Fundamentals of Conduction and Radiation
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Lecture - 08
1 - D Steady State Heat Conduction Part 3

Hello friends. Welcome to the third lecture of our module three where we are talking about the 1 - Dimensional Steady State Heat Conduction. Now in the previous two lectures, we have discussed about the Cartesian and cylindrical coordinate frame that is starting from the basic heat diffusion equation we have reduced that to a 1 - dimensional steady state version assuming in general the constant properties and also no heat generation.

And subsequently we have developed expressions for the corresponding thermal resistance associated with conduction and also possible situations if convection is also there in the picture. Accordingly we have learned that for 1-dimensional steady state heat conduction at the absence of heat generation and with constant properties any system can be reduced to a combination of series and parallel resistances and accordingly using the electrical analogy we can easily analyze such systems. So we have already got the expressions for thermal resistance corresponding to conduction and convection in Cartesian coordinate system referring to a plane wall and also to a cylindrical coordinate system referring to a hollow cylinder. In the previous lecture, I was trying to go very quickly and that is why I made couple of mistakes in situations.

But hopefully I have recovered from there and you have got the idea about how to do the calculation for cylindrical reference frame. Today we shall first be extending that one to spherical coordinate system the third system that we have in our syllabus or in our curriculum. And there we shall be trying to develop the expression for the thermal resistance corresponding to conduction and convection in spherical reference frame and then we shall be seeing a comparison for all the three and a few applications of that.

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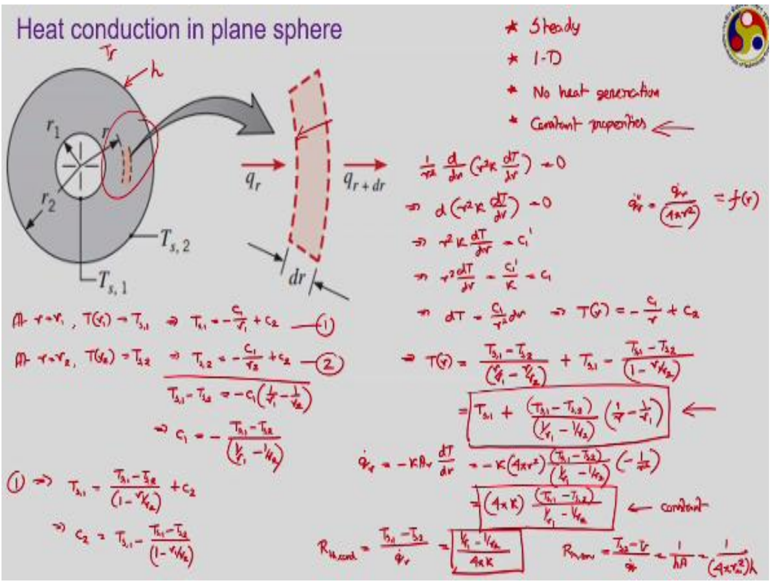

So straightway we are starting with a spherical coordinate system where our geometry is that of a plane sphere. You remember in cylindrical coordinate system initially I started with a solid cylinder and then reach to a difficult situation referring to the position r = 0 which of course need to have finite reference of temperature. But generally that kind of solid cylinder is not of that much use in practice rather we always work with hollow cylinders.

And that is why finally I went to the hollow cylinder to get the expression for corresponding thermal resistance. Similarly, I am starting with a hollow sphere here. Here we have 2 concentric circles in the plane of the screen as you can see one of radius r_1 and other of radius r_2. Basically I have 2 concentric spheres here the first one is of radius r_1 and the one having larger radius of r_2. And the temperatures are given.

$$at\ r = r_1, T = T_{s,1}$$

$$at\ r = r_2, T = T_{s,2}$$

So if we take an infinitesimally small section just like this at some radius r of thickness dr then we can assume this one to be a 1 -dimensional steady state heat conduction situation. So the standard assumptions that we have taken always we have assumed our system to be under steady state. We have assumed heat transfer in one particular direction to be much more dominant compared to the other one such that we neglect the variation in other two directions; like in this case we can neglect the variation in temperature in θ and φ direction. We are considering the temperature variation in the r direction to be the only significant one. Then we are also assuming the absence of any kind of heat generating element. And we are assuming constant properties.

So under such kind of situation we know that the heat diffusion equation the generalized heat diffusion equation in spherical coordinate system reduces to a 1 dimensional version of this form

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 K\frac{dT}{dr}\right) = 0$$

$$\Rightarrow \frac{d}{dr}\left(r^2 K\frac{dT}{dr}\right) = 0$$

$$\Rightarrow r^2 K\frac{dT}{dr} = C'_1$$

Rearranging,

$$\Longrightarrow r^2\frac{dT}{dr} = \frac{C'_1}{K} = C_1$$

$$\Rightarrow dT = \frac{C_1}{r^2}dr$$

Integrating,

$$\Rightarrow T(r) = -\frac{C_1}{r} + C_2$$

Now let us put the boundary conditions. So we have

$$at\ r = r_1, T(r_1) = T_{s,1}$$

$$at\ r = r_2, T(r_2) = T_{s,2}$$

So writing the two expressions; we have

$$T_{s,1} = -\frac{C_1}{r_1} + C_2$$

$$T_{s,2} = -\frac{C_1}{r_2} + C_2$$

If we combine both together then we have

$$T_{s,1} - T_{s,2} = -C_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Rearranging

$$C_1 = -\frac{T_{s,1} - T_{s,2}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

So that is the expression for C_1 that we have got similarly we can get C_2. We can pick any one of the expressions and get that value. Let us pick up this expression number 1. So from Equation 1 we can write

$$T_{s,1} = -\frac{\frac{T_{s,1} - T_{s,2}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}}{r_1} + C_2$$

$$\Rightarrow C_2 = T_{s,1} - \frac{T_{s,1} - T_{s,2}}{\left(1 - \frac{r_1}{r_2}\right)}$$

So we have got the expression for both C_1 and C_2 now. So if we take it back then we get

$$T(r) = -\frac{C_1}{r} + C_2$$

$$= \frac{T_{s,1} - T_{s,2}}{\left(\frac{r}{r_1} - \frac{r}{r_2}\right)} + T_{s,1} - \frac{T_{s,1} - T_{s,2}}{\left(1 - \frac{r_1}{r_2}\right)}$$

Now there are several ways you can simplify this one whatever seems convenient to you, but let me just write one form as our final objective is to get the expression for the thermal resistance from here. So let me just combine the two terms having $T_{s,1}$ - $T_{s,2}$. So I am having it as

$$= T_{s,1} + \frac{T_{s,1} - T_{s,2}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

That is the expression for temperature at any radial location using the knowledge of the two radius, radii r_1 and r_2 and also the corresponding temperatures $T_{s,1}$ and $T_{s\ 2}$.

But we start with the objective of getting the expression for thermal resistance. So to get the rate of heat conduction at any location

$$\dot{q}_r = -KA_r\frac{dT}{dr}$$

Putting the value of A_r which is the surface area of a sphere of radius r, and differentiating T from previous expression

$$= -K(4\pi r^2)\frac{T_{s,1} - T_{s,2}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}\left(-\frac{1}{r^2}\right)$$

$$= (4\pi K)\frac{T_{s,1} - T_{s,2}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Look at this expression. This is the one that we have just now developed corresponding to the heat conduction in the radial direction and what are the terms that we have in this expression? We have r_1 and r_2 we have $T_{s,1}$ and $T_{s,\ 2}$; all are known information and K is the thermal conductivity which also we have assumed to be constant here. That means this entire term is a constant one which indicates that the rate of heat transfer that remains constant in the radial direction, when you are treating with the spherical coordinate system as well. This is quite similar to the cylindrical or Cartesian coordinate frame. But one difference is there instead of $\dot{q}_r$ if our interest is to know the heat flux at any location any radial location then that will be equal to

$$\dot{q}''_r = \frac{\dot{q}_r}{4\pi r^2} = f(r)$$

Therefore it is a function of r. Therefore, while in the Cartesian frame that is in the plane wall both heat transfer rate and heat flux generally remains constant unless we are leading to a situation of parallel resistances. But in spherical and cylindrical coordinate system while heat transfer remains constant in the radial direction but heat flux keeps on varying because of the change in the area and finally to get the thermal resistance if we want to get the $R_{th,cond}$then what that will be; that will be equal to

$$R_{th,cond} = \frac{T_{s,1} - T_{s,2}}{\dot{q}_r}$$

By now you know that thermal conduction resistance can be expressed in terms of the potential difference divided by the corresponding effect.

That is the temperature difference which is the reason for this heat transfer divided by the rate of heat transmission and so we can easily rearrange the above expression to get the conduction resistances

$$R_{th,cond} = \frac{T_{s,1} - T_{s,2}}{\dot{q}_r} = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{(4\pi K)}$$

So this is the thermal conduction resistance in spherical coordinate for this hollow sphere or the spherical shell that we are dealing with of inner radius r_1 and outer radius r_2 and K being the thermal conductivity. Then we are having the corresponding conduction resistance to be $\frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{(4\pi K)}$. And if I want to calculate the convective heat transfer resistance at any location how much will be that? Let us say at the outer surface we are having some fluid flowing and with some convective heat transfer coefficient h, then how we can have this? If the outer surface or outer fluid temperature is T_∞, then

$$R_{th,cond} = \frac{T_{s,2} - T_\infty}{\dot{q}_r} = \frac{1}{hA} = \frac{1}{h(4\pi r_2^2)}$$

$4\pi r_2^2$being corresponding area and h is the convective heat transfer coefficient. So now we have done the analysis for the all the 3 coordinate system.

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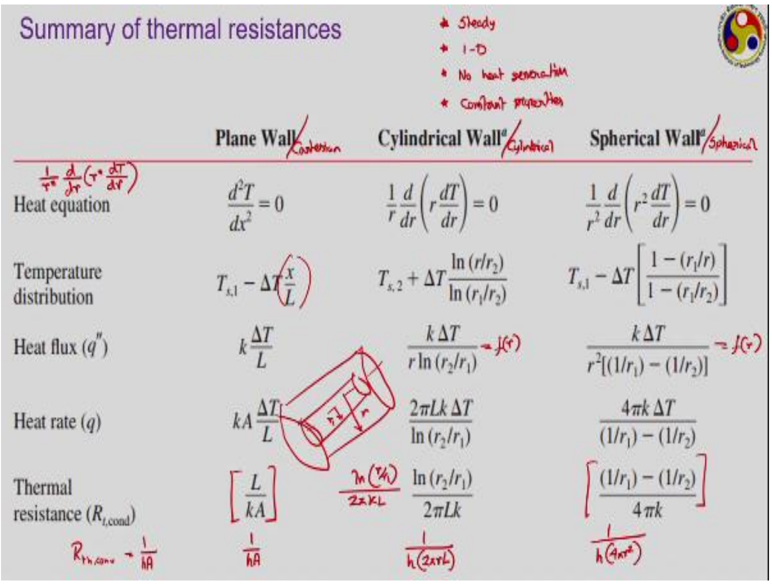

And let us have a quick summary of that. So I keep on repeating the conditions for which we are doing this. We are assuming steady state heat transfer only in one direction, then we are neglecting any kind of heat generation and we are assuming properties to be constant. The only thermo-physical property of importance here is thermal conductivity. So basically this fourth assumption refers to constant thermal conductivity.

Then in that situation we have compared the three. This is the plane wall which also refers to the Cartesian coordinate framework, this is a cylindrical wall or maybe you can say hollow cylinder which refers to the cylindrical coordinate system and finally we have the spherical coordinate system in the form of a spherical wall or hollow sphere in their case corresponding heat diffusion equations are written the reduced version of the heat diffusion equation.

$$\frac{d^2T}{dx^2}=0 \; for \; plane \; wall$$

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dT}{dr}\right]=0 \; for \; cylindrical \; wall$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right]=0 \; for \; spherical \; wall$$

So quite often just to write them in common form we can also adopt a notation something like this

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left[r^n\frac{\partial T}{\partial r}\right]=0$$

Where n =0 for plane wall; n=1 for cylindrical wall and n=2 for spherical wall. Putting the corresponding n values we can get the required equation. And of course thermal conductivity is also there inside the bracket what we are assuming that to be constant so that has come out. So these are the heat diffusion equations now temperature distribution we have developed.

This one we have developed in the first lecture in Cartesian coordinate system

$$T(x) = T_{s,1} - \Delta T \frac{x}{L}$$

For cylindrical

$$= T_{s,2} + \left[\frac{\Delta T \ln\left(\frac{r}{r_2}\right)}{\ln \frac{r_1}{r_2}}\right]$$

For spherical

$$= T_{s,1} - \Delta T \left(\frac{1 - \left(\frac{r_1}{r}\right)}{1 - \left(\frac{r_1}{r_2}\right)}\right)$$

This is shown in one particular form we can always rearrange the expressions to get this kind of forms. Then the heat flux

For plane wall

$$= K \frac{\Delta T}{L}$$

For cylindrical wall

$$= \left(\frac{K \Delta T}{r \ln \frac{r_2}{r_1}}\right)$$

For spherical wall

$$= \frac{K \Delta T}{r^2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

You can see for cylindrical wall it is actually a function of radius. Same we get in spherical coordinate system. So the heat flux keeps on varying as we move along the coordinate direction in cylindrical and spherical shells, but that remains constant in the Cartesian wall or in the plane wall. So finally the expression for heat rate for Cartesian

$$= KA \frac{\Delta T}{L}$$

For cylindrical wall

$$= \left(\frac{2\pi L K \Delta T}{\ln \frac{r_2}{r_1}} \right)$$

For spherical wall

$$= \frac{4\pi K \Delta T}{\left(\frac{1}{r_1} - \frac{1}{r_2} \right)}$$

Finally the most important one as of now at least conduction resistance that is

For Cartesian

$$\frac{L}{KA}$$

For cylindrical wall

$$\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k}$$

For spherical wall

$$\frac{\left(\frac{1}{r_1}\right) - \left(\frac{1}{r_2}\right)}{4\pi K}$$

I would also like to add here the thermal resistance corresponding to convection. In case of plane wall actually the form of convective resistance is always

$$R_{th,conv} = \frac{1}{hA}$$

Where, h is the heat transfer coefficient and A is the area which is experiencing heat transfer in its perpendicular direction. In the cylindrical wall we can easily express this area in terms of its radius.

$$R_{th,conv} = \frac{1}{h(2\pi r L)}$$

And in case of spherical wall it will be

$$R_{th,conv} = \frac{1}{h(4\pi r^2)}$$

Where, again r is the corresponding radial location. So you can see again the convective resistance also depends upon the radial location where we are calculating; for spherical and cylindrical wall but does not depend on that in case of plane walls.

Let’s look into one final important observation in cylindrical coordinate system. Suppose we consider that hollow cylinder that we have considered yesterday we have this is the inner one this is the outer one. The inner radius is r_1 the outer radius is r which can be a variable. In that case what is going to be your conduction resistance? Conduction resistance is going to be

$$\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$$

And convection resistance is whatever written. Now in both cases the radius is present and in case of the first one r is in the numerator in case of a second one r is in the denominator what does that indicate. So as the r increases as the thickness of this cylindrical shell that keeps on increasing keeping r_1 constant.

Then the value of conduction resistance that keeps on increasing whereas the value of convective resistance that keeps on reducing. So there must be some optimum location of r which is going to give you some maximum or minimum in the heat transfer rate

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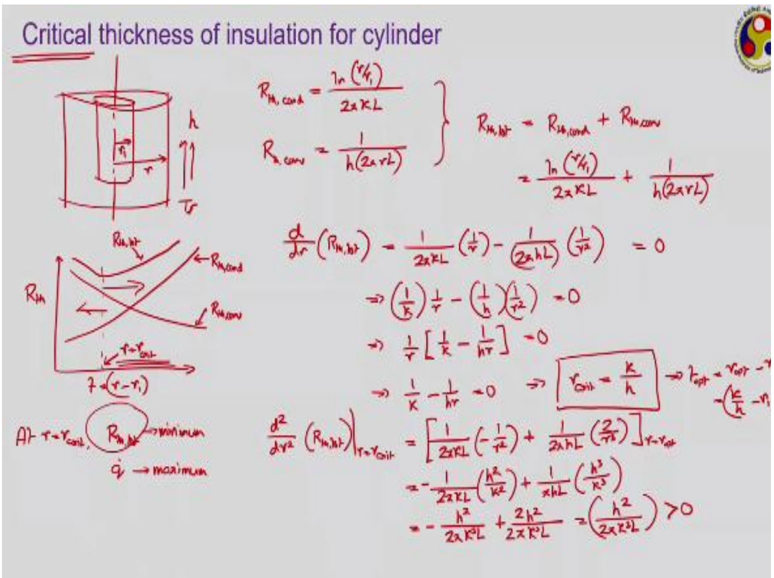

And that is what we are going to identify here in terms of this critical thickness of insulation for a cylinder. As I explained, we have one inner cylinder and this is the outer cylinder. So the inner one is having radius of r_1 outer one is having radius of r. Then our conduction resistance

$$R_{th,cond} = \frac{\ln\left(\frac{r}{r_1}\right)}{2\pi Lk}$$

And if we assume that over the outer surface of the cylindrical element some fluid is flowing in with a temperature T_∞ and the characteristic heat transfer coefficient of h then corresponding convective heat transfer resistance is going to be

$$R_{th,conv} = \frac{1}{h(2\pi rL)}$$

So the total thermal resistance is equal to

$$R_{th,tot} = R_{th,cond} + R_{th,conv}$$

$$= \frac{\ln\left(\frac{r}{r_1}\right)}{2\pi Lk} + \frac{1}{h(2\pi rL)}$$

If we now try to see the variation of these resistances in the radial direction; just what I was talking verbally at the end of the previous slide. Thermal resistance on the vertical axis and this small r on the horizontal axis which we can write this one in terms of t which is r - r_1 that is thickness of this shell then as the thickness increases that is as a value of r increases what happen to your conduction resistance?

The conduction resistance keeps on increasing following a logarithmic profile. What happens to the convective resistance? The convective resistance keeps on decreasing and therefore we must reach a position where the net has some kind of maximum or minimum; let us try to identify that. We do not yet know the profiles of $R_{th,tot}$, this is the profile for $R_{th,conv}$, this are profile for R thermal convection.

Let us see what about the $R_{th,tot}$. So for that we need to differentiate this $R_{th,tot}$with respect to r or with respect to this thickness t. So differentiating this expression we have

$$\frac{d}{dr}\left(R_{th,tot}\right) = \frac{\frac{1}{r}}{2\pi Lk} - \frac{1}{h(2\pi L)}\frac{1}{r^2}$$

To get the optimum location of this $R_{th,tot}$ let us equate this one to 0.

$$\frac{\frac{1}{r}}{2\pi Lk} - \frac{1}{h(2\pi L)}\frac{1}{r^2} = 0$$

$$\Rightarrow \frac{1}{rk} - \frac{1}{h}\frac{1}{r^2} = 0$$

$$\Rightarrow \frac{1}{r}\left(\frac{1}{k} - \frac{1}{hr}\right) = 0$$

Now r cannot be equal to 0, so the only possible solution is

$$\Rightarrow \frac{1}{k} - \frac{1}{hr} = 0$$

That means

$$r_{opt} = \frac{k}{h}$$

So this is optimum value of this r or correspondingly

$$t_{opt} = r_{opt} - r_1 = \left(\frac{k}{h} - r_1\right)$$

But this refers to a maximum or minimum? To get that let us differentiate this once more

$$\left.\frac{d^2}{dr^2}\left(R_{th,tot}\right)\right|_{r=r_{opt}} = \left[\frac{1}{2kL}\left(-\frac{1}{r^2}\right) + \frac{1}{2hL}\left(\frac{2}{r^3}\right)\right]_{r=r_{opt}}$$

Putting the value of r_{opt}

$$= \left[\frac{1}{2\pi kL}\left(-\frac{h^2}{k^2}\right) + \frac{1}{\pi hL}\left(\frac{h^3}{k^3}\right)\right]$$

$$= -\left(\frac{h^2}{2\pi k^3 L}\right) + \left(\frac{2h^2}{2\pi k^3 L}\right)$$

$$= \frac{h^2}{2\pi k^3 L}$$

So this entire quantity has to be greater than 0, what does that mean? That means that the value that we have got that refers to a minima in the total thermal resistance. Therefore, the profile for total thermal resistance will be something like this, it keeps on reducing which is a minima and then keeps on increasing. So this is the profile for $R_{th,tot}$ and this value is at r= r_{opt}.

We have the minimum thermal resistance at this particular location and as the thermal resistance is minimum at this, so what will be about what about the heat transfer? If the temperature ranges is given then the rate of heat transfer has to be maximum at this location. So this actually refers to an optimum value; I should not say an optimum that is why we generally do not use that term optimum we call it critical.

And that is why I am going to make a change now instead of using this optimum I am going to call it a critical because generally the term optimum refers to a maximum value of the thermal resistance or minimum value of heat transfer rather here we are getting the opposite and that is why we should not call this the optimum thickness or optimum radius.

Rather they should call it a more critical radius, and therefore what we are getting is

$$at\ r = r_{crit}, R_{th,tot}\ is\ minimum$$

$$\dot{q} \ is \ maximum$$

Therefore, as long as the value of r is less than this critical one then the total resistance keeps on decreasing, whereas when the value of r is greater than this greater than this critical the radius keeps on increasing with increase in r, then how can we make use of this fact.

There are certain situations where we want the heat transfer to be high; there are certain other situations where we want the heat transfer to be low. What can be the situation where we want the heat transfer to be high, just think about the situation of an electrical wire, electricity carrying wire that is generally made of some kind of metal some electrically conducting material which generally is also very good thermal conductor.

And around that we put a layer of insulation like suppose if this is your electrical conductor electrical current carrier then around this we are putting insulation. So just assume that the inner cylinder refers to your conductor and having radius of r_1, and then this outer shell refers to the insulation. Now what should be the thickness of insulation that actually depends on how the heat transfer or what heat transfer rate that we want.

Generally, as the current flows through the conductor, because of joule heating there will be some kind of thermal energy generation. And to maintain the temperature of the conductor within check we want this thermal energy to get dissipated to the surrounding. Here our objective is to have high rate of heat transfer, in order to have high heat transfer what we want, we want the radius to be small and therefore in which regime we should operate.

The choice of r should be such that your final value of r should be what, greater than r critical or less than r critical? Here we want heat transfer to be high and we want resistance to be low. Therefore, in this situation we should operate in this side. Ideally we want to be somewhere here so that we can maintain this value of r = r critical which is the minimum possible thermal resistance scenario and giving the maximum possible heat transfer.

The opposite scenario we can have in your refrigerator. The refrigerant is coming with a very low temperature and is about to enter that the chiller. Now here the refrigerant is at a much lower temperature compared to surrounding and therefore we need to insulate the refrigerant coil refrigerant coil. Now here we want heat transfer to be very low as low as possible then what we should want.

We want the thermal resistance to be high and therefore your choice of insulation thickness should be such that we operate on this side. So accordingly the choice of this critical thickness of insulation is very important. In certain applications we want the heat transfer rate to be high and thermal resistance to be low. In that situation we should choose the thickness such that final value of r should be less than r critical.

Whereas in certain other scenario we want the value of heat transfer to be low, thickness of insulation they should be chosen such that your total resistance is quite high you are operating on the zone r greater than r critical.

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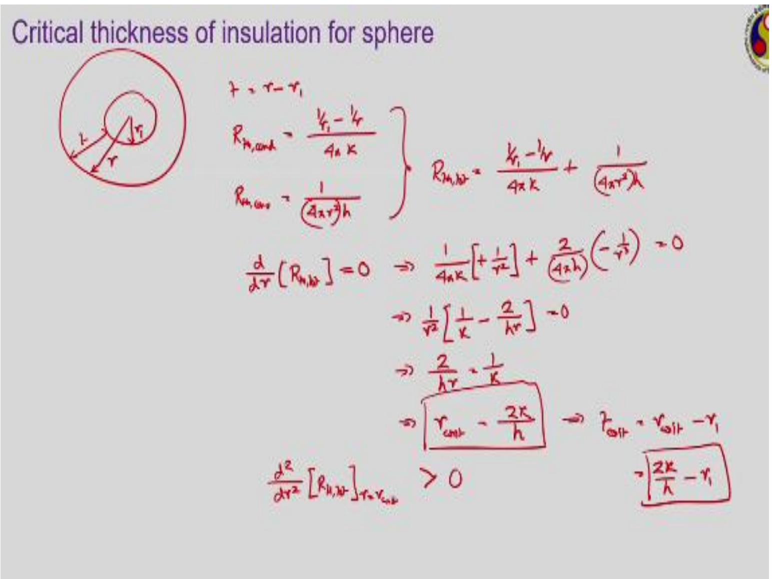

In case of spherical situation or spherical coordinate also the same situation is valid let us choose 2 spheres like we have done earlier. I am quite poor in drawing this; just assume these two as spheres. So the inner one is having a radius of r_1, the outer one is having radius of r which is our choice. This is the thickness t which is equal

$$t = r - r_1$$

Now here we know that

$$R_{th,cond} = \frac{\left(\frac{1}{r_1}\right) - \left(\frac{1}{r}\right)}{4\pi K}$$

$$R_{th,conv} = \frac{1}{h(4\pi r^2)}$$

$$R_{th,tot} = R_{th,cond} + R_{th,conv}$$

$$= \frac{\left(\frac{1}{r_1}\right) - \left(\frac{1}{r}\right)}{4\pi K} + \frac{1}{h(4\pi r^2)}$$

So now you know the procedure so to identify the critical value of this thickness or critical choice for r, we have to differentiate this R thermal total with respect to r and equate that to 0.

$$\frac{d}{dr} R_{th,tot} = 0$$

$$\Rightarrow \frac{1}{4\pi K}\left[\frac{1}{r^2}\right] + \frac{2}{4\pi h}\left[-\frac{1}{r^3}\right] = 0$$

$$\Rightarrow \frac{1}{r^2}\left[\frac{1}{K} - \frac{2}{hr}\right] = 0$$

$$\Rightarrow \frac{2}{hr} = \frac{1}{K}$$

$$\Rightarrow r_{crit} = \frac{2K}{h}$$

So in the previous case what we have got it was K/h in case of cylindrical coordinate system it is 2K/h in case of spherical coordinate system. I am leaving the next exercise to you.

You should also check whether this corresponds to a maximum or minimum. For that you need to differentiate this term once more at r = r critical and you will find this is actually coming to be greater than 0, which again corresponds to a minimum value of insulation thickness and maximum value of heat transfer rate. So the critical thickness of insulation for a cylindrical element is K/h and for spherical coordinate system it is 2K/h.

I should be very careful actually. We are not talking about the critical thickness. So the critical thickness of insulation is

$$t_{crit} = r_{crit} - r_1 = \frac{2K}{h} - r_1$$

So this is going to be the critical thickness of insulation in spherical coordinate system whereas this is the critical thickness of insulation in case of cylindrical coordinate system.

And this choice of insulation thickness is very important in several heat transfer scenarios. As I have given examples some cases like very common application of spherical coordinate system is the fuel tank of some satellite and space shuttle which are generally spherical in shape. So as we want to maintain them generally at very low temperature at cryogenic temperature as long as they are in the earth.

The outer temperature may be significantly larger than compared to the fuel temperature stored inside the tank and in order to maintain that low cryogenic level temperature we must reduce the heat transfer to the smallest possible and if the heat transfer has to be small as possible then the radius or rather the total thermal resistance should be very high.

And therefore we have to ensure that the thickness of the insulation is greater than this t_{crit} that we have just now calculated. So that is what we had to discuss about the 1 dimensional steady state scenario. There are several other smaller concepts can be developed from this which you can refer to book or you can try on your own. I shall be solving a few numerical examples now to show you the applications of whatever we have discussed.

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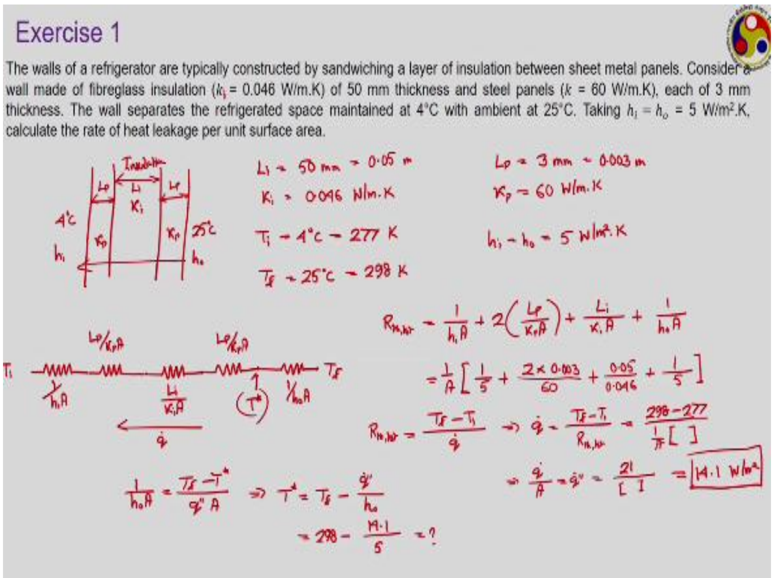

The exercise number one corresponds to a domestic refrigerator; just read the problem carefully, you can pause the video here and also read the problem statement carefully. So here I am talking about the wall of a refrigerator. It is typically constructed by sandwiching a layer of insulation between sheet metal panels. So this is a layer of insulation and on either side of that we have sheet metal panels.

This is your insulation layer and both side you have metal sheets on either side. So we are considering here a wall made of fiberglass insulation, k is equal to 0.046 or let us say K_i refers to the insulation material let us say this is K_i and 50 mm thickness, so this length is L_i

so here our L_i = 50 mm and it is safe to convert everything to SI unit that is 0.05 m and K_i =0.046 W/m K.

Now the steel panels that has K of 60 W/mK let us say K_p refers to this panel, So in this case K_p=60 W/m K, which is actually a quite high value for common stainless steel. The thermal conductivity is only in the range of 15 to 16; each of 3 mm thickness. So on both side let us say this is L_p ; this is also L_p both layers of metal panels are of the same thickness L_p = 3 mm that is 0.003 m.

The wall separates the refrigerated space maintained at 4 ºC. So this side your temperature is 4 ºC let us call this T_i = 4 ºC that is 277 K. Ambient is at 25 ºC, so our T_∞ is at 25 ºC that is 298 K and the heat transfer coefficients are given h_i and h_0 to be 5 W/m^2 K. So here we have h_i and here we have h_o.

And both are equal in this problem. Inner and outer heat transfer coefficients both are 5 W/m^2 K. So let's calculate the rate of heat leakage per unit surface area. So what will be the direction of heat transfer? Of course the ambient is at a higher temperature so this is the direction of heat transfer. We can easily visualize this as a combination of several thermal resistances.

And which kind of coordinate system we should use here? This looks like a plane composite wall so we can stick to the Cartesian coordinate system. So how many resistances you can think of? The insulation is there. Let us say this is our insulation related radius. What will be the expression for this one; L_i/K_iA. Either side of this we have sheet metals. So this side we have a layer of sheet metal and corresponding resistance this side another layer of sheet metal and corresponding resistance.

So this is having resistance of L_p/K_pA. This is also the same L_p/K_pA. Then we have convection on inner side and outer side; so this is the inner side one, and what will be the corresponding convective resistance 1/hA, outer side we have the convective resistance $1/h_oA$. Here you have this temperature of T_i temperature of T_∞. And this is the direction of heat transfer $\dot{q}$ which we have to calculate.

So quite straightforward we can do the calculation. Then

$$R_{th,tot} = \frac{1}{h_i A} + 2\left(\frac{L_p}{K_i A}\right) + \frac{L_i}{K_i A} + \frac{1}{h_i A}$$

So putting the values

$$= \frac{1}{A}\left[\frac{1}{5} + \frac{2 \times 0.003}{60} + \frac{0.05}{0.046} + \frac{1}{5}\right]$$

So this way you can get the value of R thermal total. I do not have the value written with me can you please calculate and of course the area is still included. We do not know the value of area and that is why you are going to do the final calculation as per unit surface area.

And I would also urge to calculate the values of each of these 5 individual components. Try to identify the most dominant one in determining this total resistance. I guess that has to be the insulation one. So the final total resistance will be

$$R_{th,tot} = \frac{T_\infty - T_i}{\dot{q}}$$

$$\Rightarrow \dot{q} = \frac{T_\infty - T_i}{R_{th,tot}} = \frac{298 - 277}{R_{th,tot}} = \frac{298 - 277}{\frac{1}{A}\left[\frac{1}{5} + \frac{2 \times 0.003}{60} + \frac{0.05}{0.046} + \frac{1}{5}\right]}$$

Now,

$$\frac{\dot{q}}{A} = \dot{q}" = \frac{298 - 277}{\left[\frac{1}{5} + \frac{2 \times 0.003}{60} + \frac{0.05}{0.046} + \frac{1}{5}\right]}$$

$$= 14.1\ W/m^2$$

This is answer that we are looking for.

If I add something to this, let us say please tell me what is going to be the temperature at this particular point? Let us say temperature at this point is T*, how can we calculate the temperature at the outer surface of the refrigerator? We now know the value of $\dot{q}$. Then we have to consider only that outer convective resistance and for that we can write this resistance as

$$\frac{1}{h_o A} = \frac{T_\infty - T^*}{\dot{q}"A} \Rightarrow T^* = T_\infty - \frac{\dot{q}"}{h}$$

And now putting the values

$$= 298 - \frac{14.1}{5}$$

So putting this we can easily get the value of this T* and this we can easily get any other temperatures as well. So this is a numerical problem dealing with the Cartesian coordinate system a composite wall.

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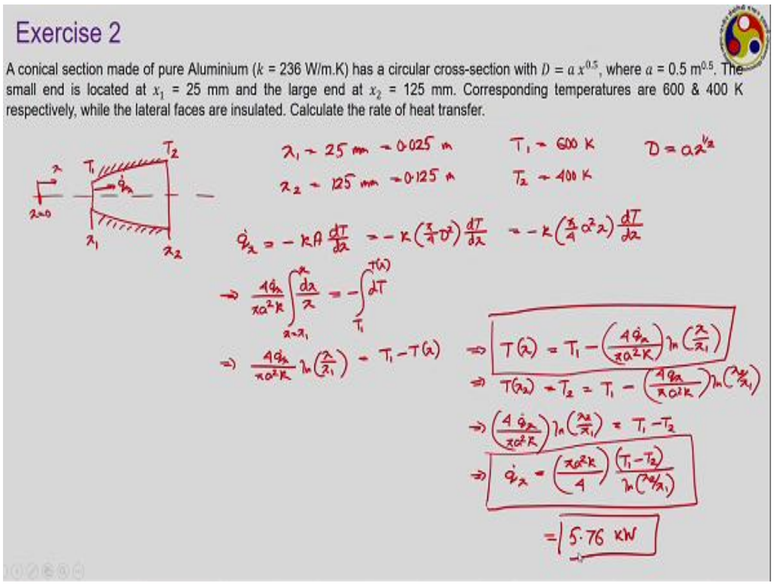

Now let us have the second problem where we have a conical section to deal with, which is made of pure aluminum. Thermal conductivity is given. It has a circular cross section with variable diameter. So the diameter is changing in the direction of heat transfer and accordingly the area corresponding to the direction of heat transfer is also changing.

The small and large ends are given so we have a conical section to deal with, something like this. Let us say this is your x = 0 location these are x direction so this is x_1 which is given as 25 mm this is x_2 given as 125 mm. Here the temperature is T_1 here the temperature is T_2. So we now given that $x_1 =$ 25 mm that is 0.025m, $x_2 =$ 125 mm that is 0.125 m. $T_1 =$ 600 K, $T_2 =$ 400 K. Thermal conductivity is also given.

Lateral faces are insulated that means these faces are insulated. Quite often we put this hatch to indicate that there is no heat transfer in that direction or perpendicular to those areas. So only possible direction of heat transfer is this $\dot{q}$. We have T_1 is at a higher temperature so direction of heat transfer will be from x_1 towards x_2. We have to calculate the rate of this heat transfer.

Now can you directly calculate the conduction resistance in this case? We cannot. Because while the thermal conductivity is constant, but the area is changing in the direction of heat transfer; so we have to go for that alternate approach. Whenever the cross section area is changing in the direction of heat transfer then it is always safe to go for this alternate approach. So let us say this is $\dot{q}_x$.

Then we know that

$$\dot{q}_x = -KA\frac{dT}{dx}$$

Assuming 1 - dimensional heat transfer in the x Direction and K is a constant, and the conical section is having a circular cross section

$$-K\left(\frac{\pi}{4}D^2\right)\frac{dT}{dx}$$

Area in this case is varying as given.

$$= -K\left(\frac{\pi}{4}a^2x\right)\frac{dT}{dx}$$

So if we rearrange the terms and take the constants like $\dot{q}_x$, a and K out, it becomes

$$\frac{4\dot{q}_x}{\pi a^2 K}\int_{x=x_1}^{x}\frac{dx}{x} = -\int_{T_1}^{T(x)} dT$$

So integrating this we have

$$\frac{4\dot{q}_x}{\pi a^2 K}\ln\frac{x}{x_1} = T_1 - T(x)$$

$$\Rightarrow T(x) = T_1 - \frac{4\dot{q}_x}{\pi a^2 K}\ln\frac{x}{x_1}$$

Now if we put $x = x_2$

$$\Rightarrow T(x_2) = T_2 = T_1 - \frac{4\dot{q}_x}{\pi a^2 K}\ln\frac{x_2}{x_1}$$

From there if we separate out this $\dot{q}_x$ then what we are going to get. Okay instead of jumping a step let us do carefully. So we have

$$\frac{4\dot{q}_x}{\pi a^2 K}\ln\frac{x_2}{x_1} = T_1 - T_2$$

$$\Rightarrow \dot{q}_x = \left(\frac{\pi a^2 K}{4}\right)\frac{T_1 - T_2}{\ln\frac{x_2}{x_1}}$$

So now we have to put the values. We know the values of x_1, x_2, T_1, T_2, and K. So we can easily put the values to get the value of heat flux. So from in this problem we have got an expression for the temperature profile and we have also got an expression for this $\dot{q}_x$.

So whatever may be our desire, well we can easily calculate. Unless we know the value of $\dot{q}_x$ we can't get the temperature profile because $\dot{q}_x$ is included here. So we need to get the value of $\dot{q}_x$. If we put all of them then the value of $\dot{q}_x$ in this problem is going to be 5.76 kW; this is the final answer that we are looking for.

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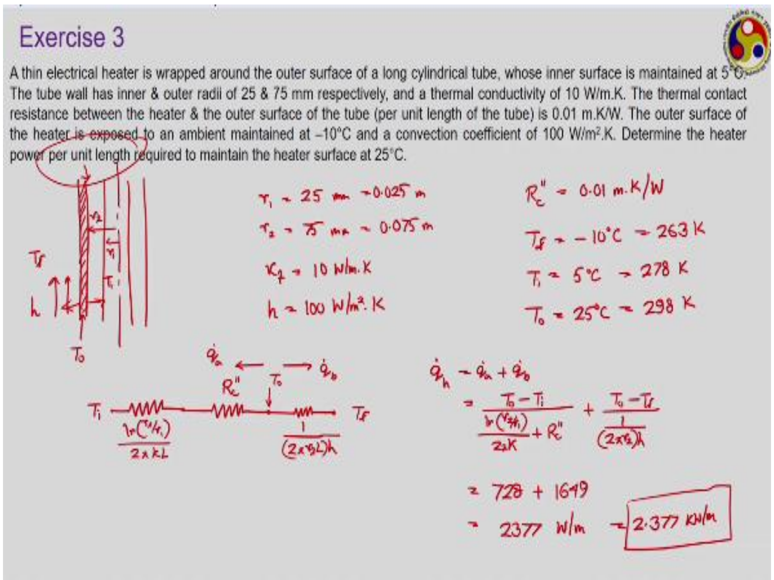

Now let us see one example of applying the cylindrical coordinate system. Again a big problem statement you can pause the video to read it carefully and then again restart the video. A thin electrical heater is wrapped around the outer cylinder outer surface of a long cylindrical tube whose inner surface is maintained at 5 ºC. The tube wall has inner and outer radii of 25 and 75 mm respectively.

And thermal conductivity is 10 W/m K. So we are talking about a long cylindrical tube. So this is a center line this and basically we are talking about a cylindrical shell the inner radius is r_1 and outer radius is r_2 and as for the given problem r_1 = 25 mm that is 0.025 m, r_2 =75 mm =0.075 m.

Thermal conductivity for this material say let us take K_t = 10 W/m K. So we have the heater mounted somewhere here. No information is given about the dimensions of the heater so we assume the heater to having a zero thickness. But there is a thermal contact resistance between the heater and the outer surface of the tube per unit length of the tube which is R_c" given as 0.01 mK/W. The outer surface of the heater is exposed to an ambient so this is your T_∞. T_∞ is -10 ºC that is 263 K. And the inner surface temperature T_i that is given at the beginning to be equal to 5 ºC which is 278 K. As the inner surface at a higher temperature you can expect heat transfer from inner surface to outer surface. But there is a heater in between so we have to be careful about that, and there is a convective heat transfer coefficient also. So some fluid is flowing here with temperature T_∞ and convective transfer coefficient is h = 100 W/m^2 K. Determine the heater power per unit length required to maintain the heater surface at 25 ºC.

So the heater surface, that is this portion the temperature let us say T_o has to be maintained at 25 ºC that is 298 K. So it is a very interesting scenario here both T_i and T_∞ are less than this T_o that is heat will get transferred from this heater to this inner surface by conduction and also by convention to this direction. So if we plot the corresponding resistances then how we can assume this?

So let us take this is a temperature T_i at this node, if we move from this innermost position towards the ambient first we are facing a conduction resistance, this is the conduction resistance. This conduction resistance is having a value of the K_t is this cylindrical wall you have to remember. So this conduction resistance will be as per our notation

$$\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk}$$

Then we have a contact resistance R_c" on the outer surface somewhere here. Then we have the convective resistance

$$\frac{1}{2\pi r_2 Lh}$$

Nothing is given about the thickness of the heater. So whatever is a thickness or whatever is the radius of the outer side of the cylinder that same can be taken as a final radius. So this contact resistance is given as per unit area, so it needs to be multiplied the corresponding area. So how we can do this? We have to determine the heater power per unit length required to maintain the heater surface at the desired temperature.

So this is the point where we are having the temperature $T_{o.}$ Outside we have the temperature of $T_{\infty.}$ Then what is the direction of heat transfer? As I have mentioned heat is being transferred in this direction and also in this direction. Let us say this direction is $\dot{q_a}$and this is $\dot{q_b}$ and therefore whatever power the heater is producing let us say $\dot{q_h}$ under steady state that has to be

$$\dot{q_h} = \dot{q_a} + \dot{q_b}$$

Now writing the expression for both the heat transfers per unit length with corresponding temperature differences and resistances

$$\dot{q}_h = \frac{T_o - T_i}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk}} + \frac{T_o - T_\infty}{\frac{1}{2\pi r_2 Lh}}$$

So you can easily calculate both the components as the values are given. So I have got the numbers as 728 for the first one and 1649 as the second one giving a total of 2377 W/m or maybe 2.377 kW/m. This per m coming because we are doing this calculation for per unit length of the cylinder. So it's a very interesting problem where somewhere in between the temperature is higher. This problem can also use visualized in form of volumetric heat generation only in that part. But as in this entire model we have neglected any kind of heat generation so we are solving this one not using any volumetric heat generation and completely neglecting the dimension of heater rather we are assuming a point source of energy which is getting dissipated by conduction and convection on both sides and accordingly we are solving this problem.

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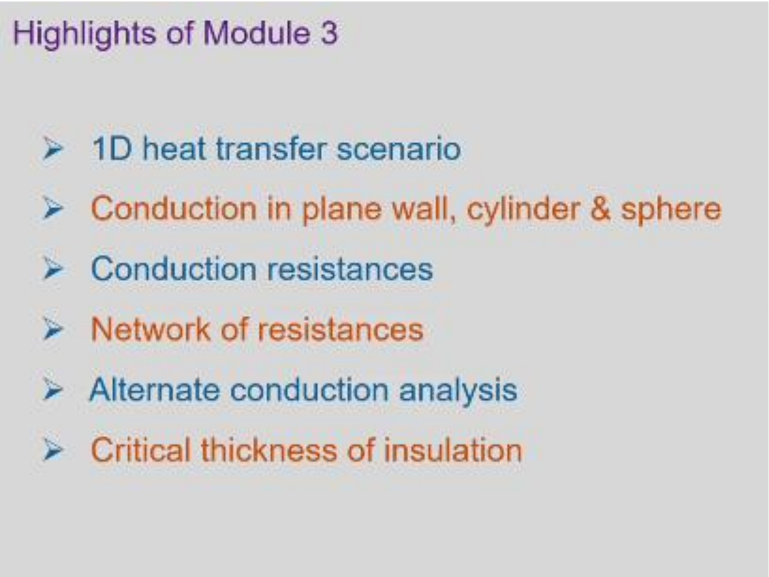

So this is where I would like to complete this module number three where we discussed about the 1D heat transfer scenario in all the three coordinate directions. Assuming plane wall, cylindrical shells and spherical shells and the conduction and convection resistance concept was introduced; thermal resistances that is. Then we have discussed about the networks of resistances connected in series and parallel to solve any kind of 1-D steady state heat transfer scenario.

The alternate conduction analysis was also introduced when the area keeps on varying in the direction of heat transfer or actually in case of plain cylinder and sphere also the area also the area varies in the direction of heat transfer. So this alternate conduction analysis is more prevalent in case of plane wall scenario the Cartesian coordinate. And finally the critical thickness of insulation for spherical and cylindrical coordinate system was also discussed in detail.

And we have also solved quite a few numerical examples you please try to solve a few more by referring to the books and also try to solve the problems that are given in the assignment corresponding to this module. If you have any query, please write back to me I shall be very happy to answer to you. Thank you very much.