Fundamentals of Conduction and Radiation
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Lecture - 31
Radiation Exchange between Surfaces - Part 3

Hello everyone. We are continuing the module number 11, radiation exchange between surfaces. In last class, we have seen the radiation exchange between black surfaces in an enclosure and we have studied for a real surface what is the radiation exchange with certain assumptions. So let us recapitulate what we have done in last. We made the following assumptions you can see here.

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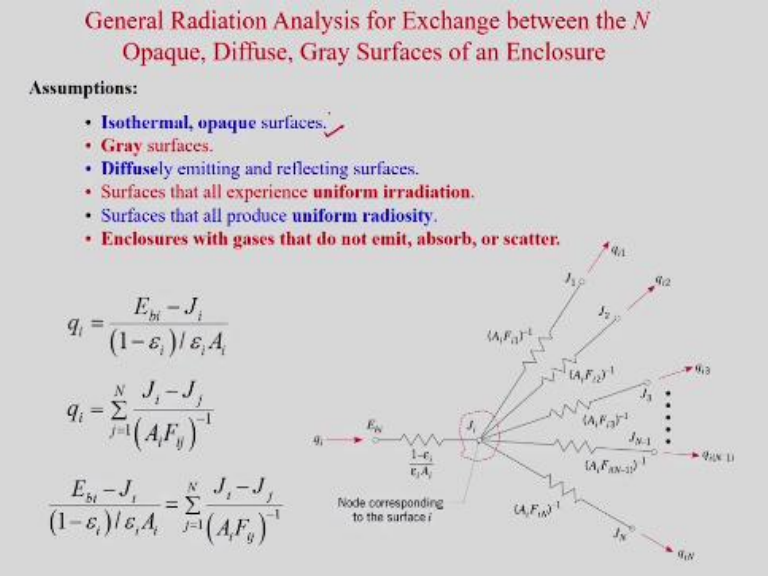

We have considered isothermal and opaque surfaces, then we considered gray surface okay. Then diffusely emitting and reflecting surfaces and surfaces that all experience uniform irradiation as well as surfaces is that all produce uniform radiosity. And also enclosures with gases that do not emit or absorb or scatter. So the gases inside this enclosure does not participate in radiation.

So it is a nonparticipating medium. So with those assumptions we have calculated what is the deduction exchange between the surfaces in an enclosure and you can see here we have carried out this analysis and we have written

$$q_i = \frac{E_{bi} - J_i}{\left(\frac{1 - \epsilon_i}{A_i \epsilon_i}\right)}$$

$\frac{1-\epsilon_i}{A_i\epsilon_i}$ is your surface resistance, and also we have carried out

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

$\frac{1}{A_i F_{ij}}$ is the space resistance okay. So obviously J is the radiosity okay and E_{bi} is the emissive power for a black body. So obviously now both qi if you equate it then you are going to get this relation.

$$\frac{E_{bi} - J_i}{\left(\frac{1-\epsilon_i}{A_i\epsilon_i}\right)} = \sum_{j=1}^{N} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

So obviously $E_{bi} - J_i$ divided by the surface resistance equal to whatever number of surfaces you have been in an enclosure that you are actually summing it up $J_i - J_j$ divided by space resistance.

So here you have seen that this is known as radiation network okay. So in the radiation network we have seen that this is your driving force, $E_{bi} - J_i$ that is the driving potential for transferring q_i ; and it is the resistance $\frac{1-\epsilon_i}{A_i\epsilon_i}$. And now you have N number of surfaces, so obviously it is connected with J_1, J_2, J_3, so these are different surfaces.

So now you can see it is having different space resistance okay, $\frac{1}{A_i F_{i1}}$, $\frac{1}{A_i F_{i2}}$; so with surface i this are connected with N number of surfaces 1, 2, 3 you can see and N surfaces. So this radiation network we have seen and after that we have seen that the simplified case, if it is a 2 surfaces enclosure then this is the simplified case, for that you have seen what is the radiation exchange.

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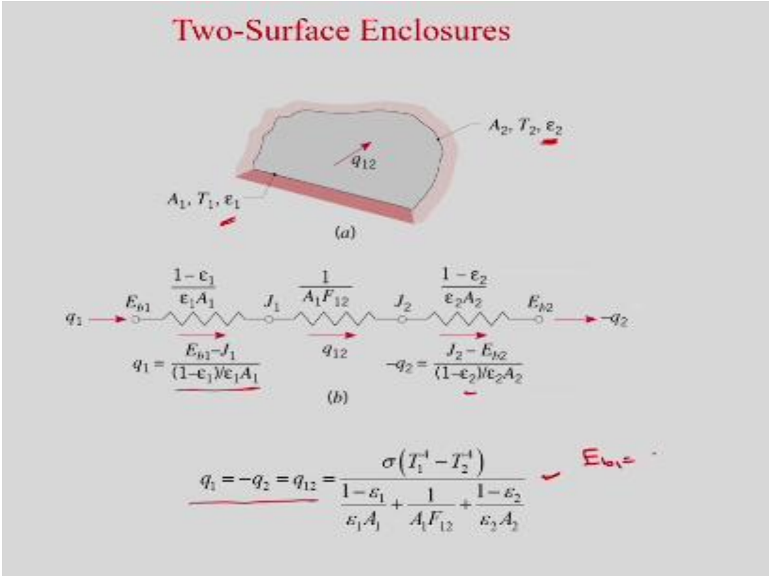

And if you can recall that we consider this 2 surfaces okay with emissivity ϵ_1 , the other surface emissivity is ϵ_2 and this is the radiation network. So you can see q_1 obviously is $\frac{E_{b1}-J_1}{\left(\frac{1-\epsilon_1}{A_1\epsilon_1}\right)}$ and this is $-q_2$ which is $E_{b2}-J_2$ divided by this. So obviously you can write $q_1 = -q_2 = q_{1,2}$ $E_{b1} - E_{b2}$ okay.

E_{b1} is your σT_1^4 to the power 4 and E_{b2} is σT_2^4. So this potential difference divided by summation of all the resistances; so in 2 surfaces enclosure, how many resistances you have; 2 surfaces, so obviously 2 surface resistances and 1 space resistance, okay. We discussed when you draw the network first you draw the surface resistances connecting to the each surfaces after that those you connect with the space resistance.

So as it is 2 surfaces enclosure obviously first you draw 2 surface resistances, then you join this 2 with one space resistance that we have done here. You can see that E_{b1}, J_1, J_2, E_{b2} and this are connected with surface resistance, base resistance and the surface resistance okay. Now let us take few simplified case in the next slide.

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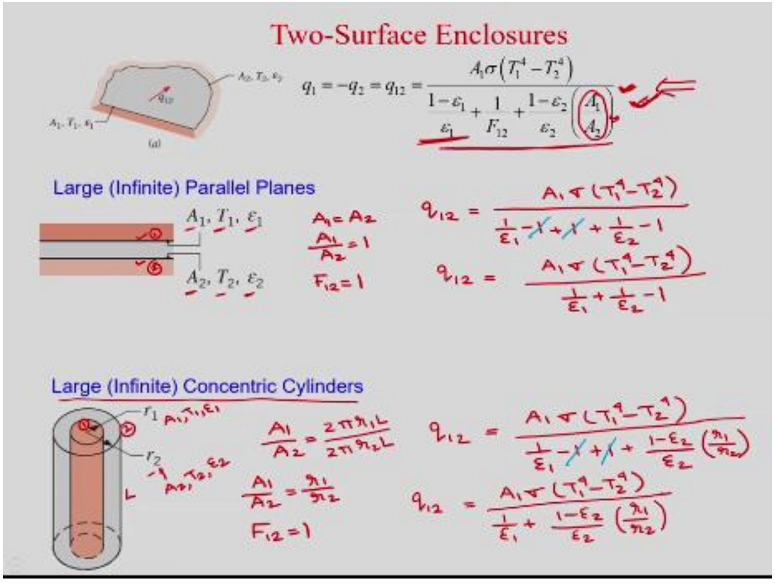

So you can see this are 2 parallel infinite planes okay. So you can neglect the radiation from the 2 ends. So obviously this is the surface 1 okay and this is your surface 2. So this are surface area A_1 maintained at surface temperature T_1 and emissivity is ϵ_1 and for surface 2, your area is A_2, temperature is T_2 and emissivity is ϵ_2 .

So as this is radiation exchange between 2 parallel planes obviously here $A_1 = A_2$ okay. So in this case you can see that it is $A_1 = A_2$ or you can write

$$\frac{A_1}{A_2} = 1$$

And what about the view factor $F_{1,2}$? So obviously it is a flat surface so $F_{1,2}$ is obviously 1, because $F_{1,1}$ is 0, so $F_{1,2}$ should be 1. So in this case your view factor $F_{1,2}$ is 1 okay. And we have already in generalised case for 2 surfaces enclosure we have carried out this expression

$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{A_1}{A_2}\left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

Now you put it here and get the expression for q_{12} in this case. So q_{12} you will get

$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} - 1 + 1 + \frac{1}{\epsilon_2} - 1}$$

So just putting the view factor as 1 and A_1/A_2 equal to 1 we have just simplified this expression. Then you will get the expression for large parallel planes as

$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

So whatever total resistances, here 3 resistances are there and that you have simplified and you have written for 2 parallel planes as $\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1$ okay. So this is the general expression. Similarly, you can do for large infinite concentric cylinders okay.

So you consider 2 cylinders, so the inner cylinder is 1 and it is radius is r_1 and outer cylinder is 2, and it is radius is r_2. Obviously this is maintained at let us say surface area is A_1, T_1 and ϵ_1 and this 2 surface area is A_2 maintained at isothermal temperature T_2 and emissivity is ϵ_2, okay. And obviously as it is infinite concentric cylinders then you can neglect the radiation loss from the end. So with that assumption, now what will be your A_1/A_2? You know the radius, so what is the area? So it is, if L is the length of the cylinder then

$$\frac{A_1}{A_2}=\frac{2\pi r_1 L}{2\pi r_2 L}=\frac{r_1}{r_2}$$

Similarly, now you tell me what is the view factor F_{12}? Obviously, 1 is your convex surface, so your F_{11} is 0, so obviously F_{12} will be 1. So in this case also F_{12} is 1, okay. So now you put it in a general expression,

$$q_{12}=\frac{A_1\sigma(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}-1+1+\frac{r_1}{r_2}\left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

So in this case now again this 1, 1 will cancel out okay. So you will get the final expression for this particular case as

$$q_{12}=\frac{A_1\sigma(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{r_1}{r_2}\left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

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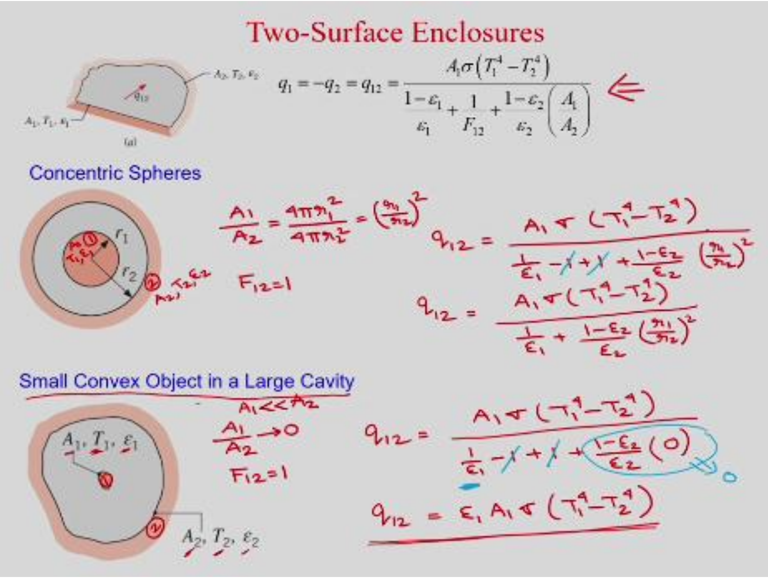

So next, we will consider 2 concentric spheres. So the inner sphere is 1, okay outer sphere is 2 and inner surface temperature is T_1 okay and ϵ_1 is the emissivity and A_1 is the area, and for outer surface area is A_2, T_2 is the surface temperature and the emissivity is ϵ_2 . So similarly, you find first what is the area ratio A_1/A_2 and what is the area for a sphere, it is $4\pi r^2$. So obviously your A_1/A_2 will be

$$\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$$

And what about the view factor? In this case, obviously your surface 1 is convex, so F_{11} is 0, so F_{12} will be 1. So you can write

$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} - 1 + 1 + \left(\frac{r_1}{r_2}\right)^2 \left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

So this is similar like expression like concentric cylinder case. However, here A_1/A_2 is $\left(\frac{r_1}{r_2}\right)^2$. So finally, you can write this as

$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \left(\frac{r_1}{r_2}\right)^2 \left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

So all this we have considered as 2 surfaces enclosure and lastly now we will consider small convex object in a large cavity okay. So you have a small convex object you can see. 1 is your small convex object whose area is A_1, T_1 is the temperature and ϵ_1 is the emissivity and the surface 2 is having area is A_2, surface temperature T_2 and emissivity is ϵ_2 . So in this case obviously we are considering a small object inside an enclosure or large enclosure then obviously $A_1 << A_2$ right. So in this case we can write

$$\frac{A_1}{A_2} \to 0$$

And we have considered one convex surface, A_1 as a convex surface, so obviously F_{12} will be 1. So in this case,

$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} - 1 + 1 + 0 \times \left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

$$\Rightarrow q_{12} = \epsilon_1 A_1 \sigma(T_1^4 - T_2^4)$$

So it is very simplified form because as you can say that A_1/A_2 here as tends to 0. So we have considered several cases for 2 surfaces enclosure and now we will consider 3 surfaces enclosure. If you consider 3 surfaces enclosure obviously now the complexity will increase and more than that if you go then it will be more complex.

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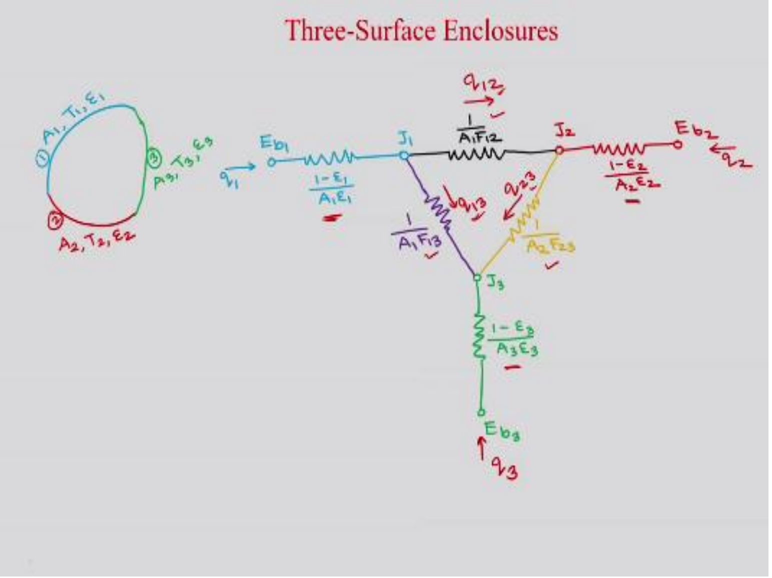

So now you first consider 3 surfaces enclosure of gray diffuse, uniform radiosity, uniform irradiation and no participating medium. With those assumptions if you take 3 surfaces enclosure having the isothermal surfaces okay, radiation network, let us first draw. Then we can calculate what will be the radiation heat transfer. So first, let us draw this 3 surfaces enclosure okay.

So let us say this is one surface okay, maintained at A_1 is the area, temperature is T_1 and emissivity is ϵ_1 , another surface let us consider that this is your another surface, okay, it is area A_2 and maintained at temperature T_2 and emissivity is ϵ_2 . Now it is enclosure, so we have to close it. So now another surface is there, okay.

So this surface is 3 okay and its area is A_3, temperature is T_3, it is isothermal surface and emissivity is ϵ_3 okay. So now, it is 3 surfaces enclosure. To draw the radiation network, first you draw the surface resistances. So how many surface resistance will be there. As there are 3 surfaces, so 3 surface resistances will be there.

So first you draw 3 surface resistances, then you connect this with space resistances okay. So if you draw first, the surface resistances; so now let us draw okay. So for the first one you can see this is the resistance. So this will be your E_{b1}, okay, this is your J_1 and what is your resistance, surface resistance okay, so it will be $\left(\frac{1-\epsilon_1}{A_1\epsilon_1}\right)$.

And whatever it is going that is your q_1. So you understood right. So now, another 2 resistances let us draw. So if you draw the second surface resistance, so this is your surface resistance of 2, so it is your E_{b2} to J_2 and what is the resistance, it is $\left(\frac{1-\epsilon_2}{A_2\epsilon_2}\right)$, and the third resistance now you draw.

So this is your third resistance. So it will be E_{b3} okay and it will be J_3. So J obviously your radiosity and E_{b3} is the black body emissive power and what is the resistance? It is surface resistance, so $\left(\frac{1-\epsilon_3}{A_3\epsilon_3}\right)$. So now, for the 3 surfaces enclosure we have just drawn 3 surface resistances. Now we will connect this 3 with space resistances. So obviously, how many space resistances will be there, 3 space resistances because 1 to 2 then 1 to 3 and another 2 to 3.

So 3 space resistances will be there. So here, three space resistances and three surface resistances are there. So let us connect it. So let us connect with this one space resistance from 1 to 2 and what will be the space resistance in this case, it will be $\frac{1}{A_1F_{12}}$. Now you connect the other one.

So now, surface 1 to 3 we have connected with the space resistance and space resistance is $\frac{1}{A_1F_{13}}$. Now you connect the last one. So that if you connect from surface 2 to 3 we have connected and the space resistance will be $\frac{1}{A_2F_{23}}$. So obviously from reciprocity relation you can write $\frac{1}{A_3F_{32}}$, that also you can write.

So you can see now in this case as we have considered 3 surfaces enclosure, so obviously you can see that these are the 3 surface resistances and these are the 3 space resistances. So now you can see that this will be actually your q_2 and here it will flow from q_{12} and here it will flow q_{13} and here in this direction if you consider then it will be q_{23} and this you have q_3 okay.

So q_1 q_2 q_3 and between 1 to 2 surface you can have q_{12} 2 but if you remember when we have considered 2 surface enclosure, as there are 2 surfaces only, then $q_1 = -q_2 = q_{12}$, but in this case it is not true, because there are 3 surfaces, so radiation exchange between this surfaces will be different. So q_{12} will be different, q_{13} will be different, q_{23} will be different okay.

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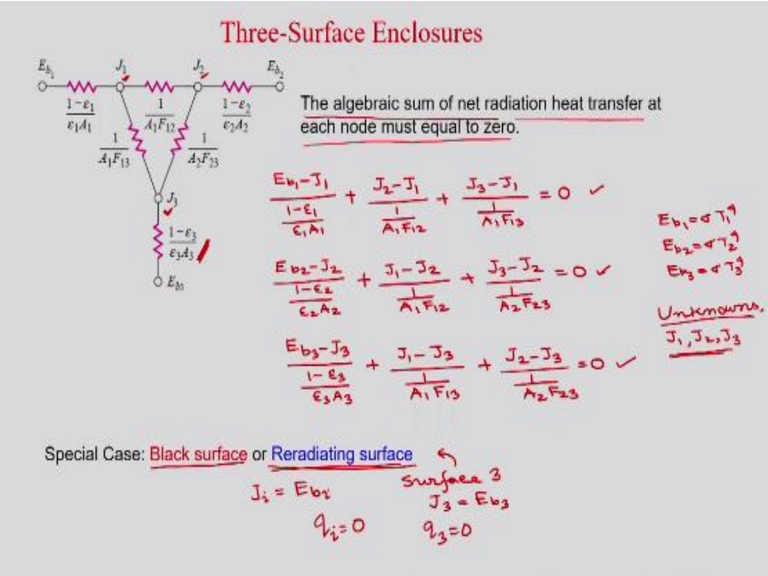

So this is the radiation network, whatever we have drawn right now. So now the algebraic sum of net radiation heat transfer at each node must equal to be 0 okay. So if you consider this node J_1, so what you can write? You can write

$$\frac{E_{b1} - J_1}{\left(\frac{1-\epsilon_1}{A_1\epsilon_1}\right)} + \frac{J_2 - J_1}{\frac{1}{A_1F_{12}}} + \frac{J_3 - J_1}{\frac{1}{A_1F_{13}}} = 0$$

So that means the algebraic sum of net radiation heat transfer at each node must equal to 0. Now for 2, if you write then it will be

$$\frac{E_{b2} - J_2}{\left(\frac{1-\epsilon_2}{A_2\epsilon_2}\right)} + \frac{J_1 - J_2}{\frac{1}{A_1F_{12}}} + \frac{J_3 - J_2}{\frac{1}{A_2F_{23}}} = 0$$

Similarly, if you consider the third node,

$$\frac{E_{b3} - J_3}{\left(\frac{1-\epsilon_3}{A_3\epsilon_3}\right)} + \frac{J_1 - J_3}{\frac{1}{A_1F_{13}}} + \frac{J_2 - J_3}{\frac{1}{A_2F_{23}}} = 0$$

So now you can see that at each node we have done the algebraic sum of net radiation and that will be equal to 0. So with this now we can see we have got 3 equations right. And if the temperatures are known at each surfaces then obviously

$$E_{b1} = \sigma T_1^4$$

$$E_{b2} = \sigma T_2^4$$

$$E_{b3} = \sigma T_3^4$$

So if you know the temperature at each surfaces then you can calculate E_{b1}, E_{b2}, E_{b3} means those are known. So what are the unknowns in this case? Only J_1, J_2, J_3 okay, if the temperatures are known okay. Then if J_1, J_2, J_3 are unknown, then you have 3 equations and 3 unknowns okay, then you can solve this and you can find J_1, J_2, J_3.

Because the geometric properties like A_1, A_2, A_3 and F_{12}, F_{23} and F_{13} you can calculate and the emissivities are known. So all the resistances you can calculate. So this 3 unknowns J_1, J_2, J_3, you can calculate from this 3 equations. Now there are some special case, already we have discussed, let us again discuss that, that if you have one surface is black surface or reradiating surface right.

What is the reradiating surface we have discussed earlier? So reradiating surface is where net radiative heat transfer will be 0, that means q will be 0. And black surface emissivity is 1, so obviously your surface resistance will be 0 okay. In both the cases, you will have this if surface i you have then

$$J_i = E_{bi}$$

Okay, so in this case if it is a 3 surface, if you consider the surface 3 is either black surface or reradiating surface then obviously you can write $J_3 = E_{b3}$. Here you note one thing, although $J_3 = E_{b3}$ for this case where you have black surface or reradiating surface, but the reasons are different okay. For black surface the reason is that your emissivity is 1, for that reason your surface resistance is becoming 0.

So if this resistance become 0 then obviously your J_3 will become E_{b3} and for reradiating surface your $q_i = 0$ or $q_3 = 0$ here. For that reason you are getting $J_3 = E_{b3}$. So you should remember that although in both the cases for black surface as well as the reradiating surface your $J_3 = E_{b3}$, but the reasons are different because one when your black surface it is surface resistance is 0 since your emissivity is 1. And for reradiating surface your J_3 is equal to E_{b3} as you have q is 0, $q_3 = 0$. So in the next slide we will discuss more about it.

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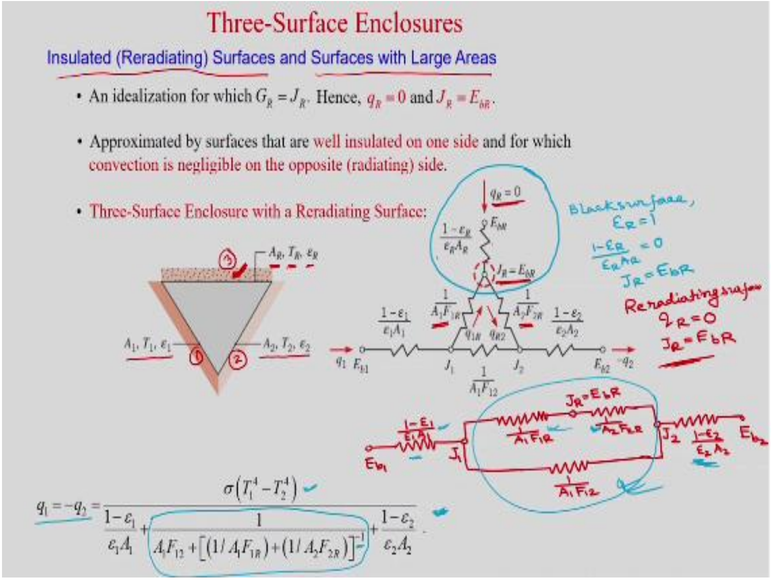

Okay so you can see that you have a 3 surfaces enclosure. But you have either insulated reradiating surface or surface with large areas. Surfaces with large areas we can also consider as a black surface as we have already discussed earlier. So you can see you have 3 surfaces where you have A_1, T_1, ϵ_1 for the surface 1, for the surface 2 you have A_2, T_2 and ϵ_3 .

And for third surface if the area is A_R, T_R and ϵ_R which is your reradiating surface or the black surface. Surface with large areas also considered black surface. So in this case you can see that it is the radiation network right. In this radiation network for surface 3 you consider this particular case okay.

Here you can see if it is black surface then your $\epsilon_R = 1$ then

$$\frac{1-\epsilon_R}{\epsilon_R A_R} = 0$$

And hence you have $J_R = E_{bR}$. This is for black surface. And if you consider it is a reradiating surface

$$q_R = 0 \Rightarrow J_R = E_{bR}$$

So now, you can see that if the third surface is either insulated or reradiating surface or surfaces with large area or black surface then this resistance part actually will vanish. So $J_R = E_{bR}$ only. So now you can see that you can have the $q_1 = -q_2$. So if you draw it in simple way then you can see what is happening actually.

So these are in series, this one and this one you can consider as in series. So now you can see this is your E_{b1} okay. This is your J_1, this is your J_R okay, which is equal to E_{bR} in this case because it is a reradiating surface and this is your J_2 and this is your E_{b2} okay. So if you consider that third surface is reradiating surface or black surface then your radiation network will look like this.

Where the resistance is, this is your surface resistance, this is your space resistance, this is also space resistance, this is also space resistance and this is your surface resistance. So now you can see from here if you have to calculate the equivalent resistance this 2 are in series and this 2 are parallel. So now you consider the equivalent resistance and then this 3 will be in series.

So you can write $q_1 = -q_2 = \sigma(E_{b1} - E_{b2}) = \sigma(T_1^4 - T_2^4)$ divided by the summation of all the resistances. So this is $\frac{1-\epsilon_1}{A_1\epsilon_1}$. Now this equivalent resistance is this one okay the middle one. And then you add $\frac{1-\epsilon_2}{A_2\epsilon_2}$ okay.

$$q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12} + \left[\left(\frac{1}{A_1F_{1R}}\right) + \left(\frac{1}{A_2R_{2R}}\right)\right]^{-1}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

So now if you consider 3 surfaces enclosure with 1 surface as black surface or reradiating surface then with this actually you can simplify the resistances and you can calculate with this expression okay. So now first to attack some problems related to this radiation exchange you should first draw the radiation network, then it will be very easier. So then you first draw the surface resistances then you connect to it those with space resistances then it will be easy to calculate the radiation exchange. However, it is having very good application.

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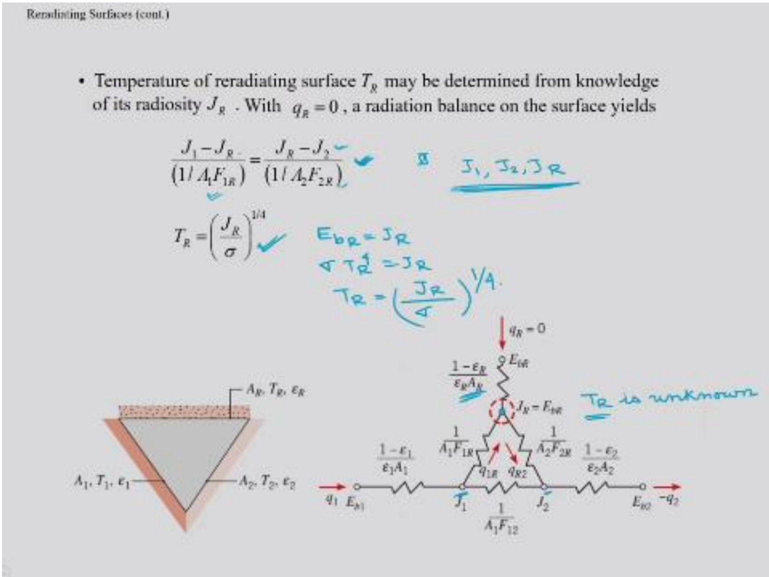

So in the next slide we will just discuss. Before that let us see if T_R is unknown, which is your reradiating surface temperature that you can calculate from here. So at this point now if you do the energy balance it will be

$$\frac{J_1 - J_R}{\frac{1}{A_1F_{1R}}} = \frac{J_R - J_2}{\frac{1}{A_2F_{2R}}}$$

As $\frac{1-\epsilon_R}{\epsilon_R A_R} = 0$. So from here for the other nodes also you can relate the expression. Then after that you find J_1 ,J_2 and J_R from 3 equations which already we have written.

You can just go back and see. So these are the 3 equations. For reradiating surface it is special, so this expression you can write and find J_1, J_2, J_R. And once J_R is known then obviously you know $J_R = E_{bR} = \sigma T_R^4$. And J_R is known so T_R you can now calculate.

$$T_R = \left(\frac{J_R}{\sigma}\right)^{\frac{1}{4}}$$

So that way you can calculate the temperature of that insulated or reradiating surface. Now I was talking about the application of this that is in radiation shields.

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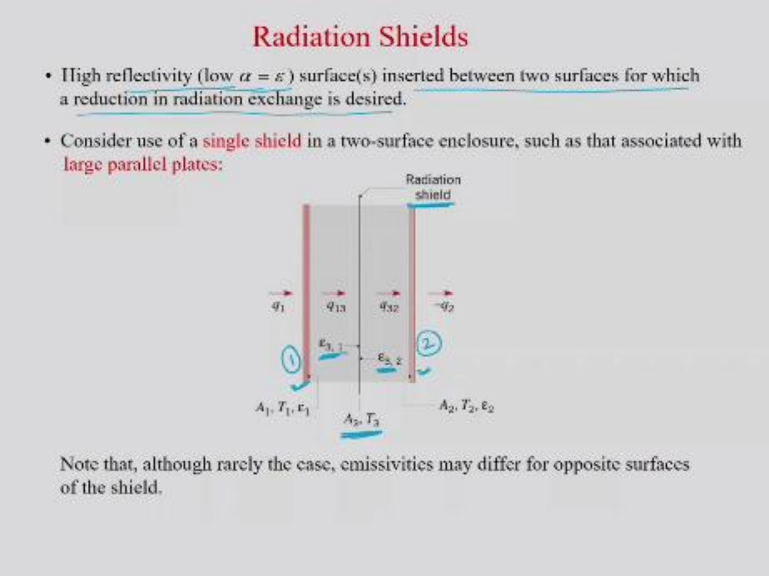

So what are radiation shields? So first let us discuss that you want to decrease the radiation between 2 surfaces. So how you will do it? So generally, it is done inserting a thin plate with high reflectivity or low emissivity material okay, and those thin sheets are known as radiation shields. So if you consider 2 surfaces where the radiation exchange is taking place so if you want to decrease the radiation exchange then you can increase the resistances so that your radiative heat transfer will decrease.

So just by increasing the resistances, you are actually decreasing the radiative heat transfer okay. So that is done just by inserting a thin plates okay with high reflectivity and low emissivity. Obviously if it is an opaque body, then obviously reflectivity is very high then your absorptivity will be very low and hence emissivity will be very low. Therefore, you can see that in this slide that high reflectivity means low absorptivity or emissivity surfaces inserted between 2 surfaces for which a reduction in radiation exchange is desired.

Then you can see here. If we consider use of single shield in a 2 surfaces enclosure then there are 2 parallel plates okay 1 and 2. This is surface 1 and this is surface 2. Without putting this radiation shield there will be some heat transfer. Now if you insert one radiation shield in between this 2 parallel plates which is actually of area A_3 and surface temperature T_3.

And if its emissivity maybe in this side ϵ_{31} and the other side ϵ_{32}, then you can draw the radiation network and obviously after putting this your resistance will increase because you can see here 2 surface resistance will increase okay.

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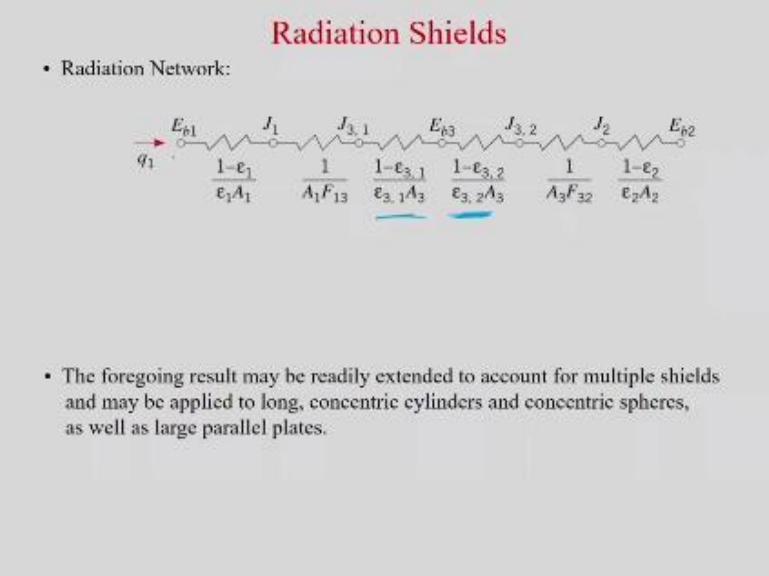

So now if you draw it then you will understand. So you can see these are the 2 surface resistance that increased. If you have 2 parallel plates so obviously you can see that this is E_{b1}, J_1 , the resistance is $\frac{1-\epsilon_1}{\epsilon_1 A_1}$, this is the surface resistance for surface 1. Then you have a space resistance so that is $1/A_1F_{13}$ and this is now $J_{31}$to E_{b3}, E_{b3} is for the surface 3 at temperature T_3.

So there will be surface resistances and this is the surface resistance. So this 2 surface resistances actually are additionally added just by inserting the radiation shield okay. And it is having very low emissivity and hence the resistances will be higher and therefore your radiation heat exchange will be less. And obviously there was another space resistance $1/A_3F_{32}$ and another surface resistance for surface 2 that is $\frac{1-\epsilon_2}{\epsilon_2 A_2}$.

So if you insert another surface radiation shields then obviously another 2 surface resistance will increase. So that way you can actually decrease the radiation heat transfer. So it is having some applications, generally in cryogenics and space, where you have a very low temperature region where radiation is the very important heat transfer mode, in that case just inserting the radiation shield you can decrease the radiation heat transfer.

This is also applicable for any other situations like between 2 concentric cylinders or spheres. So this foregoing results may be readily extended to account for multiple shields and may be applied to long concentric cylinders and concentric spheres as well as large parallel plates. So actually, by increasing resistances you are decreasing the radiative heat transfer.

So now let us solve one problem, and then you will understand that how important it is.

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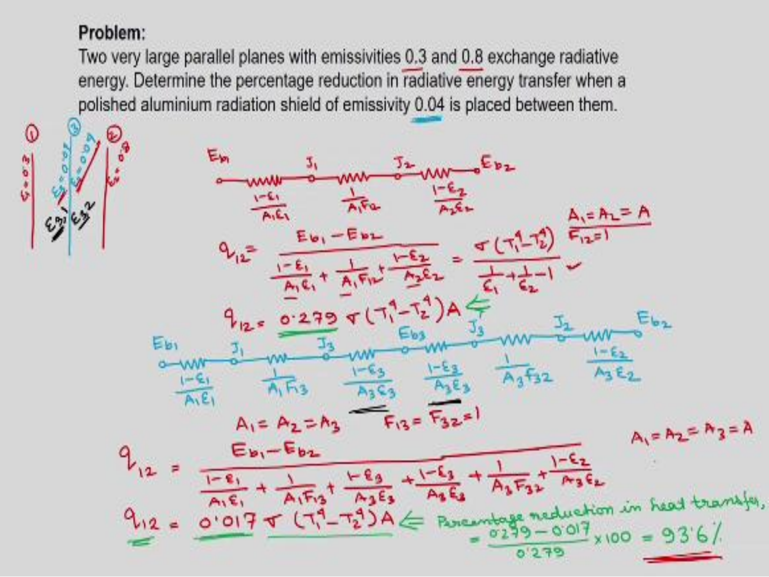

So one problem we are considering that 2 very large parallel planes with emissivities 0.3 and 0.8 exchange radiative energy. So you have 2 parallel plate and they are exchanging the radiation. Determine the percentage reduction in radiative energy transfer when a polished aluminium radiation shield of emissivity 0.04 is placed between them.

So now to decrease the heat transfer one radiation shield is placed in between and its emissivity is 0.04. So you can see the emissivity is very low. So what will be your percentage reduction in radiative energy transfer? So let us solve this problem. First, you understand the problem.

So you have 2 parallel plates initially, infinite parallel plates, this is 1, this is 2, your ϵ_1 is 0.3 and ϵ_2 is your 0.8. So what will be your resistances in this case? So there will be one surface resistance, one space resistance and another surface resistance. So this is your E_{b1}, E_{b2}, J_1, J_2 and resistances are $\frac{1-\epsilon_1}{\epsilon_1 A_1}$, $\frac{1}{A_1 F_{12}}$ and $\frac{1-\epsilon_2}{\epsilon_2 A_2}$.

So you can write

$$q_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

So in this case obviously $A_1 = A_2 = A$ and $F_{12} = 1$. So this actually you can write with this simplification

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

So already you have derived this okay, heat exchange between 2 infinite parallel planes, so that is the expression. So if you put the ϵ_1 and ϵ_2 value, you are going to get

$$q_{12} = 0.279\, A\sigma(T_1^4 - T_2^4)$$

So this is without inserting the radiation shield.

Now you insert one radiation shield in between. So this is the radiation shield you are inserting. And both side you have ϵ_3 is 0.04. So now 2 surface resistances will increase, so your new radiation network will look like this. This is your E_{b1}, now it is J_1 then you have J_3, this is your E_{b3} okay because of the surface 3, then your J'_3, then you can write J_2 then E_{b2}. And what are the resistances? $\frac{1-\epsilon_1}{\epsilon_1 A_1}$, this is now space resistance, $1/A_1F_{13}$, then it will be again surface resistance okay, so that will be your $\frac{1-\epsilon_3}{\epsilon_3 A_3}$, then again this is surface resistance $\frac{1-\epsilon_3}{\epsilon_3 A_3}$, then this is your space resistance so it will be $1/A_3F_{32}$ and this is again surface resistance, $\frac{1-\epsilon_2}{\epsilon_2 A_2}$. Now obviously we are considering all are parallel plane, so in this case $A_1 = A_2 = A_3 = A$ okay, and also $F_{13} = F_{32} = 1$. So with this assumption now if you put then you are going to get

$$q_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1F_{13}} + \frac{1-\epsilon_3}{\epsilon_3 A_3} + \frac{1-\epsilon_3}{\epsilon_3 A_3} + \frac{1}{A_3F_{32}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

ϵ_3 is same and ϵ_1 is known, ϵ_2 is known so, you can calculate this, finally, you will get it as

$$q_{12} = 0.017 A\sigma(T_1^4 - T_2^4)$$

So now you can see this 2 expressions okay, one is this one, another is this one. So which is less? Obviously, after inserting the radiation shield you can see q_{12} decreases, here the coefficient is 0.017 but here it is 0.279 okay.

Rest are same right, $A\sigma(T_1^4 - T_2^4)$. So you can write percentage reduction in heat transfer because you have already calculated the radiation transfer, so you can see that it will be

$$= \frac{0.279 - 0.017}{0.279} = 93.6\%$$

So you note the value here, okay. So you can imagine that just by inserting 1 radiation shield, your reduction in heat transfer takes place by 93%. Just with low emissivity material one thin plate if you insert.

And obviously it will have some high reflectivity, with that you can consider. So in this case although here the emissivities are same in both sides, ϵ_3 but in some cases you may consider different emissivities like in this case you can consider ϵ_{31} and here you can have ϵ_{32}. So in both side you might have different emissivity okay.

So then in this case just here this expression will change okay, it will be ϵ_{31} and it will be ϵ_{32} and you can find the total resistances and find the heat transfer okay. So let us solve another problem. So first, I will read the problem.

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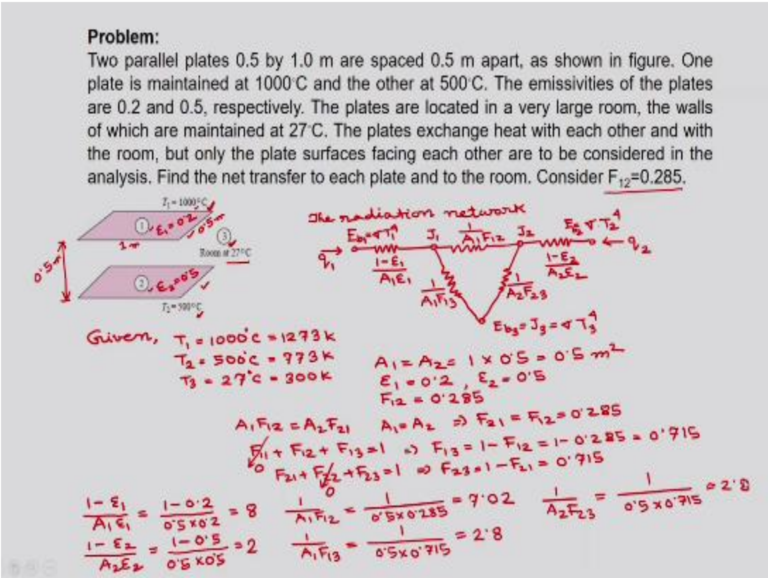

Two parallel plates 0.5X1 m are spaced 0.5 m apart. So you can see these are 2 plates having dimensions of 0.5 mX 1 m and the distance between this 2 plates is 0.5 m. One plate is maintained at 1000 ºC and the other at 500 ºC. So you can see the upper plate is maintained at 1000 ºC and the bottom plate is maintained at 500 ºC.

The plates are located in a very large room, the walls of which are maintained at 27 ºC. So you can see that this room is having a temperature of 27 ºC and that we are considering as the third surface. The plates exchange heat with each other and with the room, but only the plate surface is facing each other are to be considered in the analysis.

Find the net heat transfer to each plate and to the room, consider F_{12} = 0.285. So the view factor is given as 0.285. So this is a 3 body problem, so 3 surfaces, surface 1 is maintained at 1000 ºC and surface 2 is maintained at 500 ºC and each surface is having the dimensions of 0.5X1 m and the distance between this 2 plates are each 0.5 m.

So you can see that it is surrounded by the ambient which is having 27 ºC and this room now we will consider as third surface. Obviously, you can see that earlier we have discussed that it is considered a black surface because its area is very large. So if area is very large, obviously the surface resistances will be very small so obviously you can consider that all the radiation whatever is coming to it, it will be absorbed.

So ϵ you can consider to be 1 or absorptivity as 1. So obviously, it is considered a black surface. So with this now first let us draw the radiation network. So there are 3 surfaces, one surface which is the room, obviously there is no surface resistance, so we have 2 surface resistances for plate 1 and plate 2 and we have 3 space resistances.

Because from 1 to 2, 1 to 3 and 2 to 3, so 3 space resistances we will have. So let us draw the radiation network first. The radiation network, okay, so first we will draw the 2 surface resistances. So these are 2 surface resistances. q_1 is coming from here, from surface 1 and this is your surface 2 and q_2 , and this is your radiosity J_1 and this is J_2. So this surface resistances is $\frac{1-\epsilon_1}{\epsilon_1 A_1}$ and this surface resistance is $\frac{1-\epsilon_2}{\epsilon_2 A_2}$.

And this third surface resistance will not be there as we are considering it is a large enclosure. And now we are connecting with space resistances. So there will be 3 space resistances, so you can consider this as $E_{b1} = \sigma T_1^4$ and this is your $E_{b2} = \sigma T_2^4$ and this in this case is $E_{b3} = J_3 = \sigma T_3^4$ because there is no surface resistance. In this case, you please remember that when we do the radiation analysis, the temperature will be in absolute temperature. So you convert the ºC to K. So now what are the surface resistances? So this is your $1/A_1F_{12}$. This is your $1/A_1F_{13}$ and this is $1/A_2F_{23}$ and this is $E_{b3} = J_3$obviously we have considered.

So this is the radiation network. So let us first see the given quantities. Let us write down then we will solve the problem. So what are the given data?

$$T_1 = 1000℃ = 1273\ K$$

$$T_2 = 500°C = 773\ K$$

$$T_3 = 27°C = 300K$$

$$A_1 = A_2 = 1 \times 0.5 = 0.5\ m^2$$

$$\epsilon_1 = 0.2;\ \epsilon_2 = 0.5$$

$$F_{12} = 0.285$$

So these are the given data. So with this given data you can calculate all the resistances okay. So once you know all the resistances then for each node you can do the energy balance and solve for J_1 and J_2 because you know the temperature T_1, T_2, T_3 so obviously you know J_3 and E_{b1} and E_{b2} are known only unknowns are J_1 and J_2. So we will write 2 equations, okay, at 2 nodes so obviously you can solve for 2 unknown variables using those 2 equations. So first, let us find all the resistances. So now, let us find the view factors. So F_{12} is 0.285, so you know reciprocity relation

$$A_1F_{12} = A_2F_{21}$$

But $A_1 = A_2$ right, so obviously

$$F_{12} = F_{21} = 0.285$$

Now you find F_{13}. So we will use the summation rule. What is the summation rule, $F_{11}+F_{12}+F_{13}=1$, but $F_{11}=0$ because it is a flat surface, so obviously

$$F_{13} = 1 - F_{12} = 1 - 0.285 = 0.715$$

Similarly, also you can write $F_{21} + F_{22} + F_{23} = 1$ and F_{22} is 0, so obviously you will get

$$F_{23} = 1 - F_{21} = 0.715$$

So all the view factors now we have found out. Now let us calculate all the resistances. So first, let us calculate 2 surface resistances connecting to plate 1 is

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.2}{0.5 \times 0.2} = 0.8$$

Then the second surface resistance is

$$\frac{1-\epsilon_2}{\epsilon_2 A_2} = \frac{1-0.5}{0.5 \times 0.5} = 2$$

So now you calculate 3 space resistances.

$$\frac{1}{A_1F_{12}} = \frac{1}{0.5 \times 0.285} = 7.02$$

$$\frac{1}{A_1F_{13}} = \frac{1}{0.5 \times 0.715} = 2.8$$

$$\frac{1}{A_2F_{23}} = \frac{1}{0.5 \times 0.715} = 2.8$$

So now, all the resistances are known.

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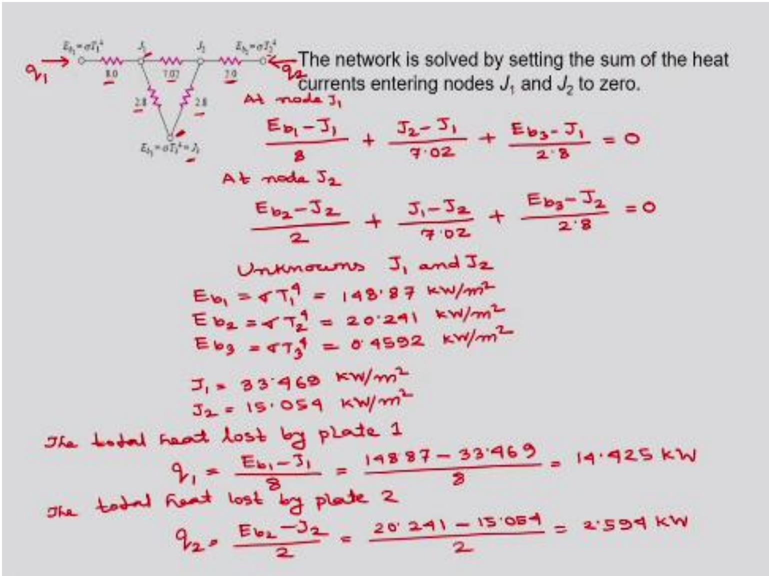

You can see this figure, obviously this is your E_{b1}, J_1, J_2 E_{b2} and all this resistances whatever we have calculated, so this is first surface resistance, second surface resistance and these are the space resistances and here $E_{b3} = J_3$. So now what we will do, the network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero.

So first if you consider J_1, then what you can write we have already done you know,

$$\frac{E_{b1} - J_1}{8} + \frac{J_2 - J_1}{7.02} + \frac{E_{b3} - J_1}{2.8} = 0$$

So this is at node J_1 okay. Similarly, at node J_2 you would write

$$\frac{E_{b2} - J_2}{2} + \frac{J_1 - J_2}{7.02} + \frac{E_{b3} - J_2}{2.8} = 0$$

So here, in this 2 equations you can find that E_{b1}, E_{b2} and E_{b3} are known as you know the temperature of those surfaces. So only unknowns are J_1 and J_2. So you have 2 equations and 2 unknowns, so you can find J_1, J_2. First calculate what are the E_{b1}, E_{b2}, E_{b3}.

$$E_{b1} = \sigma T_1^4 = 148.87\ kW/m^2$$

$$E_{b2} = \sigma T_2^4 = 20.241\ kW/m^2$$

$$E_{b3} = \sigma T_3^4 = 0.4592\ kW/m^2$$

Here σ is Stefan Boltzmann constant. So in the above 2 equations you put this values and do the algebra and find J_1, J_2. So now J_1, you will get as 33.469 kW/m^2 and J_2 you will get 15.054 kW/m^2. So these are known, so now you can find easily the q_1 and q_2.

So obviously the total heat transfer or heat lost by plate 1 is q_1 . So you can see this is your q_1 and through which it is going? Through this resistance circuit it is going. So you can write this as $E_{b1} - J_1$ divided by the resistance.

$$q_1 = \frac{E_{b1} - J_1}{8} = \frac{148.87 - 33.469}{8} = 14.425\ kW$$

Similarly, you can write the total heat lost by plate 2 will be $E_{b2} - J_2$ divided by the resistance.

$$q_2 = \frac{E_{b2} - J_2}{2} = \frac{20.241 - 15.084}{2} = 2.594\ kW$$

So obviously you can see that the total heat lost by plate 1 and plate 2 will be actually received by the room. So you can actually calculate.

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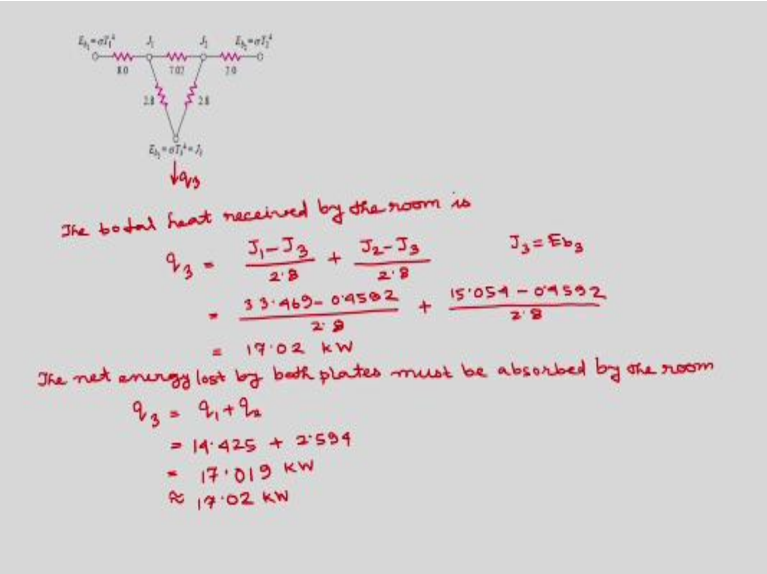

So the total heat received by the room is q_3; so from the radiation network you can see it is coming from $J_1$and from J_2. So if you add it, you are going to get q_3 .

$$q_3 = \frac{J_1 - J_3}{2.8} + \frac{J_2 - J_3}{2.8}$$

But $J_3 = E_{b3}$. So if you put the values

$$q_3 = \frac{33.469 - 0.4592}{2.8} + \frac{15.054 - 0.4592}{2.8} = 17.02\ kW$$

So other way also it is true. If you do the energy balance then whatever heat loss is happening from the plate 1 and plate 2, those are coming to room, so obviously $q_3 = q_1 + q_2$. So the net energy lost by both plates must be absorbed by the room. So

$$q_3 = q_1 + q_2 = 14.425 + 2.594 = 17.019 \cong 17.02\ kW$$

So this type of problem you solve from this Incropera and DeWitt heat transfer book and using this radiation network you solve some problems from this radiation shield. Already we have solved one problem in last class, so few problems you just solve from this book and we will some assignments too. So with this today we will stop. Thank you.