Fundamentals of Conduction and Radiation
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Lecture – 30
Radiation Exchange between Surfaces

Hello everyone, so in last class we have seen the view factor which takes cares about the radiation exchange for different geometrical effect and geometrical orientation. So today we will study the radiation exchange between surfaces in an enclosure. So first, we will see that for black surface, what is the radiation exchange, then we will study for real surface with certain assumptions.

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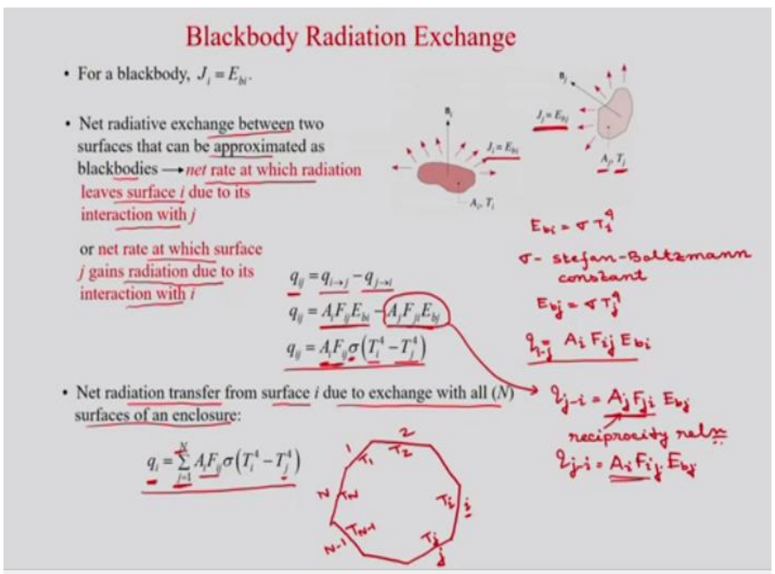

So, let us see first the blackbody radiation exchange. So here you can see in this figure. So it is one surface; isothermal surface maintained at temperature T_i and area is A_i. So obviously it will radiate and as it is black surface, there will be no reflection because whatever radiation will come to the surface, all will be absorbed, so there will be no reflection, so the radiosity will be whatever it is emitting only. So, you can see here that

$$J_i = E_{bi} = \sigma T_i^4$$

J_i is the radiosity, so it is equal to whatever the radiation emitted from this surface $E_{bi} = \sigma T_i^4$. T_i is the temperature of the surface and σ you know that is Stefan Boltzmann constant. Similarly, another surface if you consider at isothermal surface with temperature T_j and surface area is A_j, so it will also radiate with

$$J_j = E_{bj} = \sigma T_j^4$$

So these are the black surfaces A_i and A_j and we are trying to find the radiation exchange between this 2 surface A_i and A_j. So obviously you can see that net radiation exchange between these 2 surfaces approximated as blackbodies, can be net rate at which radiation leave surface i due to its interaction with j. Or net rate at which surface j gains radiation due to its interaction with i. So obviously you can see that surface i is radiating and surface j is also radiating, so the difference between this 2 will be the radiation exchange between this 2 surfaces. So we can calculate q_{ij} , as whatever going out from surface i minus whatever coming in to it.

$$q_{ij} = q_{i\to j} - q_{j\to i}$$

So now considering the geometrical effects that are the view factors we can write corresponding radiation exchanges. Hence,

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}$$

Using reciprocity relation

$$A_i F_{ij} = A_j F_{ji}$$

Hence,

$$q_{ij} = A_i F_{ij} \sigma \left(T_i^4 - T_j^4\right)$$

Where, T_i and T_j are the temperature at surface A_i and surface A_j respectively. Here, the assumption is that the surfaces are isothermal okay. So the temperature is constant for any given surface. So, now it is just between this 2 surfaces we have considered from i to j. Now, if you consider one enclosure consisting of N surfaces, so obviously, it will radiate and exchange the radiation with each surface of that enclosure.

So, we can write now net radiation transfer from surface i due to exchange with all N surfaces of an enclosure is

$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma \left(T_i^4 - T_j^4\right)$$

So if you have an enclosure with N surfaces which are maintained at constant temperatures we can write this. So this is one enclosure, let us say each 1, 2 and this is your surface N, this is surface N-1 and all are maintained at some temperature constant temperature and let us say it is surface j. And T_j is the temperature, T_{N-1} is the temperature T_N, and so you can see that each surfaces are maintained at a constant temperature okay, either T_1, T_2, T_j, T_{N-1}, or T_N. So obviously if it is surface i, it is maintained at temperature T_i, then from surface i, there will be exchange of all the surfaces from 1, 2, 3 similarly j, N-1 and N and that we are just summing it up. So q_i is the total net radiation transfer for any black surface enclosure.

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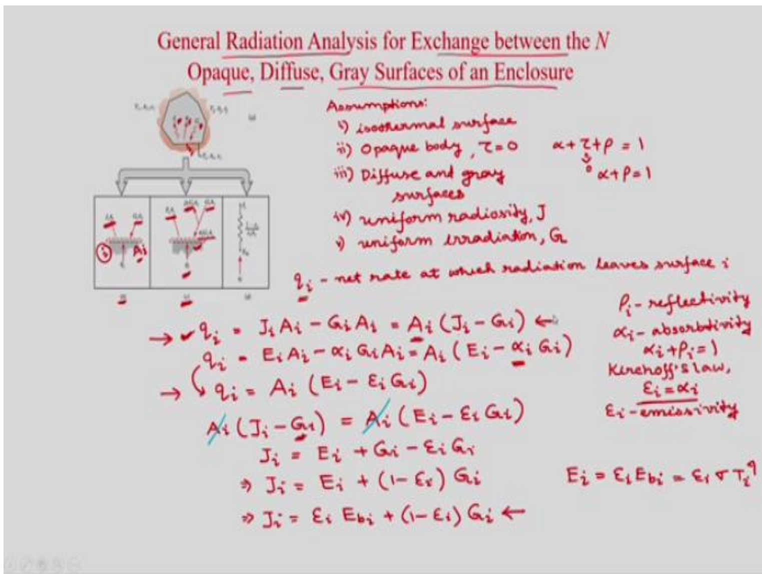

So, now let us consider real surface okay. So, real surface again, it is difficult to analyse. So we will have some assumption. So first assumptions obviously whatever we have taken for the black surface that each surface are maintained at constant temperature that means, those surfaces are isothermal okay. So that is the first assumption.

Here also, we will consider first radiation exchange between two surfaces, then we will consider one enclosure and the interaction of the radiation from surface j to 1 to N, all the surfaces we will calculate. So first assumption used is isothermal surface. Second is opaque body. So opaque body means transmittivity will be 0 that means τ =0.

Because there will be no radiation transmitted through that body. So that means as τ =0,

$$\alpha + \rho + \tau = 1 \Rightarrow \alpha + \rho = 1$$

Now, we will consider that the surfaces are diffuse and gray. You know the diffuse and gray surfaces. Diffuse means independent of direction and gray is independent of wavelengths. And we will consider uniform radiosity and uniform irradiation. So what is radiosity? The radiosity already you have learned that the total radiation leaving the surface means whatever it is emitted at well as the reflected portion of the irradiation.

And what is irradiation; total radiation falling to the surface is known as irradiation. So obviously you know the radiosity is denoted by J and irradiation is denoted by G. so these are the assumptions we are taking because then it will be easier to analyse this radiation exchange for a real surface. So that is why we are considering that general radiation analysis for exchange between N opaque diffuse gray surfaces of an enclosure, okay.

So, now you look into this figure. So you can see that it is one enclosure considering N surfaces and all the surfaces are maintained at constant temperature and those are gray and diffuse surface and opaque. So you can see this surface I and obviously your irradiation is G_i. So in the enclosure from other surfaces all the radiation coming to the surface i is G_i.

So, G_i is obviously considering all the radiation which is coming from other surfaces and J_i is the radiosity. So from the surface i, your radiosity is J_i. So obviously it will be the reflected portion of the G_i and whatever it is emitting, right. So the body itself is emitting so, obviously emitted portion plus the reflected portion of the G_i is your J_i, radiosity. And now what will be the net rate of radiation exchange from the surface? That will be your q_i, okay.

So, q_i we can write as net rate at which the radiation leaves surface i. So q_i is actually given in this surface i to maintain a constant temperature T_i, when the radiation exchange is taking place. So, now you can see that from here this figure b you can see. So obviously if the area is A_i of the surface i, then total radiation falling on the surface that will be $A_i G_i$. And whatever is leaving from this surface is reflected portion of the G_i as well as whatever it is emitting. So that will be $A_i J_i$. So we can write the energy balance that is whatever energy going out minus coming in from this figure b to be

$$q_i = A_i J_i - A_i G_i = A_i (J_i - G_i)$$

Now, you consider this figure c. Whatever $A_i\ G_i$ is coming, so one portion is reflected, $\rho_i A_i\ G_i$. ρ_i is the reflectivity of surface i, so already it is reflected, so it has not taken part in changing the temperature of this surface i.

And only this $\alpha_i A_i\ G_i$, α_i is the absorptivity, so that is actually taken by surface i and whatever it is emitting that is your $A_i E_i$. So that is the emitted portion of the radiation from surface i. So if you now do the energy balance from here, you can see that this portion does not participate in surface i for the radiation, okay. So whatever is absorbed and whatever it is emitted okay, so this difference will be your q_i.

So, another way from this figure c, we can write

$$q_i = A_i E_i - \alpha_i A_i\ G_i = A_i(E_i - \alpha_i G_i)$$

So now you can see 2 different energy balances we have considered. In first case, we are telling whatever total irradiation is happening and whatever is going out that is the radiosity.

So, radiosity minus the irradiation that is your q_i. So we are writing that from here. You can see $q_i = A_i\ (J_i - G_i)$. And from figure c, you can see from here, here you can see that now whatever is absorbed by the surface and whatever is emitted, so this difference will give you the q_i. So that we can write $q_i = A_i(E_i - \alpha_i G_i)$.

And from here we have shown that $\alpha_i + \rho_i = 1$, okay because it is an opaque surface. Now, from the Kirchhoff's law, what you know? That Kirchhoff's law tells that whatever is absorptivity that is equal to your emissivity.

$$\epsilon_i = \alpha_i$$

So in this case we are considering real surface, okay. So, obviously there will be some emissivity. Earlier case, we have considered black surface, so there will be no emissivity or emissivity is equal to 1. So, in this case now we are considering real surface, so for any surface i the emissivity is ϵ_i and from the Kirchhoff's law, you can write $\epsilon_i = \alpha_i$. So, now here, now we will substituted with ϵ_i so,

$$q_i = A_i(E_i - \epsilon_i G_i)$$

So, now as both are equal right, so this and this are equal q_i; so we can equate it, so we can write

$$A_i\,(J_i - G_i) = A_i(E_i - \epsilon_i G_i)$$
$$\Rightarrow J_i = E_i + (1-\epsilon_i)G_i$$

Now in this case, E_i is the emission from real surface. So it will be $\epsilon_i E_{bi}$ where, E_{bi} is the blackbody emission. So

$$E_i = \epsilon_i E_{bi} = \sigma T^4$$

So, in this case now, you can write,

$$J_i = \epsilon_i E_{bi} + (1-\epsilon_i)G_i$$

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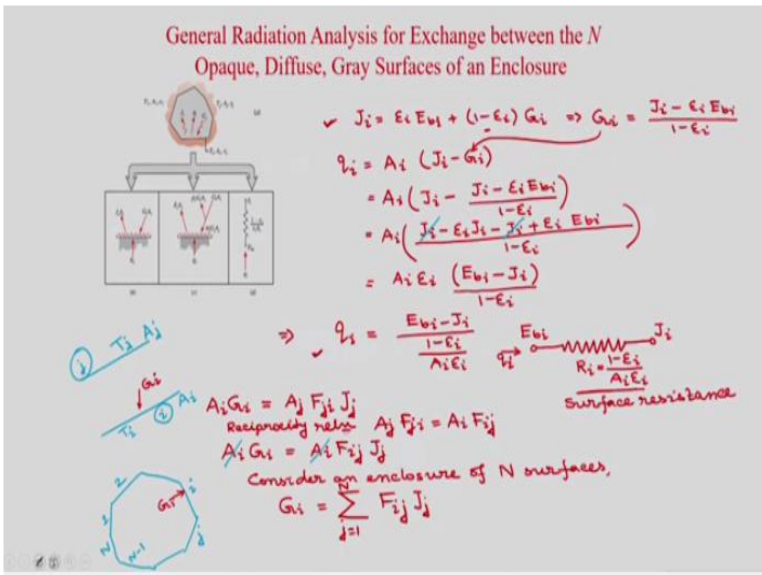

So, now this relation of J_i you put it in this equation. So in the next slide, let us do it. So, we have written that

$$J_i = \epsilon_i E_{bi} + (1-\epsilon_i)G_i$$

So this is the total radiosity for any real surface, we have just evaluated. So from here

$$G_i = \frac{J_i - \epsilon_i E_{bi}}{1-\epsilon_i}$$

Now, we know q_i.

$$q_i = A_i\,(J_i - G_i)$$

So this G_i now you put here.

$$q_i = A_i\left(J_i - \frac{J_i - \epsilon_i E_{bi}}{1-\epsilon_i}\right)$$

So, now you just do simple algebra and find the simplified form, okay.

$$= A_i\left(\frac{J_i - \epsilon_i J_i - J_i + \epsilon_i E_{bi}}{1-\epsilon_i}\right)$$

$$= A_i\epsilon_i\left(\frac{E_{bi} - J_i}{1-\epsilon_i}\right)$$

Or we can write

$$q_i = \frac{E_{bi} - J_i}{\left(\frac{1-\epsilon_i}{A_i\epsilon_i}\right)}$$

So now we can see that net rate of radiation leaving the surface i; q_i is just this. So E_{bi} is the total emission from the surface i and J_i is the radiosity. So we can write with the electrical analogy like this. So, if you consider this is the resistance okay, so the potential difference is $E_{bi} - J_i$, and what is going, so that is your q_i okay and what is the resistance; so this R_i we can write $\left(\frac{1-\epsilon_i}{A_i\epsilon_i}\right)$.

So this resistance is known as surface resistance or surface radiative resistance, because due to the radiation it is coming and this resistance is occurring at the surface, so this is known as surface resistance and which is nothing but $\left(\frac{1-\epsilon_i}{A_i\epsilon_i}\right)$ okay. So this is one analysis we have done.

So now, we are trying to see another form of q_i, okay. So now you considered 2 surfaces okay. We are trying to find what is G_i okay. So you consider 2 surface, let us say this is your j surface and this is surface i okay and these are isothermal surfaces, so temperature maintained is T_j and T_i, okay.

And the area is A_i and this is your A_j, okay. So between this 2 surfaces, if you consider the G_i okay, so what will be your G_i? so G_i is the radiation falling on this surface I, right. So now you consider surface i and your irradiation is happening is G_i but from where this G_i is coming; G_i is coming from the surface j. If you are considering only 2 surfaces, let us say i and j, then the G_i is actually your irradiation on surface i.

But it is actually coming from surface j due to reflection as well as its own emission. So that means it is radiosity of surface j, right, it is J_j. So this J_j is actually coming to surface i as irradiation. I think you have understood. So we are considering only 2 surfaces, so whatever the radiation leaving the surface j actually is reaching to surface i.

So, whatever leaving is in the form of radiosity, which is actually combination of both emission as well as reflection. And whatever is coming to the surface i that is your irradiation and that is G_i. So if you consider only this two surfaces, so you can say

$$A_i G_i = A_j F_{ji} J_j$$

F_{ji} is coming because it is the geometrical configuration you are considering, the view factor and whatever is coming to i is leaving the surface j, so that is the radiosity J_j.

So now you use the reciprocity relation again.

$$A_j F_{ji} = A_i F_{ij}$$

So putting this

$$A_i G_i = A_i F_{ij} J_j \Rightarrow G_i = F_{ij} J_j$$

So now you consider that there are N surfaces okay. Now from N surfaces if you consider G_i the irradiation for the surface i, will be equal to all the radiosity from all the surfaces 1 to N. So now, in general if you consider one enclosure of N surfaces okay, let us say this is your surface 1, 2, this is your i, this is your j, this is your N-1 and this is your N. So now you are considering whatever G_i is coming. So, that will be

$$G_i = \sum_{j=1}^{N} F_{ij} J_j$$

Because from all the surfaces 1 to N whatever radiosity is there that is actually adding to the irradiation to the surface i okay.

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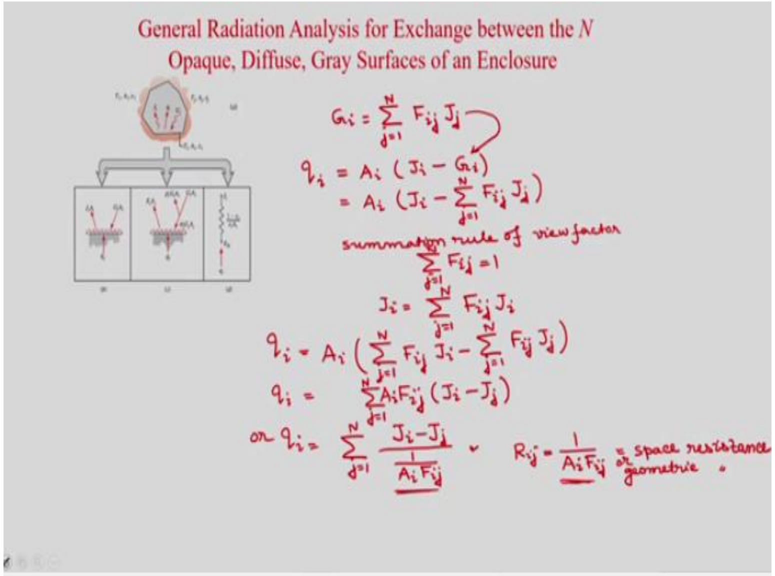

So we have written that G_i. So

$$G_i = \sum_{j=1}^{N} F_{ij} J_j$$

Now

$$q_i = A_i\,(J_i - G_i)$$

So putting the value of G_i

$$q_i = A_i\left(J_i - \sum_{j=1}^{N} F_{ij} J_j\right)$$

So, now this is your q_i, net rate at which the radiation leave the surface i. So, we can write in terms of your radiosity of surface i J_i and radiosity of surface j J_j. So here now we will use the summation rule of view factor okay; so what is that? It is

$$\sum_{j=1}^{N} F_{ij} = 1$$

So this J_i, whatever J_i is actually radiosity from surface i and going to N number of surfaces okay. So summation of all the fraction will be actually one. So this J_i actually we can write as

$$J_i = \sum_{j=1}^{N} F_{ij}\, J_i$$

So that we can write using the summation rule okay, because summation of F_{ij} is equal to 1. Or whatever J_i is leaving the surface i, it is reaching to all the surfaces 1 to N. So, now here if you put it

$$q_i = A_i \left(\sum_{j=1}^{N} F_{ij} J_i - \sum_{j=1}^{N} F_{ij} J_j \right)$$

So again simplification if you do, then you will get

$$q_i = \left(\sum_{j=1}^{N} A_i F_{ij} (J_i - J_j) \right)$$

Or,

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

So, here now we can do the electrical analogy. So here this resistance is

$$R_{ij} = \frac{1}{A_i F_{ij}}$$

This is called space resistance okay because when it is going from surface i to j, this resistance actually is coming into picture. So this is space resistance okay or geometric resistance also you can write okay.

So 2 types of radiative resistances we have seen, one is your surface resistance due to the emissivity and another is space radiative resistance which actually comes when radiation is going from surface i to j, so due to the geometrical configuration this resistance is coming into picture.

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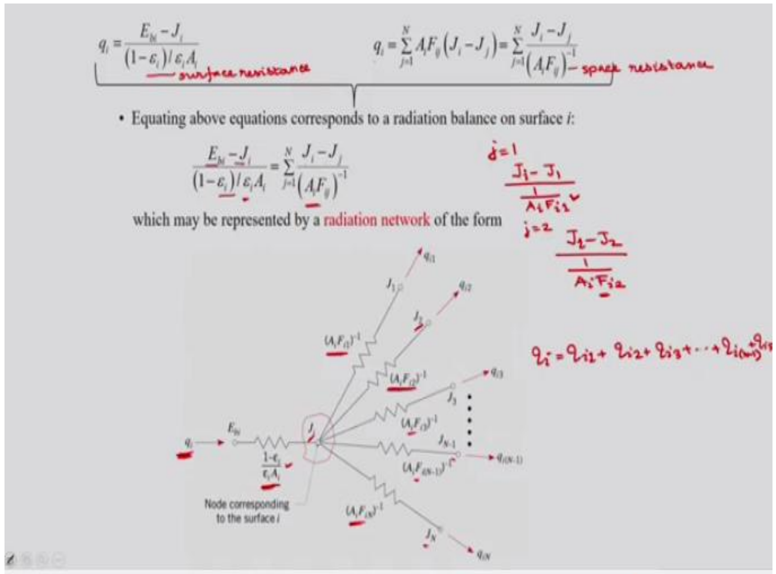

So one relation we have shown

$$q_i = \frac{E_{bi} - J_i}{\left(\frac{1-\epsilon_i}{A_i \epsilon_i}\right)}$$

And another we have written

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

So now, this q_i you can equate it,

$$\frac{E_{bi} - J_i}{\left(\frac{1-\epsilon_i}{A_i \epsilon_i}\right)} = \sum_{j=1}^{N} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

Now, what will be the radiation network in this case now? Now we can see that from Ebi to J_i one surface resistance is occurring, so this is q_i is your net rate of radiation which is leaving the surface i, so that is equivalent to electric current. Here the potential difference equivalent will be $E_{bi} - J_i$. And the resistance which is surface resistance will be $\frac{1-\epsilon_i}{A_i \epsilon_i}$. Now the right hand side is summation j=1 to N because we have considered N surfaces and your this radiation exchange is happening to N surfaces from i surface. So if you put j=1 this is your $J_i - J_1$. That is the potential difference and in the denominator $\frac{1}{A_i F_{i1}}$, which is your space resistance for j=1.

Similarly, j =2 if you put, so you will get $\frac{J_i - J_2}{\frac{1}{A_i F_{i2}}}$. So F_{i2} is the view factor when a radiation is going from surface i to 2 and space resistance is $\frac{1}{A_i F_{i2}}$, so this is your q_{i2} and similarly, you can do for q_{i2}, $q_{i,N-1}$, q_{iN} etc. So this is the radiation network and you can see it is very much complicated.

Because if you have N number of surfaces you will get this N number of space resistances but one surface resistance which is actually occurring at the surface i. So, now obviously from the energy balance

$$q_i = q_{i1} + q_{i2} + q_{i3} + \cdots + q_{i,N-1} + q_{i,N}$$

This is obviously from the energy balance you can do but you can see here that it is very much complicated when you have N number of surfaces.

So, now you can see when the radiation exchange is happening for a real surfaces from the surface at which the radiation is taking place there you have one surface resistance which is equal to $\frac{1-\epsilon_i}{A_i \epsilon_i}$ and now when it is interacting with the other surfaces or participating with the radiation, so that is obviously it is having N number of space resistance.

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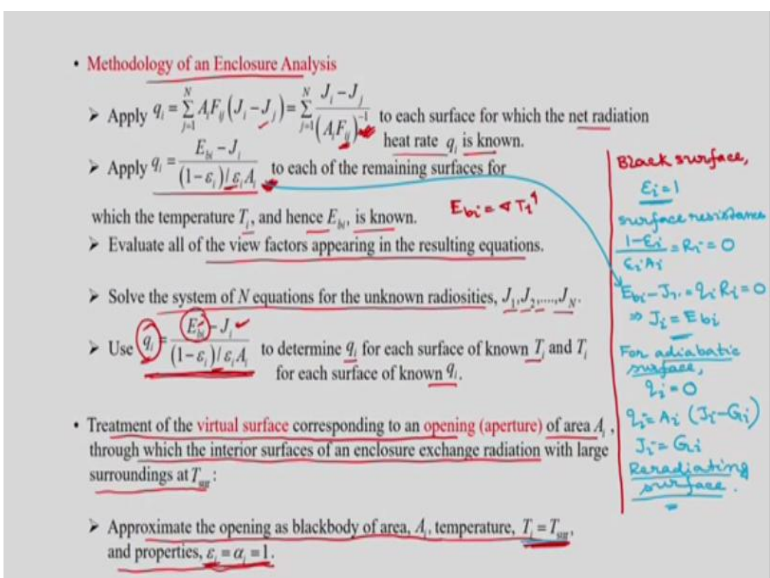

Now how you will solve any problem using this analysis, so that let us discuss. So you can see methodology of an enclosure analysis. We have already derived this equation you can see that

$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$, so that you apply to each surface for which the net radiation heat rate q_i is known. So for a surface if your net radiation q_i is known, then you apply it. And if you know this temperature T_i, then you can calculate the E_{bi}. In that case we apply this equation $q_i = \frac{E_{bi} - J_i}{\left(\frac{1-\epsilon_i}{A_i \epsilon_i}\right)}$ to each of the remaining surfaces for which the temperature T_i, is known, because $E_{bi} = \sigma T_i^4$. So, now this 2 equations for either q_i known or your E_{bi} known you can apply.

Now, evaluate all the view factors appearing in the resulting equations because in this equation you can get F_{ij}, so whatever view factors appearing here, you just calculate. Then solve the system of N equations for the unknown radiosities; J_1, J_2, J_3 to J_N. So all the unknown radiosities you can calculate from this 2 equations knowing the view factors.

Now, once you know all the radiosities, now you will use this $q_i = \frac{E_{bi} - J_i}{\left(\frac{1-\epsilon_i}{A_i \epsilon_i}\right)}$e to determine q_i for each surface of known T_i. If T_i is given, so that means your temperature is known, then you can calculate from this relation q_i because J_i already you have evaluated. Or you find T_i for each surface of known q_i. So, if the heat transfer rate is given, then q_i is known, then you can calculate E_{bi}.

And from E_{bi} you can calculate the surface temperature T_i. So now methodology is like this. So first you try to find the radiosities from N equations. So J_1 to J_N you find by solving this 2 equations. And if you find the view factor, then you will be able to find the radiosities J_1 to J_N. Once J_1 to J_N are known, then you use this equation, $q_i = \frac{E_{bi} - J_i}{\left(\frac{1-\epsilon_i}{A_i \epsilon_i}\right)}$.

So, now if T_i is given that means temperature is known, you can calculate q_i; and if q_i is known, you can calculate E_{bi}, hence temperature T_i. So this way we will calculate. Now, you consider that you have an open enclosure, okay, so one side it is open to the atmosphere. So in that case you can consider one virtual surface. So what we are telling here you can see; treatment of the virtual surface corresponding to an opening of area A_i through which the interior surfaces of an enclosure exchange radiation with large surrounding at T_{sur}.

Approximate the opening as blackbody of area A_i, temperature $T_i = T_{sur}$ and properties $\epsilon_i = \alpha_i = 1$. So what we are telling; that if have one open enclosure, so this enclosure you use some virtual surface and that is surface temperature you consider as surrounding temperature T_{sur} and you consider it is as the blackbody. So in case of black body you know whatever radiation will come, it will be absorbed.

So, $\alpha_i = 1$, then emissivity from Kirchhoff's law you can write, $\epsilon_i = 1$ is, because $\epsilon_i = \alpha_i$. So you can see that here $T_i = T_{sur}$ you consider and your surface property this emissivity ϵ_i you consider $= \alpha_i = 1$ in that case. Now, from this if you have the black surface, okay, this we have considered for the real surface, now, you consider if it is the black surface. okay if you consider black surface then obviously, $\epsilon_i = 1$. Then what will be your surface resistance? so surface resistance will be your

$$R_i = \frac{1 - \epsilon_i}{A_i \epsilon_i} = 0$$

So the surface resistance will be 0 in case of a blackbody or black surface. So, now obviously you can write

$$E_{bi} - J_i = q_i R_i = 0$$

$$\Rightarrow J_i = E_{bi}$$

So that already we have shown earlier but from this equation also you can see $J_i = E_{bi}$. And now, if you consider adiabatic surface, that means, there will be no heat transfer across this surface. So $q_i = 0$. And from previous derivations

$$A_i (J_i - G_i) = q_i = 0$$

$$\Rightarrow J_i = G_i$$

So whatever is radiosity is equal to the irradiation in case of an adiabatic surface. So whatever is coming in that is actually going out, so that means $J_i = G_i$ and this is known as reradiating surface.

So, you can see that if it is adiabatic surface for the radiation then this surface is known as reradiating surface. You should remember that for any problem if it is written it is a reradiating surface that means, it is adiabatic surface and $q_i = 0$. And if it is an open enclosure, then you consider it as a black surface and T_i you write as T_{sur} and $\epsilon_i = 1$.

So, now we have shown for N number of enclosures. If you simplify 2 surface enclosure or 3 surface enclosures then it will be easier to evaluate all those resistances and do the analysis.

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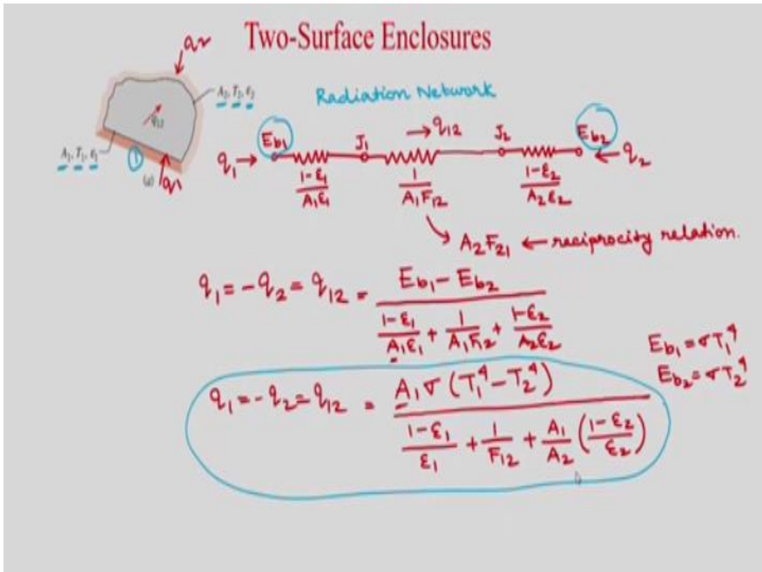

So, first we will consider 2 surface enclosure. So you can see here one surface is this one which is your surface having the area A_1, constant temperature T_1 and the emissivity is ϵ_1 and another surface you can see this is your surface A_2, okay and temperature T_2 and your emissivity is ϵ_2. So we can see there will be net radiation exchange between only this 2 surfaces.

Because it is 2 surface enclosure, so radiation exchange will be between this two surfaces. So how you will do it; so first you draw the radiation network. So if you draw the radiation network, how many surface resistances and how many space resistances will be there? As there are 2 surfaces; surface 1 and surface 2, obviously there will be 2 surface resistances and between this 2 surface resistances, now you connect with the space resistance from 1 to 2.

So, first you draw the 2 surface resistances because you can see that there are 2 surfaces and their emissivity ϵ_1 and ϵ_2, so obviously there will be 2 surface resistances. So first you draw this 2 surface resistances then you connect with space resistance. So from surface 1, you can see it is $\frac{1-\epsilon_1}{A_1\epsilon_1}$ and how it is going? It is going from E_{b1} to J_1. This is the potential difference. Another surface resistance will be there, so it will be E_{b2} and J_2. And what will be the surface resistance;

$\frac{1-\epsilon_2}{A_2\epsilon_2}$. Now you connect it this 2 with the space resistance. It will be between this J_1 to J_2 and it is $\frac{1}{A_1F_{12}}$ okay. you can write also $\frac{1}{A_2F_{21}}$, because from the reciprocity relation that also we can write. So now we have drawn the radiation network and obviously from the configuration

$$q_1 = -q_2 = q_{12} = \frac{(E_{b1} - E_{b2})}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

$E_{b1} - E_{b2}$ is the potential difference and in the denominator summation of all resistances. Here you have 2 surface resistances and one space resistance. Now you can write putting the expressions for the E_{b1}, E_{b2}

$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{A_1}{A_2}\left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

So if you have 2 surface enclosure, it is the simplest form you can actually do the radiation analysis. And if you first draw the radiation network because you know that there are 2 surfaces, so with 2 surfaces , you have 2 surface resistances that you draw first, then you join with the space resistance, then you calculate what will be the net rate at which the radiation leave the surface i and that is your $q_1 = -q_2 = q_{12}$ and that is nothing but the potential difference; $E_{b1} - E_{b2}$ divided by the summation of all the resistances, 2 surface resistances and one space resistance and with some simplification we have written this expression.

And with this, we have done the radiation analysis for 2 surface enclosure and in the next class, we will do few simplification of 2 surfaces enclosure and that will study, thank you.