Fundamentals of Conduction and Radiation
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Lecture – 33
Radiation Exchange with Participating Media

Hello everyone, so today is the last lecture of this course. Today's module is radiation exchange with participating media. So when we consider the participating media and radiation is taking place, then we need to consider absorption, emission, in-scattering and out-scattering and with that we will do the energy balance and we will derive the radiative transfer equation.

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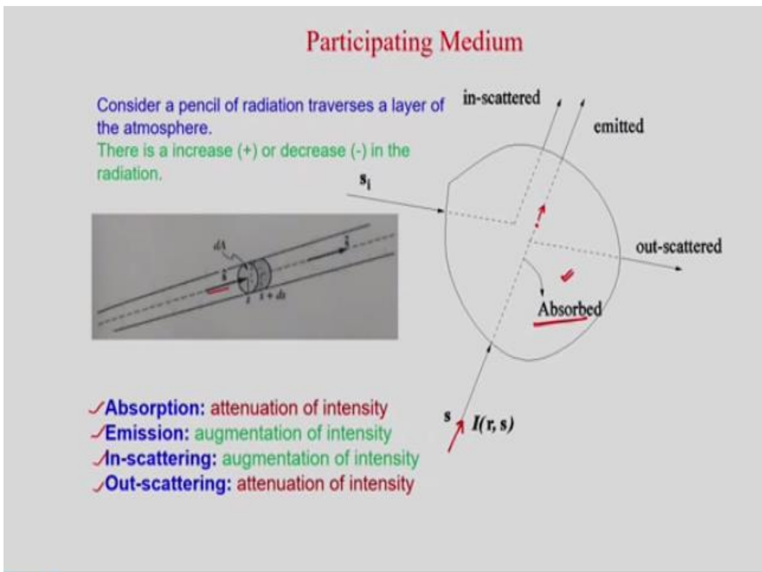

So, consider a pencil of radiation traverses a layer of atmosphere. So you can see that one beam is travelling in this direction s and it is travelling through this participating medium of distance ds. So when it is passing through this participating medium of distance ds obviously the absorption, emission, in-scattering and out-scattering will take place. So you can see that absorption and emission is the attenuation of intensity and in-scattering out-scattering are augmentation of the intensity. So you consider here let us say this is one participating media and in this direction s, your ray is passing through. So obviously, when it is passing through some energy will be absorbed by this media, so this is the absorbed part, some will be out scattered and obviously, it will attenuate the energy in that direction.

But obviously, it will increase the radiation in other direction. And obviously, due to its own temperature, it will emit energy so, it will augment. And from other direction, some radiation will come and will go as an in-scattering in this direction and it will gain the energy in the direction of s.

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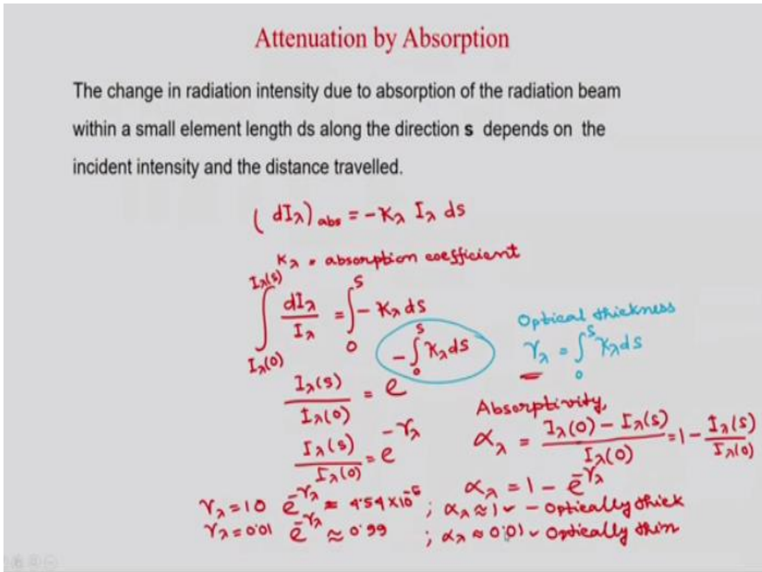

So, let us see that attenuation by absorption. So how the absorption takes place that we have already known that is the Beer's law. So the change in radiation intensity due to absorption of the radiation beam within a small element length ds along the direction s depends on the incident intensity and the distance travelled. So what we can write from the Beer's law? We know that

$$(dI_\lambda)_{abs} = -\kappa_\lambda I_\lambda ds$$

dI_λ which is part of absorption is directly proportional to the magnitude of the intensity and the distance travelled by the ray. So κ_λ is the proportionality constant and negative sign is coming as your radiation is attenuated. So κ_λ is your absorption coefficient. So, now what we can write

$$\frac{dI_\lambda}{I_\lambda} = -\kappa_\lambda ds$$

$$\Rightarrow \int_{I_{\lambda,0}}^{I_{\lambda,s}} \frac{dI_\lambda}{I_\lambda} = -\int_0^s \kappa_\lambda ds$$

$$\Rightarrow \frac{I_\lambda(s)}{I_\lambda(0)} = e^{-\int_0^s \kappa_\lambda ds}$$

$\int_0^s \kappa_\lambda ds$ is known as optical thickness. So it is denoted by γ_λ, which is known as optical thickness. So, you can write

$$\Rightarrow \frac{I_\lambda(s)}{I_\lambda(0)} = e^{-\gamma_\lambda}$$

So, γ_λ is known as optical thickness. Now we will see the absorptivity. So absorptivity is defined as the ratio of intensity of radiation absorbed till a distance s, to the actual incident radiation at s=0.

$$\alpha_\lambda = \frac{I_\lambda(0) - I_\lambda(s)}{I_\lambda(0)} = 1 - \frac{I_\lambda(s)}{I_\lambda(0)} = 1 - e^{-\gamma_\lambda}$$

So you can see how the absorptivity and the optical thickness are related. Now

$$if\ \gamma_\lambda = 10 \Rightarrow e^{-\gamma_\lambda} = 4.54 \times 10^{-5} \ \Rightarrow \alpha_\lambda \approx 1; optically\ thick$$

$$if\ \gamma_\lambda = 0.01 \Rightarrow e^{-\gamma_\lambda} \approx 0.99 \ \Rightarrow \alpha_\lambda \approx .01; optically\ thin$$

So, you can see now the relation between these two. So if you have the optical thickness is high, then absorptivity is almost 1. And if it is optically thin, then absorptivity is very low. That means, if you have optical thick media, then most of the radiation will be absorbed by the medium, it will not pass and if it is optically thin medium, it is kind of transparent medium, so intensity without absorption will actually pass through that medium. So that is why your α_λ is becoming 0.01.

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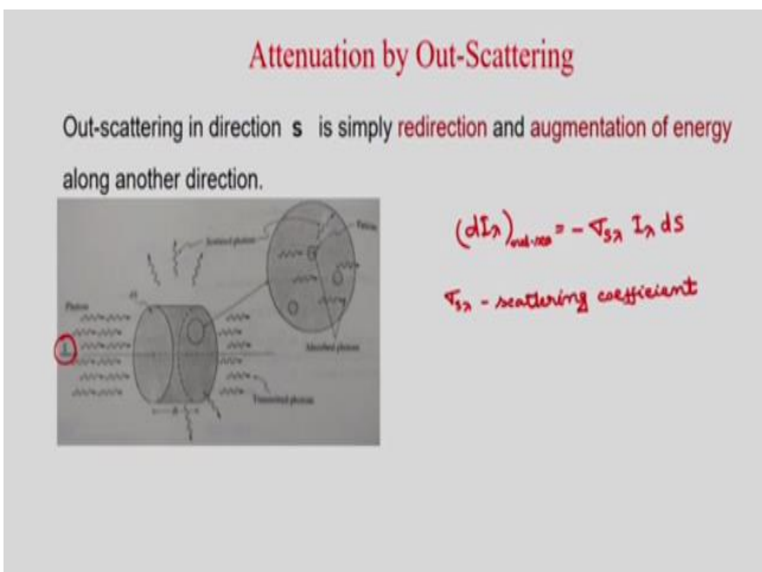

Now, you see the attenuation by out-scattering. You see this figure, when this energy is coming as photons, in this medium obviously it will be scattered to other directions. So it is coming in the s direction but when it will pass through the participating media, it will be scattered in other direction. So obviously, it will lose the energy, but it will enhance or it will increase the energy in other direction.

But what is the difference between these two; one is absorption; in absorption the energy actually increases the internal energy of the system; but when it is out scattering it will actually increase the energy in the other direction, so that is the difference. So, in this case now, if you see that dI_λ for out scattering will be

$$(dI_\lambda)_{out\ scatter} = -\sigma_{s\lambda} I_\lambda ds$$

So, it is again directly proportional to the magnitude of the intensity and the distance travelled by the ray and $\sigma_{s\lambda}$ is known as scattering coefficient.

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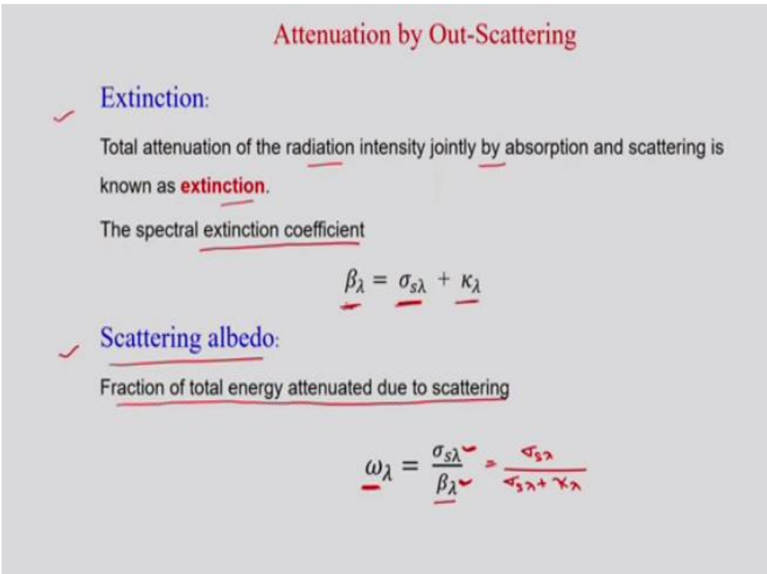

Let us now define what is extinction? Total attenuation of the radiation intensity jointly by adsorption and scattering is known as extinction. So you can see that your attenuation is taking place due to absorption and the out scattering. So obviously, you can see that $\sigma_{s\lambda}$ is the scattering coefficient and κ_λ is your absorption coefficient. So, together this is known as β_λ which is called extinction coefficient.

$$\beta_\lambda = \sigma_{s\lambda} + \kappa_\lambda$$

And scattering albedo is defined as the fraction of total energy attenuated due to scattering. So it is the ratio of energy attenuation due to scattering to the total energy attenuated. Or it is the ratio of scattering coefficient to extinction coefficient. It can be defined as

$$\omega_\lambda = \frac{\sigma_{s\lambda}}{\beta_\lambda} = \frac{\sigma_{s\lambda}}{\sigma_{s\lambda} + \kappa_\lambda}$$

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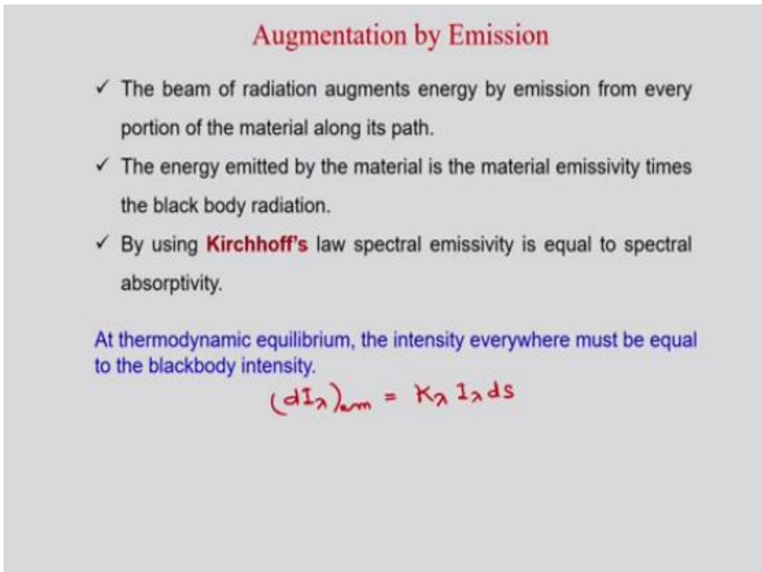

Now, we will learn the augmentation by emission. So the beam of radiation augments energy by emission from every portion of the material along its path. The energy emitted by the material is the material emissivity times the blackbody radiation. And obviously, using Kirchhoff's law, spectral emissivity is equal to the spectral absorptivity. So now at thermodynamic equilibrium, the intensity everywhere must be equal to the blackbody intensity.

And we can write,

$$(dI)_{emission} = \kappa_\lambda I_{b\lambda} ds$$

But here it is not negative, because it is actually enhancing the intensity or energy in the direction s, and here κ_λ is the same coefficient as absorptivity but here you can see it is a plus sign because it is augmentation.

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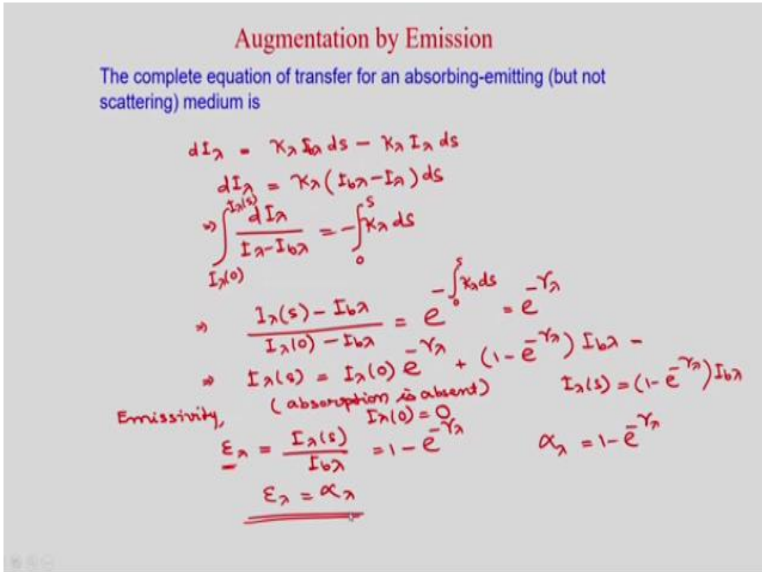

So, now if scattering is absent and only you see absorption and emission, then you can write the complete equation of transfer for an absorbing and emitting medium excluding the scattering as,

$$dI_\lambda = \kappa_\lambda I_{b\lambda} ds - \kappa_\lambda I_\lambda ds$$

$$\Rightarrow dI_\lambda = \kappa_\lambda (I_{b\lambda} - I_\lambda) ds$$

$$\Rightarrow \frac{dI_\lambda}{(I_\lambda - I_{b\lambda})} = -\kappa_\lambda ds$$

So again we can integrate from 0 to s,

$$\Rightarrow \int_{I_\lambda(0)}^{I_\lambda(s)} \frac{dI_\lambda}{(I_\lambda - I_{b\lambda})} = -\int_0^s \kappa_\lambda ds$$

$$\Rightarrow \frac{I_\lambda(s) - I_{b\lambda}}{I_\lambda(0) - I_{b\lambda}} = e^{-\int_0^s \kappa_\lambda ds} = e^{-\gamma_\lambda}$$

Here γ_λ is your optical thickness. So if you rearrange you can write

$$I_\lambda(s) = I_\lambda(0) e^{-\gamma_\lambda} + (1 - e^{-\gamma_\lambda}) I_{b\lambda}$$

So now you can see, this is the intensity due to absorption and the emission when the scattering is absent. So, now we will define the emissivity, so what is emissivity? So, let us say that your absorption is absent, so you can write $I_\lambda(0)$ is 0, because there is no absorption. So emissivity now you can write,

$$\epsilon_\lambda = \frac{I_\lambda(s)}{I_{b\lambda}}$$

So, from the previous equation now, if $I_\lambda(0) = 0$, what you can write?

$$I_\lambda(s) = (1 - e^{-\gamma_\lambda}) I_{b\lambda}$$

Hence

$$\epsilon_\lambda = \frac{I_\lambda(s)}{I_{b\lambda}} = (1 - e^{-\gamma_\lambda})$$

So now you can see that we have already derived the absorptivity, so what absorptivity we derived?

$$\alpha_\lambda = 1 - e^{-\gamma_\lambda}$$

And here also in the absence of scattering and absorption, if emission is taking place, we have derived the emissivity as $1 - e^{-\gamma_\lambda}$, where γ_λ is your optical thickness. And due to Kirchhoff's law, you know that you can see here, $\epsilon_\lambda = \alpha_\lambda$, so that we have wrote.

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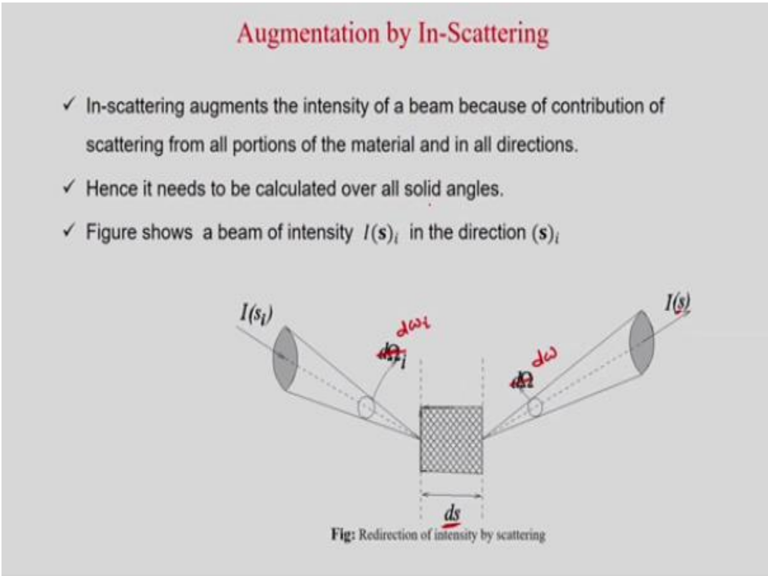

So, now you see augmentation by in-scattering from the other direction. So you can see that here you have this participating media of distance ds and a random direction s_i. This intensity is coming and falling in this participating medium and let us say this is your $d\omega_i$, the solid angle. And this is $d\omega$, your solid angle in the direction where we are considering the pencil of rays going out in direction s.

So, now you can see that in-scattering augments the intensity of a beam because of contribution of scattering from all portions of the material and in all directions. Because in this direction, whatever will come from all other direction, it will fall and some portion will come into this direction s. Hence it needs to be calculated about all solid angles.

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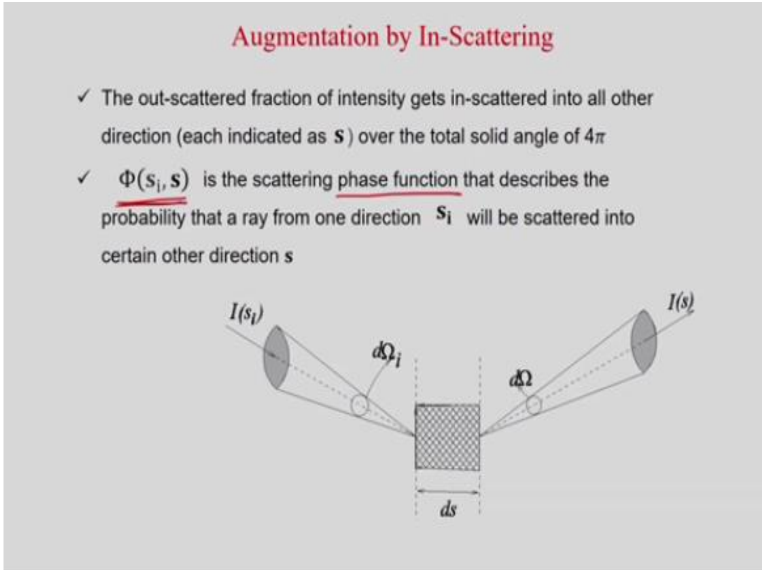

So, you can see that out-scattered fraction of intensity gets in-scattered into all other direction over the total solid angle 4π. So now we will define one phase function ϕ, which is a function of s_i and s and describes the probability that a ray from one direction s_i will be scattered into certain other direction s. So now we are defining the phase function because from any direction s_i, whatever radiation is coming, some portion will go into the direction s of interest. So, in that direction we are considering the radiative heat transfer balance. So this phase function is the probability that a ray from one direction s_i will be scattered into certain other direction s, so that is the probability function. With that we will derive what is the intensity by in-scattering.

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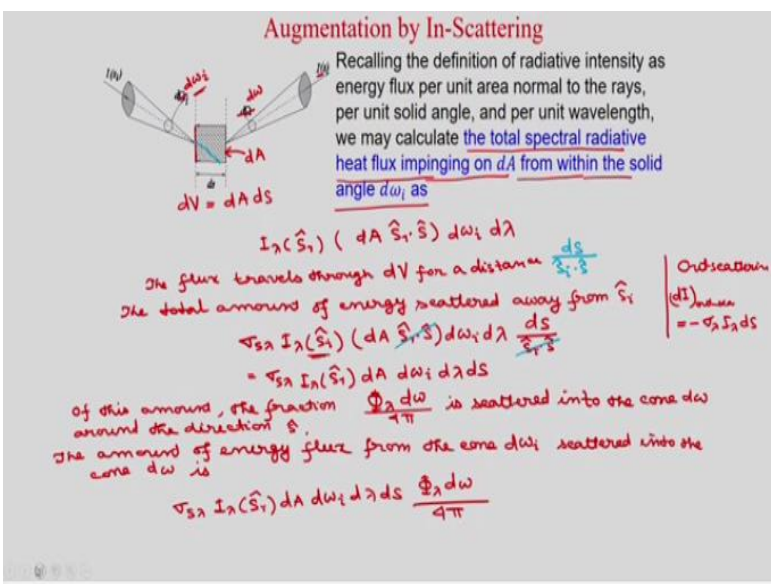

So, we can recall this definition of radiative intensity as energy flux per unit area normal to the rays per unit solid angle and per unit wavelength, we may calculate that total spectral radiative heat flux impinging on dA from within the solid angle $d\omega_i$. So we can see this is your ds and this is your dA, so what is the volume of this?

$$dV = dA.ds$$

And we are considering these as $d\omega_i$, which is the solid angle and this is your $d\omega$ and this is the direction s in which we are interested in, and from any direction i we are considering and we will consider for all solid angle and what is the fraction it is going through the s that will consider. So, if you see the total spectral radiative heat flux impinging on dA from within the solid angle, $d\omega_i$, then what you can write; you can write as

$$I_\lambda(\hat{s}_i)(dA\ \hat{s}_i.\hat{s})d\omega_i\ d\lambda$$

In $\hat{s}_i$ direction area will be $dA\ \hat{s}_i.\hat{s}$. So this is the total spectral radiative heat flux. Now, the flux travels through dV for a distance $\frac{ds}{\hat{s}_i.\hat{s}}$. So with this now you can write the total amount of energy scattered away from s_i,

$$= \sigma_{s\lambda} I_\lambda(\hat{s}_i)(dA\ \hat{s}_i.\hat{s})d\omega_i\ d\lambda \frac{ds}{\hat{s}_i.\hat{s}}$$

It is just similar to what you consider for out-scattering that is $(dI_\lambda)_{out\ scatter} = -\sigma_{s\lambda} I_\lambda ds$. So, I_λ equivalent is $I_\lambda(\hat{s}_i)(dA\ \hat{s}_i.\hat{s})d\omega_i\ d\lambda$ and ds equivalent is $\frac{ds}{\hat{s}_i.\hat{s}}$. Now after cancelling $\hat{s}_i.\hat{s}$ from both numerator and denominator

$$= \sigma_{s\lambda} I_\lambda(\hat{s}_i)(dA\ )d\omega_i\ d\lambda ds$$

So, now you can see that this is coming from any direction i, so obviously its some fraction will go in the direction of s. So that probability function whatever we have considered as phase function, so that we have to consider that what is the fraction it is going to the s direction and this is only solid angle, $d\omega_i$, we have considered, so we have to integrate over whole solid angle.

So, of this amount, the fraction $\frac{\Phi_\lambda d\omega}{4\pi}$ is scattered into the cone $d\omega$ around the direction s. So, the amount of energy flux from the cone $d\omega_i$ scattered into the cone $d\omega$ is

$$= \sigma_{s\lambda} I_\lambda(\hat{s}_i)(dA\ )d\omega_i\ d\lambda ds \frac{\Phi_\lambda d\omega}{4\pi}$$

So this is the fraction will go in the direction of s. So we considered whatever the radiative heat flux is coming from direction s_i, now this is the portion whatever we have written that is going to the direction s. Now we have to consider whole solid angle, so you have to integrate over all solid angle $d\omega$.

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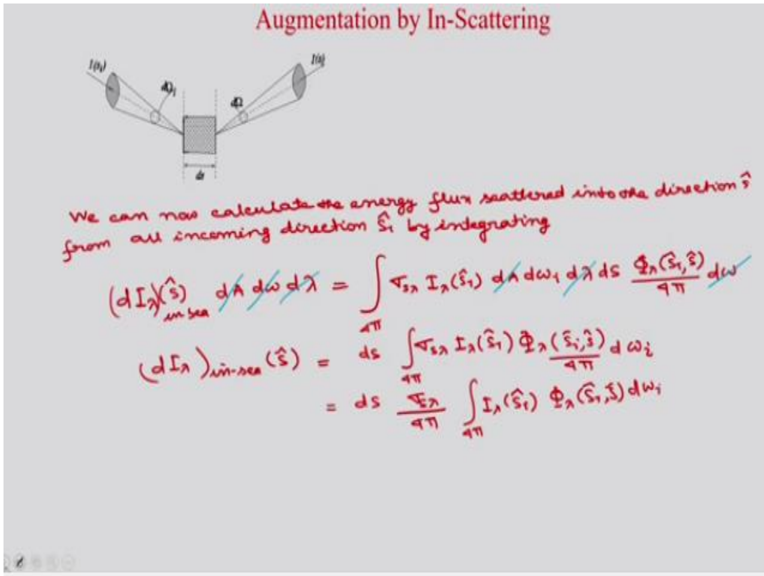

So that will now write. We can now calculate the energy flux scattered into the direction s, from all incoming direction s_i by integrating. So, in dI_λ which is going in direction s, which is your out-scattered should be equal to whatever is coming from all other directions.

$$(I_\lambda)_{in\, scatter}(\hat{s}_i)(dA)d\omega\, d\lambda = \int_{4\pi} \sigma_{s\lambda} I_\lambda(\hat{s}_i)(dA)d\omega_i\, d\lambda ds \frac{\Phi_\lambda(\hat{s}_i.\hat{s})}{4\pi} d\omega$$

So what we have done? We have just equated whatever total energy is going in the direction of s that is your $(I_\lambda)_{in\, scatter}(\hat{s}_i)(dA)d\omega\, d\lambda$ which is actually summation of all the directions, right which is coming in the direction s. So cancelling the common terms from both sides

$$(I_\lambda)_{in\, scatter}(\hat{s}_i) = ds \int_{4\pi} \sigma_{s\lambda} I_\lambda(\hat{s}_i) \frac{\Phi_\lambda(\hat{s}_i.\hat{s})}{4\pi} d\omega_i$$

$$= ds \frac{\sigma_{s\lambda}}{4\pi} \int_{4\pi} I_\lambda(\hat{s}_i)\Phi_\lambda(\hat{s}_i.\hat{s})\, d\omega_i$$

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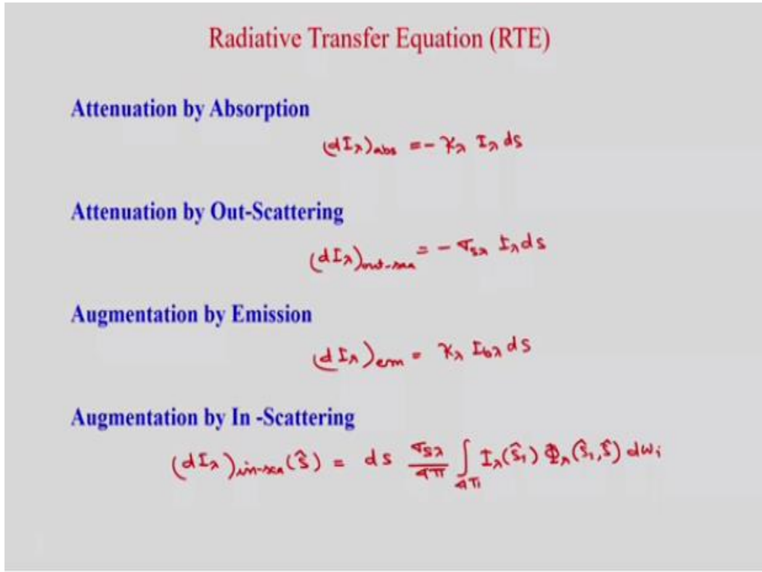

So, now we have derived all these intensities, due to you can see attenuation by absorption we have written

$$(dI_\lambda)_{abs} = -\kappa_\lambda I_\lambda ds$$

Then you have done due to out-scattering,

$$(dI_\lambda)_{out\ scatter} = -\sigma_{s\lambda} I_\lambda ds$$

And due to emission

$$(dI)_{emission} = \kappa_\lambda I_{b\lambda} ds$$

And attenuation by in-scattering just now we have derived

$$(I_\lambda)_{in\ scatter}(\hat{s}_i) = ds \frac{\sigma_{s\lambda}}{4\pi} \int_{4\pi} I_\lambda(\hat{s}_i) \Phi_\lambda(\hat{s}_i . \hat{s})\, d\omega_i$$

So we have seen now that these are the attenuation by absorption and out-scattering and augmentation by emission and in-scattering, now we will do the energy balance okay.

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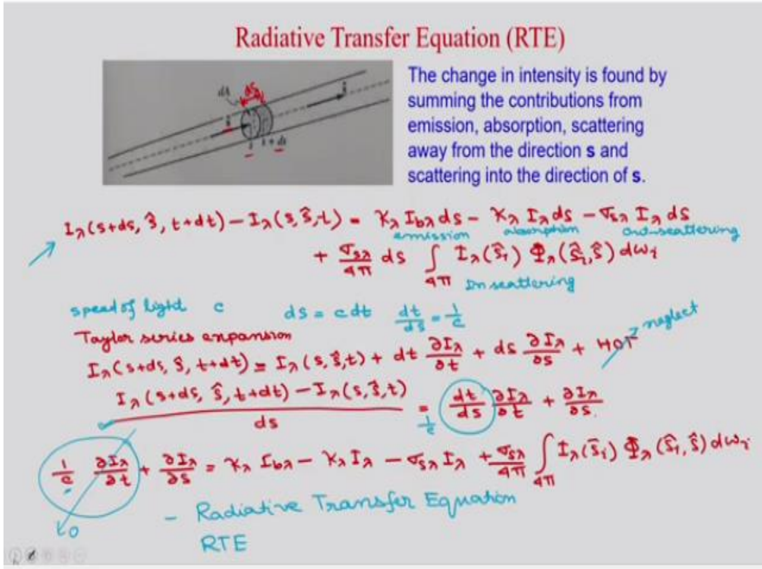

So, now you can see this beam of rays going in the direction of s through this participating medium of distance ds, it is going from s to s+ds, so the distance is ds. And this is the area is dA. The change in intensity is found by summing the contribution from emission, absorption, scattering away from the direction s and scattering into the direction of s.

So that if you can write so, you can essentially write as

$$I_\lambda(s+ds,\hat{s},t+dt)-I_\lambda(s,\hat{s},t)$$

$$=\kappa_\lambda I_{b\lambda}ds-\kappa_\lambda I_\lambda ds-\sigma_{s\lambda}I_\lambda ds+ds\frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi}I_\lambda(\hat{s}_i)\Phi_\lambda(\hat{s}_i.\hat{s})\,d\omega_i$$

The first term in the bracket represents the location at which intensity is calculated. Second term represents the direction taken and the last term is the time. So this is the simple energy balance where the difference between the intensity at s+ds and s, okay at time obviously, when it will go to s+ds, so it will be t+dt, okay.

So the difference will be the summation of all these augmentation and the attenuation. So we have written that. So first part is your emission, okay second is your absorption then, third is your out-scattering, okay and fourth is your in-scattering. So this is the energy balance we have done. And you can see that it is Lagrangian in nature because you are going in the direction s; from s to s+ds.

So, with this now you can see that the ray travels at the speed of light. So if rays travels at the speed of light let us say c, then what is the relation between ds and dt? It will be

$$ds = cdt$$

Now, we will use the Taylor's series expansion. So, if you use the Taylor's series expansion then

$$I_\lambda(s + ds, \hat{s}, t + dt) = I_\lambda(s, \hat{s}, t) + dt\frac{\partial I_\lambda}{\partial t} + ds\frac{\partial I_\lambda}{\partial s} + HOT$$

So you can neglect the higher order terms and divide by ds

$$\Rightarrow \frac{I_\lambda(s + ds, \hat{s}, t + dt) - I_\lambda(s, \hat{s}, t)}{ds} = \frac{dt}{ds}\frac{\partial I_\lambda}{\partial t} + \frac{\partial I_\lambda}{\partial s}$$

So, now in the previous equation we divide by ds and we equate that to this equation. And also we can write $\frac{dt}{ds} = \frac{1}{c}$, which we have found out earlier. Then it will be

$$\frac{1}{c}\frac{\partial I_\lambda}{\partial t} + \frac{\partial I_\lambda}{\partial s} = \kappa_\lambda I_{b\lambda} - \kappa_\lambda I_\lambda - \sigma_{s\lambda} I_\lambda + \frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi} I_\lambda(\hat{s}_i)\Phi_\lambda(\hat{s}_i.\hat{s})\, d\omega_i$$

So, now we have derived, you can see from the energy balance, we have derived this equation which is known as radiative transfer equation or it is known as RTE. So from the energy balance considering a participating media of volume dAds, we did the energy balance and using this Taylor's series expansion, we have derived this equation which is known as radiative transfer equation.

And you can see the first term in the left hand side, so if speed of light is very high, then in practical situation whatever time you considered, it is very small compared to the speed of light. So the phenomena what is happening, that time duration if you consider, so that is kind of quasi steady, right because it is very small compared to the speed of light.

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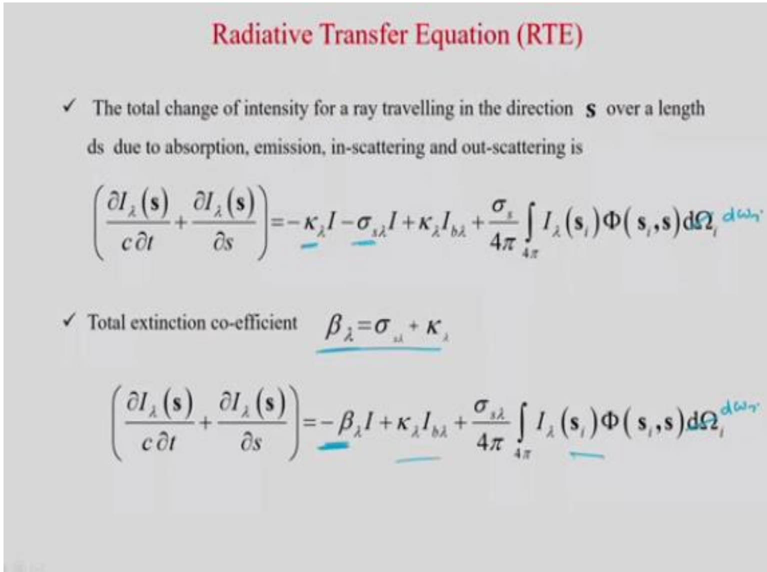

So, this term generally it is neglected okay, because it is almost 0 because it is Quasi steady. So with this now, we can see that now this 2, this $\sigma_{s\lambda}$ and κ_λ you can consider as a extinction coefficient β_λ and you can rewrite this equation.

$$\frac{1}{c}\frac{\partial I_\lambda}{\partial t}+\frac{\partial I_\lambda}{\partial s}=\kappa_\lambda I_{b\lambda}-\beta_\lambda\,(I_\lambda+I_\lambda)+\frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi} I_\lambda(\hat{s}_i)\Phi_\lambda(\hat{s}_i.\hat{s})\,d\omega_i$$

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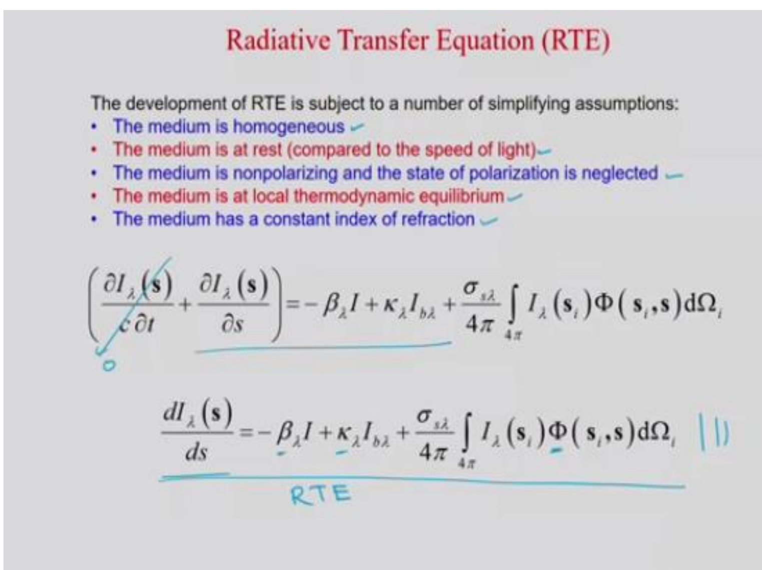

And now, what are the assumptions we have considered during the derivation? Let us summarise. So the medium is homogeneous. The medium is at rest compared to the speed of light, so that I am calling as Quasi-steady. The medium is non-polarising and the state of polarisation is neglected. The medium is at local thermal equilibrium, okay is that is very important and the medium has a constant index of refraction.

So, you can see this equation, now if you consider that medium is almost at rest, so this you can put as 0, and you can write this equation

$$\frac{\partial I_\lambda}{\partial s} = \kappa_\lambda I_{b\lambda} - \beta_\lambda\,(I_\lambda) + \frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi} I_\lambda(\hat{s}_i)\Phi_\lambda(\hat{s}_i.\hat{s})\,d\omega_i$$

So this is your RTE; radiative transfer equation. And generally, we use this equation because we neglect the first term as the speed of light is very high.

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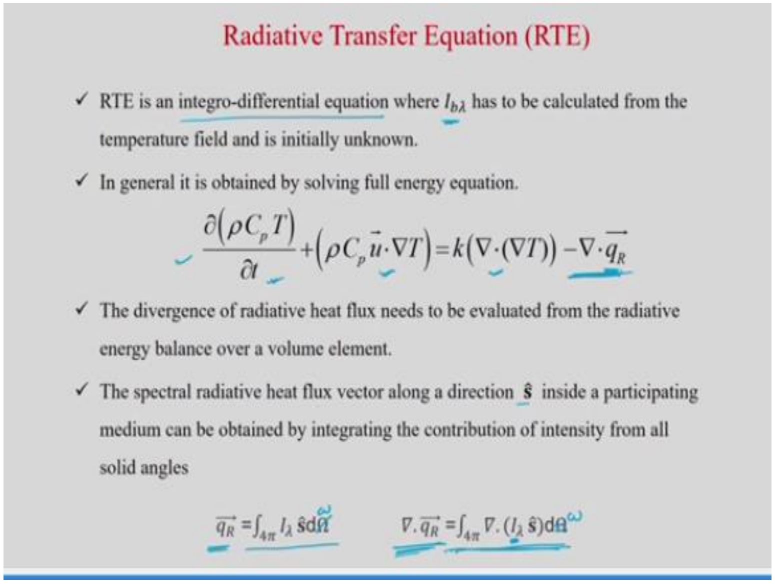

So, with these assumptions, now we can see that what is your goal? So, now you can solve this equation and you can find what is I_λ . So, you will get the intensity. But when you solve the energy equation you can see in the energy equation one term is there. So it is the energy equation, this is the temporal term, this is the convection term, this is the diffusion term and this is your radiative heat flux, q_R, okay.

And this q_R; radiative heat flux we have to calculate from the intensity. And to get the intensity you have to solve this equation, radiative transfer equation. If you solve this equation, you will get the I_λ and this I_λ you will be used to calculate the radiative heat flux, q_R. So, you can see this radiative transfer equation is an integro-differential equation okay.

Because it is a differential equation you can see as well as there is a integration. So this is known as integro-differential equation, where $I_{b\lambda}$ has to be calculated from the temperature field and is initially unknown. So $I_{b\lambda}$ is the blackbody intensity which you need to calculate from the temperature. And how you will calculate the temperature; you need to solve this full energy equation.

So, once you solve this energy equation, you will get the temperature and from this temperature, you can calculate the $I_{b\lambda}$. But at the same time, in the energy equation this is your divergence of q_R which is your radiative heat flux. So, now radiative heat flux, how will calculate? The divergence of radiative heat flux needs to be evaluated from the radiative energy balance over a volume element.

The spectral radiative heat flux vector along a direction s inside a participating medium can be obtained by integrating the contribution of intensity from all solid angles. So q_R you can write

$$\overrightarrow{q_R} = \int_{4\pi} I_\lambda \hat{s} d\omega$$

Now, this is your radiative heat flux and divergence of q_R you need to calculate and divergence of q_R you can write

$$\nabla.\overrightarrow{q_R} = \int_{4\pi} \nabla.(I_\lambda \hat{s}) d\omega$$

So you can see that to calculate the $I_{b\lambda}$, you need to know the temperature and for that you have to solve the energy equation; this equation. And at the same time, when you will solve the energy equation, you need to calculate the radiative heat transfer or radiative heat flux. So this radaitive heat flux, you will calculate in terms of the intensity.

And this intensity will calculate from the radiative transfer equation, which you derived. So these divergence of q_R now we can relate in terms of the intensity and once you solve for I_λ using the radiative transfer equation, you will able to calculate divergence of q_R. So these are coupled. So when you are solving the intensity equation which is radiative transfer equation, you need to know the temperature to calculate the $I_{b\lambda}$. And when you are solving the temperature equation, which is energy equation, you need to know the intensity.

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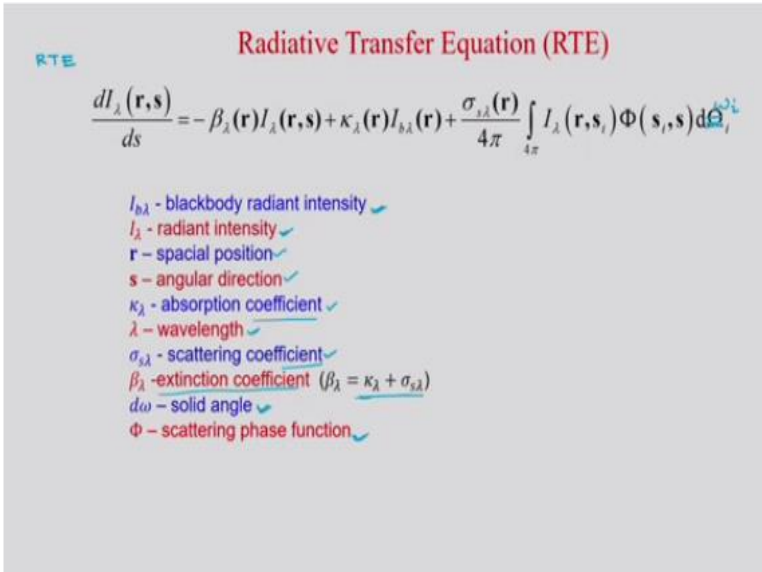

So, now let us summarise okay, the equation which is your radiative transfer equation, RTE.

$$\frac{dI_\lambda}{ds} = -\beta_\lambda\,(r)I_\lambda(r,\hat{s}) + \kappa_\lambda(r)I_{b\lambda}(r) + \frac{\sigma_{s\lambda}(r)}{4\pi}\int_{4\pi} I_\lambda(r,\hat{s}_i)\Phi_\lambda(\hat{s}_i.\hat{s})\,d\omega_i$$

So you can see dI_λ is function of space as well as the direction. The first term we have neglected, because the phenomena is at rest you can consider compared to the speed of light.

And it is your $-\beta_\lambda$ which is again maybe function of r and β_λ is your extinction coefficient. okay which is your $\kappa_\lambda + \sigma_{s\lambda}$ and what is κ_λ; κ_λ is your absorption coefficient and $\sigma_{s\lambda}$ is your scattering coefficient. Now $\kappa_\lambda I_{b\lambda}$, which is function of again space. $I_{b\lambda}$ is obviously blackbody radiant energy and κ_λ is your absorption coefficient. Plus $\frac{\sigma_{s\lambda}(r)}{4\pi}$, where $\sigma_{s\lambda}$ is your scattering coefficient and $\int_{4\pi} I_\lambda(r,\hat{s}_i)\Phi_\lambda(\hat{s}_i.\hat{s})\,d\omega_i$. So Φ_λ is your scattering phase function and $d\omega$ is solid angle, and obviously, I_λ is radiant intensity, i is special position, s is your angular position and λ is wavelength.

So this is one integro-differential equation because you have the integration from also in this equation. So, with this now we will conclude that already initial classes of radiative heat transfer you have learned how the radiation exchange take place when you have vacuum or nonparticipating medium. Now, in last 2 or 3 classes, we have learned if there is a participating medium that means the medium participates in the radiation then you need to consider absorption, then emission, then in-scattering and out-scattering. And the intensity equation which is actually radiative transfer equation that today we have derived. So this is the last lecturer as I told. If you have any doubt then we will discuss during the discussion time and I hope that you will enjoy this lecture, thank you.