Fundamentals of Conduction and Radiation

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Lecture - 27
Radiative Properties of Real Surfaces

Hello friends. Welcome to our module number 10, where we are going to talk about the radiative properties of real surfaces. In the previous week, you have been introduced to the fundamental definitions and concepts of radiation heat transfer, like different kinds of intensities and heat fluxes. You have been taught about or you must have learned about the spectral intensity of incidence and emission and also different heat fluxes, like the spectral directional emissive power, spectral hemispherical emissive power and total hemispherical emissive power like the three kinds of definitions that we commonly use for each of the heat fluxes. I hope you have understood that. Similar definitions are possible in terms of irradiation radiosity and also for net radiative heat flux. And also like I have mentioned earlier also the net radiative heat flux we have here defined in terms of net amount of energy received by a surface minus net amount of radiation energy that is leaving the surface.

But it is also possible to define it in the other way that is in terms of net amount of energy leaving the surface. Now in the third lecture of the previous week, you have been introduced to the concept of black body or black surfaces, which actually is the most idealized kind of surface that we can have in terms of radiation heat transfer. You know that a surface to be categorized as the black surface or black body it must satisfy 3 conditions.

Firstly, it should be a diffuse emitter. That is whatever emission that is coming out of this that should not have any kind of directional dependence. It should depend only on the wavelength and the temperature of the surface. Secondly whatever irradiation that falls on a black surface, 100% of that should get absorbed, that means it has an absorptivity of 1 and reflectivity of 0 for opaque surfaces that is.

And thirdly which is the most important one probably; that for a given wavelength and temperature, the amount of energy emitted by a black surface is the highest among all possible surfaces. That is any real surface; any non black surface should emit radiation less then what is emitted by a black surface for a given combination of temperature and wavelength. And this particular point is very important because there is no black surface in nature.

Ideally black surface is concept is a highly hypothetical one. And because as the black surface is assumed to be a diffuse emitter, so we can completely eliminate the direction dependence and can talk about the dependence on wavelength and temperature. And we have also learned about the Planck's's distributions etc, so that we can very easily calculate the spectral intensity of emission for a black surface and accordingly the spectral hemispherical emissive power and the total hemispherical emissive power.

However, if we are talking about a real surface, then we know that once the wavelength and temperature is given its emission is going to be less than or equal to that black surface. In the limiting case it is going to be equal to the black surface, but practically it should always be less than that. But how can you quantify that? If I provide you one value of wavelength and one value of temperature, then you know the upper limit of the emission that we can expect from a real surface.

But what should be the actual value, how can we quantify this? That is precisely the thing that we are going to learn in this particular module. Similarly, when you are talking about irradiation, that is radiation energy being incident on the surface, we know that for a black surface everything is going to get absorbed. That means it has an absorptivity of 1 and reflectivity of 0. But real surfaces always have absorptivity less than 1 and certain value of reflectivity as well.

For certain surfaces reflectivity can be extremely high also, absorptivity can be quite low. Then how to quantify that? Once some energy is being a incident on a surface we know that if the surface is a black one then everything is going to get absorbed. But if the surface is a real one only a fraction of the incident energy is going to get absorbed and rest is going to get reflected.

And if it is a non-opaque surface, if it is a transparent surface some part is also going to get transmitted.

Then how to quantify those factors? That is the topic that we are going to discuss in this module via two lectures today and the next one. And in this lectures, we shall be learning about different kind of properties of real surfaces or radiative properties for real surfaces. Now today we are going to talk about the emission and in the next lecture we shall be talking about the irradiation and corresponding phenomena like absorption, reflection and transmission.

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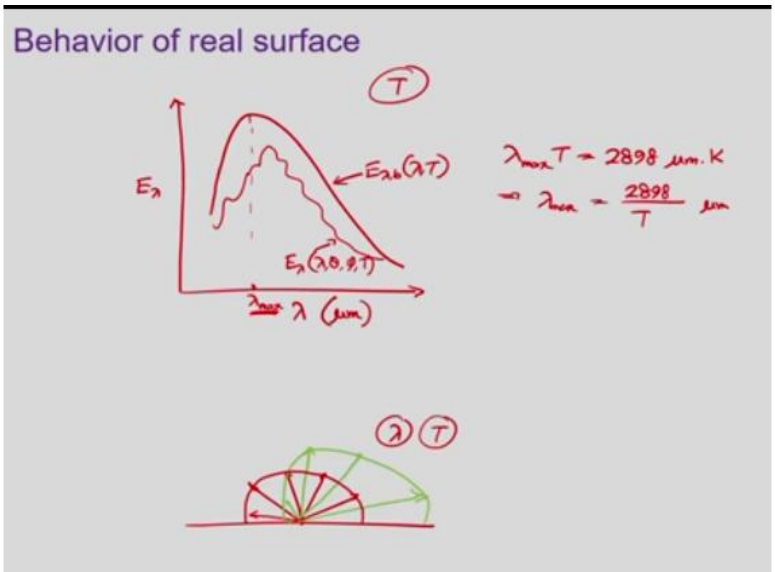

When we are talking about an emission from real surface, we plot the Planck's distribution curve for a black surface, which we have plotted in the previous week that is wavelength on this axis which is generally plotted in terms of μm and E_λ. What is this symbol suggest? E corresponds to emissive power and we have a subscript λ, so what it signifies? It is wavelength dependent that is a spectral quantity but there is no directional dependence. So this is spectral hemispherical emissive power. So it is going to depend on λT. Or to be more precise, let us write this as$E_{\lambda,b}\,(\lambda, T)$. And this b suggests that we are going to talk about a black surface or blackbody. Now the value of this spectral emissive power for a blackbody or its dependence on the wavelength can be given by the Planck's distribution, once the temperature is specified.

And we know that the profile will be somewhat like this. For different temperatures we can get different lines. As the temperature becomes lower the line gets shifted towards this side, but that is not of our objective. Our objective is to plot for a single temperature. So a temperature value is given and then the spectral emissive power for a black surface is plotted here with respect to the wavelength.

We know that there will be certain value corresponding to which you are going to get the maximum, which we have identified as the λ_{max}. Though truly speaking this is an optimum value of λ which corresponds to the maximum value of this $E_{\lambda,b}$. We also know how to identify this point. How to identify this λ_{max}? Using the Wien's displacement law we know that

$$\lambda_{max}T = 2898\ \mu m.K$$

So once we know the surface temperature, then we can also calculate this λ_{max}. Generally it is given in terms of micron, if you are using T in terms of K. But this is for a black surface or an ideal surface. What we are going to get for a real surface? For a black surface this is a function of only λ and T. Now I am going to remove this restriction.

Now I am going to plot only E_λ for any surface. Already we have the profile for a black surface, now I am going to add the profile for a real surface. For real surface the direction dependence also comes into play and also its wavelength dependence is not precisely what we get from the Planck's distribution. It can be quite random. It can be something in that particular form.

This is the spectral emissive power or spectral hemispherical emissive power that we are getting for real surface, which is a function of λ, then direction and temperature as well. And therefore for real surfaces for every different wavelength, we may get a different fraction of actual emission to the corresponding blackbody emission. Similarly, the emission can also have direction dependency.

Like suppose on a surface we are talking about emission coming out of this point. Then if we are talking about blackbody emission, then in every direction it is going to emit with equal intensity. Here I am talking about just one fixed wavelength only. A fixed value of wavelength, a fixed value of temperature, then there is no directional dependency. In every direction, it is going to emit with equal intensity.

But if I now plot here for a real surface, we can see a very strong directional dependence. We may have very strong emission in one direction. We may have extremely low emission in certain other direction, maybe zero emission in certain other direction and very large emission may be on one side. That means if we join the tip of these lines, you a very non-uniform kind of distribution.

If we join the tip of these black surface emissions then we are going to get it perfect hemisphere. As all of these lines are of equal length; representing equal intensity of emission. So the emission from real surfaces should have both spectral and directional dependence and therefore we must quantify this emission from real surfaces by taking the spectral and directional dependence into consideration.

And the parameter we used for that purpose is known as the emissivity. Emissivity I have defined in the very first week itself, but that was a very crude definition. You have used the value of emissivity also while dealing with the thermal resistance concept of conduction heat transfer, if there is any radiation or convection heat transfer is present.

But that was the only crude idea, I repeat again. But we have just used a value of emissivity. We actually were talking more in terms of the total hemispherical behavior. But now we know that it can have both spectral and directional dependence and accordingly emissivity can also have different definition.

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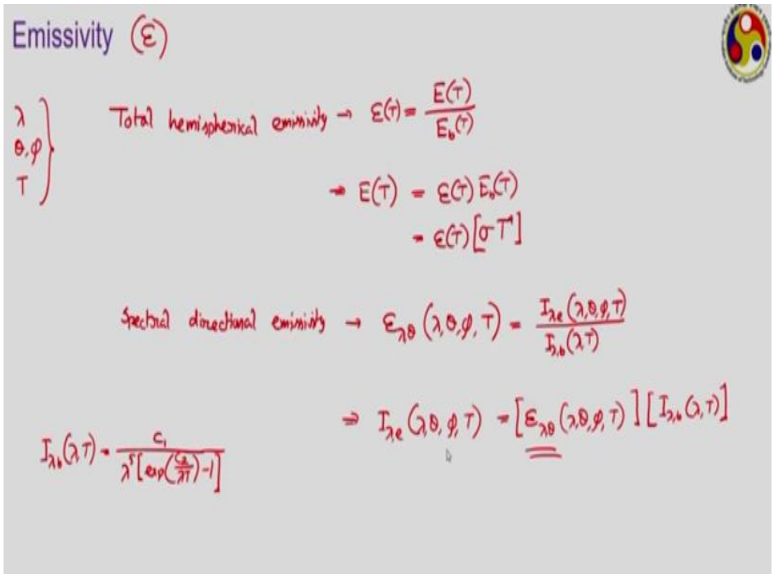

The definition that you have used earlier that is known as the total hemispherical emissivity. As the name suggests, it is total. That means there is no wavelength dependence. It is hemispherical, so there is no directional dependence. And the symbol emissivity is represented by ε. We define this ε to be the emission that you are getting from the real surface divided by corresponding blackbody emission.

$$\epsilon = \frac{E}{E_b}$$

Now there is no wavelength dependence, no direction dependence, so ε is a perfect constant; is it total hemispherical emissivity? No, it is a function of temperature, because we know that there are 3 kinds of dependence we can have in terms of radiation. In first two lectures of our previous week, we talked about the dependence on the wavelength and direction. And in the third week when we came to the black surface, you also mentioned about the contribution coming from temperature.

So there are three possible dependencies. The term total indicates that it does not depend on λ; the term hemispherical indicates that it does not depend on the θ, φ combination, but temperature dependence is always there. Accordingly, here the numerator and denominator they are also functions of temperature. So if we want to know the total hemispherical emissive power from a real surface that is going to be corresponding emissivity multiplied by corresponding black body emissive power or the total hemispherical emissive power from a black body. And following Stefan-Boltzmann law,

$$E(T) = \epsilon(T)E_b(T) = \epsilon(T)[\sigma T^4]$$

And this is the relation that we have used so far, whenever talked about radiation, where we have just taken this total hemispherical definition of emissivity. But truly speaking emissivity can have dependence on both direction and wavelength because the emission characteristics can vary quite randomly. And accordingly it is important that we should consider both these dependence into definition. So the most basic definition of emissivity is generally given in terms of the spectral directional emissivity. Spectral directional emissivity is again denoted by ε, but there are subscripts that we put in. We put a subscript λ to denote the spectral dependence and also put a subscript θ to indicate that there is a directional dependency ($\epsilon_{\lambda\theta}$).

While defining the heat fluxes, we never use this θ. But generally while defining the properties we use the θ to exclusively indicate the directional dependence. Now what will be the definition of this spectral directional emissivity? Before that it should be a function of which quantities? Spectral dependence is there, so this is the function of λ, directional dependence is there, so this is a function of θ, φ. And temperature dependence is also there. So it depends upon all three kinds of parameters that we have to consider.

$$\epsilon_{\lambda\theta} = \epsilon_{\lambda\theta}(\lambda, \theta, \phi, T)$$

It is defined in terms of the ratio of intensities. The spectral intensity of emission coming out from this real surface for that particular wavelength and in that particular direction for the given temperature divided by the corresponding spectral intensity of emission for a black surface.

$$\epsilon_{\lambda\theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda e}(\lambda, \theta, \phi, T)}{I_{\lambda b}(\lambda, T)}$$

Now we know that for a black surface or for a blackbody, there is diffuse emission. There is no directional dependency. And therefore θ, φ should go off. So the denominator is a function of only λ and T. This is the definition of the spectral directional emissivity. Accordingly, the spectral intensity of emission from real surface can be defined as

$$I_{\lambda e}(\lambda, \theta, \phi, T) = \epsilon_{\lambda\theta}(\lambda, \theta, \phi, T) I_{\lambda b}(\lambda, T)$$

And how we can estimate this $I_{\lambda b}$? We have learned in the previous week that $I_{\lambda b}$ can be expressed in terms of the Planck distribution.

I am just trying to recall the relation that we have used earlier, which was of a form like this

$$I_{\lambda,b}(\lambda,T) = \frac{c_1}{\lambda^5\left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]}$$

I guess I am writing it correctly. Value of c_1 and c_2 was also given in the last lecture that we had in the week number 9. And from there once the λ and T combination is known, you can calculate the spectral intensity of emission from the black surface.

And then once you have some idea about this quantity, you can calculate this spectral intensity of emission for the real surfaces. Now this is the most fundamental definition you can have where you are considering the variation of wavelength, direction and temperature. But in most of the applications we generally go for averaged kind of definitions, where I will take either the wavelength as a gross quantity or integrate over the entire direction. Accordingly, we can have two kinds of definition.

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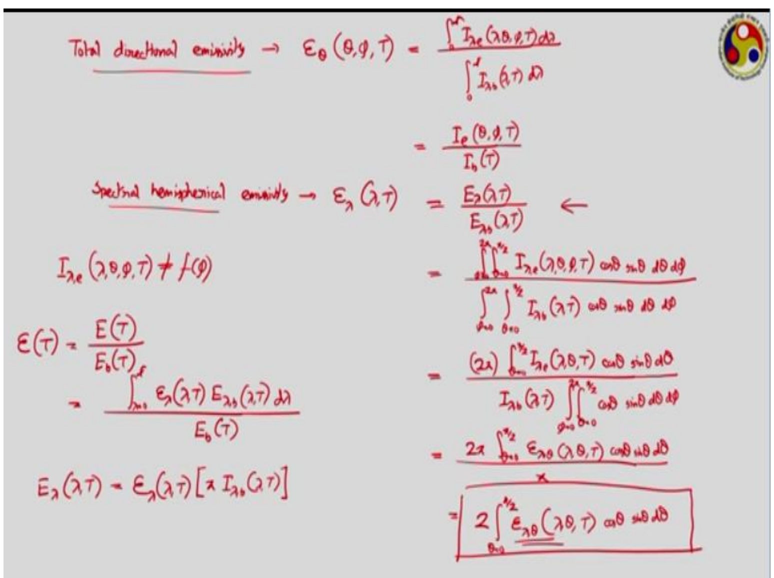

One definition is of total directional emissivity. What should be the subscript here? It is total so no spectral dependence, so λ is not coming here, but θ dependence is there, it is a directional dependence quantity. So accordingly it will be a function of θ, φ and T. Total directional emissivity is defined as when you integrate this one over the entire range of wavelength. That is

$$\epsilon_\theta(\theta,\phi,T) = \frac{\int_0^\infty I_{\lambda e}(\lambda,\theta,\phi,T)d\lambda}{\int_0^\infty I_{\lambda b}(\lambda,T)d\lambda}$$

Or as we have already integrated over the wavelength range commonly it is represented as

$$\epsilon_\theta(\theta, \phi, T) = \frac{I_e(\theta, \phi, T)}{I_b(\lambda, T)}$$

These are total directional emissivity and in engineering application instead of this, we generally go for the other one which is called the spectral hemispherical emissivity. Here we integrate over the directions over a full hemisphere. So its notation should be, ε to denote its emissivity, then suffix λ to denote it is spectral and it will be a function of λ and T. So what will be the definition of this one? Its definition should be then the ratio of spectral hemispherical emissive power for the real surface to the spectral hemispherical emissive power for the black surface at a given combination of wavelength and temperature.

$$\epsilon_\lambda(\lambda, T) = \frac{E_\lambda(\lambda T)}{E_{\lambda b}(\lambda T)}$$

So if we write that using the full integrated form, then this is going to be

$$\epsilon_\lambda(\lambda, T) = \frac{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,e}(\lambda, \theta, \phi, T) \cos\theta \sin\theta \, d\theta d\phi}{\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,b}(\lambda, T) \cos\theta \sin\theta \, d\theta d\phi}$$

This is the spectral hemispherical emissivity. Now most of the cases for most of the surfaces, we generally can consider the dependence of this $I_{\lambda,e}$, the numerator to be very less on the azimuthal direction and accordingly what I am trying to say is that this $I_{\lambda,e}$ for most of the practical cases is not a function of φ or rather it has very weak dependence on φ.

$$I_{\lambda,e}(\lambda, \theta, \phi, T) \neq f(\phi)$$

Accordingly, the φ term can be removed. And from this we can integrate over the dφ to have in the numerator as

$$\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,e}(\lambda, \theta, \phi, T) \cos\theta \sin\theta \, d\theta d\phi = (2\pi) \int_{\theta=0}^{\frac{\pi}{2}} I_{\lambda,e}(\lambda, \theta, T) \cos\theta \sin\theta \, d\theta$$

What about the denominator? Now $I_{\lambda b}$ the spectral intensity mentioned for the black surface is completely independent of both θ and φ because a black surface is direction independent. Or I should say black surface a diffuse emitter. Accordingly we can take this $I_{\lambda b}$ out. So

$$\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,b}(\lambda, \theta, \phi, T) \cos\theta \sin\theta \, d\theta d\phi = I_{\lambda,b}(\lambda, T) \iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} \cos\theta \sin\theta \, d\theta d\phi$$

Hence,

$$\epsilon_\lambda(\lambda, T) = \frac{(2\pi)\int_{\theta=0}^{\frac{\pi}{2}} I_{\lambda,e}(\lambda,\theta,T)\cos\theta\sin\theta\, d\theta}{I_{\lambda,b}(\lambda,T)\iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi}\cos\theta\sin\theta\, d\theta d\phi}$$

Now we have this $I_{\lambda,e}$ in the numerator and $I_{\lambda b}$ in the denominator. Look at what we have in the previous slide, if we use this one then $I_{\lambda b}$ in the numerator and denominator cancels and we can express it as

$$\epsilon_\lambda(\lambda, T) = \frac{(2\pi)\int_{\theta=0}^{\frac{\pi}{2}} \epsilon_{\lambda\theta}(\lambda,\theta,T)\cos\theta\sin\theta\, d\theta}{\pi}$$

$$\epsilon_\lambda(\lambda, T) = 2\int_{\theta=0}^{\frac{\pi}{2}} \epsilon_{\lambda\theta}(\lambda,\theta,T)\cos\theta\sin\theta\, d\theta$$

So once we know the spectral directional emissivity and its dependence on the directions and wavelength and temperature, we can also calculate the spectral hemispherical emissivity. Similarly, the total hemispherical emissivity ε which is a function of temperature only as we have defined in the previous slide which is the total hemispherical emissive power from the real surface divided by total hemispherical emissive power from the corresponding black surface.

$$\epsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \epsilon_\lambda(\lambda,T)E_{\lambda b}(\lambda,T)d\lambda}{E_b(T)}$$

From there we can easily calculate. Again once we know this particular quantity, then we can also calculate the total hemispherical emissivity. And once you know the total hemispherical emissivity, like I have shown here we can also calculate the total emissive power and we can calculate any other quantity as well.

So using this, we can also calculate the spectral emissive power from any real surfaces. There are several definitions then we are taking like emissivity has been defined in terms of the spectral directional emissivity. Then we can integrate this one over the wavelength to get the total directional emissivity or over the direction to get the spectral hemispherical emissivity.

And finally we have the total hemispherical emissivity. This is the one that we have kept on using earlier where we had only the concept of the Stefan-Boltzmann law, but have no idea about this direction and spectral dependence. But once we have any idea about this particular quantity, the spectral directional emissivity, then each of the others can be calculated just like shown here.

Once we know this using this relation, you can calculate the spectral intensity of emission in that particular θ, φ direction. You can also calculate the total emissive power from this. If we want to calculate the spectral hemispherical emissive power that is that also you can calculate from here. Like if our objective is to calculate the spectral hemispherical emissive power for this surface, then this should be

$$E_\lambda(\lambda,T) = \epsilon_\lambda(\lambda,T)E_{\lambda b}(\lambda,T)$$

And how to get this one this $E_{\lambda,b}$. Blackbody being a diffused emitter, we can make use of this $I_{\lambda b}$ and correspondingly we can change this $E_{\lambda,b}$.

$$= \epsilon_\lambda(\lambda,T)[\pi I_{\lambda b}(\lambda,T)]$$

And $I_{\lambda b}(\lambda,T)$ can be calculated using previous relation, again from only wavelength and temperature. Accordingly different definitions of emissivity are possible. You have to choose which one is relevant to your case.

While the spectral directional emissivity is the most fundamental one, for most of the engineering applications we may not have to go to that level. We generally can stick to the spectral hemispherical emissivity in most of the cases and there also we are seeing that it is being hemispherical, there is both θ, φ dependence can be taken care of. And in certain cases if you have to go for the total directional emissivity quite often the dependence of the azimuthal direction can be written off for this.

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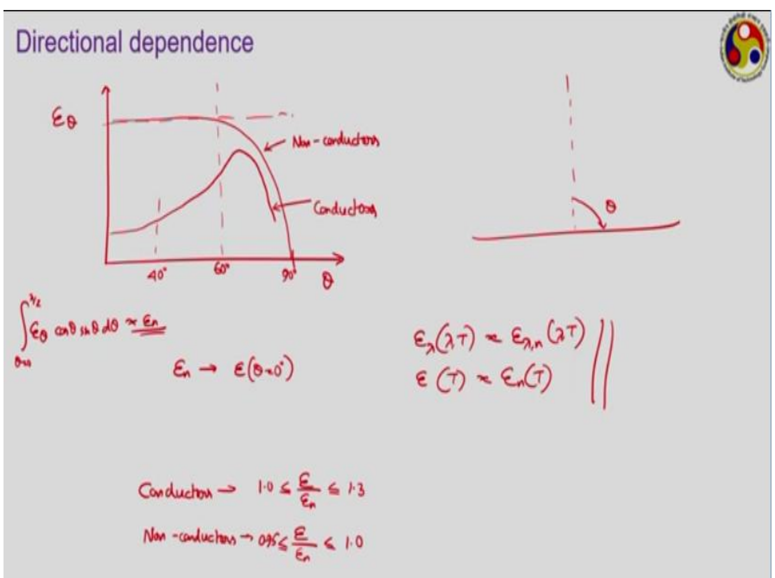

So if we quickly talk about the directional dependence of real materials then it has been observed that like for certain materials if we plot θ here and this ε_θ, that is the total directional emissivity, then it has been found that for certain non conductors it generally shows a very flat line and then drops quite sharply, generally beyond a θ value of about 60 degree to go to 0 at about 90 degree. These are generally for non conducting materials. This is only an indicative curve.

Certain materials can have slightly different kind of behavior. But this is more or less the general trend. It remains constant almost up to 50, 60 degrees and then decays very sharply. Or it can be even up to 70 degree also and then decays very sharply and goes to 0 around the 90 degree. Whereas for conductor materials it generally is constant, then increases and again decreases towards 90 degree. This line is for conductors.

It has been observed that for conductors again it is approximately constant up to a range of about 40 degree, it does not change too much. This is not to scale of course this diagram; up to 40 degrees, there is not much change but then increases sharply and then beyond that 60 degrees about 70, 75 degree it again drops sharply towards 0 at 90 degree. So accordingly we can see there is a very strong directional dependence and we need to have this ε_{θ} or its variation with θ, and to know that mathematical relation to perform any kind of integration. But one thing generally we can find that the value of ε_n, which actually refers to ε at θ =0º. ε_n refers to normal direction. Like if this is your surface and this is the normal to that surface, then we have defined θ just like this. This particular angle is the θ.

We are measuring θ this way, starting from this going toward this side. So θ =0º actually refers to the vertical line where θ =90º refers to the horizontal line. Means the surface actually is a vertical one and θ =0º refers a horizontal surface with this normal pointing upwards. Or at least the configuration that is shown here, θ =0º refers to the normal direction.

So ε_n accordingly refers to the normal direction. But it has been found that there is a strong directional dependence. But once we perform the integration that is once we perform this integration $\int_{\theta=0}^{\frac{\pi}{2}} \epsilon_\theta \cos\theta \sin\theta \, d\theta$ like the one that we had in the previous side in terms of $\epsilon_{\lambda\theta}$, this one does not vary that much from this ε_n. It remains quite close to this ε_n.

And accordingly over a large range, we can almost use this value of ε_n with certain corrections to replace the value of ε over the entire range. Means what I am talking about this for a nonconductor this is your ε_n. So up to about 60 to 70 degree, it is exactly equal to ε_n and then varies a bit towards the later side. Accordingly, we can define ε_λ, which is a function of both λ T

$$\epsilon_\lambda(\lambda, T) = \epsilon_{\lambda,n}(\lambda, T)$$

This is the spectral hemispherical emissivity. Similarly, we can define the total hemispherical emissivity also. Total hemispherical emissivity can also be approximated as the normal value of total hemispherical emissivity for most of the cases without too much error.

$$\epsilon(T) = \epsilon_n(T)$$

If we are dealing with conducting materials, generally it has been found that

$$1.0 \leq \frac{\epsilon}{\epsilon_n} \leq 1.3$$

For non-conductors, it is even better.

$$0.95 \leq \frac{\epsilon}{\epsilon_n} \leq 1$$

So it is a very small range that we generally deal with. And for most of the cases therefore this assumption is not a bad one. We can eliminate the directional dependence just by considering the normal value of emissivity or emissivity in the normal direction.

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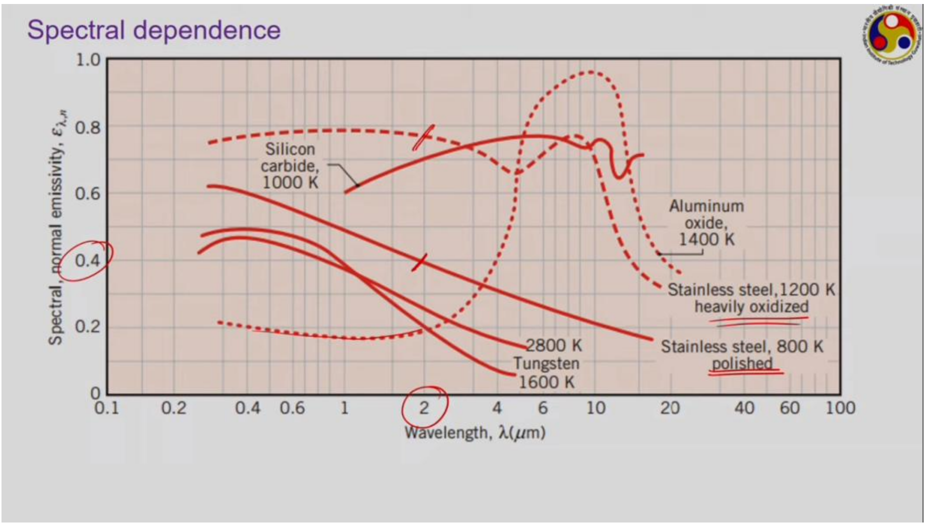

Spectral dependence is shown here for certain common material. You can see there is no specific trend in materials like stainless steel, tungsten, etc.; their spectral normal emissivity that is. Now we are going to plot only normal emissivity. So spectral normal emissivity, $\varepsilon_{\lambda n}$ continuously decreases with increase in wavelength. But not for all material, like aluminum oxide at 1400 K it generally is a flat line and then rapidly increases to indicate a maximum somewhere here and then drop sharply. Also you can see that depending upon the nature of the surface, external surface the value of emissivity can vary strongly. Like two cases are shown for stainless steel; stainless steel at 800 K when the surface is polished stainless steel at 1200 K when the surface is heavily oxidized.

You can see if we pick up any particular value of wavelength, say if we pick up this wavelength value of 2, then for polished surface while the emissivity is somewhere here, which is around 0.4, but if we go to the highly oxidized stainless steel surface this value is somewhere here, which is in the range of 0.76, 0.77. And accordingly by modifying the nature of the surface, we can easily change the value of the emissivity as well.

But in general the non-conductors have higher emissivity like we have seen in the previous slide itself. Most of the cases non-conductors have higher emissivity compared to the conductors.

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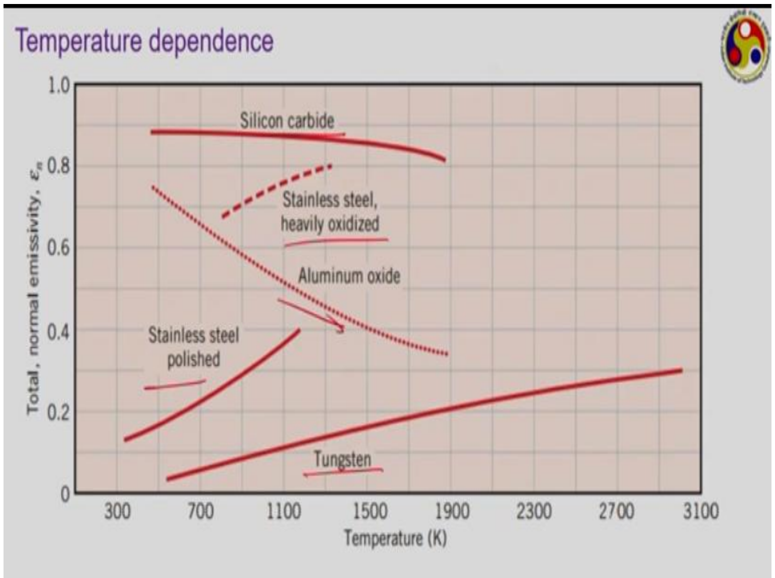

This is the temperature dependence for certain common materials again. You can see for materials like tungsten or polished stainless steel, even oxidized stainless steel also, it increases with increase in temperature the total normal emissivity. For silicon carbide it is more or less flat line, then starts decreasing beyond 1100 K, whereas for aluminium oxide it is a continuously decreasing trend. Means as the temperature increases its total normal emissivity or emissivity in the normal direction that keeps on reducing. So this way you can refer to the books to find the values of emissivity for certain other materials as well.

Like non conducting materials something like say carbon, graphite can have quite high emissivity in the range of 0.6 to 0.9 whereas our human skin, human skin is a highly emitting surface. Its emissivity will be in the range of 0.8 or even higher. And with the change in temperature as we can see from the screen, the emissivity value also can vary quite a bit.

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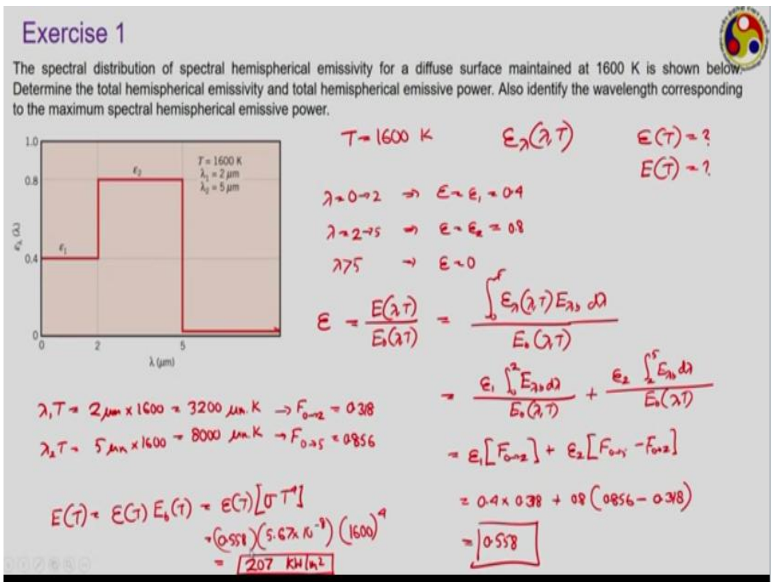

So now we shall be solving a couple of numerical problems to see whatever we have learned. The spectral distribution of spectral hemispherical emissivity for a diffused is maintained at 1600K is shown below. So we know that the temperature of the surface is 1600 K and the one given is the spectral hemispherical emissivity. So it is a function of only λ and T. Of course T is constant in this case that is your 1600 K.

Determine the total hemispherical emissivity. So you have to calculate the total hemispherical emissivity ε, which should be a function of only temperature. This one you have to find and total hemispherical emissive power. The second part I am coming later on. So the values are shown there that is

$$\lambda = 0 \rightarrow 2; \epsilon = \epsilon_1 = 0.4$$

$$\lambda = 2 \rightarrow 5; \epsilon = \epsilon_2 = 0.8$$

$$\lambda > 5; \epsilon = 0$$

So this surface is not able to emit anything once the wavelength crosses 5. So you have to perform this calculation. We have to find the total hemispherical emissivity. So how we define that? Total hemispherical emissivity it was defined in the previous slides.

So it is equal to the actual emissive power from the real surface divided by the corresponding black body emissive power.

$$\epsilon = \frac{E(\lambda, T)}{E_b(\lambda, T)} = \frac{\int_0^\infty \epsilon_\lambda(\lambda, T) E_{\lambda b} d\lambda}{E_b(\lambda, T)}$$

Accordingly we can break this one into two parts.

$$= \frac{\epsilon_1 \int_0^2 E_{\lambda b} d\lambda}{E_b(\lambda, T)} + \frac{\epsilon_2 \int_2^5 E_{\lambda b} d\lambda}{E_b(\lambda, T)}$$

Now if we just forget about the ε_1, ε_2 can you identify the other two terms? In the previous week, we introduced you to the concept of band emission. The same can be used here.

$$= \epsilon_1[F_{0\rightarrow 2}] + \epsilon_2[F_{0\rightarrow 5} - F_{0\rightarrow 2}]$$

Now we have

$$\lambda_1 T = 2\ \mu m \times 1600\ K = 3200\ \mu m\ K$$

$$\lambda_2 T = 5\ \mu m \times 1600\ K = 8000\ \mu m\ K$$

And then we can make use of the table that I introduced in a previous week for $\lambda_1 T$, the value of F, I have identified or I have noted this down. $F_{0\rightarrow 2}$ is coming as 0.318, $F_{0\rightarrow 5}$ is 0.856. So if you put the numbers

$$\epsilon = 0.4 \times 0.38 + 0.8\ (0.856 - 0.318) = 0.558$$

This is a total hemispherical emissivity for this surface. And now if you want to calculate the total hemispherical emissive power E (T) will be equal to

$$E(T) = \epsilon(T) E_b(T) = \epsilon(T) \sigma T^4$$

$$= 0.558 \times (5.67 \times 10^{-8})(1600)^4 = 207\ kW/m^2$$

So this goes the first part of the problem, where we have calculated the total hemispherical emissivity and then you we have used that to calculate the total hemispherical emissive power.

Now you have to identify the wavelength corresponding to the maximum spectral hemispherical emissive power. Now if it is a blackbody then we know that from Wien's displacement law, we can identify the solution.

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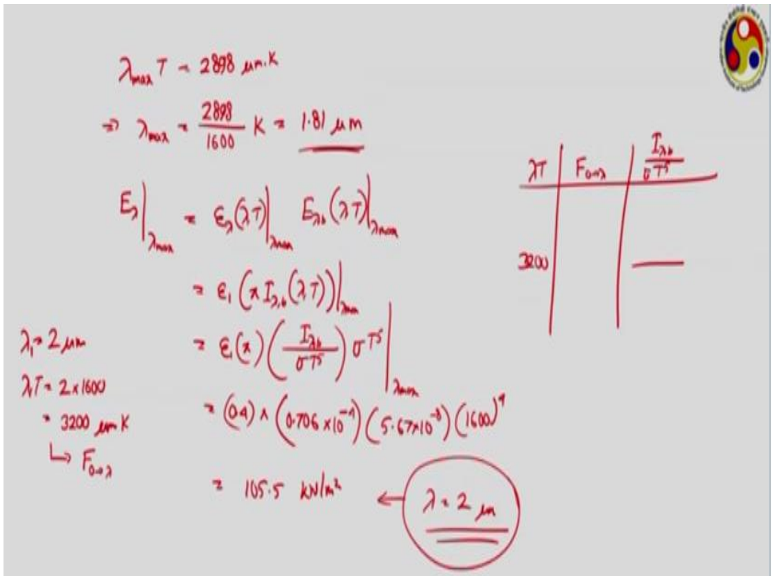

From the Wien's displacement law, we know that

$$\lambda_{max}T = 2898\ \mu m.K$$

So in this case had it been a blackbody, it would have been

$$\lambda_{max} = \frac{2898}{1600} = 1.81\ \mu m$$

So had it been a black body, the maximum value of this spectral emissive power would have appeared at this 1.81 μm. But this is a real surface and so it is not guaranteed that the optimum value appears at this particular point only. So let us try to calculate the spectral emissive power at this particular wavelength. If we want to calculate spectral emissive power E_λ for this λ_{max}, then that will be equal to

$$E_\lambda|_{\lambda_{max}} = \epsilon_\lambda(\lambda, T)|_{\lambda_{max}} E_{\lambda b}(\lambda, T)|_{\lambda_{max}} = \epsilon_1\left(\pi I_{\lambda,b}(\lambda, T)\right)$$

Now if $\epsilon_\lambda(\lambda, T)$ is given in this case that is 1.81 μm falls within this range. It will be somewhere here. For λ =1.81 we can stick to $\varepsilon = \varepsilon_1$; that is 0.4. Now how to get the $I_{\lambda,b}$?

Remember that chart that I have shown earlier. There we had columns like this. You had λT here you at this $F_{0\rightarrow\lambda}$ here and you had a quantity here as $\frac{I_{\lambda b}}{\sigma T^5}$. So once you know the λT combination, then we can calculate that $I_{\lambda b}$ as well. And from there you can get this solution of E_λ. So

$$\epsilon_1\left(\pi I_{\lambda,b}(\lambda,T)\right) = \epsilon_1\pi\left(\frac{I_{\lambda b}}{\sigma T^5}\right)\sigma T^5$$

Now you have to identify this quantity or rather we have to just bring this from the chart. So what value should we use here? Corresponding to 2 micron, if we use then

$$\lambda_1 T = 2 \times 1600 = 3200\ \mu m\ K$$

So in this particular case for λT= $3200\ \mu m\ K$, the value of $\frac{I_{\lambda b}}{\sigma T^5}$ is coming to be $0.706\text{X}10^{-4}$. So putting it

$$E_\lambda|_{\lambda_{max}} = 0.4 \times (0.706 \times 10^{-4})(5.67 \times 10^{-8})(1600)^5 = 105.5\ kW/m^2$$

So this is the way we can solve one problem, use and identify the maximum value of this. Here we can easily get compare the value corresponding to this one and we have calculated here for λ=2 micron. And this value is coming to be higher.

And accordingly it is proved that this particular system is going to be having its maximum at λ=2 micron.

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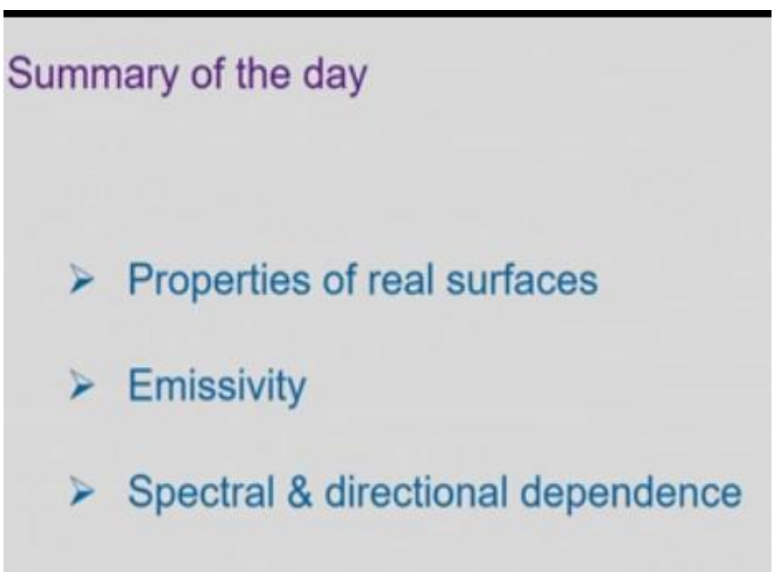

So that is where I would like to stop for the day, where I have introduced the properties of real surfaces. Actually only one property we have discussed which is the emissivity and different definitions of emissivity was proposed, considering the spectral and directional dependence. We have also solved one numerical problem. In the next class, I shall be solving another numerical problem.

And then I shall be discussing about different properties of real surfaces in relation with the irradiation. So till then please revise this lecture and if you have any confusion, please write to me. Thank you very much.