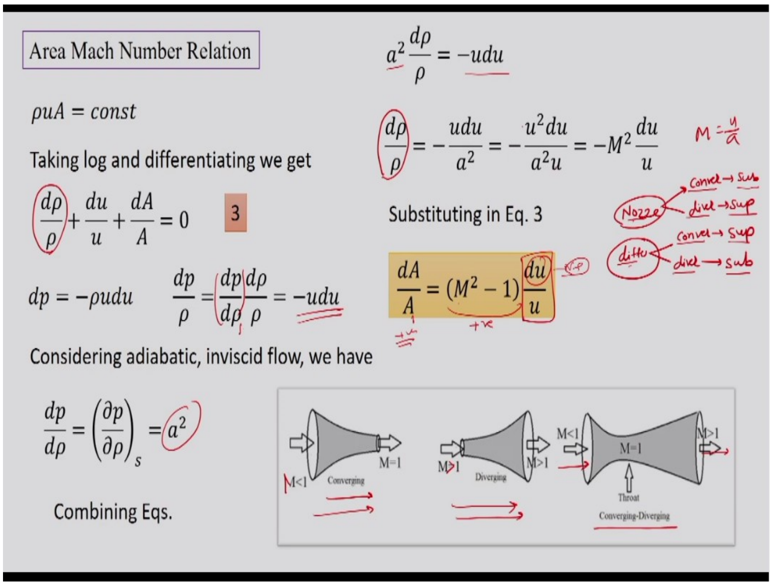

Now, let us try to find out what is the area Mach number relation. We now know that for quasi one-dimensional case, $\rho uA=const$ is mass conservation equation. Let us take log and differentiate this equation which says that $\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0$, but from momentum equation, we know that $dp=\rho\,u\,du$, so we have $\frac{dp}{\rho}=\frac{dp}{d\rho}\frac{d\rho}{\rho}$ but that from momentum equation is equal to $-u\,du$.

So, considering adiabatic, inviscid flow, we have this term at isentropic case is equal to a square. So, we get $a^2\frac{d\rho}{\rho}=-u\,du$. So, we have $\frac{d\rho}{\rho}=\frac{-udu}{a^2}$. We will multiply numerator and denominator by u, so we will have $-M^2\frac{du}{u}$ where we know that $M=u/a$. Substituting this $\frac{d\rho}{\rho}$ term in equation number 3 which is this and then we can get $\frac{dA}{A}=\left(M^2-1\right)\frac{du}{u}$.

So, we have 3 things in our disposal. First is this $\frac{dA}{A}$ is the area geometry variation and now let us consider a converging section where $\frac{dA}{A}$ is a negative term. So, if this is a negative term on left hand side, we should have right hand side also to be a negative term. So, for that term to be negative, let us consider the Mach number to be less than 1. So, if Mach number is less than 1, then this term will be negative okay.

If this term is negative, then we have to have total product to be negative, so this term becomes positive. That means when area is decreasing in a subsonic flow, velocity increases so this becomes the nozzle for subsonic flows. Now, let us take an example of supersonic flows or diverging section and in case of diverging section like this, let us feel that Mach number at the inlet is more than 1.

So, Mach number at the outlet is more than 1. So, this term will be positive and now if $\frac{dA}{A}$ is a positive number, then this term also needs to be positive. So, this term will be positive if du is equal to positive, that means again diverging section acts as nozzle in case of supersonic flow. So, converging section acts as nozzle for subsonic flow and diverging section acts as nozzle for supersonic flow.

Further, if we add converging and diverging section together, then we have convergent and divergent nozzle which acts as nozzle where at the inlet we have subsonic flow and at the outlet we have supersonic flow. So, what we can see over here that the variation of fluid flow properties across the duct depends upon the Mach number. So, if Mach number is less than 1 and Mach number less than 1 means velocity of the flow is less than the local speed of sound means flow is subsonic.

So, for such subsonic flows we have convergent section as nozzles. So, we have 3 things, here we have converging section and we have diverging section and we have nozzle and we have diffuser okay. So, converging section for nozzle, nozzle and converging section, this combination is going to suit for subsonic part or subsonic flow. So, in the subsonic flow, converging section acts as nozzle.

But in case of supersonic flow, converging section acts as diffuser. In the similar line, we have supersonic flow, which acts as nozzle when we have diverging section. Similarly, supersonic flow so since the flow with flow variation depends upon Mach number. We have some criteria which says that we have suppose we have nozzle, then this nozzle will be converging section if the flow is subsonic and this nozzle will be diverging section if the flow is supersonic.

Similarly, we have diffuser and then this diffuser will be converging section if the flow is supersonic but this diffuser is diverging portion if the flow is subsonic. So, nozzle and diffuser, these 2 are the philosophies where flow is going to have certain fixed variation of pressure and velocity. In case of nozzle, velocity will increase and pressure will decrease and in case of diffuser, pressure will increase and velocity will decrease.

So, this equation says that converging section for subsonic flow acts as nozzle, diverging portion for supersonic flow acts as nozzle, converging portion for supersonic flow acts as diffuser and diverging portion for subsonic flow acts as diffuser. Rest of the relations and their derivations, we will see in the next class. Thank you.