Fundamentals of Conduction and Radiation

Prof. Dipankar N. Basu
Department of Mechanical Engineering
Indian Institute of Technology - Guwahati

Lecture - 28
Irradiation of Real Surfaces

Hello friends, welcome to the second lecture of our module number 10 where we are talking about the radiative properties of real surfaces. In the previous week you have been introduced to the concept of blackbody or black surface which in a way you can think about the most ideal surface that you can visualize from radiation point of view, because it has an emissivity of 1 or I should say the definition of emissivity is based upon the black surface.

So for a given temperature and wavelength it gives you the maximum possible emission. At the same time it has an absorptivity of 1 or reflectivity of 0. But truly speaking no practical surface behaves like a black surface and therefore we always need to know the behaviour of real surfaces and we always try to compare the behaviour of real surfaces in terms of the black surface.

Accordingly, in the previous lecture you have been introduced to the concept of emissivity. Both the spectral and directional dependence of emissivity we have discussed. In all the cases we are talking about the ratio of actual emission from the real surface to the corresponding emission from a corresponding or equivalent blackbody may be at the same temperature; if you are talking about a spectral situation then at the same wavelength.

If you are talking about the directional situation then in the same direction but always we are comparing with a black body or black surface. Now today we shall be talking about the radiative properties associated with the irradiation.

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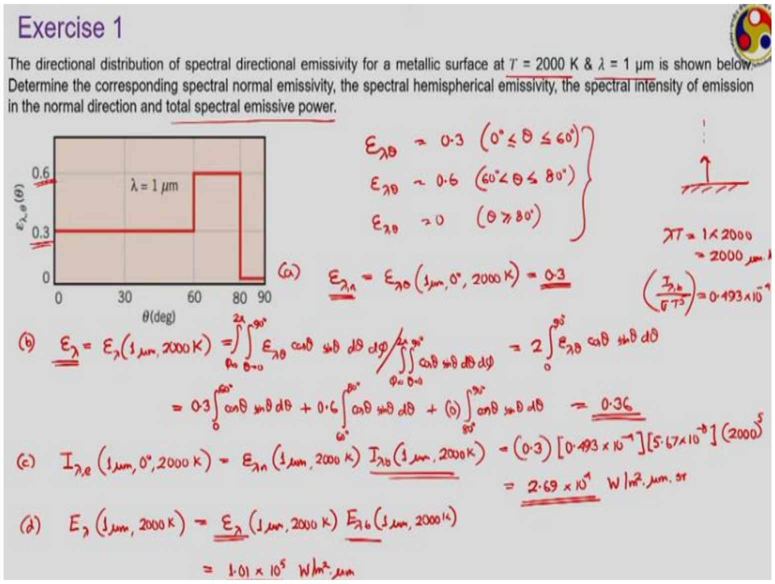

But before that let us quickly solve one numerical example associated with emission. In the previous lecture I was slightly short of time that is why I just discussed one numerical problem. Today I shall be discussing about a second one associated with the emissivity and then moving to the irradiation related properties.

So just read the question carefully. Here you are given it the directional distribution of spectral direction and emissivity for a metallic surface maintained at temperature T = 2000 K and we are talking about a particular wavelength that is λ =1 micron. So look at the graph. Here you have been given the variation from theta is equal to 0 to 90 degree. And variation of $\varepsilon_{\lambda\theta}$ that is spectral directional emissivity with polar angle θ is given. you can see over 0 to 60 degree its value is 0.3 and from 60 to 80 degree it is equal to 0.6 and after 80 degree it is equal to 1. That is

$$\epsilon_{\lambda\theta} = 0.3 \ (0° \le \theta \le 60°)$$
$$\epsilon_{\lambda\theta} = 0.6 \ (60° \le \theta \le 80°)$$
$$\epsilon_{\lambda\theta} = 0.3 \ (\theta \ge 80°)$$

80 to 90 you can say because we do not need to mention 90 here because the range of polar angle is always 0 to π/2 or 0 to 90 degree. Now we have to determine the corresponding spectral normal emissivity, the spectral hemispherical emissivity, spectral intensity of emission in the normal direction and total spectral emissive power. So several quantities we have to identify using this. So the first thing that we have to identify is the spectral normal emissivity.

So let us say A. So spectral normal emissivity ε_λ in the normal direction what does that mean? That means emission that is taking place say if this is your surface then we are talking about emission taking place in the direction normal to the surface. Now in this normal direction what is the value of θ, how we measure θ; that you always have to understand. We always take a normal to the surface and θ is measured with respect to this normal.

So the normal actually corresponds to this $\varepsilon_{\lambda\theta}$ at θ equal to 0 degrees or in this particular case to be proper we are talking about the wavelength of 1 micron and the angle of 0 degree and a temperature of 2000 K.

$$\epsilon_{\lambda n} = \epsilon_{\lambda\theta}(1\ \mu m\ ,0°, 2000K) = 0.3$$

For real surfaces the temperature information you can omit from here. But when you are trying to compare this one with an equivalent black surface this will also come into play.

So now just directly from the graph we can write this to be equal to 0.3. So the normal emissivity is equal to 0.3 for this particular configuration. So now you have to get the spectral hemispherical emissivity. So the second part this spectral hemispherical emissivity for this we have to integrate this one over the entire range. So spectral hemispherical emissivity what is the notation that you should use? It is hemispherical, so the θ will not come into picture but it is spectral and we are trying to get this corresponding to this 1 micron and 2000 K. So,

$$\epsilon_\lambda = \epsilon_\lambda(1\ \mu m, 2000K)$$

Again this 2000 K you can omit from here. This is of course integrated over the entire range of θ that we are considering that is

$$\epsilon_\lambda = \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \epsilon_{\lambda\theta} \cos\theta \sin\theta \, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \cos\theta \sin\theta \, d\theta d\phi}$$

Now in this case the directional distribution is given only in terms of this θ. So we can assume the φ dependence to be negligible. In that case the dφ can directly come out. So you know the denominator will become π and 2π comes out from the numerator. So it becomes just

$$= 2\int_{\theta=0}^{\frac{\pi}{2}} \epsilon_{\lambda\theta} \cos\theta \sin\theta \, d\theta$$

Now for this 1 μm case there are two different values you can see or rather over this entire range of 0 to 90 degree there are 3 values of $\varepsilon_{\lambda\theta}$ that you can see. Accordingly we can break this integration into three parts.

$$= 2 \times 0.3 \times \int_{\theta=0}^{60^\circ} \cos\theta \sin\theta \, d\theta + 2 \times 0.6 \times \int_{60^\circ}^{80^\circ} \cos\theta \sin\theta \, d\theta + 2 \times 0 \times \int_{80^\circ}^{90^\circ} \cos\theta \sin\theta \, d\theta$$

$$= 0.36$$

So corresponding to this 1 micron wavelength your spectral hemispherical emissivity is coming to 0.36. Now we have to get the third part the spectral intensity of emission in the normal direction. In the normal direction we are talking about so that normal emissivity $\epsilon_{\lambda n}$ that we have calculated earlier that will come into picture.

So how we can get this? We have to get the spectral intensity of emission. So how we represent the spectral intensity of emission? Intensity is represented by I. spectral you are talking about so $I_{\lambda e}$ and e for emission we are talking about and this is at this wavelength of 1 micron. Here you are talking about the normal direction. So this 0 degree is coming into picture here. We can include the temperature because here we have to compare this one with the black surface to get this. So this one will be equal to

$$I_{\lambda e}(1\ \mu m, 0^\circ, 2000K) = \epsilon_{\lambda n}(1\ \mu m, 2000K) I_{\lambda b}(1\ \mu m, 2000K)$$

$I_{\lambda b}$is the spectral intensity of emission from corresponding blackbody at this wavelength of 1 micron and this temperature of 2000 K.

So you have to get the value for this particular quantity now. How to get this value? We know the $\epsilon_{\lambda n}$that we have calculated to be 0.3 that we have we are getting from the chart. Then we just need to know the spectral intensity of emission for this black surface. So how can we do this? Here the value of λT is given to be

$$\lambda T = 1 \times 2000 = 2000\ \mu m\ K$$

Then if you look at the table that we have been introduced in the previous week then you would find there is a quantity $\frac{I_{\lambda b}}{\sigma T^5}$ .and for this particular value of λT, here it is coming to be $0.493 \text{X} 10^{-4}$. So now you know how to make use of this.

$$\epsilon_{\lambda n} I_{\lambda b} = \epsilon_{\lambda n}\left(\frac{I_{\lambda b}}{\sigma T^5}\right)(\sigma T^5)$$

$$= 0.3 \times (0.493 \times 10^{-4})(5.67 \times 10^{-8})(2000)^5 = 2.69 \times 10^4 \frac{W}{m^2.\mu m\ .sr}$$

So if you put this in picture then the spectral intensity of emission in the normal direction in this case is coming to be this. Now what will be the unit for this? We have discussed about how to get the unit because this is the intensity we are talking about so definitely W/m^2. But it is spectral, so this μm is coming into picture. And also a spectral intensity means there is a directional dependence as well so the steradian is also coming into picture.

And then we have to get the last one which is the total spectral emissive power. Total spectral emissive power is integrated over the entire range. I am leaving this calculation to you. You please try to find this spectral emissive power corresponding to this 1 micron and 2000 K.

We have already got this quantity ϵ_λ. So you have to make use of this quantity somehow, that is

$$E_\lambda(1\ \mu m, 2000K) = \epsilon_\lambda(1\ \mu m, 2000K) E_{\lambda b}(1\ \mu m, 2000K)$$

You just think about how to get this quantity spectral emissive power for the black surface $E_{\lambda b}$ because ϵ_λ is already calculated to be 0.36. You need to get the spectral emissive power for the black surface using the information of the spectral intensity of emission from this. And I am giving you the final answer it is going to be

$$E_\lambda = 1.01 \times 10^5 \frac{W}{m^2.\mu m}$$

Remember it is spectral but total. So there is no direction dependence but the spectral dependence is there so the μm is coming in the denominator of the SI unit for this.

So this way we can do the calculation just from the knowledge of emissivity or rather the distribution of emissivity and the knowledge of the temperature and a particular wavelength for this.

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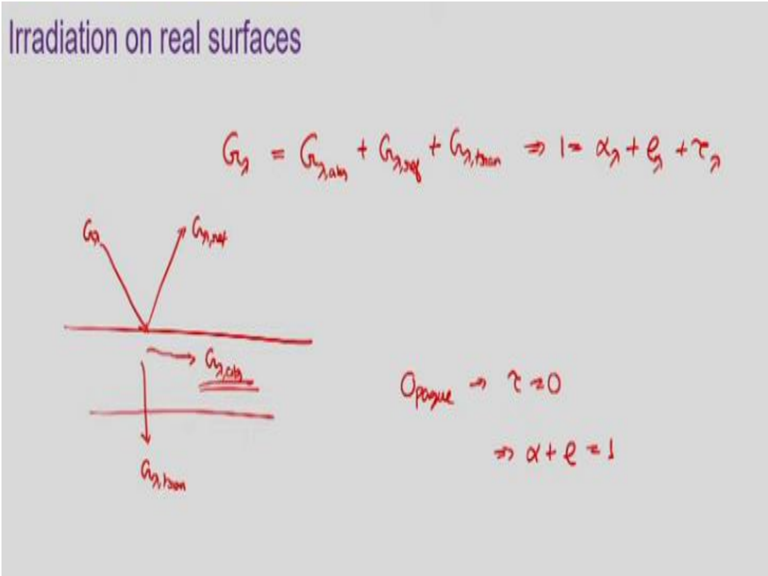

So now let us move to the irradiation part. Irradiation on real surfaces we have to discuss. Now for black surfaces we know that when some irradiation falls on the black surface entire part of that gets absorbed, nothing is reflected. But that is not true for the real surfaces because for real surfaces whenever some emission falls on this say if you are talking about this spectral irradiation say G_λ is the spectral irradiation for a given value of λ whenever it falls on a surface then there are three possibilities.

Like if these are surface you are talking about and say this is the irradiation that is falling on this then it is possible that a part of that may get reflected, say that part we write as $G_{\lambda, ref}$, a part may get absorbed in this surface which is $G_{\lambda,abs}$ and if this surface is a semi-transparent or transparent one then a part also may get transmitted.

So there are three different fates that are possible and accordingly this G_λ can be written as

$$G_\lambda = G_{\lambda,ref} + G_{\lambda,abs} + G_{\lambda,trans}$$

And accordingly we have defined three properties earlier. Like here we are writing in terms of spectral total definitions. So we can write that as an absorptivity or spectral total absorptivity plus reflectivity plus transmittivity.

$$1 = \alpha_\lambda + \rho_\lambda + \tau_\lambda$$

There are three properties that are possible. And similar to emission each of these properties has their own spectral and directional dependence and therefore we can have the spectral directional definition, spectral total definition or spectral hemispherical definition as well as the total hemispherical definition. So we have to get the definitions of each of these quantities.

Now for most of the surfaces that we deal with in engineering situations they are opaque in nature. Therefore the transmissivity part is equal to 0 which gives us

$$\alpha + \rho = 1$$

Here I am not writing any subscript because that is true for all kind of definition; spectral directional, Spectral hemispherical as well as total hemispherical.

And transmittivity comes into picture only for certain substances. And also out of these three quantities the absorbed part is the only one that contributes towards the change in the temperature of this surface or the body that we are talking about. Because reflected part just goes as it is coming in transmitted part also just passes through without disturbing the energy content of the molecules.

But it is a absorbed part it what that causes a change in the internal energy content of the molecule on the surface and accordingly the absorbed part needs to be considered if we are looking for some kind of energy balance for the surface. But if you are looking only for the radiative energy balance then of course all the three components comes into picture, comes into discussion.

Before moving on to the definition of each of these properties, I would like to discuss a bit about the perception of colour. The perception of colour that is whatever I can see in front of like the camera that I am seeing in front of me that is black in colour. Or the shirt that I am wearing that you can see it is slightly pinkish in colour. Now this interpretation of colour comes from our sense of irradiation or exactly what is happening after the irradiation.

Unless the temperature of the surface is more than 1000 K, colour is nowhere related to the emission from a surface. Because as we have discussed again in the previous lecture that when the surface temperature is less than 800 K or 1000 K then the entire emission happens in the IR part of the spectrum.

Only when the temperature crosses 1000 K or goes to something in the range of 1800 or 2000 K then only a significant part of its emission comes within the visible spectrum. And therefore we are able to visualize that, but for most of the bodies that we encounters are in our surrounding their temperatures are much lower and accordingly whatever colour that we can see from them the emission has no role to play.

It comes entirely from the reflection, entirely from the reflection part or the reflectivity property of the surface. Like this shirt that I am wearing it is looking like pink. What does that mean? That is whatever light is getting reflected from this surface or rather I should say whatever irradiation is being received by the surface of my shirt or the fabric of the shirt; it is reflecting only the portion which corresponds to that pinkish kind of colour. That means something in the range of red and yellow or in that range of the visible spectrum. Similarly something if we are seeing the colour to be blue like the sky; that basically corresponds to the blue or greenish part of the spectrum. That is the other part of the spectrum is absorbed but only the blue green part of the spectrum is being reflected. If we are seeing something to be black that means actually it is absorbing the entire part of the visible spectrum. It is not reflecting any part of the visible spectrum. But that does not mean that that black colour surface is having reflectivity of 0.

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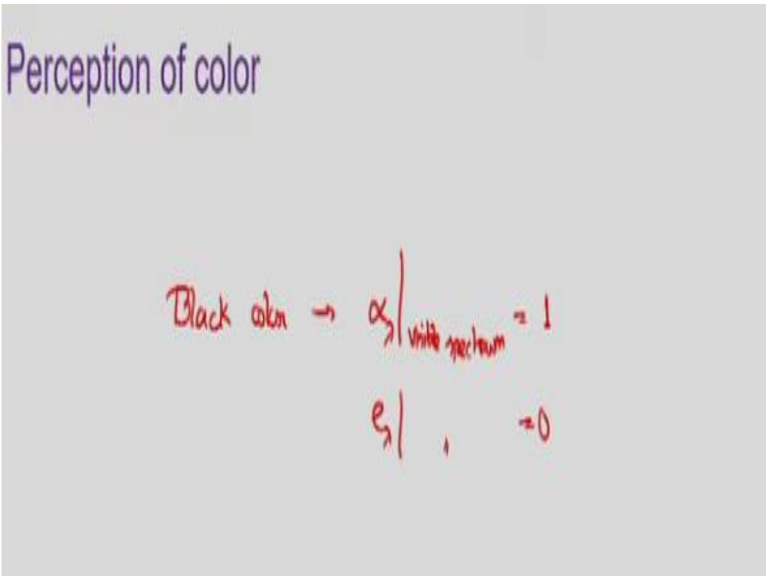

So if I am talking about a surface having black colour that means that its absorptivity within the visible spectrum is 1. Its reflectivity in the same spectrum is equal to 0. I should write the subscript λ because I am talking about certain wavelengths. For all the wave lengths within the visible spectrum it is having an absorptivity of 1 reflectivity of 0.

I am talking about opaque surfaces only thereby neglecting transmittivity. However it is very much possible that once we go to the IR range, the surface may have quite high value of reflectivity. One very common example of such kind can be eyes. Eyes have a very high reflectivity within the visible spectrum and therefore it reflects almost all possible colours or all possible wavelengths within the visible spectrum.

Therefore we can see that as to be white. However eyes are excellent absorber of the IR radiation. And in the IR side its absorptivity is very close to 1 and reflectivity is very close to 0. But in the visible spectrum its absorptivity is very low. Its reflectivity is very close to 1. When within the visible spectrum something is having very high reflectivity we can see it to be white, whereas when its absorptivity is very high reflectivity is very low within the visible spectrum we see it to be black.

When you see something of some colour, like we see the grass or tree leaves to be green because like the example that I have mentioned. Their reflectivity corresponding to that blue or green kind of or a spectrum part of the spectrum or corresponding wavelengths is very high but it absorbs the reddish part of the visible spectrum. And therefore we can see them to be green or blue or dark blue.

Therefore the entire perception of colour is associated with this absorptivity and reflectivity, because reflectivity is the one that governs this perception of colour. Only when the temperature becomes a higher than 1000 K then emission also starts to have its own contribution towards the colour of a surface. Therefore whenever you are talking about a surface or substance to have a certain colour then try to identify the value of its absorptivity and reflectivity corresponding to the wavelength corresponding to that particular colour. You would invariably find that it has a absorptivity of almost 0 and reflectivity of close to 1 and that is why you are seeing that to have that particular colour. Now let us talk about absorptivity. As I have mentioned all these properties has their own spectral and directional dependence.

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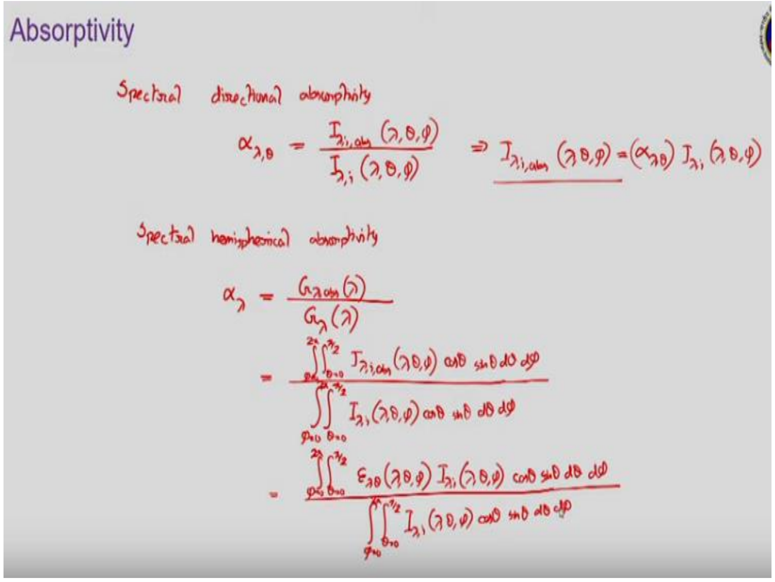

And accordingly for absorptivity we are starting with the definition of spectral directional absorptivity. α is the symbol. Now what should be the subscript? Spectral dependence is there, so λ is there and θ is also there because it is directional. So what will be the definition we have already defined the emissivity. Quite similar way we can also define. In the denominator we are talking about the intensity corresponding to the entire irradiation corresponding to the particular wavelength and direction. So it is spectral intensity of incidence corresponding to that wavelength and that direction. And in the numerator it is the absorbed part of that irradiation corresponding to the same direction and wavelength. And accordingly the absorbed part of the spectral intensity of incidence can be calculated as

$$\alpha_{\lambda,\theta} = \frac{I_{\lambda i,abs}(\lambda,\theta,\phi)}{I_{\lambda i}(\lambda,\theta,\phi)} \Rightarrow I_{\lambda i,abs}(\lambda,\theta,\phi) = \alpha_{\lambda,\theta} I_{\lambda i}(\lambda,\theta,\phi)$$

So that is a spectral directional absorptivity. What should be the next one? We are going to talk about the spectral hemispherical absorptivity. So α is the symbol for absorptivity and what will be the subscript? Spectral dependence is there but no directional dependence, so it is only λ. So in the denominator we should have the spectral irradiation or spectral irradiation flux that is G_λ and in the numerator absorbed part of G_λ.

$$\alpha_\lambda = \frac{G_{\lambda,abs}(\lambda)}{G_\lambda(\lambda)}$$

I hope you remember the definition of this spectral irradiation. It is the one integrated over the entire range of directions or over a hemisphere. And in the numerator only the absorbed part of that same intensity.

$$\alpha_\lambda = \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i,abs}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}$$

$$= \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \epsilon_{\lambda\theta}(\lambda,\theta,\phi) I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}$$

So this is the spectral hemispherical absorptivity. This way you can define the spectral hemispherical absorptivity. If we are talking about certain situations where the directional dependence of this incidence can be eliminated or if we say that the $I_{\lambda i}$ is a not a function of this; that is if I am talking about a diffused irradiation, this is a special condition that is I am talking about the surfaces has been diffusely irradiated and also sometimes which is a very logical assumption

$$\alpha_{\lambda,\theta} \neq f(\phi)$$

Subjected to these particular two conditions α_λ can take a special form. As the irradiation that is the spectral intensity of incidence is not a function of θ and φ you can take it outside. So accordingly that cancels out. And the denominator becomes only π. Remember we are talking about diffuse irradiation that is a special case. But in several situations, we can take this assumption. Like if I talk about the surface of this particular pen then the intensity or rather irradiation on this one can be assumed to be diffused, because it is getting light from all possible directions and therefore the irradiation hardly has any directional dependence. And therefore it is not a very bad assumption. Similarly the absorptivity is again very weakly dependent on the azimuthal angle.

But it may have quite strong dependence on the polar angle. And accordingly we are assuming this $\alpha_{\lambda,\theta}$ to be independent of φ. And the implication of that one will be this $I_{\lambda i}$ is coming out of the integration from both numerator and denominator. And accordingly it is getting cancelled in the denominator we just have now cosθsinθdθdφ remaining inside the integrations or double integrations therefore their value will be equal to π.

Similarly that dφ can be integrated within the numerator giving it 2π. Accordingly for this situation this becomes

$$\alpha_\lambda = 2\int_{\theta=0}^{\frac{\pi}{2}} \epsilon_{\lambda\theta}(\lambda,\theta)\cos\theta\sin\theta\, d\theta$$

This is a form of this spectral hemispherical absorptivity which is quite commonly taken as long as we are sticking to a surface which has been diffusely irradiated because the second assumption can be taken almost in most of the cases or for most of the surfaces. Accordingly this is the equation that we shall be using for spectral hemispherical absorptivity.

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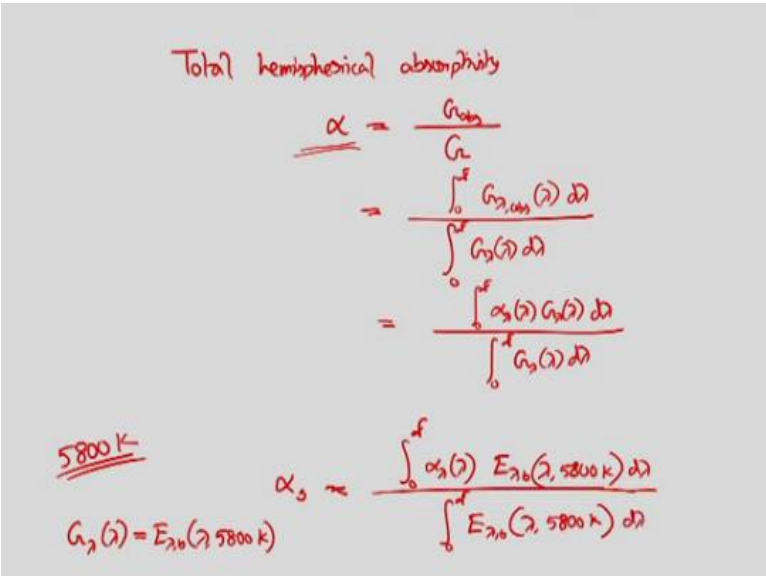

Now we shall be defining the other one. So we have already integrated over direction now we have to integrate over all possible wavelengths and accordingly we are getting the total hemispherical absorptivity which is α which is not a function of λ or θ. So accordingly this will be equal to total hemispherical irradiation that is in the denominator and its absorbed part in the numerator and then we can write this one to be

$$\alpha = \frac{G_{abs}}{G}$$

$$= \frac{\int_0^\infty G_{\lambda,abs}(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

Now what is $G_{\lambda,abs}$? Look at the previous slide we can get this α_λ into picture. Accordingly this becomes

$$= \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

This is the total hemispherical absorptivity. So the value of this total hemispherical absorptivity depends upon the spectral distribution of the incident irradiation and also on the directional dependence coming to the dependence of α_λ on the directions for this.

So we can see that this property α or any of its variant they are properties of the surface. And also they are dependent on the direction of incident irradiation. So there are several things that come into picture. Quite often we are interested in knowing the absorptivity of a surface subjected to solar irradiation, because on the earth everything is subjected to solar irradiation and therefore particularly when you are doing any calculation associated with the solar energy, solar energy falling on a surface or the application of solar energy in the form of solar water heaters, solar collectors, solar photovoltaic cells etc.; we need to know the absorptivity with respect to the solar irradiation. Like just the name mentioned solar photovoltaic cell, here the current that you are getting out of the photovoltaic cell is directly proportional to the total amount of energy that has been absorbed. And therefore the absorptivity is the property that plays the governing role.

And hence quite often we need to know a particular variant of this total hemispherical absorptivity which is corresponding to the solar radiation only. That is corresponding to only a particular source. Now we know that the sun can be visualized as something like a blackbody having a surface temperature of about 5800 K.

So accordingly quite often we define something called total hemispherical absorptivity corresponding to solar irradiation represented by α_s. Total hemispherical absorptivity or total absorptivity corresponding to solar irradiation, sometimes just called the solar absorptivity. In this particular case we are taking the irradiation G_λ as the irradiation coming from the sun. Or spectral irradiation coming from the sun which can be replaced by the emissive power of a black body or spectral emissive power of a black body at the same wavelength and it is at temperature of 5800 K.

$$G_\lambda(\lambda) = E_{\lambda b}(\lambda, 5800)$$

So this one directly goes here. This G_λ gets replaced by the spectral emissive power or spectral hemispherical emissive power of a black body having this temperature of 5800 K.

$$\alpha_s = \frac{\int_0^\infty \alpha_\lambda(\lambda) E_{\lambda b}(\lambda, 5800) d\lambda}{\int_0^\infty E_{\lambda b}(\lambda, 5800) d\lambda}$$

This is called the total hemispherical absorptivity corresponding to solar irradiation. In several practical cases particularly when you are looking for the application of solar energy or utilization of solar energy, this is the one that can directly be taken. Because here you are looking up for irradiation related information associated only with a special source which is the sun which can be visualized as a blackbody of this 5800 K temperature.

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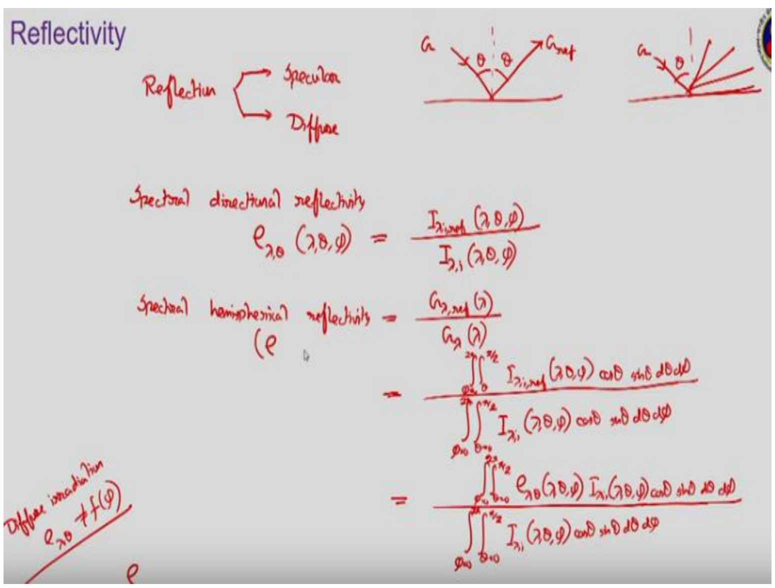

So after that let us move to the next property which is reflectivity. Now problem with reflectivity is that while everything remains quite same to absorptivity but reflectivity is bidirectional in nature. Bidirectional means in case of absorptivity we are just bothered about the direction from which the irradiation is coming in. But in case of reflection we have to consider both the direction from which the irradiation is coming in and also the direction in which the reflection is going into. And accordingly the estimation of reflection is much more complicated compared to absorption.

Because once irradiation falls on a surface from a certain direction it can go in any possible direction after the reflection. Accordingly the reflection can be classified into two categories; one is the specular other is the diffuse reflection. Specular reflection talks about something that you have learned at your school level that is, if say this is the surface and these are normal to the surface.

Then if irradiation falls on this with a certain angle θ then it should also get reflected with the same angle θ. So this is your G this is G_{ref}. This is the specular reflection. But diffuse reflection talks about if the irradiation is coming with an angle θ then it can reflect it in any possible direction, that depends upon on the orientation of the surface and also the nature of the surface. That is what we call the diffuse reflection.

So for specular reflection the angle of incidence and angle of reflection remains equal to each other but that is not true in case of diffuse reflection. Now this bidirectional nature makes it very difficult to define reflectivity as a property and its different variants. And therefore to simplify this we are going to neglect the bidirectional nature that is the angle of reflection we are going to neglect while defining reflectivity.

So if we neglect the angle of reflection or the angle made by the reflected rays then we can have similar definitions. We are starting with spectral directional reflectivity which is defined as, if ρ is the symbol that we are using for reflectivity then subscript will be spectral directional. So it is a function of λ, θ, φ. So the same definition that we used for absorptivity only instead of absorption it will be reflection.

$$\rho_{\lambda\theta}(\lambda,\theta,\phi) = \frac{I_{\lambda i,ref}(\lambda,\theta,\phi)}{I_{\lambda i}(\lambda,\theta,\phi)}$$

So in the denominator we shall be having the spectral intensity of incidence, and in the numerator we shall be having the reflected part of the same thing. Then we can define the spectral hemispherical reflectivity as the spectral irradiation in denominator and reflected part of the same in the numerator.

$$\rho_\lambda = \frac{G_{\lambda,ref}(\lambda)}{G_\lambda(\lambda)}$$

So accordingly we can write in the denominator we have integration over a hemisphere. We have this spectral intensity of incidence and in the numerator its reflected part.

$$= \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i,ref}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}$$

Remember the angles θ and φ that we are talking about that is orientation of the surface and here we are neglecting the bidirectional nature. So using the definition of the spectral directional reflectivity

$$\rho_\lambda = \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \rho_{\lambda\theta}(\lambda,\theta,\phi) I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}$$

And like in the previous case we have done the special condition of diffuse irradiation and this spectral directional reflectivity that is $\rho_{\lambda\theta}$ is not a function of φ, then for this special condition we can write

$$\rho_\lambda = 2\int_{\theta=0}^{\frac{\pi}{2}} \rho_{\lambda\theta}(\lambda,\theta)\cos\theta\sin\theta\, d\theta$$

This is a special case when the surface is subjected to diffuse irradiation and the spectral directional reflectivity is independent of the azimuthal angle.

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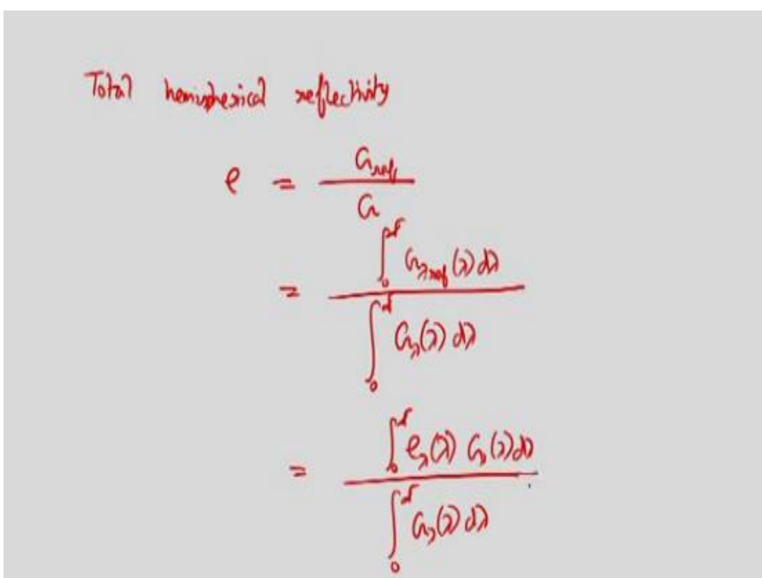

And finally we have to define the total hemispherical reflectivity. That is just ρ. That will be equal to total irradiation in the denominator and reflected part of that irradiation in the numerator.

$$\rho = \frac{\int_0^\infty G_{\lambda,ref}(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

$$= \frac{\int_0^\infty \rho_\lambda G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

So it's the total hemispherical reflectivity. So similar to absorptivity we can have three different definitions of reflectivity. But I keep on repeating all these definitions you are writing neglecting the bidirectional nature of reflection.

Finally transmittivity; transmittivity is a property which is relevant only to transparent or semi-transparent surfaces. While most of the engineering surfaces are opaque in nature and transmittivity does not need to be considered; still for the academic purpose we can have all similar definitions of transmittivity.

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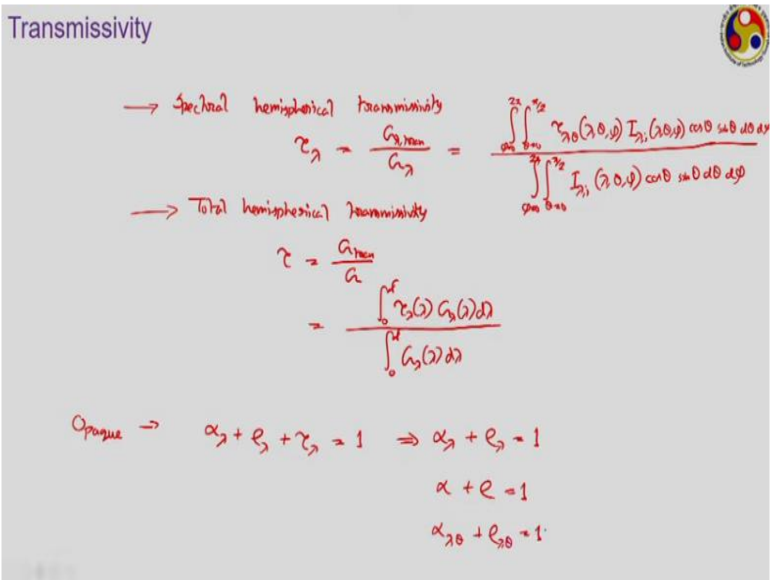

Just like if we write the spectral hemispherical transmissivity then τ is the symbol that is used to denote transmissivity. And if we are writing spectral hemispherical then it will be equal to the spectral irradiation and the transmitted part of that same spectral irradiation.

$$\tau_\lambda = \frac{G_{\lambda,trans}}{G_\lambda}$$

Similarly the total hemispherical transmissivity will be equal to total radiation flux in the denominator and its transmitted part in the numerator.

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

Following the previous practices

$$\tau_\lambda = \frac{G_{\lambda,trans}}{G_\lambda} = \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \tau_{\lambda\theta}(\lambda,\theta,\phi) I_{\lambda i}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta d\phi}$$

Here $\tau_{\lambda\theta}$ is spectral directional transmittivity which we have not defined yet. For most of the cases actually the direction dependence of transmissivity is neglected. Only we can deal with this spectral and the total definition of transmittivity and their hemispherical quantities only by neglecting the direction dependence. In most of the cases of course as I mentioned we are going to deal with the opaque surfaces. So if you are talking about the spectral definition then

$$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$$

It is always true for a given wavelength and for opaque surfaces we have

$$\alpha_\lambda + \rho_\lambda = 1$$

Similarly, if we are talking about the total hemispherical definitions then

$$\alpha + \rho = 1$$

If we are talking about a particular direction a particular wavelength then

$$\alpha_{\lambda\theta} + \rho_{\lambda\theta} = 1$$

This way we can for opaque surfaces we can get all the spectral directional spectral hemispherical and total hemispherical definitions to work with.

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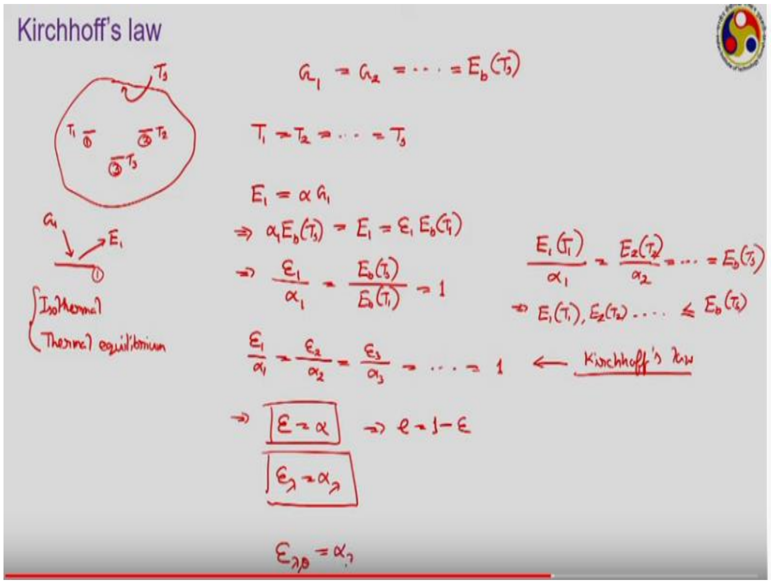

Now I am moving on to something known as the Kirchhoff's law which allows us to find a relation between these properties. That is to calculate the relation between this emissivity and the properties associated with the irradiation for an opaque surface. So to do that let us assume a large enclosure and this inner surface of enclosure is maintained at temperature of T_s and inside this enclosure we have several small bodies like this. So this is the first body which is maintained at temperature T_1 second body is maintained at temperature T_2 the third body is maintained at a temperature of T_3. There are several bodies that are inside but each of the body is very small compared to the size of the enclosure.

As each of these bodies are very small compared to the enclosure itself so we can almost neglect the presence of the body in the radiation field. And in such situation as we have done earlier also we can assume that whatever irradiation these small surfaces are receiving that is coming from a black surface maintained at this temperature of T_s.

I mean the irradiation that has been received by each of the surfaces say G_1 is the irradiation received by first surface G_2 is the irradiation received by the second surface, this way for all the surfaces whatever irradiation that we are receiving we can assume it to be coming from a black surface maintained at this T_s that is at the enclosure temperature. So now if we talk about the surface number 1.

So it is being subjected to this G_1 amount of irradiation and it is also emitting something. Say E_1 is the emission coming out of this particular one. Then what can be the energy balance that we can write for this? Assuming this surface to be isothermal that is we are assuming each of these small surfaces as well as the enclosure to be under isothermal condition and the thermal equilibrium is also maintained.

If the thermal equilibrium has to be maintained then whatever amount of energy that is coming to the surface maintained a temperature T_1, the same amount of energy should go out is not it? Whatever amount of energy is being received by this surface the same amount of energy is also leaving the surface, so that the temperature T_1 can be maintained. And that is possible only when the surface temperature T_1 is equal to the temperature T_2 is equal to T_s that is then only you can have a perfect thermal equilibrium.

That is there is radiation exchange going on from the enclosure surface to all the small surfaces. But none of them are changing their temperature because each of the small surfaces whatever in amount of energy they are absorbing from the irradiation part the same amount of energy they are emitting. So writing an energy balance then the amount of energy emitted by surface number 1 is E_1 and that should be equal to what should be equal to G_1?

No that should be equal to the amount of energy that has been absorbed. Remember what I have told. The amount of energy that gets absorbed by surface 1, that is the only portion that causes a change in the energy content of molecules. Whereas the reflected and transmitted part just goes as it is. And accordingly

$$E_1 = \alpha G_1$$

Now this E_1 is coming from where? E_1 is coming from a black surface maintain a temperature T_s, and hence we can write

$$\alpha E_b(T_s) = E_1 = \epsilon_1 E_b(T_1)$$

As these smaller surfaces are not black surfaces. So it will be equal to $\epsilon_1 E_b(T_1)$ using the definition of emissivity. So what we can write is then

$$\frac{\epsilon_1}{\alpha_1} = \frac{E_b(T_s)}{E_b(T_1)}$$

But if thermal equilibrium has to be maintained then T_1 and T_s they are same temperature. So

$$\frac{\epsilon_1}{\alpha_1} = 1$$

That is true for all the intermediate surfaces or all the small bodies. Accordingly we can write

$$\frac{\epsilon_1}{\alpha_1} = \frac{\epsilon_2}{\alpha_2} = \frac{\epsilon_3}{\alpha_3} = \cdots = 1$$

That is true for all the surfaces and this is the one that is known as Kirchhoff's law. Actually there are several other variations also we can write. Like we could have also written the ratio of emission coming from surface to the absorptivity α as

$$\frac{E_1(T_1)}{\alpha_1} = \frac{E_2(T_2)}{\alpha_2} = \cdots = E_b(T_s)$$

This is another alternate formula of Kirchhoff's law. As the α_1 ,α_2 and all of them can have a maximum value of 1 in fact they are less than 1. Therefore

$$E_1(T_1), E_2(T_2) \leq E_b(T_s)$$

This is one of the things that we have assumed for a black surface that is for a given temperature a black surface will give you the maximum possible emission that is the proof of that.

So from Kirchhoff's law then we are getting that for any surface maintained inside an enclosure we have ε = α. Remember we are talking about opaque surfaces so the transmittivity is not there and reflectivity now we can easily get. So

$$\rho = 1 - \epsilon$$

So if we know only one of them the other two properties are also known. But you have to remember the conditions that we are taking. Here we are assuming that the entire irradiation that has been received by this surface is coming from a black surface maintained at this temperature T_s. If the same procedure we are repeating in spectral sense that is we are talking about a particular wavelength. Then the same way it can be proved that

$$\epsilon_\lambda = \alpha_\lambda$$

That is a while this one says that the total hemispherical emissivity is equal to total hemispherical absorptivity, if you repeat the same procedure for a given wavelength then it can be proved then the spectral hemispherical emissivity is equal to spectral hemispherical absorptivity. And again repeating the same process for a particular direction we can say that spectral directional emissivity is equal to spectral directional absorptivity.

$$\epsilon_{\lambda\theta} = \alpha_{\lambda\theta}$$

Now these two things that are the spectral directional emissivity and spectral directional absorptivity, these are surface properties. And therefore this is always true there are no conditions imposed on this. But there are a couple of conditions imposed on this one and this one from the concept of this Gray surface and let us try to check that out.

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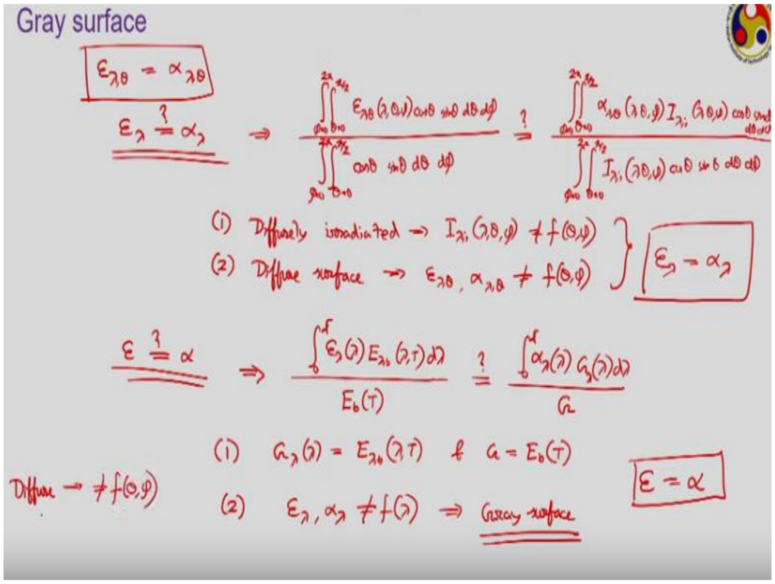

We have seen that from the previous case that if we assume small surfaces maintained inside a large enclosure then it can be proved that for a given wavelength and given direction

$$\epsilon_{\lambda\theta} = \alpha_{\lambda\theta}$$

This is always true for any surface spectral direction and emissivity and spectral directional absorptivity has to be equal to each other.

Now from there we are going in the upward direction that is we are trying to see $\epsilon_{\lambda} = \alpha_{\lambda}$ what is the condition corresponding to this? For which condition this is also true that is a spectral hemispherical emissivity is also equal to spectral hemispherical absorptivity. Now let us check out the definition. We are trying to find that spectral hemispherical emissivity.

What is the definition of that? The definition of spectral hemispherical emissivity is the definition that I put yesterday

$$\epsilon_\lambda = \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \epsilon_{\lambda\theta}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \cos\theta\sin\theta\, d\theta d\phi}$$

This is the definition of spectral hemispherical emissivity and spectral hemispherical absorptivity what is the definition? It is

$$\alpha_\lambda = \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \alpha_{\lambda\theta}(\lambda,\theta,\phi) I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi}$$

In which situation these two are equal? Assuming this to be always true, this is actually always true. The spectral directional values are always equal. Then for which situation the spectral hemispherical values also will be equal? Look at the integration that I have written. There are two situations where they can be equal. Situation number one if it is subjected to a diffused irradiation. That is the surface that I am talking about that surface has been diffusely irradiated then what will happen?

If the surface is diffusely irradiated, then the spectral intensity of incidence is independent of θ and φ that will come out of the integration limits. And as $\epsilon_{\lambda\theta} = \alpha_{\lambda\theta}$, so this two will be equal to each other. That is one condition.

$$I_{\lambda i}(\lambda,\theta,\phi) \neq f(\theta,\phi)$$

What can be the other condition when they are equal? Other is if you are talking about a diffused surface. Diffused surface means

$$\epsilon_{\lambda\theta}, \alpha_{\lambda\theta} \neq f(\theta,\phi)$$

The diffuse term always corresponds to independence of direction. So if you are talking about a diffused surface then these properties are also independent of direction and accordingly the spectral values and hemispherical values also have to be equal to each other.

So only for diffusely irradiated surface or for a diffused surface we can have spectral hemispherical values of emissivity and absorptivity can be equal. Now assuming this to be true, that is when these two conditions are satisfied then only we can have the spectral hemispherical emissivity to be equal to spectral hemispherical absorptivity.

$$\epsilon_\lambda = \alpha_\lambda$$

Now assuming if this happens let us check out for which situation their total hemispherical values are also equal? That is

$$\epsilon = \alpha$$

So let us check out their definitions. Emissivity will be equal to

$$\epsilon = \frac{\int_0^\infty \epsilon_\lambda(\lambda) E_{\lambda b}(\lambda, T) d\lambda}{E_b(T)}$$

Here $E_b(T)$ is the total hemispherical emissive power coming from the black surface maintained at same temperature. And what is the definition of total hemispherical absorptivity?

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda) d\lambda}{G}$$

So for which situation this two will be equal to each other? Of course we are assuming their spectral hemispherical values to be equal. That is you are assuming that we are talking about either a diffusely irradiated surface or a diffused surface. Then in which situation they can be equal to each other?

Again you can identify 2 conditions. Condition number one is when the irradiation the surface is receiving is coming from a black surface maintained in the same temperature. Just like the previous slide the example that we took of an enclosure, whatever irradiation the surface is receiving is coming from a black surface. That means this G_λ is actually equal to the spectral emissive power coming from a black surface maintained at the same temperature. And accordingly the total irradiation is also equal to the total hemispherical emissive power of a black surface maintained at the same temperature.

$$G_\lambda(\lambda) = E_{\lambda b}(\lambda, T) \; and \; G = E_b(T)$$

That is one possibility that is when the entire irradiation that the surface is receiving coming from a black surface maintained at the same temperature. And condition number two is

$$\epsilon_\lambda, \alpha_\lambda \neq f(\lambda)$$

In that case they will come out of the corresponding integration and hence these two quantities become equal to each other. This particular condition is called a Gray surface. So Gray surface means independence of wavelength, whereas diffused surface we are talking about as independence of direction. Similarly we are talking about gray surface to be independence of wavelength.

So if we are talking about a diffusely irradiated surface or a diffused surface which is either being subjected to irradiation coming from a black surface maintained at the same temperature or if you are talking about a Gray surface then for their case total hemispherical emissivity will be equal to total hemispherical absorptivity. So then I just write the final term. Diffuse refer to not functions of direction and Gray refers to not functions of wavelength.

$$diffuse \rightarrow \neq f(\theta, \phi)$$

$$Gray \rightarrow \neq f(\lambda)$$

So if you are talking about a diffuse and Gray surface then there is no need to go for all these definitions because there is no spectral or directional dependency. A diffused surface does not have any direction dependency; a Gray surface does not have any spectral dependency. And hence a diffuse Gray surface is independent of any kind of dependency. And therefore their total hemispherical values and spectral directional values as well as the spectral hemispherical values they all are equal to each other.

So in certain situations or rather in most of the radiation analysis we would like to go for surfaces which are diffuse and Gray in nature which simplifies our analysis to a great deal. We do not have to consider any kind of directional dependence any kind of wavelength dependence. We can just work with single values of all these properties. And also if the surface is opaque in nature, this diffuse Gray and opaque is very common condition.

If they are diffused Gray and opaque in nature then just one value sufficient because we know that transmittivity $\tau = 0$, $\varepsilon = \alpha$. And here is the reflectivity $\rho = 1-\varepsilon = 1-\alpha$. Just one of measurement is sufficient to provide information about the other three as well.

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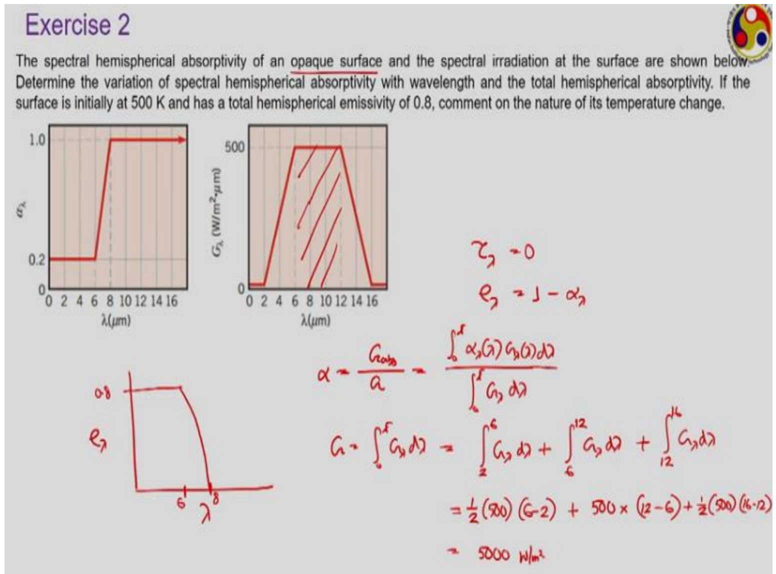

So I would like to round off this module by solving one problem on absorptivity. So the spectral hemispherical absorptivity of an opaque surface and the spectral irradiation at the surface are shown below. So we are talking about an opaque surface that means its transmittivity τ is equal to 0. But of course the spectral dependence is there. We are talking about hemispherical quantities so directional dependence is not required. We just need to consider only the spectral dependence.

So we have to first determine the variation of spectral hemispherical absorptivity. So hemispherical quantities we are talking about. So let me write

$$\tau_\lambda = 0$$

$$\rho_\lambda = 1 - \alpha_\lambda$$

As we already know the definition of this α_λ accordingly can easily put this λ here ρ_λ here. Then from this graph you can easily identify that 0 to 6 your α_λ absorptivity is 0.2.

So your ρ_λ will be equal to 0.8. So if this value is 0.8, then up to a value of λ = 6 μm it will be constant at 0.8. And beyond 8, if this value is 8 it will be equal to 0, because there absorptivity is equal to 1, reflectivity has to be equal to 0. So it will follow a straight line coming to 0 after this. This is that variation of this spectral hemispherical absorptivity.

Now we have to calculate the total hemispherical absorptivity. So α we have to calculate which is absorbed part divided by the total irradiation.

$$\alpha = \frac{G_{abs}}{G} = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

Now let me calculate the denominator first. So we can break it into several parts. Up to 0 to 2 it is 0, we do not need to consider. 2 to 6 there is a linear variation that is 2 micron to 6 micron it is following a straight line. Then we have the variation from 6 to 12 to be constant. From 12 to 16 it is again following a straight line going back to 0. And beyond 16 micron it does not need to be considered.

$$\int_0^\infty G_\lambda(\lambda) d\lambda = \int_2^6 G_\lambda(\lambda) d\lambda + \int_6^{12} G_\lambda(\lambda) d\lambda + \int_{12}^{16} G_\lambda(\lambda) d\lambda$$

So what will be the areas for each of this? You can easily check it out. So from the graph we can calculate

$$= \frac{1}{2}(500)(6-2) + 500 \times (12-6) + \frac{1}{2}(500)(16-12)$$

$$= 5000 \frac{W}{m^2}$$

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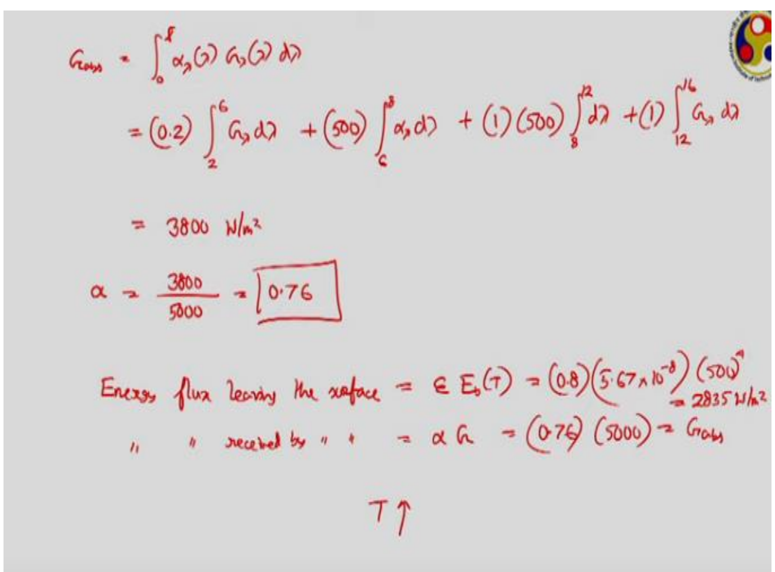

And the numerator which is the absorbed part is

$$G_{abs} = \int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda) d\lambda$$

So we have to integrate this over the range of wavelength that we are considering. Now there are again several groups that we can consider, considering the distribution of this value of α and G. See, up to 0 to 2 G_λ is 0. So there is no need to consider this integration.

So your integration will start from 2. If you look at this graph up to 2 irradiation is 0. So we do not need to consider it. Then 2 to 6 there is a linear variation during which range α is constant at 0.2. So during 2 to 6 we can take α out of integration because during 2 to 6 α is constant. And G_λ is varying following a straight line coming going from 0 to 500.

Then next variation in α varies from 6 to 8 following a straight line going from 0.2 to 1. During which it remains constant. So during 6 to 8 your G remains constant at 500. So we can take that out of integration. Then for the next part α remains constant after 8 micron to a constant value of 1 but there are two groups that we can consider now.

We have already come up to 8. So 8 to 12 both of them remains constant. So between 8 to 12, α remains constant at 1 and it remains constant at 500. And in the last part beyond this 12 α again remains constant at 1 but it follows a straight line going from 500 to 0 up till 16. So it writing it after breaking the limits

$$G_{abs} = (0.2)\int_2^6 G_\lambda(\lambda)d\lambda + (500)\int_6^8 \alpha_\lambda(\lambda)d\lambda + (1\times 500)\int_8^{12} d\lambda + 1\times\int_{12}^{16} G_\lambda(\lambda)d\lambda$$

$$= 3800\frac{W}{m^2}$$

So we are breaking this one into 4 parts and the integration becomes very easy. You can easily perform all these integrations. So we have got the total hemispherical irradiation to be 5000 and and the absorbed part of that to be 3800. Accordingly the total hemispherical absorptivity coming out to be

$$\alpha = \frac{3800}{5000} = 0.76$$

And the final thing we have to check that if the surface is initially at 500 K and has a total hemispherical emissivity 0.8.

You have to comment on the nature of its temperature change. So if we perform energy balance let me write as energy flux leaving the surface in the form of emission

$$energy\ flux\ leaving\ the\ surface = \epsilon E_b(T) = (0.8)(5.67 \times 10^{-8})(500)^4 = 2835\,\frac{W}{m^2}$$

And energy flux received by the surface that is by the absorption which is

$$energy\ flux\ received\ by\ the\ surface = \alpha G = (0.76)(5000) = G_{abs}$$

So the total amount of energy leaving the surface is less compared to the amount of energy reaching the surface. So the temperature of the surface will increase from the initial value of 500K because it is giving less amount of energy than what it is absorbing. So its temperature is going to increase. But how much increase it will take place that by doing further energy analysis we can always perform.

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That takes us to the end of this module. In module number 10 we have talked about the properties of real surfaces. There are several properties we talked about. The emissivity is the one which characterizes the emission from a surface and compares with equivalent black surface. We talked about the absorptivity and reflectivity and very little about transmittivity as well which are associated the irradiation received by a surface.

We talked about the spectral and directional dependence of all these properties and different definitions that is the spectral directional dependence or rather spectral directional definition, spectral hemispherical definition and total hemispherical definition all we have discussed and found their relations. Then the perception of colour in relation with the reflection or reflectivity has been discussed in detail.

Then we talked about the Kirchhoff's law which allows us to calculate or find a way of equating the emissivity and absorptivity. And finally the concept of Gray surface was introduced which actually talks about the independency of wavelength for a given surface. So that is it for module 10 where you have learned about the properties of real surfaces.

In the next week the radiation exchange between practical surfaces will be talked about mostly in context of Gray, Opaque and diffuse surfaces, because as I mentioned in those cases we just need to know one properties others do not need to be considered because they can be easily be calculated just from one. And that is also end of my role in this course Prof. Dalal will be coming in to take care of the next two weeks.

So I would like to say goodbye from my part. And if you have any query please keep on writing back to me in the portal or to my personal mail address and I shall be very happy to answer to you. Thank you very much.