Fundamentals of Conduction and Radiation

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Lecture – 03
Introduction to Conduction

Good morning everyone. Welcome to week number two, where we are going to talk about the conduction heat transfer or basically I am going to introduce you to the topic of conduction heat transfer. In the previous week, we just had a very brief introduction starting from the fundamentals of thermodynamics or the laws of thermodynamics. I have tried to make you understand what the needs of heat transfer are, or where is the scope for heat transfer study.

And there we have seen that in thermodynamics we broadly focus on the overall quantity of heat that gets transferred to or from the system. In case of heat transfer, we focus on the nature of the heat transfer, the mode of the heat transfer and also we primarily try to estimate the rate of heat transfer or the heat flux associated with any particular thermal energy transmission process.

And there the idea of conduction, convection, radiation was introduced. We have also learned the basic laws for each of those heat transfers. Now, the basic Law for convection and radiation will also be used while studying conduction. But let us start with a bit more detailed about the conduction heat transfer to be precise.

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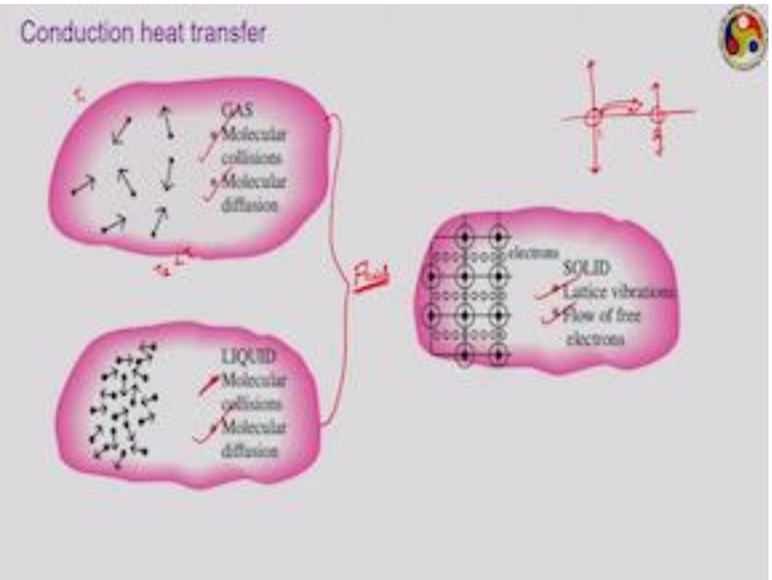

Conduction heat transfer

Now, as we have discussed in the previous week, in conduction heat transfer the nature of heat transmission or the primary mode of heat transmission comes from the molecular activity because of the random motion of the molecules in gases, or because of the lattice vibration of the molecules in a solid, we have the energy transmission.

Just to repeat on the same thing, like the situation is shown here for gasses. In gases, you know that the volume of the domain available to us is extremely large compared to the volume occupied by the molecules. Accordingly, each of the molecules have sufficient, significant amount of space or significant distance can be covered by each molecule between two successive collisions. Accordingly, the molecules can keep on moving freely, and as they move occasionally they collide with the neighbouring molecules.

Now, if there is any temperature gradient, say like the diagram shown here, if suppose this side we have a high temperature, let us call this T_1 and this side we have a low temperature, let us call this T_2 and this T_2 is less than T_1. Then the molecules, I probably have mentioned that in the previous lecture also still I am repeating the same thing; the molecules which are close to this high temperature zone they are expected to have higher energy content compared to the molecules you are close to this low temperature zone.

Accordingly, the molecules in this zone will be having higher kinetic energy because at the molecular level, kinetic energy is the only form of energy that we need to consider. So they will be moving randomly or rather they will be moving with a higher kinetic energy compared to the second group of molecules. And if I draw one arbitrary plane somewhere here, then the number of molecules crossing this particular plane from top to bottom will be more compared to the number of molecules crossing it from bottom to top.

Accordingly, there is a net energy transmission, from the high temperature side to the low temperature side. And this particular thing we refer as the molecular diffusion. And also when these high energy content molecules collide with the low energy content molecule, then they also transfer a larger amount of momentum to the low energy content molecule and accordingly by molecular collision also the low energy level molecules can get some additional energy from the high energy level molecules.

So, that is what they are referring as molecular collisions. So, molecular collisions and molecular diffusions are the two types of mechanism that we can have in gas conduction. But generally the number of molecules being extremely small compared to the overall space available to us. The total conduction or capacity for a gas to transfer heat by conduction generally extremely small compared to the convection or radiation mode of heat transmission. And also if we think about the effect of temperature on this, as the temperature of the gas increases, their overall energy level increases and accordingly their molecular activity also increases. So you can expect the capacity for the gas to transmit energy by conduction or heat by conduction, you should keep on increasing with temperature. We shall be seeing that again shortly.

Next let’s talk about the liquids. In liquids molecules are allowed to move freely too, but not as freely as the gases; rather than molecules are generally bound together in a somewhat close form clusters and these clusters are a group of molecules generally float around with respect to each other. And therefore both the mechanisms and molecular diffusion and molecular collisions are still valid. But as molecules are not allowed to move as freely as in case of gases, so the effect of molecular collision may not be as much prominent as compared to gases. But molecular diffusion is the primary mechanism of heat conduction in case of liquids, where the molecules can have bulk motion. Accordingly as they move from high temperature zone to the low temperature zone, they carry energy with them as well. And then we can come to the solids.

In case of solids, we know that solids are generally bound by solid crystal structure or solid Lattice structure, and therefore the molecules are not allowed to move at all; or at least are not allowed to move, just similar to the gas or liquid molecules. But of course they can move while maintaining their own position. That means they can vibrate with respect to their base position and that lattice vibration is the primary mode of heat conduction in case of solids. As I probably had mentioned, say if this is one molecule which is at a higher energy level and this is molecule number two which is at a lower energy level, then we can expect that both the molecules will be vibrating, but the vibration amplitude for molecule number one is expected to be larger than the vibration amplitude for molecule number two. And they are vibrating and in lattice structure as they are quite close to each other.

So during vibration this molecule number one will also try to induce the same amount of vibration amplitude to molecule number two and accordingly, it will transmit some amount of energy to this molecule number two. And this lattice vibration is the primary mode of mechanism. There is a secondary mode of mechanism that is applicable particularly for conducting solids, the flow of free electrons.

In a conductor there are lots of free electrons available and as they keep on moving from one position to another position, maybe because of some kind of potential difference, they will also lead to the transmission of conduction energy or thermal energy. In case of gases and liquids, we have primary mechanism as molecular diffusion and also molecular collision, most prevalent in case of gases; in fact gases and liquids together we can also refer them as fluid.

So while in case of fluids, we have molecular diffusion and molecular collisions as the primary mechanism behind conduction heat transfer, in case of solid we have lattice vibration and the flow of free electrons in case of conducting solids as the primary mechanisms.

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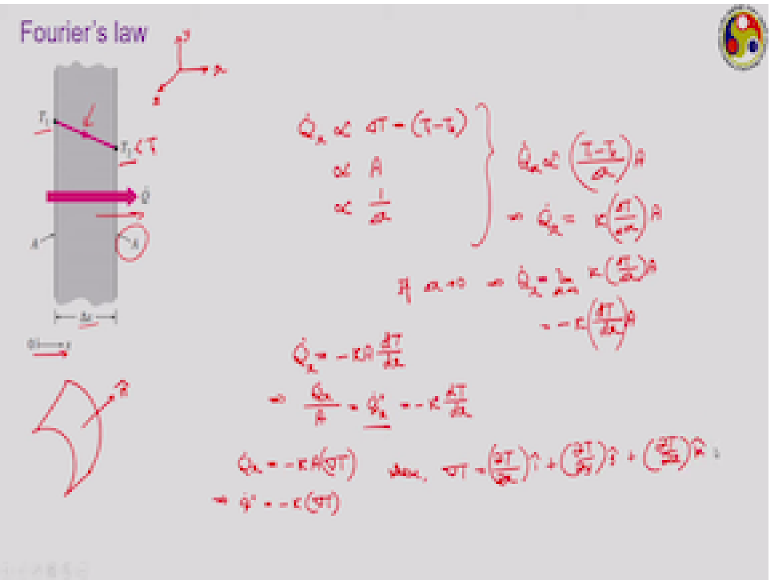

Fourier's law

$\dot{Q}_x \propto \Delta T = (T_1 - T_2)$

$\propto A$

$\propto \frac{1}{\Delta x}$

$\dot{Q}_x \propto \left(\frac{T_1 - T_2}{\Delta x}\right)A$

$\Rightarrow \dot{Q}_x = k\left(\frac{\Delta T}{\Delta x}\right)A$

If $\Delta x \to 0 \Rightarrow \dot{Q}_x = \lim_{\Delta x \to 0} k\left(\frac{\Delta T}{\Delta x}\right)A$

$= -k\left(\frac{dT}{dx}\right)A$

$\dot{Q}_x = -kA\frac{dT}{dx}$

$\Rightarrow \frac{\dot{Q}_x}{A} = \dot{q}''_x = -k\frac{dT}{dx}$

$\dot{Q}_x = -kA(\nabla T)$ where, $\nabla T = \left(\frac{\partial T}{\partial x}\right)\hat{i} + \left(\frac{\partial T}{\partial y}\right)\hat{j} + \left(\frac{\partial T}{\partial z}\right)\hat{k}$

$\Rightarrow \dot{q}'' = -k(\nabla T)$

Next is the Fourier's law of heat conduction, which I have already introduced, but I am now trying to go into a bit more detail on that. Here, I am showing you one block. Just assume it's a block of some thickness Δx and its dimensions in the other two directions are extremely large. Let us apply a coordinate system somewhat like this. Say, this is our x direction, this is our y direction and this is the z direction.

Now, we are assuming that the dimension for this particular solid in the y and z direction is infinite or practically speaking, you can say that the dimensions in the y and z directions are extremely large compared to this Δx. And then it becomes very much a one dimensional kind of scenario. Why one dimension; that I shall be coming in the next class, but for the moment you just consider that we have a block where we have temperature T_1 at one end and temperature T_2 at the other end.

Here this T_2 is less than T_1 and the distance between these two planes is Δx. A refers to the area of the plate, that is the area of the y,z surface. So its a situation of pure conduction because inside the domain, the space inside these two plates has been filled by a solid and as there is no fluid flow available and then we have conduction only.

There is no convection, and we are also neglecting any kind of radiation mode also. Again, as I have mentioned in the previous week, convection and radiation are primarily related to the surface of a solid, whereas here we are talking about transfer inside the surface; so conduction is the mode of heat transfer.

In this kind of scenario it has been observed that if suppose $\dot{Q}$ refers to the total rate of heat transmission by conduction. It has been found that, it is proportional to the temperature difference ΔT which is nothing but T_1 minus T_2 in this case. It is also proportional to the area of the plates. But it is, it has been found to be inversely proportional to the Δx.

$$\dot{Q} \propto \Delta T = (T_1 - T_2)$$
$$\propto A$$
$$\propto \frac{1}{\Delta x}$$

All these three observations are purely from experiments or observation. The Fourier's law of heat conduction, which we are trying to frame here, has not come from any kind of mathematical background; rather it is a purely phenomenological law which has been observed just from experiments. Like the laws of thermodynamics as well, these are all phenomenological law. So, if we combine these three together now, then what we can write, the rate of heat transmission then is proportional T_1 minus T_2 upon Δx into the area of the plate.

$$\dot{Q} \propto \frac{T_1 - T_2}{\Delta x} A$$

Or, if we introduce a proportionality constant, then we can write this as

$$\dot{Q} = K \frac{T_1 - T_2}{\Delta x} A$$

where this K is the constant of proportionality. Now, if the thickness of the plate is extremely small, that is if Δx tends to 0, then how we can modify this. So, $\dot{Q}$ in that limiting case is limit of Δx tends to 0, K ΔT upon Δx into A, becomes equal to

$$\Delta x \to 0$$

$$\dot{Q} = \lim_{\Delta x \to 0} K \left(\frac{\Delta T}{\Delta x}\right) A$$

$$= -K\left(\frac{dT}{dx}\right)A$$

and I am putting a negative sign here.

So, this dT/dx basically refers to the temperature gradient, because we know that limit delta x tends to 0, ΔT/ Δx can be represented by this. And I am putting the negative sign because as we are moving in the positive x direction temperature is reducing like shown by this particular profile, temperature is continuously reducing. So, dT/dx actually is a negative quantity. And therefore to make this heat transfer positive or rate of heat transmission positive, we are putting this negative sign.

So the one dimensional form of Fourier's law of heat conduction then comes as $\dot{Q}$ is equal to

$$\dot{Q}_x = -K\left(\frac{dT}{dx}\right)A$$

And I should put the suffix x here to indicate that here we are talking about heat transmission in the x direction only. Or, if I divide this by the area, then we get the heat flux in the x direction to be equal to

$$\frac{\dot{Q}_x}{A} = \dot{q}_x'' = -K\left(\frac{dT}{dx}\right)$$

This is the equation that I wrote in the last class as well, but we have as such not used this so far.

So, this is the form of the Fourier's law of heat conduction in x direction. However, one thing we have to understand or couple of things I would like to mention here. This particular heat flux quantity, can you tell me it is a scalar or a vector? Whether you know from your basic definitions on from physics that scalar refers to quantity which has only magnitude but no directions, whereas vector has both magnitude and directions. Now, your entire study of thermodynamics you never bothered about any quantity being scalar or vector, because thermodynamics never talks about any rate or the directions as such. Of course in second law you get a direction of a heat transfer from high temperature to low temperature but that does not talk about the direction in terms of space coordinates.

But here we are talking about proper directions and therefore whether it is scalar or vector it is important to know. Like the temperature is a scalar quantity, this K that I am talking about that can also be taken as a scalar quantity for the moment. But this heat flux is a vector quantity, because here we are talking about the heat flux in a particular direction. And what is the direction? The direction is perpendicular like shown here perpendicular to the area A that we are talking about.

So, the expression that we have got here from Fourier's law of heat conduction is the mathematical expression for the heat flux in the x direction or you can say it is the expression for heat flux acting normal to this area A. Here the area A we have taken on the y z plane, that this is perpendicular to the x direction, but it is very much possible that the area may have a three dimensional orientation as well.

Something hypothetical area that we can draw something like this, which, which has its normal. Like if say this is the normal of this, this normal can have all three coordinate directions. And in that case, there is a more generalised form of Fourier's law of heat conduction, where we write that

$$\dot{Q}_x = -KA(\nabla T)$$

But this grad is a vector operator or we can write the heat flux to be equal to

$$\dot{q}" = -K(\nabla T)$$

where grad of T refers to

$$\nabla T = \left(\frac{\partial T}{\partial x}\right)\hat{\imath} + \left(\frac{\partial T}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial T}{\partial z}\right)\hat{k}$$

with i j k being the corresponding unit vectors.

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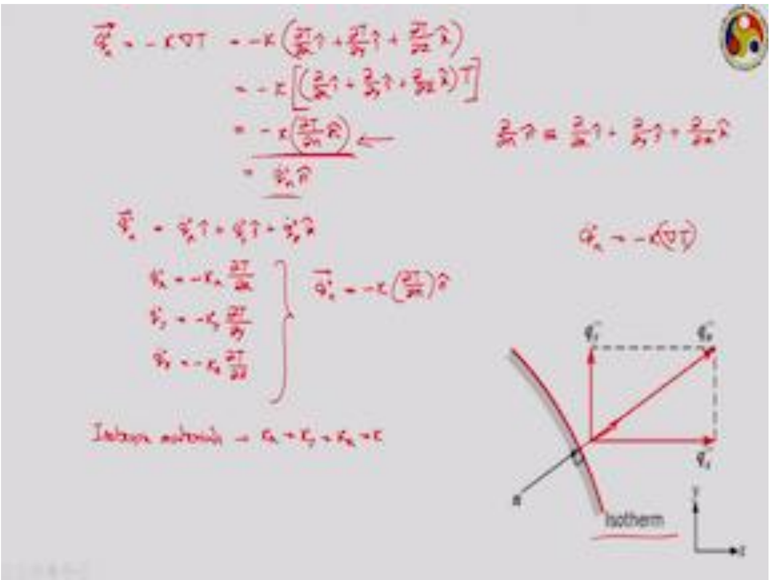

So, we can then write that the generalized form Fourier's law of heat conduction quite often we put a script n to indicate that here talking about heat transfer in this particular normal direction, direction normal to the area is equal to

$$\dot{q_n}" = -K(\nabla T) = -K\left\{\left(\frac{\partial T}{\partial x}\right)\hat{\imath} + \left(\frac{\partial T}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial T}{\partial z}\right)\hat{k}\right\}$$

or maybe just in terms of operator

$$= -K\left\{\left(\left(\frac{\partial}{\partial x}\right)\hat{\imath} + \left(\frac{\partial}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial}{\partial z}\right)\hat{k}\right)T\right\}$$

and sometimes just to indicate the direction of n, as everything comes from the direction of n; then we sometimes also write this one

$$= -K\frac{\partial T}{\partial n}\hat{n}$$

in the nth direction but this $\frac{\partial T}{\partial n}$ is the representative of this distribution.

Or

$$\frac{\partial}{\partial n}\hat{n} = \left(\frac{\partial}{\partial x}\right)\hat{\imath} + \left(\frac{\partial}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial}{\partial z}\right)\hat{k}$$

Here this n is the normal to the area that we are talking about, and this particular representation is directly giving you the value of heat flux in the nth direction.

There is another way we can write this

$$\dot{q_n}"\hat{n}$$

Because, here we are talking about the vector quantity, whereas in this expression, this is the just magnitude of the heat flux in the nth direction and $\hat{n}$ is the orientation of the surface. So, these heat flux being in vector quantity, we can easily classify or rather separate this one into three components as well.

So, we can easily write this

$$\overrightarrow{\dot{q_n}"} = \dot{q_x}"\hat{\imath} + \dot{q_y}"\hat{\jmath} + \dot{q_z}"\hat{k}$$

Where, this x component is given as

$$\dot{q_x}" = -K_x\frac{\partial T}{\partial x}$$

the y component is given as

$$\dot{q_y}" = -K_y\frac{\partial T}{\partial y}$$

And the z component is giving as

$$\dot{q_z}" = -K_z\frac{\partial T}{\partial z}$$

So, this is a more complete representation of the Fourier's law of heat conduction. Here K_x, K_y and K_z are the proportionality constants, which may have some direction depending on nature in certain materials. Thankfully for most of the materials K_x, K_y, K_z all are equal to each other and we call this kind of materials to be isotropic materials. So, for isotropic materials we have

$$K_x = K_y = K_z = K$$

And therefore for isotropic materials, we can easily write this heat flux in the nth direction to be equal to

$$\overrightarrow{\dot{q_n}"} = -K\left(\frac{\partial T}{\partial n}\right)\hat{n}$$

But actually in this form we have assumed K of the material to be isotropic. But in more proper sense, K should be a part of the derivation that I have mentioned here. For most of the materials, as I have mentioned, are isotropic in nature, but certain materials we may have to consider this variation.

So this is the scenario that I am talking about, this is of course drawn in a two dimensional plane. Now what else we can say about the surface? Here, we are talking about heat transfer in the direction normal to the area. Then what is the temperature along the area? Here, heat is being transferred normal to the area and we can get components in x, y, or z directions. But what about the component along the surface?

The tangential component of course has to be zero. And what does that mean? That tangential component of heat flux along this particular surface is zero, and that is possible only when the temperature along this surface is equal throughout. That means a surface that you are talking about is an isotherm or a constant temperature surface. So in Fourier's law, basically we are getting the expression for heat flux, in the direction normal to an isotherm.

And that is given by, for the last time I write the expression, the heat flux, in the direction normal to an isotherm is expressed as

$$\overrightarrow{\dot{q}_n''} = -K(\nabla T)\hat{n}$$

or maybe just

$$\overrightarrow{\dot{q}_n''} = -K(\nabla T)$$

where the grad itself will take care of that direction part. Now, the K that I have mentioned about is known as the thermal conductivity and is a very important property of material because it is the property which dominates the entire phenomenon of heat conduction.

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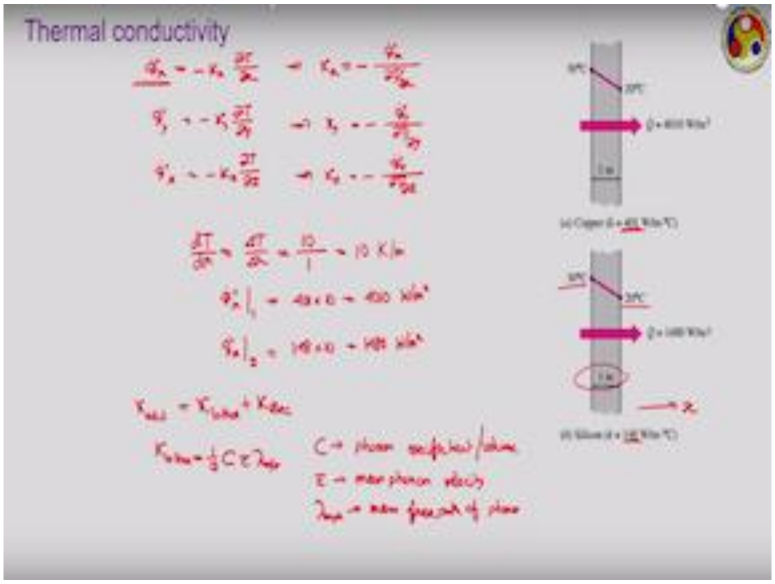

Thermal conductivity

So, if I just go back to the basic representations, there we have found that the transmission in the x direction was given as

$$\dot{q}_x'' = -K_x \frac{\partial T}{\partial x}$$

in the y direction it was

$$\dot{q}_y'' = -K_y \frac{\partial T}{\partial y}$$

and in the z direction this was

$$\dot{q}_z'' = -K_z \frac{\partial T}{\partial z}$$

So, from there we can write

$$K_x = -\dot{q}_x''/\frac{\partial T}{\partial x}$$

$$K_y = -\dot{q}_y''/\frac{\partial T}{\partial y}$$

$$K_z = -\dot{q}_z''/\frac{\partial T}{\partial z}$$

Again, as I mentioned for isotropic materials all three are equal to each other. But the important thing here is that, the magnitude of heat transfer, like if we talk about any one of them; let us say pick up the heat flux in the x direction, here the magnitude is dependent not only on the temperature gradient but on the K_x also. Like the situation shown here. Here I am showing situation of two different materials.

We have two blocks of identical dimension. Both of them are having thickness of one meter. For both blogs temperature of the left face is 30 ºC and on the right face is 20 ºC. So the temperature gradient that is acting assuming one dimensional heat conduction here; that is

$$\frac{dT}{dx} = \frac{\Delta T}{\Delta x} = \frac{10}{1} = 10\ K/m$$

That is 10 Kelvin per meter. Here this capital K is important capital K denotes Kelvin, whereas if we write small k that refers to the kilo prefix. So, the effective temperature gradient is equal for both the cases. However, in one case we have a material of copper which has a thermal conductivity of this. And this case we have silicon which has the thermal conductivity of this. Then what will be the corresponding rate of heat transfer?

So the heat flux in the x direction for the first material will be equal to

$$\dot{q_x}''\Big|_1 = 401 \times 10 = 4010 \frac{W}{m^2}$$

In the previous week we have discussed on the unit, so I am sure that is clear to you. Similarly, the conduction heat flux in the x direction for the second situation.

$$\dot{q_x}''\Big|_2 = 148 \times 10 = 1480 \frac{W}{m^2}$$

Accordingly, different materials can show different rate of heat transfer despite being subjected to the same temperature gradient, just depending upon the value of its thermal conductivity. Some of the materials like copper, aluminium etc. are extremely good conductors. Generally the materials which are good electrical conductors, solid materials that is, they are also good thermal conductors because they have lots of free electrons and the free electrons contribute towards their transmission of both electricity and thermal energy, whereas certain materials can have extremely low thermal conductivity as well.

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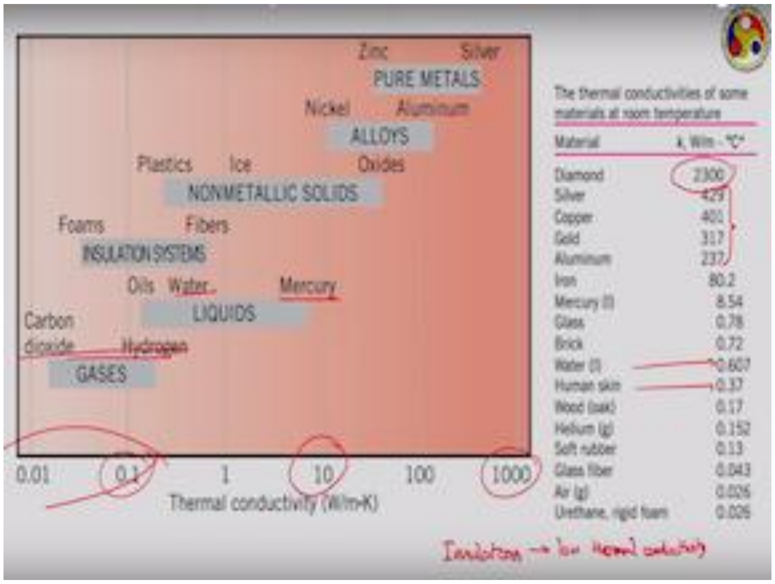

The thermal conductivities of some materials at room temperature

Material	k, W/m · °C*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

Insulation → low thermal conductivity

If we just visualize the normal chart of thermal conductivity for different materials; for gases generally the thermal conductivity level is very low just shown here, for liquids we are more on this sides, such as water. Mercury is one of the conducting metals which is remains a liquid state. So its conductivity is significantly higher. Look at the level, for mercury it is 10 W/m.K , whereas for gases of hydrogen etc it is just 0.1.

So that is 2 order difference. When you go to something like aluminium or silver, it is near 1000. Not 1000 actually but silver will be in the range of copper only. So that is extremely high conductivity that we are talking about. Now if we are talking about the source or reason behind conductivity, generally as we have seen particularly for solids, there are two ways heat can get transmitted by conduction.

One is the lattice vibration, other is the molecular activity. Accordingly, conductivity also can have two components. So the thermal conductivity for solids that is,

$$K_{solid} = K_{lattice} + K_{elec}$$

the first part refers to the lattice vibration, second part corresponds to the electrical activity.

Now from the kinetic theory, it generally can be written as the value of K for any solid is equal to

$$K = \frac{1}{3} C \bar{c} \lambda_{mfp}$$

λ_{mfp} is the mean free path. Here, when we are talking about a conducting material here, this C refers to the electron specific heat per unit volume, $\bar{c}$ refers to the mean electron velocity, where λ_{mfp} refers to that mean free path of the electron.

So you know the idea of mean free path, mean free path refers to the average distance travelled by each electron between two successive collisions. So when we are talking about the conductor, this is basically the expression that I have written that corresponds to the electrical part. So once we know these quantities, following kinetic theory we can get some idea about this K electron. Now when you are talking about a conductor, the electron velocity is quite high.

The mean free path is also high and accordingly we get this component to be extremely high. However for non-conductors, this component can be quite low. Then the lattice part becomes more significant. Certain non-conductors like say diamonds, which are electrically non-conductor or Beryllium. For them this part is more significant. If we want to calculate the $K_{lattice}$, then we can use the same relation following kinetic theory. But instead of electron, we are going to have phonons in this case.

So C becomes a phonon specific heat per unit volume, $\bar{c}$ becomes mean phonon velocity and λ_{mfp} refers to the mean free path of the phonons. So for different materials we can have different values for this. But in general, for conductors that K_{elec} is extremely large. $K_{lattice}$ is almost negligible for materials like copper, silver, whereas for non-conductors the lattice part is more important.

So for non-conducting solids, particularly if they are arranged in certain structure, then the value of $K_{lattice}$ will also be quite high. So amorphous materials has the $K_{lattice}$value to be quite small compared to the crystalline materials. Like the example of diamond. Another excellent example can be graphite. You know that in case of graphite, the carbon molecules are arranged in certain layers.

So if we think about conductivity along a layer, then that will be very high because we have particular lattice structure. Whereas if we are going from one layer to another layer, then conductivity is quite low, because in between there is no lattice structure. The layers are more or less floating over each other or placed in a very crude way. So this is the chart that I have given and these are the values for thermal conductivity of some of the common materials under normal temperature of solid liquid and gas. For all of them it is given. Look at the values for silver it is 429, copper 401, gold and aluminium 237. Aluminium and copper are two very common materials that are used in the cases where you want high conduction. Silver is also used in precise electrical instruments and also in certain thermal instruments.

But look at the other thing diamond, it is having an extremely large value of this thermal conductivity and that is because of its crystalline structure. This entire value of this 2300, almost entire part of this is coming only from the $K_{lattice}$part. Because this is a non-conductor, so K_{elec} is zero. Whereas for mercury and iron, K_{elec} is the one that is prevalent.

As mercury is liquid, the molecular diffusion plays a role but still mercury being a metal; it has certain amount of free electrons which also contributes to its high conductivity. Of course not high compared to copper or aluminium; but still higher than several other materials. Stainless steel which we commonly used as utensils etc. will be having a value in the range of 13 to 15. There are different grades of stainless steel depending on the grade their values varies, but generally in the range of 13 to 15.

Now look at thermal conductivity of certain other materials, the brick or wood 0.17, rubber 0.13 and glass fibre 0.043, glass wool or foam 0.026. These values are extremely small and as this materials and having extremely small thermal conductivity, so they are actually used as insulators. Insulator is opposite of conductors or the objective of insulators is to reduce the thermal conduction.

So for insulators we want low thermal conductivity. And the materials which I have encircled here, they all can be very good thermal insulators. Like wood or this glass fibre or rigid forms. They are very commonly used. Foams are also used for suppression of fire because of its low thermal conductivity. Now if you move to some other materials like for water, liquid water; that is in the range of 0.6.

So water is also a poor thermal conductor but much better than something like glass wool, etcetera. At least one order difference. And this is interesting, human skin 0.37. So our skin is also not a very good conductor and that actually helps us, when we are exposed to very high or very low temperatures like when you are exposed to a very high temperature, skin will act something like an insulator and it will not allow the heat to pass through very quickly.

Whereas when you are exposed to very low temperature, again the skin will allow us to retain our body heat for longer period compared to any material having higher conductivity. Also the dresses that we wear, we want the conductivity to be low so that we can retain the body heat, particularly in cold environment. Like you know in the cold environment we go for woollens. Now wool has lower thermal conductivity compared to cotton or polyester materials, and therefore it is able to restrict the leakage of heat from the body to the surrounding by conduction.

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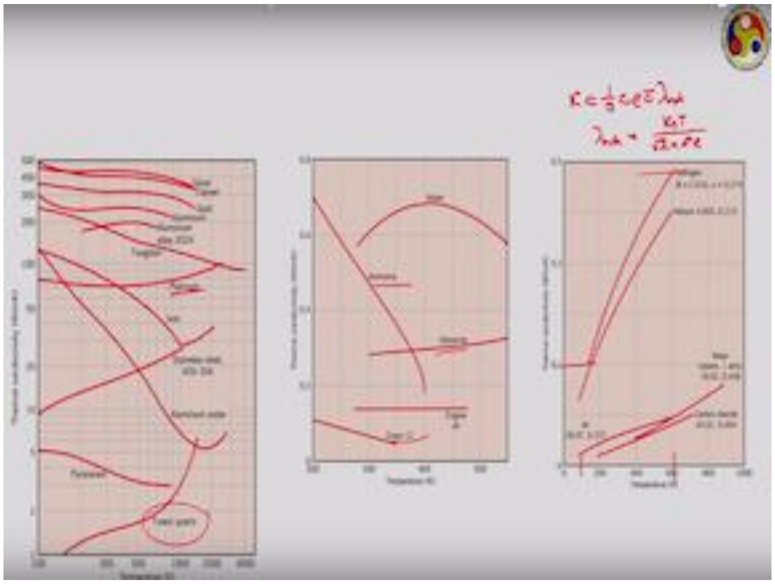

$K = \frac{1}{3} c_v \rho \bar{c} \lambda_{mfp}$

$\lambda_{mfp} = \frac{k_B T}{\sqrt{2}\pi d^2 P}$

Now this is a chart which shows the temperature dependence of thermal conductivity for solids. As the temperature increases, the electronic movement part that does not get affected that much. But lattice structure, the $K_{lattice}$ part generally keeps on reducing. Accordingly you can see there is a little bit of reduction for most of the materials thermal conductivity. But exception can be platinum at high temperature or certain kinds of quartz.

If you go to liquids now; for liquids of course we have molecular diffusion as the primary mechanism and that's why generally for liquids as the temperature increases, their conductivity reduces. Like ammonia or refrigerants. For water it generally increases a bit and then once you go to higher temperature, it keeps on reducing because water has gone to the vapour phase. Glycerine or certain degrees of engine oil can be exception, so thermal conductivity can increase a bit. For gases, as I mentioned, as the temperature increases, the molecular activity increases for gases. The source of conduction heat transfer is molecular diffusion and also molecular collision. Now as temperature increases, the level of both molecular collision and molecular diffusion keeps on increasing. Accordingly their thermal conductivity also keeps on increasing.

Just shown here, as the temperature changes from something around 150 K to 600 K, thermal connectivity of hydrogen you can see, has gone from 0.1 to about 0.3; so there is 200% increase in the value of thermal conductivity for hydrogen. For gases if we want to calculate this, quite often we follow a relation again from the Kinetic theory, I am just writing it from the source.

You do not have to remember this relation. This is more to give you an idea on how to estimate that thermal conductivity for gases from kinetic theory. It is about

$$K = \frac{1}{3} c_v \rho \bar{c} \lambda_{mph}$$

Here $\bar{c}$ and λ_{mph} are just the definition that we use in the previous slides, ρ is the density c_v is a specific heat at constant volume and the λ_{mph} for ideal gases can generally be expressed as

$$\lambda_{mph} = \frac{K_B T}{\sqrt{2}\,\pi d^2 \rho}$$

Where K_B is that Boltzmann constant, whose value definitely you know, which is $1.38 \times 10^{-23} \frac{J}{K}$ and d is the average diameter of the gas molecules, or statistically estimated diameter of the gas molecules and ρ is the density. T is the absolute temperature. So once we know the temperature and pressure, we can calculate its density using ideal gas equation of state.

Also you can get some idea about the value of its diameter, which generally is related to the molecular weight and then once we know thatλ_{mph}, then we can calculate that thermal conductivity as well. So this is about thermal conductivity.

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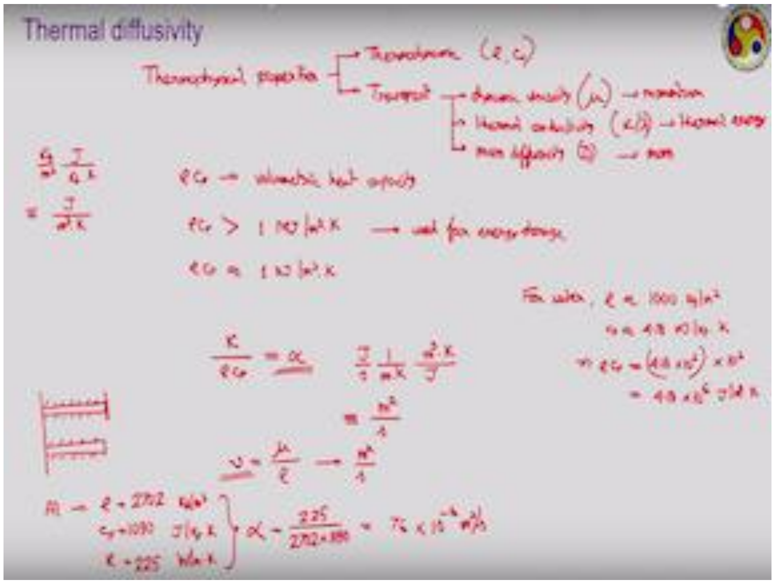

Thermal diffusivity

There is another property that is also very important that is called the Thermal diffusivity. Now the thermo physical properties for any matter are generally classified into two categories. So thermo physical properties can be classified as one group, which we call the thermodynamic properties and other is the transport properties. Thermodynamic properties referred to properties like density, specific heat, which are used extensively in thermodynamic analysis.

Because they are used more for relating that gross behaviour of a system and they are primarily associated with the equilibrium state of the system. So when the system reaches equilibrium, we can easily calculate the value of these quantities. And accordingly we call them thermodynamic quantities. But transport quantities are associated with transport or movement or certain parameter crossing the boundary of the system.

Then the transport properties come into picture. Already couple of transport properties you must know. One transport property is viscosity or dynamic viscosity, which you already introduced in your fluid mechanics. Generally the symbol μ is used for this. Dynamic viscosity is associated with the transport of what? It is associated with the transport of momentum. Another example that just we have discussed, that is the thermal conductivity.

We are using K but in certain literature, you may see the symbol λ also, which is associated with the diffusion of thermal energy, or transport of thermal energy. Another quite popular transport property is the mass diffusivity. Generally the symbol D is reserved for this, which is associated with mass transfer. So this way we can have several kinds of transport properties. Say the electrical materials or something like electron is getting transported accordingly we can also define more transport properties.

So thermal conductivity is a transport property quite similar to the dynamic viscosity, whereas ρ and C_p or C_v, these are thermodynamic properties. Now this product ρC_p is generally known as the volumetric heat capacity. Why we are giving such a name? What is ρ? Mass per unit volume, so ρC_p is the capacity of the material or concerned substance to store energy per unit volume and that's why we are calling it volumetric heat capacity.

Of course ρ and C_p can vary widely for different materials. But quite often this product ρC_p can be of the same order. Now when we are dealing with a certain substance whose ρC_pvalue is quite high saying that it is of the order of say one. Okay, what should be the unity of this ρC_p product? ρ has a unit of kg/m^3 and what is the unit of C_p? C_p is J/kg.K.

So its unit becomes then, J/m^3.K, its capacity to store energy. Now when the rho Cp value is greater than 1 MJ/m^3.K, that is greater than 10^6; these kinds of materials are used for energy storage. Whereas when this ρC_p is quite low, say in the order of 1 kJ/m^3.K, then this kind of materials cannot be used.

They are not suitable for thermal energy storage as well. What is the value of ρC_p for liquid water, can you guess? For water under atmospheric condition that is, we have rho approximately equal to1000 kg/m^3. And what is C_p for water? It is about 4.18 kJ/kg.K. So ρC_p product for water will be equal to

$$\rho C_p = 1000 \times 4.18 \times 10^3$$
$$4.18 \times 10^6 \text{ J/m}^3\text{.K}$$

So it is greater than 1 MJ/m^3.K. So water can also be a good material for energy storage. You must have seen it in our houses also, like once we heat water, to a certain higher temperature than it takes long time for the water to dissipate the heat and come back to the lower temperatures.

But now, the ratio of one transport property which is a thermal conductivity, and product of two thermodynamic properties, which is this ρC_p, the volumetric heat capacity is known as the thermal diffusivity. It is α, generally the symbol α is reserved for this. K is a thermal conductivity in the numerator and this volumetric heat capacity in the denominator which we are calling the thermal diffusivity.

$$\alpha = \frac{K}{\rho C_p}$$

It gives us an idea about the ability of a material to transmit energy compared to its ability to store. Because in your numerator you have thermal conductivity which characterizes the ability of the material to transport energy by conduction; in the denominator we have this ρC_pproduct who determines the ability for this material to store thermal energy. Accordingly this gives us a ratio of ability of the material to transmit energy compared to its capacity to store.

So if we are talking about two materials having different values of α, then the one having higher value of α is expected to transmit energy more. Like in your school level physics, you have probably heard about this experiment. Say we are having a surface which is maintained at a certain high temperature and then we are having two rods of different material.

They are having identical dimension, both of them are exposed to the surrounding or at least the tip is exposed to the surrounding. These surfaces maybe insulated; only the tip is exposed to the surrounding which is at a temperature much lower than the surface. Now their dimensions are also identical and they are exposed to the same temperature difference then the time it requires for the temperature of this end to attain the temperature of the base will depend upon the value of the α. For which case the time will be larger?

Just think. This is something that I will ask again in the next lecture. But what will be the unit for this thermal diffusivity. K is the thermal conductivity in the numerator which we have identified as a unit of W/m.K and in the denominator we have the volumetric heat capacity, which has a unit of J/m^3.K, so Joule and Kelvin cancels out so we are left with m^2/s.

Have you seen this unit earlier? Do you remember ν, the dynamic viscosity which is defined as the ratio of dynamic viscosity and density? That is also having a unit of meters square per second because this α and this ν they are analogous to each other. The α characterises the thermal diffusion where this one characterizes the momentum diffusion, and that's why they are very much analogous to each other.

So once we know the value of these properties, we can usually calculate the value of α. Let us take some examples. Let us say I have data for aluminium, pure aluminium that is; for pure aluminium rho is given as 2702 kg/m^3. Its specific heat is equal to 903 J/kg.K. Remember aluminium we are talking about solid materials, so C_p and C_v are identical for them and thermal conductivity K for aluminium, we have seen in the previous slide 237 W/m.K.

So if you combine them then α will be equal to

$$\alpha = \frac{237}{2702 \times 903} = 97.1 \times 10^{-3} \frac{m^2}{s}$$

All are expressed in basic assigned units. So we do not have to talk about the unit conversion. So the value of our α for aluminium is extremely small. Whereas if we talk about the same values of aluminium at say higher temperatures.

Now initial values I have given at 300 K that is 27 ºC. now I have the data at 700 K. For 700 K, it has been found to be the density remains more or less the same 2702, specific heat changes and increases to 1090 and as we know for solids, thermal conductivity decreases with temperature i.e. 225.

$$\alpha = \frac{225}{2702 \times 1090} = 76 \times 10^{-3} \frac{m^2}{s}$$

Accordingly its thermal diffusivity reduces even more. So this is purely dependent on the materials and also with temperature as the property changes. The value of thermal diffusivity also changes accordingly modifying the capacity of the material to transmit heat by conduction.

So in today's class you have been introduced to two different thermo physical properties, transport properties that is; and in the next class we shall be talking about using the Fourier's law of heat conduction for development of the heat diffusion equation.

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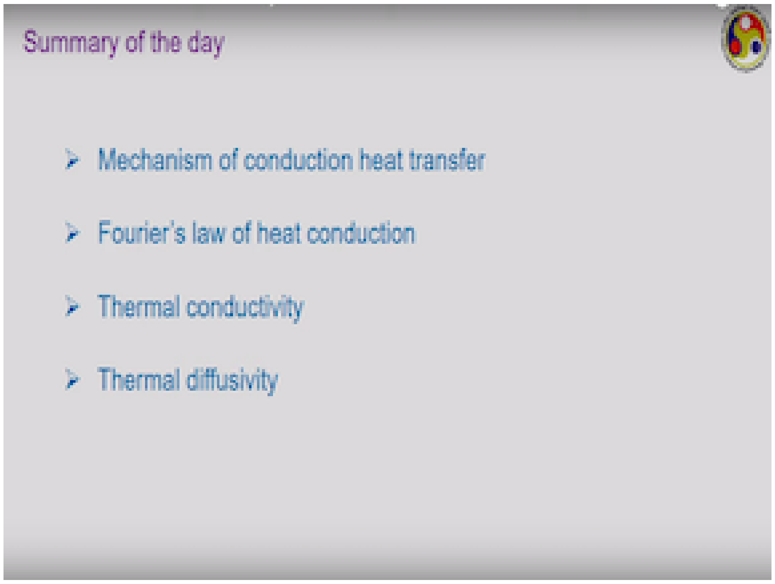

Summary of the day

- Mechanism of conduction heat transfer
- Fourier's law of heat conduction
- Thermal conductivity
- Thermal diffusivity

So just to summarize our discussion today, we discussed briefly the mechanism of conduction heat transfer, then the generalised form of Fourier's law heat conduction in three dimensional form in terms of the generalised temperature gradient or acting in that direction normal to an isothermal surface. Then we discussed in detail about the thermal conductivity its contribution coming from the lattice structure and also from the free electrons, particularly in case of solids.

And then the concept of thermal diffusivity was introduced, which is a ratio of thermal conductivity to a volumetric heat capacity. And accordingly, it gives an idea about the relative importance or relative significance of thermal conduction to the thermal energy storage. So that is where I would like to stop today. In the next class as I mentioned, I shall be developing the generalized version of the heat diffusion equation for in a solid medium to start with; till then, you please revise this lecture and once you are confident about this, then only please move to the next video. Thank you very much.