**Fundamentals of Conduction and Radiation**
**Amaresh Dalal & Dipankar N. Basu**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Guwahati**

**Lecture - 06**
**Concept of Thermal Resistance**

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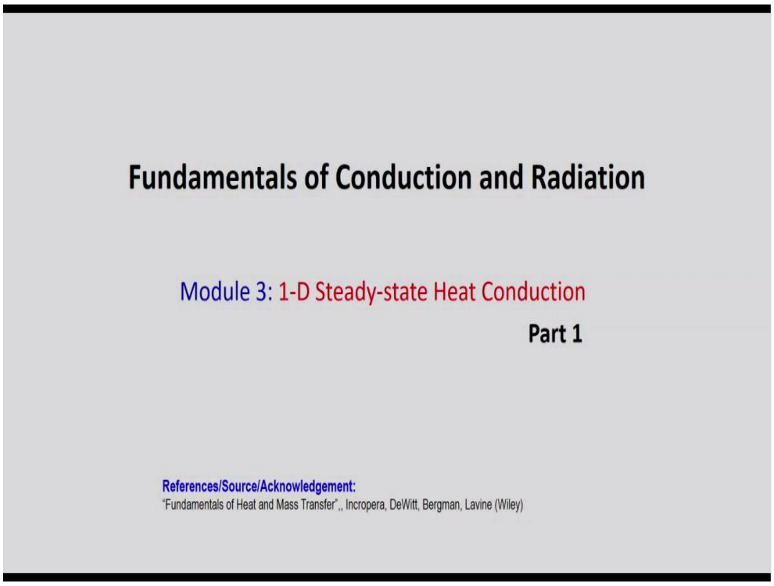


Hello friends. Welcome back to week number 3 of our course on fundamentals of conduction and radiation and this week we are going to talk about the one dimensional steady state heat conduction situations.

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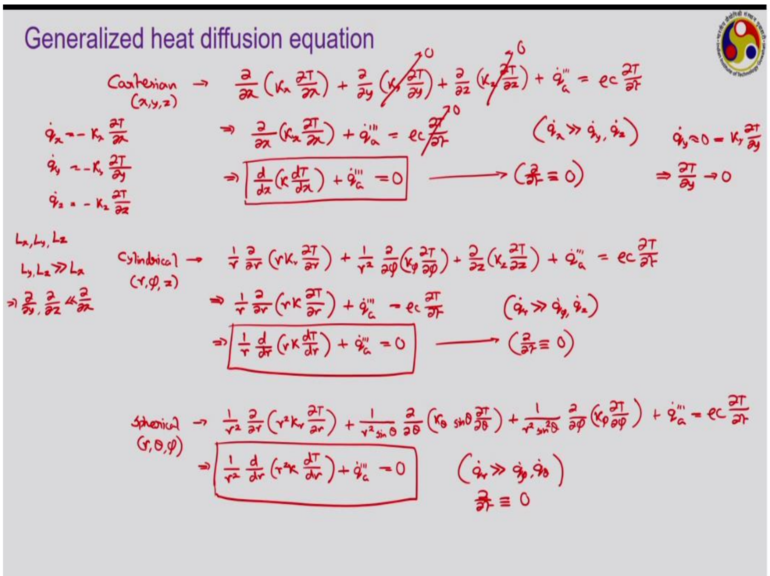


In the previous week, we have developed the basic heat diffusion equation corresponding to all the 3 coordinate systems, the Cartesian, cylindrical and spherical coordinate systems.

Now each of them has their own domain of applications and the complete form of equations which we developed that of course has its own applicability. Before proceeding further, let us just summarize the equations that I derived in the previous week.

If I write first in the Cartesian or rectangular coordinate system; see in the Cartesian coordinate system, our basic heat diffusion equation was of the form

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right) + \left(\dot{q}'''_G\right) = (\rho c)\frac{\partial T}{\partial t}$$

Where $K_x$, $K_y$ and $K_z$ refers to the thermal conductivity in all the 3 coordinate direction and $\dot{q}'''_G$ is the volumetric rate of energy generation or rate of heat generation inside the system per unit volume.

And the term on the right hand side is the transient term which refers to the time rate of change of energy content of the system. If we write the same thing in cylindrical coordinate system, then you will be getting the one that we have developed as

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left[rK_r \frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[K_\varphi \left(\frac{\partial T}{\partial \varphi}\right)\right] + \frac{\partial}{\partial z}\left[K_z \frac{\partial T}{\partial z}\right] + \left(\dot{q}'''_G\right) = (\rho c)\frac{\partial T}{\partial t}$$

While in Cartesian coordinate system, our coordinate directions are x, y and z; in cylindrical coordinate system, it is r which is radius, then the azimuthal angle φ and of course the axial direction z. And accordingly we have written this equation. $K_r$, $K_\varphi$ and $K_z$ refer to again the thermal conductivity in the 3 directions and then we have the general form of the equation. And finally in the spherical coordinate system where our correlations are r, θ and φ along with radius we have both the polar and azimuthal angles. Then, it becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 K_r \frac{\partial T}{\partial r}\right] + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left[K_\theta sin\theta \frac{\partial T}{\partial \theta}\right] + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \varphi}\left[K_\varphi \frac{\partial T}{\partial \varphi}\right] + \left(\dot{q}'''_G\right) = (\rho c)\frac{\partial T}{\partial t}$$

Now, these are the 3 equations that we have developed in the last week. But we have not talked about their applications or we have not solved them to simulate any real life situation because these equations are very complicated. Firstly, they are partial differential equations. Secondly, you can see them they are dependent on both space and time. So, there are 4 independent variables; one is time and then the 3 space coordinates.

Then, we have the direction dependence of thermal conductivity, we have the volumetric energy generation and overall it is a very complicated situation and solving them is definitely very complicated. Thankfully, in several real life situations in several engineering applications, we can visualize the system to be something like having a one dimensional heat transfer.

While practically all systems are three dimensional in nature but one dimensional refers the rate of heat transmission in one direction is significantly larger than the rate of heat transmission in the other two directions. Like if I talk about the Cartesian one, then from the general equation we know that there are 3 heat conduction equations we can get.

$$\dot{q}_x = -K_x \frac{\partial T}{\partial x}$$

$$\dot{q}_y = -K_y \frac{\partial T}{\partial y}$$

$$\dot{q}_z = -K_z \frac{\partial T}{\partial z}$$

Now, these are the 3 components of the power transmission or heat conduction in the 3 directions you can say. Now, if the situation is that your $\dot{q}_x$ is way greater than $\dot{q}_y$ and $\dot{q}_z$, then it is very logical that we can neglect the second two terms, we can neglect any heat conduction in the y and z direction and consider only the heat conduction in the x direction. And if that is a scenario$\left(\dot{q}_x \gg \dot{q}_y, \dot{q}_z\right)$, the heat diffusion equation becomes

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right) + \left(\dot{q}'''_G\right) = (\rho c)\frac{\partial T}{\partial t}$$

And along with that if we assume it to be steady state, then what we mean by steady state, there is no variation with time; that means any $\frac{\partial}{\partial t}$quantity can be said to be equal to 0.

In that case, your equation becomes even simpler. Now, see in the first line when we have written the generalized heat diffusion equation in Cartesian coordinate, we had 4 independent variables x, y, z and also time. Now, as we have neglected the variation in y and z directions, then there is only two independent variables in the second equation x and t. And now once we are going for steady state, the time is also not there. Then, we are left with only a single independent variable which is x and accordingly this equation which is a partial differential equation gets converted to an ordinary differential equation of the form

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) + \left(\dot{q}'''_G\right) = 0$$

This is when we are talking about a steady state situation. Here, we have dropped the subscript x in the definition of K because $K_y$ and $K_z$ are not coming into picture.

We have to bother about heat conduction only in the x direction and therefore $K_x$ is the only one that is of our importance. If K is independent of x coordinate, that becomes even simpler but we do not need to go into that situation for the moment. Now, the question is in what kind of scenario, we can go for such kind of approximation. There are several cases like as I have mentioned heat conduction in y and z directions are negligible compared to that in the x direction.

Now, that is possible when the temperature gradient is 0. Now, in what kind of situation, temperature gradient is 0? Let us say we talk about $\dot{q}_y$. For this $\dot{q}_y$to be equal to 0, we need to have this $K_y\frac{\partial T}{\partial y}$ to be equal to 0. Now, $K_y$ is a property that is thermal conductivity, so it cannot be 0. Then, in that situation, we must have this temperature gradient in the y direction to be tending to 0.

Now, when that is possible, one possibility is there is a uniform temperature field acting in the y direction, other possibility is the y itself or the length scale in the y direction itself is extremely large. The length scale in the y direction is extremely large compared to the length scale in the x direction. Then, this is also possible. Like if we are talking about a system having length of $L_x$ in the x direction, $L_y$ in the y direction and $L_z$ in the z direction.

And if suppose $L_y, L_z \gg L_x$, then we can virtually write that any $\frac{\partial}{\partial y}$ quantity and $\frac{\partial}{\partial z}$quantity will be extremely small compared to any gradient in the x direction$\left(\frac{\partial}{\partial x}\right)$ because of the difference in the length scale. And that is the most common situation where we can neglect any variation, particularly temperature in this case in the y and z direction. So the temperature gradient in the x direction becomes the only important one to us.

And accordingly, heat conduction in x direction is the one that we have to consider in detail. A very common example just look about, look into the wall of the room where you are sitting. Just think about the wall. Commonly your wall may be having a height of 3 m, may be having a width of 3 m, 5 m, 10 m depending upon where you are sitting. Like common houses can easily have a 3 mX3 m walls.

And if you are sitting in a big classroom, then the length can be even larger, 8 to 10 m can easily be the length of the wall on one side and similarly the height is 3 m or something like that but what about the width? What about the thickness of the wall. The thickness of the wall can be something like say 25 cm, 50 cm.

That is we are talking about a dimension of $L_y$ say in the y direction, your length is 5 m; in the Z direction that is in the vertical direction, your length this 3 m whereas in the x direction your length is say 50 cm that is 0.5 m, which is significantly smaller compared to the other two. And 50 cm is also quite thick walls that we are talking about. Common household walls may be even thinner than this, something like 25 cm or even lighter.

And in that case, therefore the length scale in the x direction is substantially smaller compared to the length scale in the y and z direction. Accordingly, we can say for this kind of scenario, the gradient in the x direction has to be much larger than the gradient in the y and z direction and therefore heat conduction through your building wall can easily be treated as a one dimensional problem.

Similarly, several other examples also we shall be seeing as we shall be solving numerical problems later on and you also be getting several scenarios in your books.

Now let's look about the cylindrical coordinate system. The general equation that we have, suppose we are talking about a situation where heat transmission in the radial direction is the only one of importance. Like just think about a common scenario, we are having a cylindrical rod; just a common rod or a pipe which is having a length of something like 10 m but its diameter is something like say 10 cm. Then, we are having 10 m on one side and 0.1 m on the other side and definitely the gradient in the r direction, the radial direction has to be significantly larger than the gradient in the z direction and quite often we neglect the $\varphi$ variation also.

I have probably mentioned the term, we call it axisymmetric. That is a variation in the φ direction can be neglected. So, in that scenario, the one dimensional unsteady form of the heat diffusion equation in cylindrical coordinate becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left[rK\frac{\partial T}{\partial r}\right] + \left(\dot{q}'''_G\right) = (\rho c)\frac{\partial T}{\partial t}$$

Again, I am dropping the subscript in the expression for K because I am talking about conduction only in the r direction. This is possible when the heat conduction in the r direction is significantly larger than the heat conduction in the φ and z direction. If we are further talking about a steady state situation, then we can convert this one to an ordinary differential equation which is of the form

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left[rK\frac{dT}{dr}\right] + \left(\dot{q}'''_G\right) = 0$$

Similarly, in the spherical coordinate system, again we can neglect the variation in the φ and θ direction. Just consider a ball, a solid sphere may be. In that case, quite often the variation in the φ and θ direction may be neglected and the major variation that you get is only in the radial direction. So, if I directly write the expression for spherical coordinate system, heat conduction only in the radial direction; and under steady state then this equation becomes

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2K\frac{dT}{dr}\right] + \left(\dot{q}'''_G\right) = 0$$

This is when your $\dot{q}_r$ is significantly greater than heat conduction in the φ and θ direction and any time variation can be neglected. And in this particular chapter in this module, we shall be talking primarily about this one dimensional steady state form of the heat conduction equation because they are extremely important in several engineering applications and secondly they are very easy to analyze or they are very easy to model mathematically.

Of course, you can see that the partial differential equation has now got converted to an ordinary differential equation, so it can be solved very easily. Secondly, there is an alternate approach where we shall be using the concept of thermal resistance where we do not need to go for the solution also; we can do it in even easier way. So, let us start with the Cartesian coordinate system.

Another point I am mentioning here, the $\dot{q}'''_G$, the volumetric heat generation rate I have retained in all the 3 equations here (Cartesian, cylindrical, spherical). I have written this one here, the volumetric heat generation rate but I am not going to consider this one for this week's lecture. That is this $\dot{q}'''_G$ also be will be set to 0 for the moment. Next week, I shall be including this one back and doing further analysis for that.

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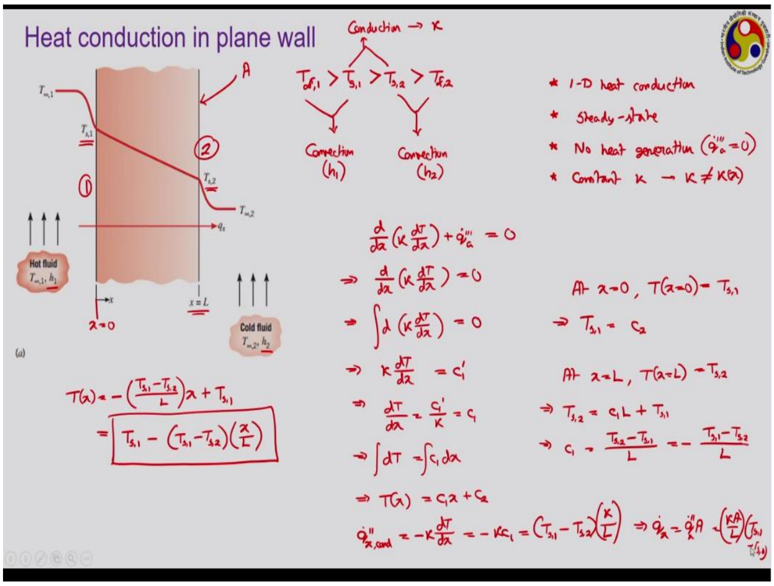


So, we are going to start with heat conduction in plane wall. Plane wall is synonymous to the Cartesian coordinate system and we are talking it as plane wall because the wall, building wall is the most simple example or straight forward example of 1-D heat conduction in Cartesian coordinate. So, look at the situation what is given. We are talking about a wall, let us say this is your x = 0, this is x = L. So, the thickness of the wall is L.

Let us say the area of each of the faces of the wall is equal to A. One side of the wall is kept at temperature $T_{s,1}$, other side is kept at the temperature $T_{s,2}$ and fluids air or some other kind of fluid is flowing on both the surfaces, one side you are having a hot fluid flowing with temperature $T_{\infty,1}$ and other side it is flowing with temperature $T_{\infty,2}$. It is given that the hot fluid side temperature $T_{\infty,1}$ is greater than cold fluid side temperature $T_{\infty,2}$. $(T_{\infty,1} > T_{\infty,2})$

So, you can expect the heat to get transferred from the hot fluid to the left face of the wall. Let us say the face number 1, and what should be the mode of heat transfer from the hot fluid to the face 1 of the wall? That has to be convection because we are talking about the flowing fluid over the solid surface. So, via convection heat gets transferred from the hot fluid to the face 1 of this, then 1 to face 2 via conduction because it is a solid wall.

And then from face 2 to the cold fluid which is maintained at temperature $T_{\infty,2}$ via convection again. Corresponding convective heat transfer coefficients are given as $h_1$ and $h_2$. But before taking into account the heat convection with hot and cold fluids, let us just stick to the conduction part. $T_{s,1}$ and $T_{s,2}$ are the temperatures of the two faces of the wall. So, using our common sense, we can say that as heat always gets transferred from high temperature to low temperature.

So, $T_{\infty,1}$ has to be greater than surface temperature for face 1, $T_{s,1}$ which has to be greater than $T_{s,2}$ and that has to be greater than $T_{\infty,2}$.

$$\left(T_{\infty,1} > T_{s,1} > T_{s,2} > T_{\infty,2}\right)$$

So, from this to this, the mode of heat transmission is convection. Similarly, from this to this, the mode of heat transmission is again convection. The second one is characterized by heat transfer coefficient of $h_2$. This is characterized by heat transfer coefficient of $h_1$. However, our first interest is this one $T_{s,1}$ to $T_{s,2}$, ;which is done by conduction.

So, you have considered A to be the area of the wall and we are considering a thermal conductivity say K in this x direction. I am not putting any subscript x because we are talking again about 1- D heat conduction. Then, what are the conditions that we are imposing? Firstly, 1- D heat conduction; secondly, steady state. So, considering this, we have already developed the equation. Then, I am putting a third condition which I verbally mentioned; there is no heat generation and fourth we are considering the thermal conductivity to be constant, which refers that K is not a function of x. That is an additional constraint that I am putting in. So, what was the equation that I developed in the previous slide? That was

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) + \left(\dot{q}'''_G\right) = 0$$

There is no heat generation. So, we have now

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) = 0$$

So, we have to integrate this equation with respect to x to get the solution. Then, what we have?

$$\Rightarrow \int d\left(K\frac{dT}{dx}\right) = 0$$

$$\Rightarrow K\frac{dT}{dx} = C'_1$$

$$\Rightarrow \frac{dT}{dx} = \frac{C'_1}{K} = C_1$$

$$\Rightarrow \int dT = \int C_1 dx$$

$$\Rightarrow T(x) = C_1 x + C_2$$

Here, $C'_1, C_1, and\ C_2$ are constants. A very straightforward linear form of equation that is temperature variation within the body in the x direction will be linear and exact nature of the profile will be dependent upon these two constants $C_1$ and $C_2$. Now, how to identify $C_1$ and $C_2$? We are given with the temperature $T_{s,1}$ and $T_{s,2}$. So, let us make use of them.

$$At\ x = 0, \qquad T = T_{s,1}$$

So, if I put this one then

$$\Rightarrow T_{s,1} = C_2$$

Similarly,

$$At\ x = L, \qquad T = T_{s,2}$$

So, we have

$$\Rightarrow T_{s,2} = C_1 L + C_2$$

Now, just we have got $T_{s,1} = C_2$. That is C1 now becomes

$$C_1 = \frac{\left(T_{s,2} - T_{s,1}\right)}{L}$$

We know that $T_{s,1}$ is higher than Ts, 2. So, it is better that we write this one as

$$C_1 = -\frac{\left(T_{s,1} - T_{s,2}\right)}{L}$$

So, what becomes our temperature profile then,

$$\Rightarrow T(x) = -\left(\frac{T_{s,1} - T_{s,2}}{L}\right) x + T_{s,1}$$

And there are several ways this one can be rearranged but probably it is better to express this one as

$$\Rightarrow T(x) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\frac{x}{L}$$

So, this is the temperature profile that we are getting through this wall and once we know that two end temperatures $T_{s,1}$ and $T_{s,2}$ then we shall easily be able to calculate this one.

It is a straight line joining $T_{s,1}$ and Ts, 2. Now, if our interest is to know the heat transmission rate, the rate of conduction heat transfer, that also we can easily calculate. Say, if we want to calculate the rate of heat transfer, then we know that

$$\dot{q}"_{x,cond} = -K\frac{dT}{dx}$$

$\dot{q}_{x,cond}$ is in the x direction following conduction only. Then, we are having this as

$$\dot{q}"_{x,cond} = -KC_1$$

putting expression for $C_1$

$$\dot{q}"_{x,cond} = -(T_{s,1} - T_{s,2})\frac{K}{L}$$

This is the heat flux, and finally if we want to have the total heat conduction, then that is equal to heat flux into the area that is

$$\dot{q}_{x,cond} = A \times \dot{q}"_{x,cond} = -(T_{s,1} - T_{s,2})\frac{KA}{L}$$

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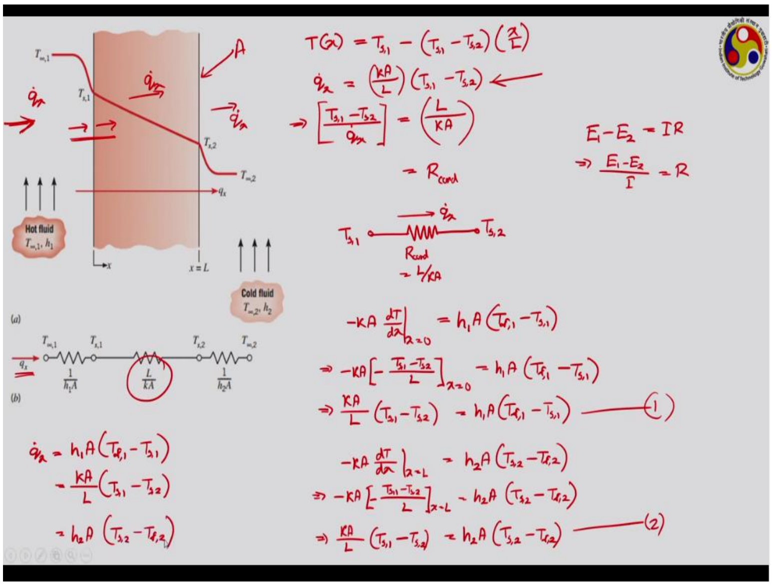


Let us proceed with this. So, we have just developed that the temperature profile inside your solid has become

$$\Rightarrow T(x) = T_{s,1} - (T_{s,1} - T_{s,2})\frac{x}{L}$$

And the conduction heat transmission in the x direction we have got to be equal to

$$\dot{q}_x = -(T_{s,1} - T_{s,2})\frac{KA}{L}$$

If we rearrange this second equation a bit and write this one as

$$\Rightarrow -\frac{(T_{s,1} - T_{s,2})}{\dot{q}_{x,cond}} = \frac{L}{KA}$$

Now, how this particular form is looking like. In the left hand side in the numerator we have the temperature difference. In the denominator, we have the conduction heat transmission rate or conduction heat transfer rate. Then, how the numerator is looking like? That is the potential difference because of which the heat transfer is taking place. What about the denominator? Denominator is the result of this potential difference. Then how this form is looking like? In the first week itself, we have introduced the concept of thermal resistance. So, can you relate this one to that? On the left hand side, we have the potential difference on the numerator and the effect of that in the denominator and we can easily relate this to the Ohm's law of electricity where we can easily write.

Suppose, if $E_1$-$E_2$ refers to a potential difference between two locations 1 and 2, then corresponding result will be equal to

$$E_1 - E_2 = IR$$

Where, I is current and R is resistance. Or,

$$\frac{E_1 - E_2}{I} = R$$

Just similarly whatever we are getting there this is also called the conduction resistance.

$$\Rightarrow -\frac{(T_{s,1} - T_{s,2})}{\dot{q}_{x,cond}} = \frac{L}{KA} = R_{cond}$$

where L is the length scale associated with this conduction heat transfer. That is the thickness of this particular block, K is the thermal conductivity and A is the area of this that is perpendicular to the plane of this screen. This area basically is we are talking about something perpendicular to the screen. So, we can virtually think about using an electrical analogy that on one side of the wall you are having potential $T_{s,1}$; on the other side having potential $T_{s,2}$.

And in between you are having a thermal resistance of $\frac{L}{KA}$ and accordingly some heat transmission $\dot{q}_x$ will be happening between $T_{s,1}$ and $T_{s,2}$. And just like shown here then we can easily draw an electrical circuit where this side you have the potential as $T_{s,1}$. This side you have the potential as $T_{s,2}$ and in between you have this conduction resistance which we have just got as $\frac{L}{KA}$in Cartesian system.

And accordingly we can get the value of $\dot{q}_x$. So, once we know that these two temperatures and we have the information about the thermal conductivity and dimensions of the wall, you can directly calculate the heat transmission that is $\dot{q}_x$without directly solving the ordinary differential equation. But now quite often the situation is that we do not know directly the value of $T_{s,1}$ and $T_{s,2}$.

Rather our information available is about hot fluid and the cold fluid. That is we know the hot fluid temperature $T_{\infty,1}$; we know the cold fluid temperature $T_{\infty,2}$ and we also know their heat transfer coefficients $h_1$ and $h_2$; but we do not know $T_{s,1}$ and $T_{s,2}$. Then, somehow we have to estimate the value of $T_{s,1}$ and $T_{s,2}$ and then only we can go for this electrical analogy to get the value of $\dot{q}_x$.

Then, how can we estimate $T_{s,1}$ and $T_{s,2}$. So, first we talk about the left hand side. So, at this particular juncture, on this side your mode of heat transfer is convection; on this side your mode of heat transfer is conduction. As we are talking about only one dimensional heat conduction, then entire energy transmission direction is this one only. So, heat is getting transmitted only in this direction.

Now, as the system is under steady state, there is no temperature variation with respect to time at a particular location. That indicates that the system has to be in some kind of balance and the wall is not having any net energy storage. Also, there is no heat generation inside the wall. Then, to maintain this energy balance, it has to be satisfied that the rate of convective heat transfer in this from the hot fluid to the wall in this direction has to be balanced by the amount of energy conducted from the wall to the interior of the wall, so that the temperature remains constant with respect to time. This is just the convective boundary condition nothing else. So, let us apply the convective boundary condition on the hot fluid side.

Then, the amount of conduction happening is

$$\Rightarrow KA\left.\frac{dT}{dx}\right|_{x=0} = h_1 A\left(T_{\infty,1} - T_{s,1}\right)$$

Using Newton's law of cooling and putting conduction equal to convection.

Now, what is $\frac{dT}{dx}$? In the previous slide, we have seen that

$$\Rightarrow \frac{dT}{dx} = C_1$$

So,

$$\Rightarrow KA\frac{dT}{dx}\bigg|_{x=0} = -KA\left[-\frac{(T_{s,1}-T_{s,2})}{L}\right]_{x=0} = h_1A(T_{\infty,1}-T_{s,1})$$

It is redundant to write x = 0 in second term because this gradient is constant. It is independent of x. So, we can now solve this, we can rearrange this entire equation.

So, we have

$$\Rightarrow \frac{KA}{L}(T_{s,1}-T_{s,2}) = h_1A(T_{\infty,1}-T_{s,1})$$

Similarly, if we write the convective boundary condition at x =L. Then we can write

$$\Rightarrow KA\frac{dT}{dx}\bigg|_{x=L} = h_2A(T_{s,2}-T_{\infty,2})$$

$$\Rightarrow KA\frac{dT}{dx}\bigg|_{x=L} = -KA\left[-\frac{(T_{s,1}-T_{s,2})}{L}\right]_{x=L} = h_2A(T_{s,2}-T_{\infty,2})$$

Or just arranging the same way, we have

$$\Rightarrow \frac{KA}{L}(T_{s,1}-T_{s,2}) = h_2A(T_{s,2}-T_{\infty,2})$$

So, now we have got two equations. Once we have idea about the information say $h_1$, $h_2$, K, A, L and also $T_{\infty,1}$ and $T_{\infty,2}$ then we can solve this equations to get $T_{s,1}$ and $T_{s,2}$ from this. But is there at all any need to go for such complicated expressions?

Look at this; both your equation 1 and equation 2 are having the same term on the left hand side and what is this term? If I go back this term is this one only, that is the rate of heat conduction which is also logical because whatever amount of heat that gets transmitted from the hot fluid, the same amount of heat under steady state will be flowing to the other wall, other face of the wall.

And then the same amount of heat will be passing onto the cold fluid that is this $\dot{q}_x$ remains the same as we are talking about a simple 1-D heat conduction under steady state. That means we can write this rate of heat transmission $\dot{q}_x$ on the hot fluid side, this is equal to

$$\dot{q}_x = h_1A(T_{\infty,1}-T_{s,1})$$

Then, interior to the body, interior to the solid block you have

$$= \frac{KA}{L}(T_{s,1}-T_{s,2})$$

And again on the cold fluid side, it is equal to

$$= h_2 A\left(T_{s,2} - T_{\infty,2}\right)$$

Now, if we rearrange all of them again, we know that $\dot{q}_x$ can be written this way.

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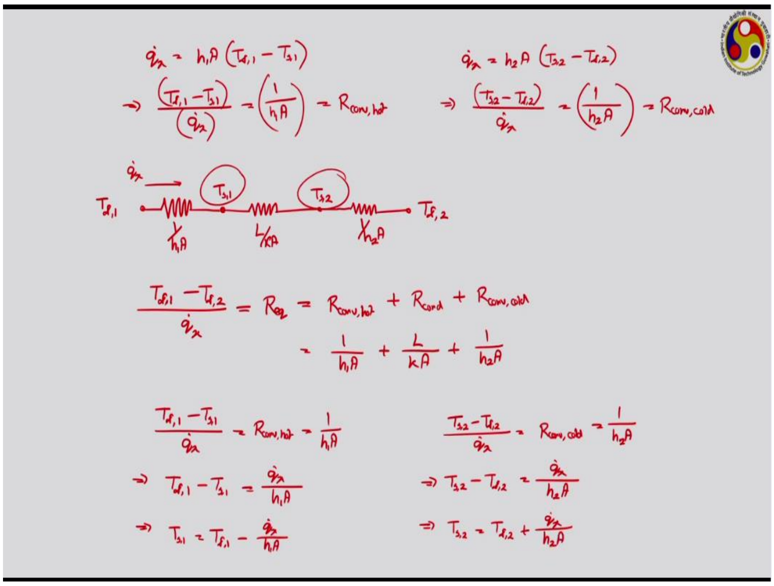

Now, for the hot fluid side, we have

$$\dot{q}_x = h_1 A\left(T_{\infty,1} - T_{s,1}\right)$$

Or if we just write the way we got the conduction resistance, it will be equal to

$$\Rightarrow \frac{\left(T_{\infty,1} - T_{s,1}\right)}{\dot{q}_x} = \frac{1}{h_1 A} = R_{conv,hot}$$

Now, what is this bracketed term? This is just the temperature difference required for the convective heat transfer. The denominator is a result of this potential difference.

Then, this is nothing but your convective resistance on the hot fluid side. Similarly, on the cold fluid side if we write

$$\dot{q}_x = h_2 A\left(T_{s,2} - T_{\infty,2}\right)$$

That is if we rearrange the same way

$$\Rightarrow \frac{\left(T_{s,2} - T_{\infty,2}\right)}{\dot{q}_x} = \frac{1}{h_2 A} = R_{conv,cold}$$

So, again this case, this is your convective resistance on the cold fluid side and in between we have the conduction resistance also which we have got here.

Then, how we can visualize the system? I have already the diagram shown here but still I am writing this step by step. So, to start with we have a potential energy source maintained at temperature $T_{\infty,1}$. Then, via convection it is transmitting energy to another node maintained at temperature $T_{s,1}$ and this heat transfer is happening through a resistance, a convective resistance which is $\frac{1}{h_1 A}$ and $\dot{q}_x$ amount of heat is getting transmitted.

So, once this heat reaches $T_{s,1}$ then this is getting transmitted through a conduction resistance to another node maintained at $T_{s,2}$ and this conduction node is having a resistance which we have got here as $\frac{L}{KA}$ and then from this $T_{s,2}$ again via convection, energy is getting transmitted to the low temperature side which is maintained at $T_{\infty,2}$ and this heat transfer is getting facilitated to another convective heat transfer resistance that is $\frac{1}{h_2 A}$.

Then, if we just want to have a direct relationship between heat transmission from $T_{\infty,1}$ to $T_{\infty,2}$ then how we can do this? Just neglect the intermediate temperatures. Then, we can directly think about that $\dot{q}_x$ amount of heat is getting transmitted from $T_{\infty,1}$ to $T_{\infty,2}$ and that is happening through 3 different resistances. That is we can write now a generalized expression of heat transmission. We can visualize that there are 3 resistances, 2 convective resistances and 1 conduction resistance in between and these resistances are connected in series here because the same heat transmission rate is applicable for all of them.

But the temperature difference across each of them that is different. That is the heat transmission rate is same for all of them but the potential difference across each of them is different. Therefore, we can say that they are connected in series. Very similar to the electrical systems and now when we are having a several resistances connected in series, how we get the equivalent resistance? We just add them up. So, just the same thing we can do here. We can say

$$\Rightarrow \frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\dot{q}_x} = R_{eq} = R_{conv,hot} + R_{cond} + R_{conv,cold}$$

$$= \frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}$$

So, once we know each of the individual resistances, we can easily get the equivalent resistance. And we can directly calculate the value of $\dot{q}_x$ without needing to know the values of $T_{s,1}$ and $T_{s,2}$. We just need to know the two end temperatures and all the resistances. Then, this $T_{s,1}$ and $T_{s,2}$ are never coming into picture. We can directly get the value of $\dot{q}_x$ using only the two end temperatures. Now, if once we know the value of $\dot{q}_x$, then if we want to calculate the value of $T_{s,1}$ how can we do this?

So, just consider the first convection part. In that case like we wrote earlier

$$\Rightarrow \frac{\left(T_{\infty,1} - T_{s,1}\right)}{\dot{q}_x} = R_{conv,hot} = \frac{1}{h_1 A}$$

we can separate out the terms from here,

$$\Rightarrow \left(T_{\infty,1} - T_{s,1}\right) = \frac{\dot{q}_x}{h_1 A}$$

$$\Rightarrow T_{s,1} = T_{\infty,1} - \frac{\dot{q}_x}{h_1 A}$$

Similarly, if we want to calculate $T_{s,2}$, then we can do that using the convective heat transfer on the cold fluid side. For that case, we can write

$$\Rightarrow \frac{\left(T_{s,2} - T_{\infty,2}\right)}{\dot{q}_x} = R_{conv,cold} = \frac{1}{h_2 A}$$

$$\Rightarrow \left(T_{s,2} - T_{\infty,2}\right) = \frac{\dot{q}_x}{h_2 A}$$

$$\Rightarrow T_{s,2} = T_{\infty,2} + \frac{\dot{q}_x}{h_2 A}$$

So, we do not need to know the values of the two faces, $T_{s,1}$ and $T_{s,2}$ to get the $\dot{q}_x$, rather using the value of $\dot{q}_x$ we are getting their values later on. So, this way any one dimensional steady state heat conduction scenario at least in Cartesian coordinate can easily be reduced to such kind of series of conduction and convection resistances.

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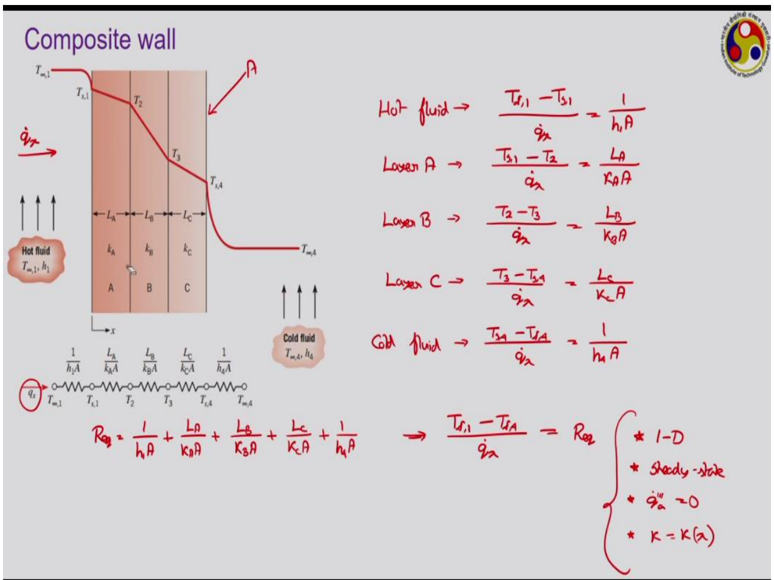

Just look at this, something known as a composite wall. Here instead of 1, there are 3 layers of solids inside the wall. Each of them are having different thermal conductivities. So, we can think about there are 3 different conduction resistances acting in series with each other. Look at the first part the block A of the solid. The temperature is $T_{s,1}$ at one side and $T_2$ on the other side.

It is 1-D and steady state. Please remember that whatever we are discussing that is applicable only for one dimensional steady state heat conduction. Then, if we consider that $\dot{q}_x$ is the amount of heat transmission that is taking place by conduction, then for block A we can directly write using the concept of thermal resistance. Let us start from the hot fluid side. So, if we write for hot fluid side, then we can write

$$\Rightarrow \frac{\left(T_{\infty,1} - T_{s,1}\right)}{\dot{q}_x} = \frac{1}{h_1 A} = R_{conv,hot}$$

Let us say A refers to the area of this wall, then the first layer, the layer number A, it is having subjected to conduction, so

$$\frac{\left(T_{s,1} - T_2\right)}{\dot{q}_x} = \frac{L_A}{K_A A}$$

Then, layer B

$$\frac{(T_2 - T_3)}{\dot{q}_x} = \frac{L_B}{K_B A}$$

Then, we have the third layer, layer C it is having

$$\frac{\left(T_3 - T_{s,4}\right)}{\dot{q}_x} = \frac{L_C}{K_C A}$$

So, this is this third conduction resistance we have. And finally on the cold fluid side, we are having convection.

$$\frac{\left(T_{s,4} - T_{\infty,4}\right)}{\dot{q}_x} = \frac{1}{h_4 A}$$

And if we want to write in overall sense, then we can directly write that R equivalent is equal to the summation of all the 5 resistances that is

$$R_{eq} = \frac{1}{h_1 A} + \frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A} + \frac{1}{h_4 A}$$

And then using that idea we can directly write the overall temperature difference

$$\frac{\left(T_{\infty,1} - T_{\infty,4}\right)}{\dot{q}_x} = R_{eq}$$

Then, from the knowledge of these two fluid temperatures $T_{\infty,1}$ and $T_{\infty,4}$ and from each of these resistances, you can calculate the value of $\dot{q}_x$.

And then using $\dot{q}_x$, you can easily calculate all these intermediate temperatures $T_{s,1}$; $T_{s,4}$; $T_2$; $T_3$ etc. They all can be calculated. So, this way we can connect several resistances in series and we can do the analysis very easily without bothering about the need to solve any ordinary differential equation even. But I keep on repeating there are several assumptions that we are considering.

The assumptions which I wrote earlier I am just repeating it here again. This is applicable strictly for one dimensional heat conduction and under steady state, no heat generation and the thermal conductivity is also a constant. Only when all these 4 conditions are satisfied, then only we can go for such kind of thermal resistance concept and we can easily form resistance circuits using the electrical analogy.

*1-D

*steady-state

*$\dot{q}'''_G = 0$

*$K = K(x)$

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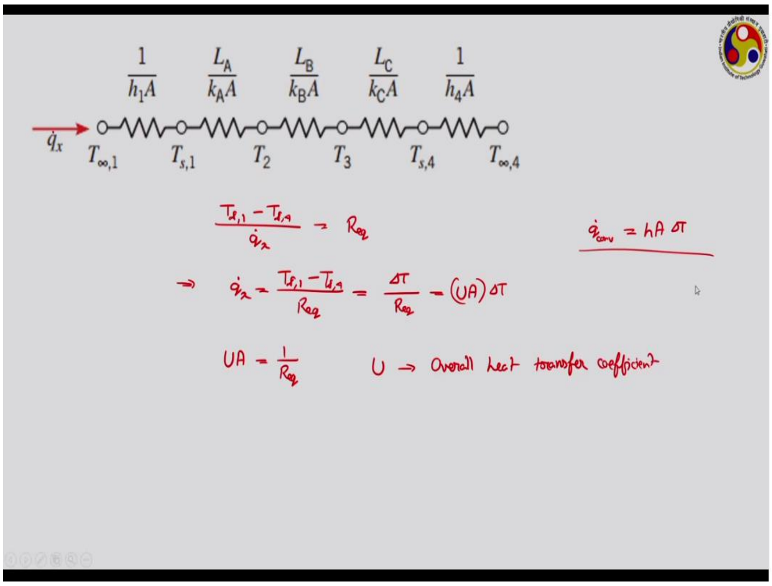


So, this is just the one that I talked about earlier. So, in this case as I have written

$$\frac{(T_{\infty,1} - T_{\infty,4})}{\dot{q}_x} = R_{eq}$$

Quite often instead of writing this way, we write in another alternate form just somewhat to be consistent with the Newton's law of cooling. In case of convection, how we represent, $\dot{q}_{conv}$ you know, we always write as

$$\dot{q}_{conv} = hA\Delta T$$

We have seen how we can represent the radiation also following this form. For conduction, we generally do not go for a form like this but when we are talking about such an equivalent circuit, occasionally we would like to have a form somewhat analogous to this particular form of Newton's law of cooling. So, if we write now

$$\dot{q} = \frac{T_{\infty,1} - T_{\infty,4}}{R_{eq}}$$

Remember what we are trying to do here that is applicable only for this combination of resistances, the total circuit, not on individual resistances because for individual cases the mode can be conduction or convection, we can analyze them separately but only for the overall circuit we are trying to identify this. So,

$$\dot{q} = \frac{T_{\infty,1} - T_{\infty,4}}{R_{eq}} = \frac{\Delta T}{R_{eq}}$$

And if we compare this one with the Newton's law of cooling, then we can write this one as something like

$$= (UA)\Delta T$$

Where,

$$UA = \frac{1}{R_{eq}}$$

And this U is known as overall heat transfer coefficient, just what I was talking about.

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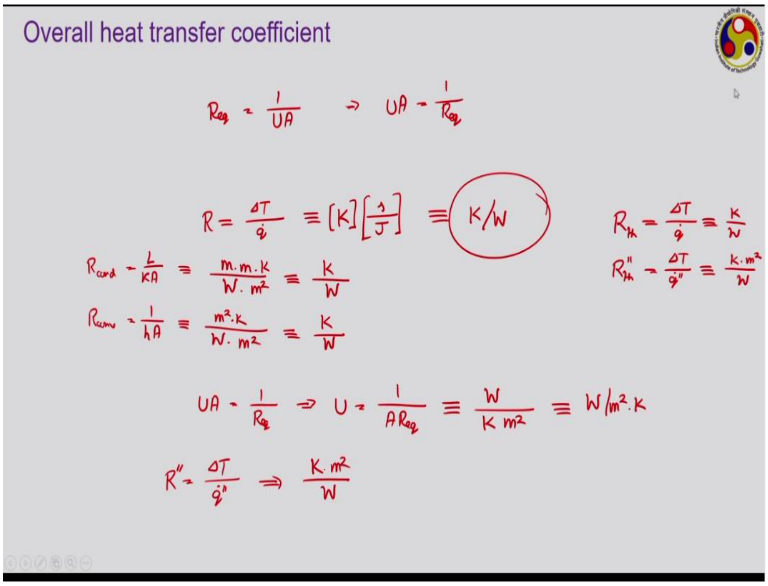


U is the overall heat transfer coefficient and A is the area at which this heat transfer coefficient is defined. As long as we are dealing with planar area like this, the choice of the area does not matter because all surfaces are having the same area.

But as we shall be moving to the cylindrical and spherical systems in the next class, then you will see that the area can keep on changing with the change in radius and therefore the value of U that you are defining that depends upon the choice of the area; while the UA product remains the same but the value of U can vary depending upon the magnitude of the area itself. Now, before I complete here what is the dimension of this U?

Okay, before going to the dimension of U, let us try to get the dimensions of this R, the thermal resistance. What will be the dimensions of R? We know

$$R_{eq} = \frac{\Delta T}{\dot{q}}$$

So, from there can we form? $\Delta T$ is Kelvin and what about $\dot{q}_x$, that is the heat transfer rate which is Watt, so that is Joule second. So, we can easily write in this form or generally go for Kelvin per Watt.

That is the dimension for this thermal resistance. You can easily form it from the others also. Like say for R conduction we know that it is $\frac{L}{KA}$. If we try to form it from there, L is a length scale so it is m, K is a thermal conductivity. What is the unit of thermal conductivity? We have developed this earlier. So, that is Watt per m Kelvin and A is area which is $m^2$.

So, equivalent from there we are getting Kelvin per Watt and if we try to do the same using the convective heat transfer thermal resistance $\frac{1}{hA}$. So, h you know has the unit of $W/m^2.K$ and area is $m^2$. So, we are getting it to be K/W. So, thermal resistance has an unit of K/W in SI scale or you can say temperature divided by heat flux as the dimension.

$$R_{cond} = \frac{L}{KA} \equiv \frac{m.m.K}{W.m^2} = \frac{K}{W}$$

$$R_{eq} = \frac{1}{hA} \equiv \frac{m^2.K}{W.m^2} = \frac{K}{W}$$

Then, what about this U? So, UA is equal to

$$UA = \frac{1}{R_{eq}} \Rightarrow U = \frac{1}{AR_{eq}}$$

So, from there if we trying to form the dimension R equivalent

$$UA = \frac{1}{R_{eq}} \Rightarrow U = \frac{1}{AR_{eq}} \equiv \frac{W}{Km^2} \equiv \frac{W}{m^2.K}$$

It's just very similar to the unit of the convective heat transfer coefficient and that is, that should be also.

Because the idea of this overall heat transfer coefficient comes only from the Newton's law of cooling and therefore this U is nothing but an analogous form of that h for overall circuit representation. So, its dimension is $W/m^2K$ and also one thing that I would like to mention here, though we do not go that route but sometimes this R, that we representing as $\frac{\Delta T}{\dot{q}}$, occasionally, instead of $\dot{q}$, we can also represent this one as $\dot{q}''$, or heat flux. In that case, the unit of R will be equal to $K.m^2/W$. Because heat flux is having an unit of $W/m^2$. But this is something that we shall not be doing. But in certain cases we use it, therefore to separate them out occasionally we also put in R". To indicate that, here the thermal resistance is defined in terms of heat flux.

So, I repeat again, R the thermal resistance generally put an $R_{th}$ or $R_{conv}$, $R_{cond}$ etc. So, this thermal resistance is defined as

$$R_{th} = \frac{\Delta T}{\dot{q}} \equiv \frac{K}{W}$$

And

$$R"_{th} = \frac{\Delta T}{\dot{q}"} \equiv \frac{K.m^2}{W}$$

So, this is about the overall heat transfer coefficient in the concept of thermal resistance and their combination. But we have so far seen how to connect the resistances in series but it is also possible to connect them in parallel, just like shown here.

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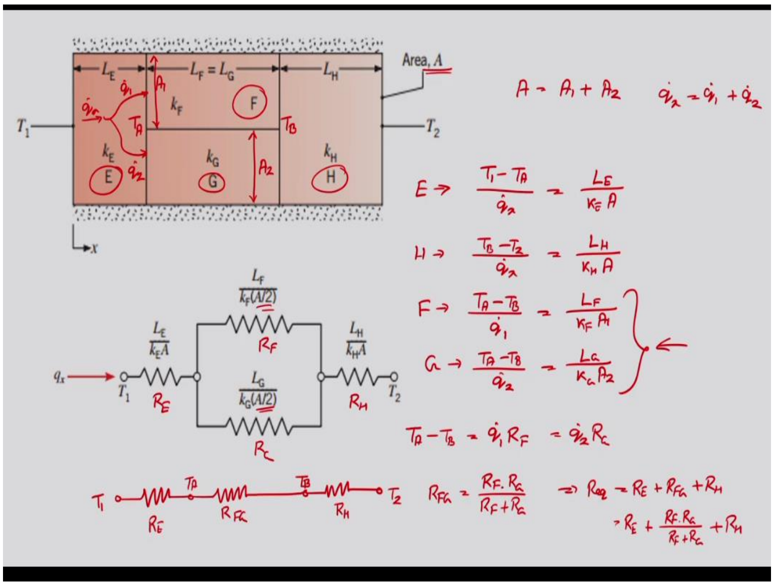


In series connection we have seen that the heat transmission rate through all the resistances will be equal but the effective temperature difference is different, whereas in case of parallel, the temperature difference has to be the same but the heat transmission rate will be different just like shown here. You can think about it is a wall having 3 layers.

This is your layer E which is having a thickness of $L_E$ and conductivity of $k_E$. The third layer the layer H is having a thickness of $L_H$ and thermal conductivity of $K_H$ but the intermittent layer that has been broken into two parts like you may have seen sometimes we are having walls say in laboratories or etc where lower part is made of concrete or made of some other material and the upper part is made of glass, it is something similar to this.

Here, the lower part and upper part are made of different materials. So, the lower part G having conductivity of $k_G$ and this is having, the layer F is having conductivity of $k_F$, the lengths are equal but their areas are different, while the overall area is this A. Let us say this area we mentioned about say A2 and this area we take as A1 such that

$$A = A_1 + A_2$$

The temperatures are given as $T_1$ and $T_2$ at the two extremes. Let us say here the temperature is $T_A$ and here the temperature is $T_B$. Then, for the layer E, how we can write the corresponding conduction resistance. In that case, the effective temperature difference is

$$= \frac{T_1 - T_A}{\dot{q}_x} = \frac{L_E}{K_E A}$$

Similarly, the last layer, layer H effective temperature difference is

$$= \frac{T_B - T_2}{\dot{q}_x} = \frac{L_H}{K_H A}$$

But the intermediate portion here the $\dot{q}_x$ now gets divided into two parts, one part flows through this F and other part flows through this G. So, if the $\dot{q}_x$ ,which is coming here that gets divided into two parts.

Let us say this is $\dot{q}_2$and this is $\dot{q}_1$ such that this $\dot{q}_x$ is equal to

$$\dot{q}_x = \dot{q}_1 + \dot{q}_2$$

Then, for the layer F what we have,

$$= \frac{T_A - T_B}{\dot{q}_1} = \frac{L_F}{K_F A_1}$$

Similarly, for the layer G

$$= \frac{T_A - T_B}{\dot{q}_2} = \frac{L_G}{K_G A_2}$$

So, these two resistances can be viewed to be in parallel just like shown here. Here, in this diagram it is shown as A/ 2 because they have assumed both $A_1$ and $A_2$ to be half of A. But I am writing in a general form which is $A_1$ and $A_2$. If we just further analyze the resistances corresponding to F and G, then from there we can write

$$= T_A - T_B = \dot{q}_1 R_F = \dot{q}_2 R_G$$

So, that same temperature difference is acting across them but the heat transmission rate is different. Now, how to get an equivalent resistance for this particular layer? Just like in electrical circuit we do that; the same we can do in this case. That is this resistance is your $R_E$, this is $R_H$, this is $R_F$ and this is $R_G$. So, first let us form an equivalent for this F and G part.

So, this is your $T_1$ resistance is $R_E$, this is $T_2$ resistance is $R_H$ and as per our notation this is TA, this is $T_B$ and this intermediate resistance is let us say $R_{FG}$. Now, what will be $R_{FG}$? We can get it from here or we can also use our electrical energy. So, from there we can directly write that it will be equal to

$$R_{FG} = \frac{R_F . R_G}{R_F + R_G}$$

Accordingly, the equivalent resistance for this entire circuit will be equal to

$$R_{eq} = R_E + R_{FG} + R_H$$

$$= R_E + \frac{R_F . R_G}{R_F + R_G} + R_H$$

So, this will be the equivalent resistance for this circuit where two resistances are connected in parallel. This way we can form conduction heat transfer circuits for one dimensional steady state scenario following any kind of series and parallel configurations. So, I would like to stop here today.

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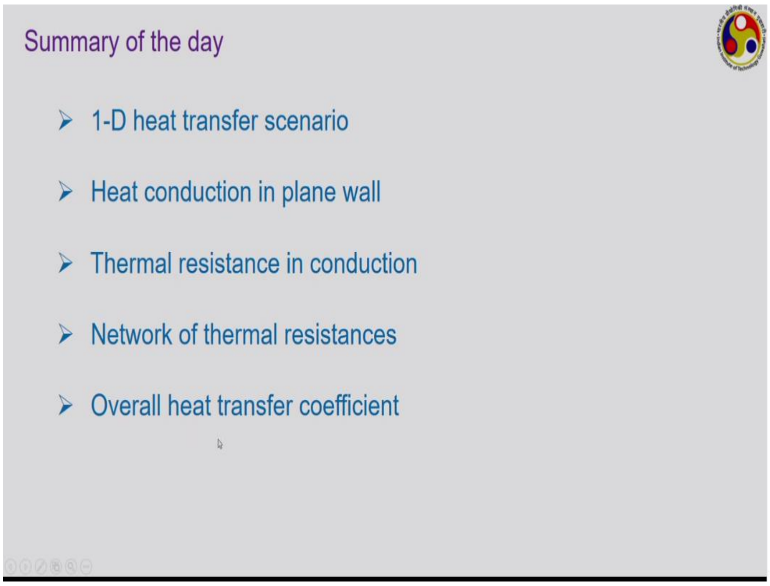


In the next class, I shall be introducing you the concept of contact resistance and then we shall be solving a few numerical problems to see the application of this conduction and convection resistances in case of Cartesian coordinate to demonstrate their application. So, to summarize our day, we have started with one dimensional heat transfer scenario, discussed the scope of that starting from the generalized heat diffusion equation in all 3 all 3 coordinate direction.

Then, we continued with the Cartesian one and discussed the heat conduction in plane wall under 1-D steady state condition. Then, the concept of thermal resistance and the network of thermal resistances by connecting the resistances in series and parallel were discussed and the overall heat transfer coefficient was also introduced. So, that is it for the day.

In the next class, as I have mentioned I shall be starting with this thermal resistances and their circuits and I shall be solving a few numerical examples to demonstrate their use. Thank you very much.