**Fundamentals of Conduction and Radiation**
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**Lecture - 29**
**View Factor**

Hello everyone. So in last few classes of radiation you have learned the radiation from a surface. But today we will study radiation exchange between surfaces. So you can see radiation exchange between surfaces. So you may have enclosures where N number of surfaces maybe there. So from one surface to another there will be radiation exchange. And this radiation exchange depends on the geometry of the surface as well as its orientation. At the same time it depends on the surface properties and the temperature.

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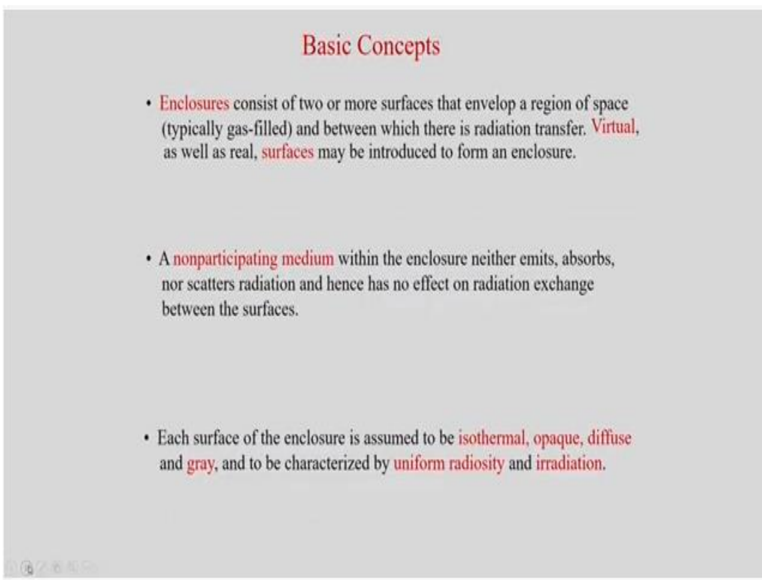


So in todays class we will study the radiation exchange between the surfaces and we will consider that the enclosure is having non-participating medium. What does it mean by non-participating medium? That means this medium does not participate in the radiation. That means it neither emits nor absorbs nor scatters. So it has no effect on radiation exchange on the surfaces. At the same time, we will do the assumptions that each surface of the enclosure to be isothermal, opaque, diffuse and Gray.

Isothermal means it is having a constant temperature, diffuse means it is independent of the angle as well as Gray means it is independent of wavelength as well as opaque means transmittivity is 0. So with this assumptions, now will study the radiation exchange between the surfaces. Now what is the objective? The first objective is that we have to find the geometrical orientation for this radiation exchange between the surfaces. For that we will consider view factor. So what is view factor?

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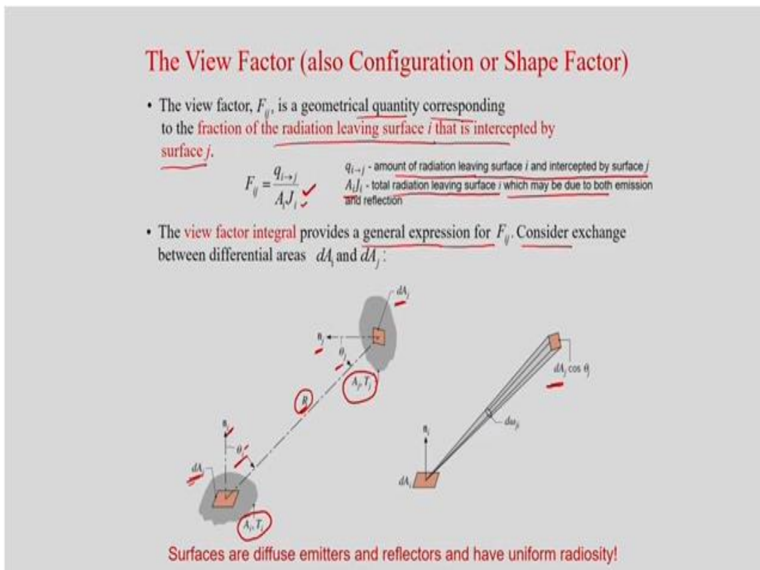


It is also known as configuration factor or shape factor. So you can see the slide the view factor $F_{ij}$ is a geometrical quantity corresponding to the fraction of radiation leaving surface i that is intercepted by surface j. So if we consider two surfaces, from one surface to another surface, your radiation will go. So whatever radiation actually living surface i and what is the fraction of it actually reaches to the surface j that is known as the view factor.

So mathematically it defines as here

$$F_{ij} = \frac{q_{i \to j}}{A_i J_i}$$

What is $q_{i \to j}$? $q_{i \to j}$ is the amount of radiation leaving surface i and intercepted by surface j. And what is $A_i J_i$? $A_i J_i$ is the total radiation leaving surface i which may be due to both emission and reflection. So already you have studied the radiosity. So that is J, so J means the total radiation leaving the surface due to emission as well as that reflection. So we have already learned that.

So the view factor integral actually provides a general expression for $F_{ij}$. Consider exchange between the differential areas $dA_i$ and $dA_j$. So you can see these two surfaces $A_i$ maintained at temperature $T_i$, another surface $A_j$ and maintained temperature $T_j$. So you can see that these are arbitrarily oriented okay. And if you consider two elemental surfaces on those areas so $dA_i$ okay and $dA_j$ , and if you join these two elemental areas by a line and that line radius is R okay. And this R actually makes a polar angle $\theta_i$ with its normal $n_i$ and $\theta_j$ with its normal $n_j$. So we are considering two elemental surfaces $dA_i$ and $dA_j$ and we are interested to know what is $F_{ij}$ which is known as view factor. So for that we have considered that the distance between these two surface is R.

So obviously if you change the $dA_i$ and $dA_j$, obviously your R will change, $\theta_i$ will change and this $\theta_j$ will change because its normal will change accordingly. So with this let us find the solid angle for this. So if you solid angle here we will define as the angle subtended by the surface $dA_j$ when viewed from the surface $dA_i$ okay. So the solid angle will be calculated about the angle subtended by the surface $dA_j$ and viewed from $dA_i$ . Because we are trying to find $F_{ij}$. So with this now let us calculate what is $F_{ij}$.

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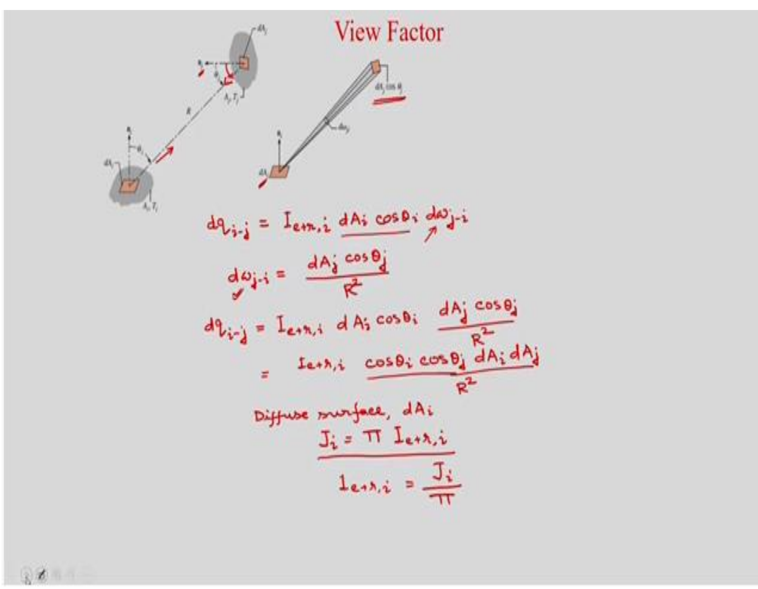


Now the radiation $dq_{i\to j}$you know the definition from the radiation intensity. So that will be

$$dq_{i\to j} = I_{e+r,i} dA_i cos\theta_i d\omega_{j\to i}$$

$e + r$ means emission plus reflection from the surface i okay. And in which direction is it? So it is in the direction of $\theta_i$. Now why cos $\theta_i$? Because this $dA_i$ , if it is in this direction, what is the area $dA_i cos\theta_i$ right. The component of this $dA_i$ area in the direction of R will be $dA_i cos\theta_i$. And the solid angle is $d\omega_{j\to i}$. So now what is $d\omega_{j\to i}$ by the definition? So it will be the angle subtended by the surface $dA_j$ when viewed from the surface $dA_i$ okay. So you can see that this is your $dA_i$ . So from here you are viewing so if you view from this $dA_i$ the component of $dA_j$ will be $dA_j cos\theta_j$ because it is the normal in this direction $n_j$. So obviously the component in this direction, it will be $dA_j cos\theta_j$. So this is the area. So it will be

$$d\omega_{j\to i} = \frac{dA_j cos\theta_j}{R^2}$$

So the distance between these two surfaces R. So it will be $\frac{dA_j cos\theta_j}{R^2}$. So now you let us put this value here okay. So this value you put it here. So what you are going to get? You are going to get

$$dq_{i\to j} = I_{e+r,i} dA_i cos\theta_i \frac{dA_j cos\theta_j}{R^2}$$

So this we can write also

$$= I_{e+r,i} \frac{cos\theta_i cos\theta_j dA_i dA_j}{R^2}$$

Also you know the relation between the intensity and radiosity for a diffuse and gray surface. So that you can write if you assume that the surface $dA_i$ is diffuse then we can write

$$J_i = \pi I_{e+r,i}$$

So this already you have derived in earlier classes okay. So for a diffuse surface $dA_i$ , you can write this relation okay. $J_i$ is the radiosity okay. And I is the intensity, total intensity, emitted plus reflected. So now

$$I_{e+r,i} = \frac{J_i}{\pi}$$

So you can substitute that.

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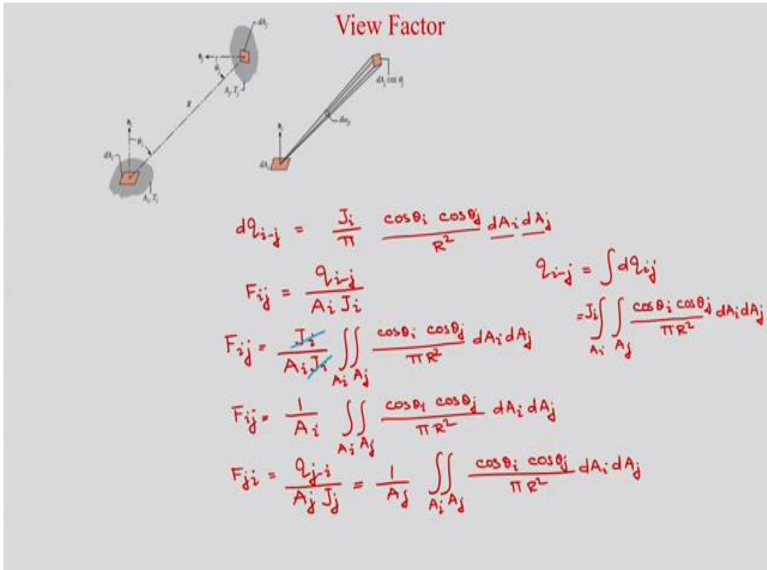


So now you can write it as

$$dq_{i\to j} = \frac{J_i}{\pi}\frac{cos\theta_i cos\theta_j}{R^2} dA_i dA_j$$

So this is the d$q_{i\to j}$ okay. So by definition the view factor is

$$F_{ij} = \frac{q_{i\to j}}{A_i J_i}$$

So now what is $q_{i\to j}$? $q_{i\to j}$ we can write integral of d$q_{i\to j}$ right. So now in this case it will be double integral because $dA_i$ is there and $dA_j$ is there. So you can write as $J_i$ is the constant because $J_i$ is the total radiosity so you can take it out of the integral. Then you can write

$$q_{i\to j} = \int dq_{i\to j} = J_i \iint_{A_i,A_j} \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_i dA_j$$

So in this case you can see that R is function of the orientation of these elemental areas $dA_i$ and dAj. So you cannot take it out of the integral, because $dA_i$ $dA_j$ changes your R will change. So you cannot take it out. So only $J_i$ is the constant because it is the total radiosity. So you can take it out of the integral. So now

$$F_{ij} = \frac{J_i}{A_i J_i} \iint_{A_i,A_j} \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_i dA_j$$

$$= \frac{1}{A_i} \iint_{A_i,A_j} \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_i dA_j$$

So this is the expression of view factor okay. So you can see that this is a geometrical parameter.

So now on the other hand, if you write $F_{ji}$ from surface $dA_j$ to $dA_i$ then you can write

$$F_{ji} = \frac{q_{j\to i}}{A_j J_j}$$

So with same analysis, you can write it as

$$F_{ji} = \frac{1}{A_j}\iint_{A_i,A_j} \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_i dA_j$$

So this you can derive it okay. Similar way you can find the solid angle subtended by the surface $dA_i$ and when viewed from the $dA_j$. So with that analysis you can find $F_{ji}$.

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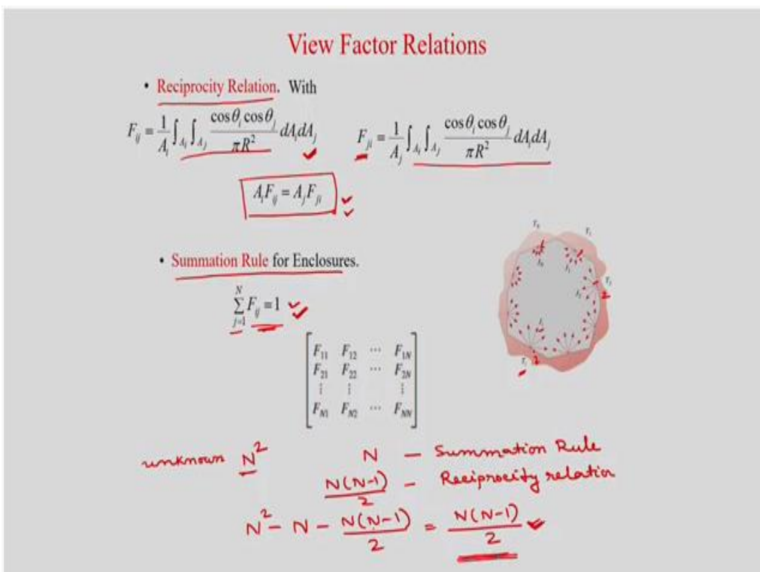


So now $F_{ij}$ and $F_{ji}$ now you can see the relation. So $F_{ij}$ we have derived this one okay. This integral will be same as this integral when you are writing for $F_{ji}$ okay. So now you write

$$A_i F_{ij} = A_j F_{ji}$$

So this you can write okay. Because inside the integral in both the expression are same, so you can write $A_i F_{ij} = A_j F_{ji}$. So this is known as reciprocity relation okay. So you should remember this. So from the analysis you can prove it right. Now you see summation rule for enclosure okay. So you consider one enclosure of N surfaces okay. And all are isothermal walls okay. All are isothermal walls you can see, so you can see now this is surface i maintain at temperature $T_i$ and the radiosity is $J_i$.

So what is this radiation; whatever fraction it will go to this surface, this surface, this surface, this surface, this surface this summation should be one. Because whatever it is actually radiating it should go to all surfaces. So that is known as summation rule and we can write it

$$\sum_{j=1}^{N} F_{ij} = 1$$

That means from surface let us say i, whatever radiation is going it will go to all the surfaces because this is an enclosure. So whatever fraction it is going to each surfaces that if you sum it up it should be equal to 1 okay. So that we are writing here and this is known as summation rule. So these two relations you should remember; one is your reciprocity relation and other is summation rule. And if you have N surfaces so how many unknowns will be there for view factor?

So this will be $N^2$ square okay. You can see for $i^{th}$ surface there will be N unknowns. Again for another surface j there will be N unknowns. So if you have N surfaces, so you will have $N^2$ unknowns okay or $N^2$ unknowns of $F_{ij}$ okay. But now you have these relations right, summation rule and reciprocity relation.

So summation rule for each surface you can write. So total N equation you will get right. For each surface if you write the summation rule then for each surface you will get one equation. So N equation you will get okay. So N unknowns you can find from N equations right. That is fine. And from reciprocity relation you will get $\frac{N(N-1)}{2}$. So this number of view factors you can find using reciprocity relation okay. So what will be total unknowns after that? So it will be

$$N^2 - N - \frac{N(N-1)}{2} = \frac{N(N-1)}{2}$$

So this many numbers of view factor you have to find from either by inspection or from the view factor integral okay.

So N you are getting from the summation rule, $\frac{N(N-1)}{2}$ number of view factor you will get from the reciprocity relation. And whatever is left from $N^2$ that you can find from the view factor integral or some inspection. So how by inspection that I will discuss okay.

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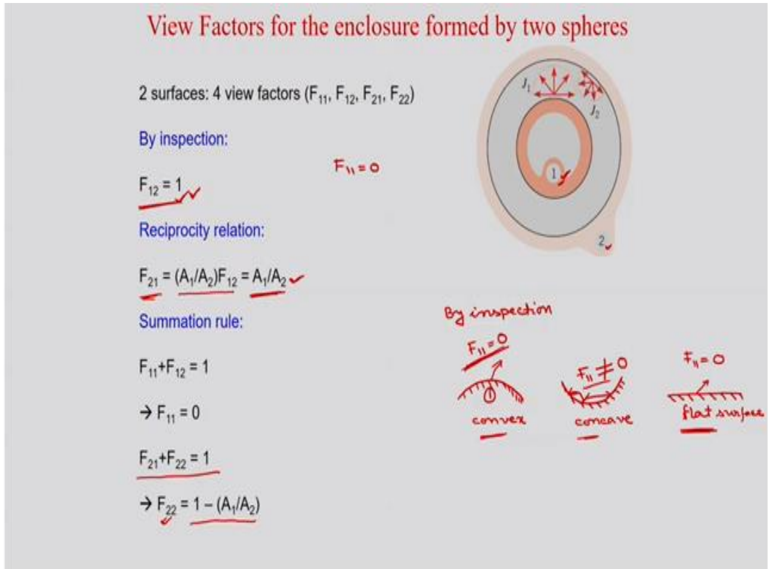


So you can see that if you have a convex surface and concave surface okay. So if you consider one surface like this okay, so it is a convex okay. One surface you can consider like this. So this is a concave. And another surface flat surface also you can consider okay. So these three configurations let us consider.

So you can see that from the convex surface whatever the radiation is going from outside that will not come to that surface right. So there if this surface is 1, then $F_{11}$ will be 0. Because whatever it is radiating, any fraction of it will not come back to the surface 1 in the case of convex. So $F_{11} = 0$ by inspection. Now if you consider concave, so whatever it is going from here radiation, so this may go to its own surface okay.

So in this case $F_{11}$ not equal to 0 okay. If you have a concave surface, it will also receive the radiation which is actually radiated from its own surface, due to this concave okay. So in this case $F_{11}$ not equal to 0. And for flat surface anyway whatever radiation is leaving that will not come back to the surface. So again in this case $F_{11} = 0$. So for a flat surface and convex surface you will get $F_{11} = 0$.

$F_{11}$ what does it mean? This view factor $F_{11}$ means whatever fraction it is going from the surface to the same surface okay. So that is $F_{11}$. So in these two cases it is 0, but in case of concave surface this is not equal to 0. Now you consider a view factor for the enclosure formed by two spheres okay. So one sphere is 1 okay, another sphere is 2 and these are concentric spheres. Now you can see what will be the view factors okay. So in this case now by inspection, you can see that whatever 1 is leaving nothing will come back to its own surface because it is a convex surface. So

$$F_{11} = 0$$

And whatever is leaving it is actually reaching to surface 2 as 2 surface is the outer sphere. So obviously whatever is emitted and reflected from the surface 1, all radiation will reach to surface 2. So

$$F_{12} = 1$$

Because $F_{11}$ is 0 and so obviously $F_{12}$ is equal to 1. By reciprocity relation now we can see

$$F_{21} = \left(\frac{A_1}{A_2}\right) F_{12} = \frac{A_1}{A_2}$$

And $F_{12} = 1$. So obviously $F_{21}$ will be $\frac{A_1}{A_2}$. And again for surface 2, the summation rule you can write

$$F_{21} + F_{22} = 1 \Rightarrow F_{22} = 1 - \left(\frac{A_1}{A_2}\right)$$

So with this by inspection or by summation rule or by reciprocity relation, you can find the view factors. Now let us solve one example problem.

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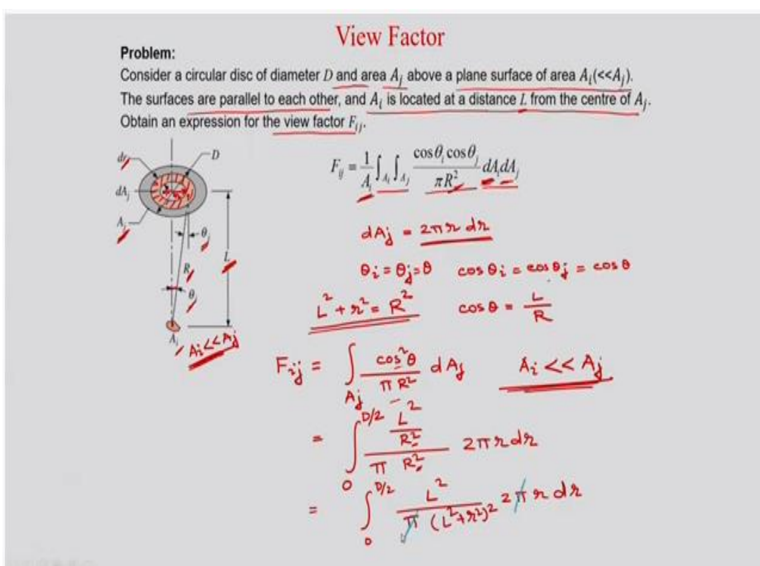

So I am first reading the problem. Consider a circular disc of diameter D and area $A_j$ above a plane surface of area $A_i$ $(\ll A_j)$. So $A_i$ is the surface which is very small surface compared to the surface $A_j$. The surfaces are parallel to each other, and $A_i$ is located at a distance L from the centre of $A_j$. So obviously these two disc are kept parallel and the distance between these two centres is L. Obtain an expression for the view factor $F_{ij}$.

Now you have to find what is $F_{ij}$? So we know the view factor integral right.

$$F_{ij} = \frac{1}{A_i} \iint_{A_i, A_j} \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_i dA_j$$

So with this relation we can find what will be the view factor $F_{ij}$. So by definition of this problem, this is your surface $A_j$. It is a circular disc. And similarly one small area is there $A_i$ okay which is $(\ll A_j)$.

And the distance between these two centres we have told that it is L. So we can see this is L. Now let us find the view factor integral $F_{ij}$ okay. So at a distance R, let us take one strip of distance dr on $A_j$ okay. So this is the elemental area $dA_j$ okay. So we are considering one strip at radius R of distance dr.

So that total area is $dA_j$ okay. So what will be the $dA_j$ in this case? So what will be the dAj?

$$dA_j = 2\pi r \ dr$$

So that is the $dA_j$. And now from this surface to this $A_i$, if you join the line, so that distance is R whatever we have defined in this view factor integral R. So that is the R. And this angles now making with the normal is $\theta_i$ and $\theta_j$.

So in this case by inspection, you can see that $\theta_i = \theta_j$ okay. That means

$$\cos\theta_i = \cos\theta_j = \cos\theta$$

So now from the geometry you see this is the distance between these two centres is L and this is the R and this is r. So what you can write? So you can write

$$L^2 + r^2 = R^2$$

So I told that your R obviously will vary. So we cannot take this out of the integral, because you see here L is constant it is the distance between the two centers. But r is actually varying when you are considering $dA_j$.

And what about the $\cos\theta$? This is the θ right, so it will be

$$\cos\theta = \frac{L}{R}$$

So with this now you can write

$$F_{ij} = \int_{A_j} \frac{\cos^2\theta}{\pi R^2} dA_j$$

As $A_i$ is very small $\left(A_i \ll A_j\right)$ you can write integral of $dA_i$ is $A_i$which will cancel out with the outside $A_i$. And $\cos\theta_i = \cos\theta_j = \cos\theta$. So we get the $\cos^2\theta$ term. Now you put the values

$$F_{ij} = \int_0^{\frac{D}{2}} \frac{\frac{L^2}{R^2}}{\pi R^2} 2\pi r dr$$

Here the integral limit will be r=0 to r=D/2 okay So that is the integral because dr is varying from 0 to D/2. So now you can write

$$F_{ij} = \int_0^{\frac{D}{2}} \frac{L^2}{\pi R^4} 2\pi r dr = \int_0^{\frac{D}{2}} \frac{L^2}{(L^2+r^2)^2} 2r dr$$

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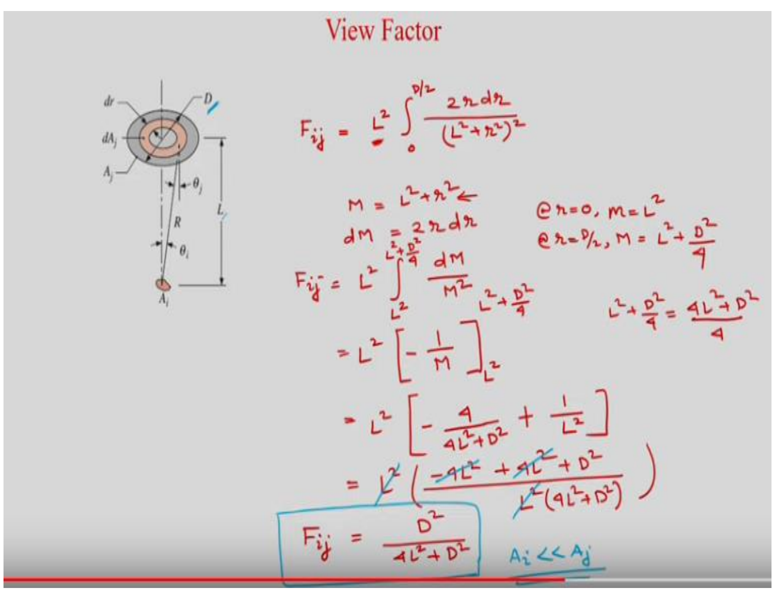

$$= L^2 \int_0^{\frac{D}{2}} \frac{2rdr}{(L^2 + r^2)^2}$$

So now this integral you have to evaluate okay. L is constant because the distance between these 2 cell centres. So that we can take it out from the integral and inside now this integral is function of all these r. So now we can evaluate okay and it is varying 0 to D/2. So the integral limit is 0 to D/2. So you can write actually now

$$M = L^2 + r^2$$

$$\Rightarrow dM = 2rdr$$

And

$$at\ r = 0\ ; M = L^2$$

$$at\ r = \frac{D}{2}; M = L^2 + \frac{D^2}{4}$$

So $F_{ij}$ you can write with the changed limits for M,

$$F_{ij} = L^2 \int_{L^2}^{L^2+\frac{D^2}{4}} \frac{dM}{M^2}$$

$$= L^2 \left[-\frac{1}{M}\right]_{L^2}^{L^2+\frac{D^2}{4}}$$

$$= L^2 \left[-\frac{4}{4L^2 + D^2} + \frac{1}{L^2}\right]$$

$$= L^2 \left[\frac{-4L^2 + 4L^2 + D^2}{L^2(4L^2 + D^2)}\right]$$

$$\Rightarrow F_{ij} = \frac{D^2}{(4L^2 + D^2)}$$

So this is the answer when $A_i \ll A_j$.

Okay so with this condition we could actually find the integral $F_{ij}$ okay and it is in terms of D. D is the diameter of this disc and L is the distance between these 2 centres.

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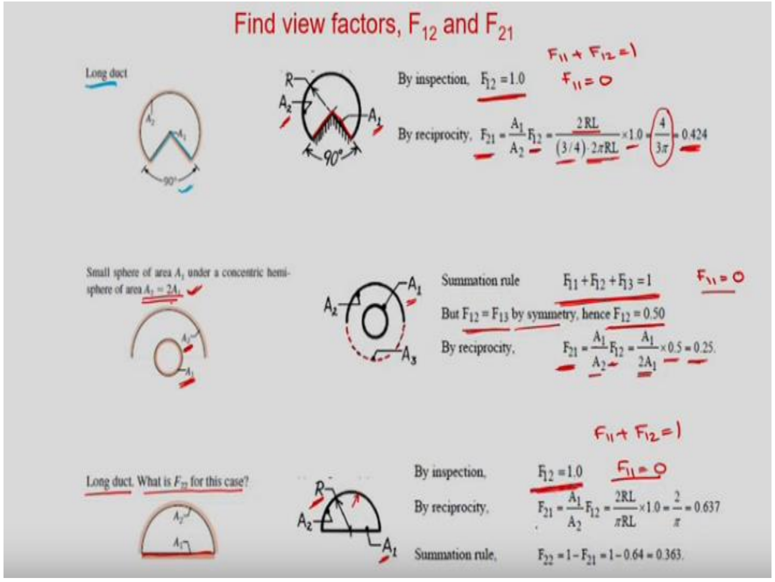


So let us find few other easy configurations for which you can find the view factors by inspection or by reciprocity relation and the summation rule. So first let us consider long duct of this shape okay. So it is a 90 degree and this surface is $A_1$, and this surface is $A_2$. So now you can see that this R is the radius and this is the 90 degree and this is the $A_1$ and this is the $A_2$.

So by inspection what is $F_{12}$? All the radiation leaving from surface 1 will reach to surface 2, because it is a convex surface. So nothing will come out or come back to this surface 1. So $F_{11}$ is 0. And obviously $F_{12}$ is equal to 1 because by summation $F_{11}+F_{12} = 1$. So for this surface now by reciprocity relation $F_{21}$ you can write

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{2RL}{\left(\frac{3}{4}\right)2\pi RL} \times 1 = \frac{4}{3\pi} = 0.424$$

$2RL$ and $\left(\frac{3}{4}\right)2\pi RL$ are areas of surface $A_1$ and $A_2$ respectively, where L is the length of the duct.

Another configuration you consider small sphere of area $A_1$ under a concentric hemisphere of area $A_2$, and $A_2 = 2A_1$. So now obviously you take one imaginary surface this one okay $A_3$. So you can see that obviously this is also 1 hemisphere so $A_2 = A_3$. And radiation leaving the surface $A_1$ will not come back to it.

So $F_{11}$ will be 0 by inspection. and by summation rule

$$F_{11} + F_{12} + F_{13} = 1$$

Now whatever radiation leaving the surface 1 whatever will reach to hemisphere $A_2$ also same fraction will reach to $A_3$ because both are symmetric. So obviously you can see that $F_{12}$=$F_{13}$ =0.5 by symmetry. So what will be $F_{21}$? By reciprocity relation

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25$$

So in this way you can actually find the view factor. Another configuration you consider; long duct okay, this is a long duct of this shape and what is $F_{22}$ for this case? So this cross section is this one okay, semicircle. Now $A_1$ is this one and this surface is $A_2$.

And radius let us say R okay. So obviously from $A_1$ whatever radiation is going it will reach to surface $A_2$, so $F_{12}$ = 1 and $F_{11}$ = 0, because summation rule $F_{11}$ + $F_{12}$ = 1. Now you use reciprocity relation

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1 = \frac{2}{\pi} = 0.637$$

If L is the length of the duct then i$A_1 = 2RL$ and $A_2$ will be the half perimeter. Now you find $F_{22}$. As surface 2 is concave so there will be $F_{22}$ also. So $F_{22}$=1-$F_{21}$=0.363 by summation rule. Okay so another configuration let us consider.

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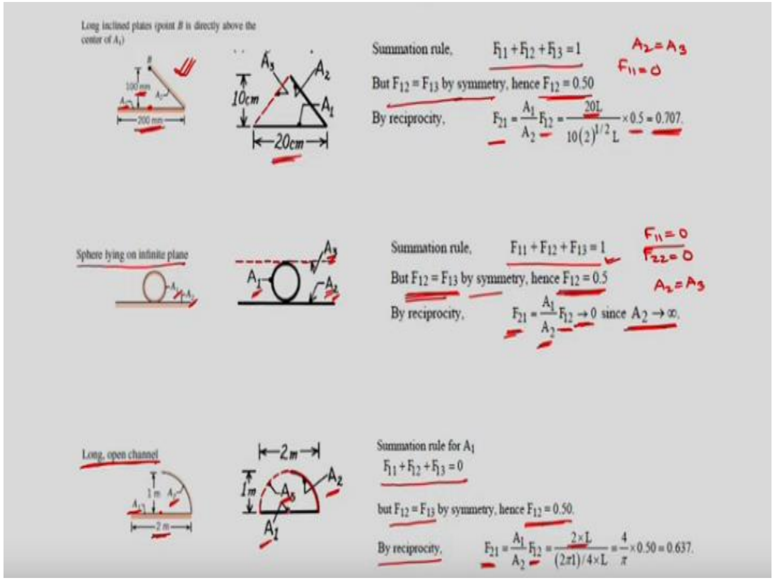


So this is the configuration, long inclined plates point B is directly above the centre of $A_1$ okay. So this surface is $A_1$ and if its centre is this one, B is just above this centre okay. And the dimensions are given. So this distance from the centre to B is 100 mm and this $A_1$ is 200 mm okay. And these are long inclined plates. So now you consider the one imaginary boundary okay? So this is the $A_3$ is the imaginary boundary okay, $A_3$ is the imaginary boundary.

So you can see that by inspection that whatever radiation is leaving surface $A_1$ it will equally divided to surface $A_2$ and $A_3$ because surface $A_2$ and $A_3$ are same okay as $A_2 = A_3$. So whatever radiation is leaving from surface 1, it will be divided equally to surface $A_2$ and $A_3$. So you can write it by summation rule

$$F_{11} + F_{12} + F_{13} = 1$$

And $F_{11}$ is 0 because it is a flat surface. And $F_{12}$ and $F_{13}$ will be equal by symmetry. So hence $F_{12}$ will be 0.5 okay. So what will be $F_{21}$? By reciprocity relation

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10\sqrt{2}L} \times 0.5 \; = 0.707$$

Okay so another configuration let us consider sphere lying on infinite plane. So we have a sphere okay and it is lying on a flat surface okay infinite flat surface that you can see that $A_1$ is the sphere and $A_2$ is the flat surface. So by inspection you can say that obviously your $A_2$ is flat surface so $F_{22}$ will be 0. And now you take one imaginary surface just above this sphere okay Like this $A_3$ and that is also infinite and having the same area as $A_2$.

So now $A_3$ and $A_2$ are same right, $A_2 = A_3$ because there are 2 parallel surfaces and the sphere is enclosed by these 2 surfaces. So now you can see that

$$F_{11} + F_{12} + F_{13} = 1$$

And whatever radiation leaving this surface 1 equally it will go to surface 2 and 3. So that means $F_{12} = F_{13}$ by symmetry okay. So hence $F_{12} = 0.5$ because $F_{11}$ is also 0, because it is a sphere and it is a convex surface. Now you use the reciprocity relation

$$F_{21} = \frac{A_1}{A_2} F_{12}$$

But $A_2$ is infinite $A_2$ tends to infinity so

$$\frac{A_1}{A_2} F_{12} \to 0 = F_{21}$$

Another long open channel you consider okay long open channel okay. So let us consider that perpendicular to the screen his length is L and this is the area $A_1$ its length is 2 m and this is curved surface and its distance from this centre is 1 m okay and this surface is $A_2$ and this is your $A_1$. So obviously you take one imaginary surface again okay you close this surface.

So this is your imaginary surface $A_3$ okay and this flat surface is $A_1$ and this curved surface is $A_2$. So it is like this problem okay, it is similar problem. So you can see that by symmetry $A_2 = A_3$ and whatever radiation leaving the surface $A_1$ equally it will go to surface $A_2$ and $A_3$. So you can write by summation rule $F_{11} + F_{12} + F_{13} = 1$ and by symmetry $F_{12} = F_{13}$ and hence $F_{12} = 0.50$.

So by now you can use the reciprocity relation. So what is $A_1$? So now you can see this $A_1$ is 2L, L is as I told that length. $A_2$ is this surface, so it will be 1/4th of this perimeter and the radius is 1 m. So,

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2L}{\frac{(2\pi \times 1)}{4} L} \times F_{12} = \frac{4}{\pi} \times 0.50 = 0.637$$

So now we have considered different configurations today where you can actually find the view factor by inspection, by reciprocity relation, by summation rule okay. And also for few cases you cannot find this view factors using these methods then you have to use this view factor integral. Okay thank you.