Fundamentals of Conduction and Radiation
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Lecture – 26
Fundamentals of Radiation Heat Transfer

Hello friends, welcome back to the module number 9 of our course, where we are talking about the fundamentals of radiation heat transfer. We already had couple of lectures on this and I hope you have gone through the corresponding video quite minutely. If you have done that then the concepts of the intensities of emission and intensity of incidence is clear to you.

And also by now you know the difference between different kinds of radiative heat fluxes that we have defined the emissive power, irradiation, radiosity and also the net radiative heat flux. If there is any doubt at the moment, if you have not gone through the previous videos carefully, then please pause this video right here, go back to the previous videos and study that minutely.

And also it is very important that you go through the books as well like we are following the book of fundamentals of heat and mass transfer by Incropera and DeWitt or like in the recent edition there are 4 authors; Incropera, DeWitt, Bergman and Lavine. This is the basic textbook that we were following but there are several others, very high quality text book on heat and mass transfer available like the book of J.P Holman, the book of Cengel and several other books also you can still go through any one of the books to clarify the concepts.

Now, once the concepts of radiative intensities and radiative heat fluxes are clear to you, then we can go for the discussion of the properties of ideal surfaces in terms of radiation.

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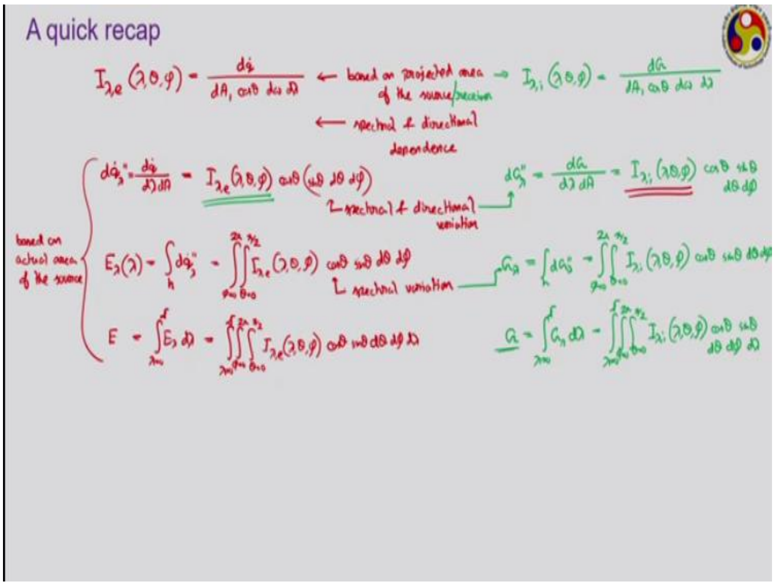

But before that let us have a quick recap of whatever I was talking about. The first term that we have defined here was the intensity of emission or I should say the spectral intensity of emission. That subscript λ denotes, it is a spectral quantity and e denotes that we are talking about emission and as I mentioned this intensity is also a directional quantity but we are not putting any subscript personal directional one, because I itself refers to directional variation. So this is a function of λ, θ and φ; polar and azimuthal angle and this is defined as

$$I_{\lambda,e}(\lambda,\theta,\phi) = \frac{d\dot{q}}{(dA_1 \cos\theta) d\omega d\lambda}$$

Where, this dA_1 refers to the area of the source surface. We know that this definition of intensity is based upon the projected area of the source. At the same time, we also have both spectral and directional dependence.

I am repeatedly mentioning about this just to clarify the concepts because there are several terms which looks quite similar and you should be absolutely sure about which term refers to what. That is why I repeatedly keep on mentioning about similar terms. From here, we have defined the spectral radiative heat flux or also which we can call the spectral directional emissive power which was defined as

$$d\dot{q}"_{\lambda} = \frac{d\dot{q}}{dA_1 . d\lambda} = \left[I_{\lambda,e}(\lambda,\theta,\phi)\right] \cos\theta \left(\sin\theta \, d\theta d\phi\right)$$

This term in the parentheses is coming from actually the solid angle dω. So this definition of this spectral directional emissive power is based by the true area not the projected area. Intensity is defined on the projected area, so that whenever we are given as a surface, we have to first identify which direction we are talking about in relation with this intensity.

And then we have to identify the projection of the source surface in terms of that direction. Like if this is your source surface and we are talking about a direction like this; it is the emission going in this direction. Then we have to find the projection of this surface normal to this direction that is may be something like this. And therefore using the angle between the normal to the surface and the direction of emission, this angle θ, we can easily calculate the project area. This θ refers to that particular angle only which you can also call the polar angle correspond with the direction. And from that spectral directional emissive power or spectral radiative heat flux, we have defined the spectral emissive power where instead of $\dot{q}$ notation, we move to the standard notation of E_λ which is a function of λ only, because here we have just integrated this $d\dot{q}"_\lambda$ over θ and φ. Or more conventionally we write this as

$$E_\lambda(\lambda) = \int_h d\dot{q}"_\lambda = \iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi$$

And in limits of integration, I hope it is clear now. We are doing this over a hemisphere, so this integration is performed over hemisphere that is why instead of writing this way. We can easily write this one just a single integration and putting the subscript h to denote that we are performing this integration over hemisphere.

And then from there, from spectral hemispherical emissive power, we define the total hemispherical emissive power which is this E_λ integrated over the λ limits from 0 to ∞.

$$E = \int_{\lambda=0}^{\infty} E_\lambda d\lambda$$

And therefore, the final definition of total hemispherical emissive power is

$$E = \int_{\lambda=0}^{\infty} E_\lambda d\lambda = \iiint_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi d\lambda$$

So, all these 3 terms that we have defined, all these three emissive powers they are based on actual area of the source and each of them has their own nature of dependence. Like the first one the spectral directional emissive power or the spectral radiative heat flux has both spectral and directional variation.

Then we are having the spectral hemispherical emissive power where we have integrated over the hemisphere and therefore, it does not have any directional variation, it has only spectral variation. And the third one is total directional emissive power which encounters or which takes care of both spectral and directional variation into consideration or integrated over all possible wavelengths and all possible directions.

So, this way we have defined for the emission. The same way we can define the characteristic corresponding to irradiation as well. And in case of irradiation, just following the same terminology, we have defined that spectral intensity of irradiation or I should say spectral intensity of incidence where we have just changed the subscript from e to i;

$$I_{\lambda,i}\,(\lambda,\theta,\phi) = \frac{dG(\lambda,\theta,\phi)}{(dA_1 \cos\theta) d\omega d\lambda}$$

Here, dG represents the infinitesimal amount of energy that is being incident on this. Again this is also based upon the projected area of the receiving surface. I should not say projected area of the source rather projected area of the receiver in this case and here also you have both spectral and directional dependence.

Then from there, we can define this spectral irradiation or you can say this is the spectral directional irradiation which is nothing but

$$dG"_{\lambda} = \frac{G}{d\lambda dA_1} = I_{\lambda,i}\,(\lambda,\theta,\phi)(\cos\theta \sin\theta\; d\theta d\phi)$$

Then integrating this over a hemisphere, we got spectral hemispherical irradiation which is this

$$G_{\lambda}(\lambda) = \int_h dG"_{\lambda} = \iint\limits_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,i}\,(\lambda,\theta,\phi)(\cos\theta \sin\theta\; d\theta d\phi)$$

And then we had this total hemispherical irradiation where we are integrating this spectral hemispherical irradiation over the entire range of wavelength

$$G = \int_0^{\infty} G_{\lambda} d\lambda = \iiint\limits_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi) \cos\theta \sin\theta\; d\theta d\phi d\lambda$$

So, first one also has the both spectral and directional variation, second one has only spectral variation and the third one is the total definition which encounters or integrates over both directions and wavelengths. And combining this, we have also defined the radiosity and their total radiative heat flux. Now, here we can see that while we are defining all this; let us just for the moment focus on the emission part.

So, to know the emission characteristic of a surface or of any radiating surface, then we actually need to know this particular quantity, $I_{\lambda e}$; the spectral intensity of emission that needs to be known. Once we know the representation of this $I_{\lambda e}$, then we can easily perform rest of the calculation. We can easily do the integration over hemisphere; we can easily do the integration over the wavelength ranges.

And we can get the values of all these heat fluxes; the spectral directional emissive power, spectral hemispherical emissive power and total hemispherical emissive power. Similarly, if we know this one somehow $I_{\lambda,i}$, the spectral intensity of incidence then again, the same way we can calculate the others. And therefore, the next objective for us should be to get an idea about how to estimate this intensities particularly, the intensity of emission.

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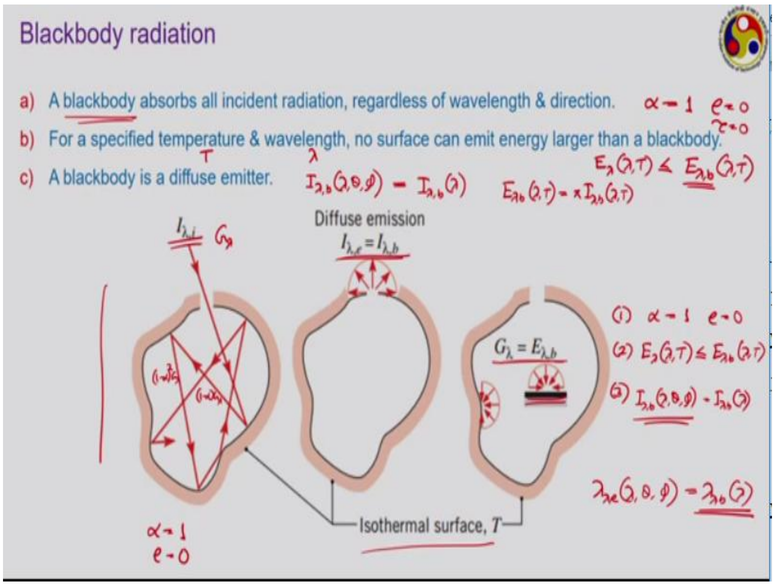

And that takes us to the concept of something known as a blackbody or black surface or the blackbody radiation. Now, the concept of a blackbody or black surface is actually a hypothetical one. There are 3 conditions that I am going to mention about a black surface. No real surface provides all these conditions to be perfect or no real surface can meet all these conditions but there are several surfaces which can approximate these conditions to certain extent.

And accordingly, they can somehow be treated as black surfaces but the concept of ideal blackbody or black surface is purely a hypothetical one, so there are 3 conditions which a surface or a body must satisfy to be identified as a blackbody. Condition number 1; a black body absorbs all incident radiation regardless of wavelength and direction.

So, whenever some radiation is incident on a black surface, it is going to absorb everything. It is not going to reflect anything; it is not going to transmit anything. And therefore, you can say the absorptivity for a black surface should be is equal to 1. And its reflectivity is equal to 0, similarly its transmissivity is also is equal to 0. Transmissivity generally comes into picture only in case of transparent surfaces.

Most of the surfaces that we deal with in heat transfer are opaque in nature and therefore, τ mostly we take it equal to 0 but reflectivity is definitely there. Only in case of black surfaces, reflectivity is also is equal to 0 and α is equal to 1. Secondly; for a specified temperature and wavelength, no surface can emit energy larger than a black body, so once we have specified one wavelength say, λ and one temperature for the surface; I mean, I am given with a surface which is at a temperature T and then I am looking to identify how much amount of emissive power or what should be a value of spectral emissive power corresponding to a particular λ; spectral hemispherical emissive power that is I am talking about. So, this second condition says that this E_λ for a given combination of λ and T should be the largest corresponding to a blackbody.

Any real surface which is a nonblack surface will always emit energy less than this particular amount corresponding to black body. So, we can say

$$E_\lambda(\lambda, T) \leq E_{\lambda,b}(\lambda, T)$$

Corresponding to a given λ, T combination. So, once we have specified the temperature and once you have specified the wavelength which is of our interest, then the maximum amount of emission or highest value of emissive power we are going to get only from a black body which is given by this $E_{\lambda,b}$.

The b subscript presents in a black surface only. So in a real surface, we will always have an spectral hemispherical emissive power value less than this quantity. So, I repeat; once I have specified the temperature of a given surface and once I also fixed up a particular value of wavelength, then the maximum value of spectral hemispherical emissive power will correspond to a black surface.

And third condition; a black body is a diffuse emitter. What do you mean by a diffuse emitter? The term diffuse as I have mentioned always corresponds to something which does not have a directional dependence. Now, here we are talking about diffuse emitter means, its emission characteristics does not have any directional dependency. Therefore, if we write this $I_{\lambda,b}\,(\lambda,\theta,\phi)$, that is the spectral intensity of emission for a black surface that is going to be a function only of λ, it does not have any dependence on θ and φ.

$$I_{\lambda,b}\,(\lambda,\theta,\phi) = I_{\lambda,b}\,(\lambda)$$

And similarly, one earlier term that we have defined or I should say one earlier derivation that we have done that is a relation between the intensity and emissive power for a black surface, do you remember that? There we have defined E_λ the emissive spectral hemispherical emissive power of a black surface for a diffuse emitter

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}\,(\lambda,T)$$

For the same λ, T combination. So these are the 3 important relations that a black surface must satisfy. The closest in reality that we get to a blackbody is something like this, where we have an enclosure just look at this picture, whose internal surfaces maintained at a constant temperature, so that we have an isothermal surface.

And then there is a very small aperture through which radiation is able to come in through this, into this enclosure. now, once the radiation enters the enclosure, just like shown here then, it will strike the surface, the first point it is striking the enclosure surface is here. At this point, a part of the radiation energy will get absorbed, remaining will get reflected.

So that reflected fraction like this $I_{\lambda,i}$ is the incidence, let us say G_λ is the corresponding magnitude of spectral hemispherical irradiation. Once it strikes at this particular point, then say αG_λ amount will get absorbed and assuming it to be an opaque surface; $(1-\alpha G_\lambda)$ will get reflected that is striking at this particular point.

There again some fraction will absorbed further, so at this point may be $\alpha(1-\alpha G_\lambda)$ will get absorbed and some $(1-\alpha)^2 G_\lambda$ will get reflected again. Now it strikes at this particular point where again some further absorption will take place, then strikes at this point. This aperture; this particular point being corresponding to a very small hole over a very large enclosure, so there is very little probability for this radiation to go out through this aperture, rather it will keep on striking the enclosure wall repeatedly and thereby getting absorbed sequentially. And it is a very much possible that the entire portion of the radiation will get absorbed finally and therefore, whatever may be the intensity of this wave irradiation, the entire amount is getting absorbed. The number of such kind of reflection it takes that may depend upon the magnitude of this wavelength λ, that may depend upon the direction also from which direction this irradiation is coming in.

But ultimately, that has to get absorbed thereby, satisfying the first condition that if we just see this entire body as a whole, then this is giving us the characteristics of $\alpha = 1$, $\rho=0$, because whatever energy, whatever intensity or whatever irradiation it is coming in, the entire portion is getting absorbed within this cavity or within this enclosure. So, the first condition is satisfied.

For a specified temperature and wavelength, no surface can emit energy larger than black body which is a condition that comes from quantum mechanics. And also from this cavity part it can be proved that if in order to maintain a thermodynamic equilibrium, then whatever energy comes in, the same amount of energy has to go out and like shown here, the $I_{\lambda,e}$ has to be is equal to $I_{\lambda,b}$ for such a cavity.

And finally, black body is a diffuse emitter because if we keep a surface like this inside, then from all direction, whatever energy gets emitted on this that ultimately, is going to get absorbed. And also from all direction this surface is going to receive energy with equal intensity which is equal to the spectral hemispherical emissive power of the black surface only.

So, such a cavity means a very large enclosure with isothermal interior surface and a small aperture or hole somewhere on the surface behaves very close to a black surface. To summarize, a black surface should satisfy 3 condition, number 1, its absorptivity should be is equal to 1, reflectivity should be is equal to 0; number 2, for a given value of wavelength and temperature, the emissive power from a black body has to be the highest among all possible surfaces maintained at the same temperature and emitting with the same wavelength.

And number 3, blackbody emission does not have any direction dependence. Therefore, spectral intensity of emission for a black surface can be written just as $I_{\lambda,b}\,(\lambda)$ as it does not have any θ, φ dependence. Now, in order to calculate the value of this one, like in the previous slide we have seen that whenever we are looking to calculate the emissive power; spectral hemispherical emissive power or total hemispherical emissive power, we need to know the value of this $I_{\lambda,e}(\lambda,\theta,\phi)$; that is the spectral intensity of emission.

Now, when you are talking about a blackbody, this one becomes$I_{\lambda,b}\,(\lambda)$, we are not putting the subscript e rather we are putting the subscript b to indicate that we are talking about a blackbody emission. The subscript λ indicates it is a spectral quantity, so there is a dependence on wavelength but there is no directional dependence. So θ, φ is not present on the right hand side.

And we are not using a subscript e, we are rather sticking to the subscript b to indicate that we are talking about blackbody emission. Now, let us try to identify the form for this one then, so that we can calculate the emissive power for a blackbody; the spectral hemispherical emissive power and total hemispherical emissive power.

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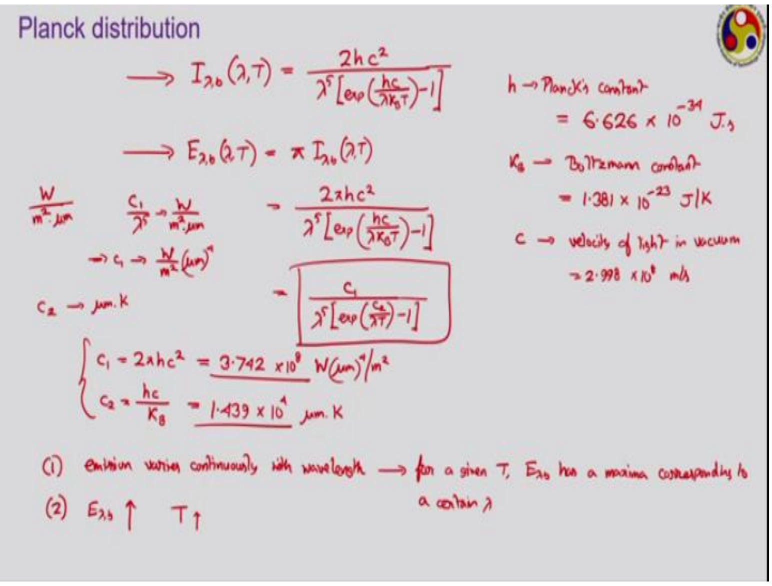

And for that we need to know about the Planck distribution. Now, Planck distribution comes from quantum mechanics and something of huge importance, Planck distribution gives us or tells us that this spectral intensity of emission from a black body which we now know is a sole function of λ or I should say, the function is λ and T. Remember in previous 2 discussions, we have talked only about the spectral and directional dependence but the temperature never came into picture.

But as we are talking about heat transfer, temperature must come into picture. And that is where the temperature is coming in through this concept of black surface. So the spectral intensity of emission from a black surface is a function of wavelength and temperature but not direction because a black surface is a diffuse emitter. Now following Planck distribution, it is given as

$$I_{\lambda,b}\,(\lambda,T) = \frac{2hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda K_B T}\right)-1\right]}$$

There are several constants that I am using here. Here, h is the famous Planck's constant. Do you remember its value? In the modern context like in this particular year of 2019, this constant has become huge importance or has gained huge importance because there is a change in the definition of the unit of kg. Like earlier you know the unit of mass that is; SI unit of mass that is kg was defined with respect to the weight of one platinum iridium sphere maintained in the museum of Paris.

However, it has been found that over several years there is some microgram decrease in its total mass and accordingly, there was a need to replace that by a universal constant and that has now been replaced by this Planck's constant. This Planck constant is a new criterion of defining the mass of kg. So, what is the value of this? It's value I am sure many of you remember, it is 6.626 X10^{-34} J.s.

What is K_B ? K_B is the Boltzmann constant. And what is the value of K_B ? It is 1.381X10^{-23} J/K. From where this one came in, can anyone say, from where this particular value came in for this? 10^{-23} comes from the Avogadro constant which is 6.023X10^{-23} and what should be the numerator? I am keeping that to you.

Just think about this what can be the possible value okay. There is another constant left that is c; c is the velocity of light in vacuum which is of course approximated 2.998X10^{8}m/s. So we have h, c and K_B are constants. So it is just a function of λ and T only wavelength and absolute temperature; T, here is absolute temperature.

In radiation we always talk about absolute temperature whereas in conduction we can still use the temperature in Celsius but we cannot use in radiation. Here everything is in terms of absolute temperature. So, now we have the expression for this $I_{\lambda,b}$, then how can you calculate this E_λ b from there; the spectral hemispherical emissive power for a given wavelength and temperature, how can we calculate?

We know that a black surface is a diffuse emitter, so therefore

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}\,(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda K_B T}\right)-1\right]}$$

Or more commonly, it is represented as

$$E_{\lambda,b}(\lambda,T) = \frac{c_1}{\lambda^5\left[\exp\left(\frac{c_2}{\lambda T}\right)-1\right]}$$

Where,

$$c_1 = 2\pi hc^2 \; and \; c_2 = \frac{hc}{K_B}$$

I have noted the values for this. You can easily combine these numbers also to get the values. What will be the unit for this? You know the unit for h is J/s and unit for c is m/s, so from there what will be a unit? There is another way of calculating the unit, even in fact that is a better way, instead of coupling this $2\pi hc^2$.

Because though π is dimensionless but here π is giving some wrong suggestion to this. The left hand side of this expression $E_{\lambda,b}$ what is the unit of that? That is spectral hemispherical emissive power. So its unit will be $W/m^2.\mu m$. As it is hemispherical, so when there is no directional dependence but it is spectral, so µm is there.

On the right hand side, then the unit must be the same. But there is a λ^5. The exponential part should be unit less, so this c_1/λ^5, this portion should have the unit of $W/m^2.\mu m$. From there then what should be the unit for c_1? So, you can easily calculate, it should be $W.\mu m^4/m^2$.

So accordingly, the value of c_1 comes to be is equal to $3.742X10^8$ $W.\mu m^4/m^2$. And c_2 again, hc/K_B, you have the value of h, you have the value of c and K_B, so from there we get it to be $1.439X10^4$. And what should be a unit for this c2? Unit for c_2 and unit for λT should be the same. Then it should be equal to µm.K.

So, these are two very standard constants. So once I specify some value of T and λ, you can easily calculate the value of corresponding $E_{\lambda,b}$ and also $I_{\lambda,b}$. So, from here then, using the

Planck distribution which is this and from there we get this one which is also known as the Planck's law.

The spectral hemispherical emissive power for a blackbody that we are getting using the Planck's distribution where we are having two constants c_1 and c_2 to deal with. Then for different temperatures therefore, like there are several things that we can note from this. Observation number 1; so, if a temperature is given, if we specify the temperature of the surface, then this intensity of emission and also emissive power that will continuously vary with λ.

So, emission varies continuously with wavelength that is the first observation. However the nature of variation is difficult to comprehend from this, either we have to plot this one or we have to do some further mathematical manipulation to get the exact nature of this dependence of $E_{\lambda,b}$ on λ but we can say that it can definitely keep on varying continuously.

Observation number 2; as the temperature increases then what will happen? Look at this expression; this was the final expression that we had. Temperature appears in the denominator of the exponential term which actually appears in the denominator of this whole expression. So, if temperature increases then what will happen? If temperature increases, then the magnitude of the exponential term will decrease. Accordingly, there will be decrease in the denominator of the overall term.

So, $E_{\lambda,b}$ will increase. So, this $E_{\lambda,b}$ continuously increases with temperature or I should say temperature increases this one also will increase. But that same thing we cannot conclude about λ. But for temperature, we can surely say that a surface with a higher temperature will lead to larger emissive power or larger black body emissive power.

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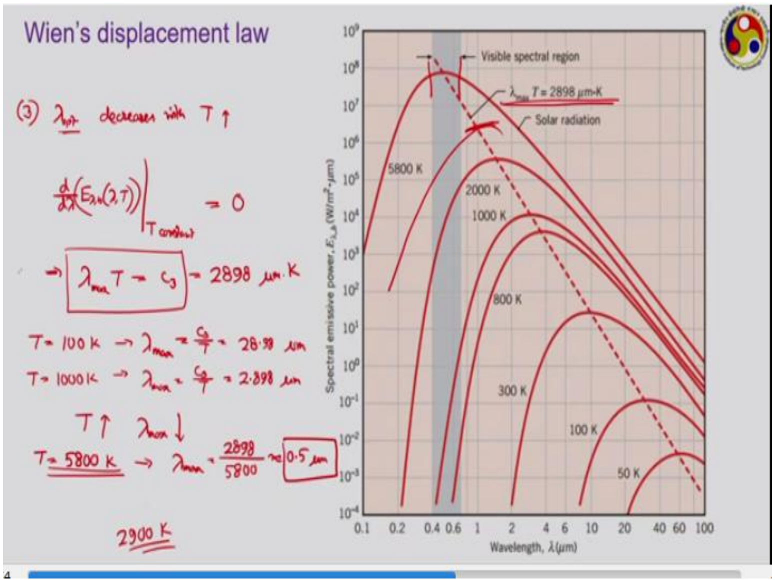

Now, if we plot this $E_{\lambda,b}$ with λ for a given temperature, then this is what we are going to get. Here we have the wavelength λ and this is the spectral hemispherical emissive power for a black surface $E_{\lambda,b}$. Often for black surface or black body, so we do not mention the hemispherical term because it is a diffuser emitter. So by default it is always giving you hemispherical idea.

And you can see there are different red lines plotted for different temperatures. Like this is for 100K, this is for 1000K. Similarly, we have different lines. And this shaded portion corresponds to the visible spectrum. Like it is from 0.4 micron to 0.7 micron of wavelengths, so this is the visible spectrum that we have. You can clearly see that for any given temperature, the spectral hemispherical emissive power for a black surface is largest corresponding to one particular λ.

It is largest corresponding to one particular λ like if you talk about 100 micron, then somewhere here it is largest. If we talk about 300 micron means, this is somewhere here it is largest. At 1000 micron this is somewhere here it is maximum. So observation number 3 we can say that for a given temperature $E_{\lambda,b}$ has a maxima corresponding to a certain wavelength. So with temperature; with increase in temperature black body emissive power or black body spectral hemispherical emissive power always increases but that is not the case with wavelength, it shows a maximum in the wavelength for a given temperature.

Then, how to identify that maximum? Before that let us observe another thing, we know that as the temperature increases, the emissive power also increases. So, if you look at this graph here, you can see for 50K, your maxima was corresponding to somewhere here, for 100K, the maxima corresponds to a wavelength of something like this, for 1000K, it corresponds to something like here.

That means as the temperature is increasing, the wavelength corresponding to the maxima in spectral hemispherical emissive power that is continuously decreasing. As the temperature is increasing the λ optimum or λ corresponding to the maxima in this emissive power that keeps on reducing. So that is our third observation; λ_{opt} decreases with increasing temperature.

As the temperature of the surface increases, the optimum value of λ decreases. Accordingly, we can say that the larger fraction of the emitted energy or I should say the wavelength band corresponding the larger fraction of emitted energy that also keeps on shifting. Like if you talk about that 100K surface temperature, then maybe we can identify band something like this within which the largest portion of emission energy is restricted.

For 800K, you may identify a band somewhat like this within which the largest fraction of energy is emitted. For 2000K, the band gets shifted further to the left, so this way as the temperature is increasing, the wavelength band correspond to the maximum emissive power that also keeps on shifting towards lower wavelength side.

Then, let us try to identify for a given temperature at which wavelength this will be maximum; this emissive power. To perform this we have to now differentiate this $E_{\lambda,b}$ functions of λ, T for a given temperature or keeping T constant and equate that to 0.

$$\frac{d}{d\lambda}\left(E_{\lambda,b}(\lambda,T)\right)\Big|_{T\ constant} = 0$$

If you do this, then you will find that

$$\lambda_{max}T = C_3 = 2898\ \mu m.K$$

Here, remember this λ_{max} does not indicate any maximum value of λ, rather it is optimum only but still I am using max because most of the textbook call it a max. It is not the maximum value of wavelength rather it is a value of wavelength corresponding to the maximum value of spectral hemispherical emissive power for a black surface.

Therefore, once we know the temperature we can straight away identify the value of the wavelength which should correspond to the maximum value of emissive power. This dotted red line shown on the graph corresponds to this particular criterion, which is known as Wien's displacement law.

So, once we know the temperature of any surface, we can easily calculate the corresponding value of λ optimum. Like say, if T=100K, then the optimum value of λ which we are calling λ_{max} will be is equal to

$$\lambda_{max} = \frac{C_3}{T} = 28.98\ \mu m$$

And you can see from this graph, the maximum corresponds to something here. This graph is plotted on a log-log scale, so be careful about the axis. So, if we put say T=1000K, then

$$\lambda_{max} = \frac{C_3}{T} = 2.898\ \mu m$$

So, your optimum will be somewhere. This dotted line passes through all these λ_{max} points or all the points correspond with the maximum emissive power for a given temperature. So, as we are increasing the temperature, your λ_{max} that also keeps on reducing continuously.

If we put now T=5800K, then

$$\lambda_{max} = \frac{C_3}{T} = \frac{2898}{5800} = 0.5\ \mu m$$

Now, what is your visible spectrum; 0.4 to 0.7 microns. So this is right in the middle of the visible spectrum somewhere here. This 5800K corresponds to the maximum emission within this 0.5 micron range. And therefore, its maximum emissive power will be emitted within this visible spectrum zone and what is this temperature then?

This is the temperature corresponding to the solar radiation. You can visualize that this 5800K is the approximate temperature of the outer surface of the Sun from where the solar radiation is originated and is able to reach the surface of the earth. As the optimum value of wavelength or wavelength value corresponds to the maxima in emissive power falls right in the middle of the visible spectrum, therefore, we are able to see the solar light and accordingly, we can make use of this. But if we are talking about a surface which is at a much lower temperature; say, any surface at a temperature lower than 1000K, will have a maximum part of its emission restricted to the IR side and therefore, we shall not be able to see that. But only when the temperature crosses 1000K or more, then only the emissions are quite close to the visible spectrum.

Like, if we talk about say a tungsten filament lamp which has a temperature of about 2900K, then if we plot it, then correspond to 2900K it will be coming somewhere here. So, you can see the maximum is still on the IR side but the curve may have a significant portion falling on this visible spectrum particularly on the red side of this.

So, we generally able to see white light from a tungsten filament lamp though the major portion or at least the maxima of its emissive power actually falls on the IR zone. So, the Wien's displacement law gives us or tells us that the value of temperature of the surface and the wavelength corresponds to the maximised spectral emissive power is a constant whose values is 2898 μm.K, which is our observation number 4 for this.

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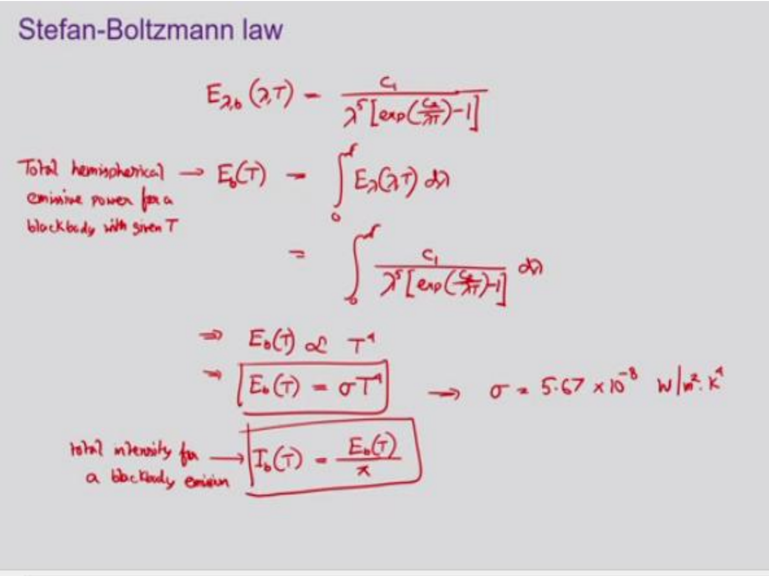

Now, we know the spectral hemispherical emissive power for a black surface. Then let us try to calculate the value of total hemispherical emissive power. So we have just seen that the spectral hemispherical emissive power $E_{\lambda,b}$ corresponding to λ T,

$$E_{\lambda,b}(\lambda,T) = \frac{c_1}{\lambda^5\left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]}$$

So, if we have to calculate the total hemispherical emissive power for the black surface for a given temperature, then what we have to do? We know that we have to integrate this spectral hemispherical emissive power over the entire range of wavelength from 0 to ∞.

$$E_b(T) = \int_{\lambda=0}^{\infty} E_{\lambda,b}(\lambda,T)d\lambda$$

$$= \int_{\lambda=0}^{\infty} \frac{c_1}{\lambda^5\left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]} d\lambda$$

This integration can easily be performed. And if you perform this, then you are going to get

$$E_b(T) \propto T^4$$

Or

$$E_b(T) = \sigma T^4$$

Where, the value of this σ is coming as $5.67X10^{-8}$ $W/m^2.K^4$.

Now, what is this? This is the Stefan Boltzmann law. So using the Planck constant and corresponding expression for spectral hemispherical emissive power for a black surface, we can calculate the total hemispherical emissive power for a blackbody with a given temperature and that comes out to be a Stefan Boltzmann law.

And that σ is the Stefan Boltzmann constant. And blackbody being a diffuse emitter, we can easily calculate corresponding total intensity

$$I_b(T) = \frac{E_b(T)}{\pi}$$

So this is the total intensity of emission associated with the black surface which again does not have any kind of spectral dependence. Directional dependence is already not there because this is a diffuse emitter. And now, we have the intensity of total radiation or maybe the total intensity for a blackbody emission.

So this is a very important relation. We can see that just from the knowledge of temperature and wavelength, we can easily calculate the total hemispherical emissive power; in fact for total hemispherical emissive power that being integrated over the range of wavelength, we do not need any information about the wavelength at all. We just need to know the temperature. And we can easily get then the total hemispherical emissive power for a black surface and total intensity of emission from a black surface, just from the knowledge of temperature.

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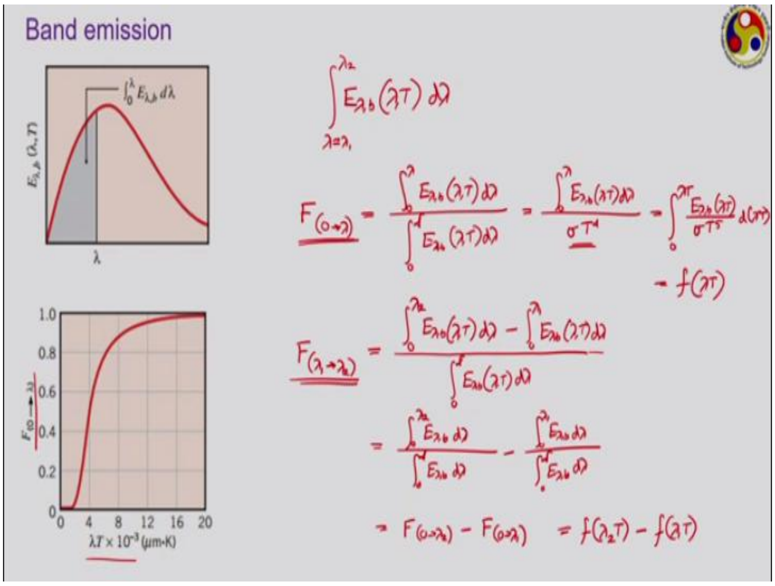

Finally, there is something known as the band emission. Quite often though the total emission we can get from black surface that is a total hemispherical emissive power. But maybe our interest is not to know the total hemispherical emissive power rather our interest is only to know a part of this. I mean we are interested only to know this $E_{\lambda,b}$ only over a range say, λ is equal to λ_1 to certain λ_2. Or

$$\int_{\lambda=\lambda_1}^{\lambda_2} E_{\lambda,b}(\lambda,T)d\lambda$$

We just want to know within a specified band of wavelengths, the total emissive power that we are getting from this black surface which is maintained at a temperature T. Then how to get that? To get this one, we have to make use of the concept of this band emission.

The band emission talks about the fraction of energy emitted within a particular band of wavelengths that is from 0 to λ. We know that total emission is given by the Stefan Boltzmann law. Then we generally use a symbol $F_{0\to\lambda}$, it indicates that the fraction of energy emitted by this black surface maintained at a given temperature T within the wavelength range interval of 0 to certain λ.

Then, what will be that? That will be equal to

$$F_{0\to\lambda} = \frac{\int_{\lambda=0}^{\lambda} E_{\lambda,b}(\lambda,T)d\lambda}{\int_{\lambda=0}^{\infty} E_{\lambda,b}(\lambda,T)d\lambda}$$

And we know that the denominator can be specified using the Stefan Boltzmann law, so getting that into picture

$$F_{0\to\lambda} = \frac{\int_{\lambda=0}^{\lambda} E_{\lambda,b}(\lambda,T)d\lambda}{\sigma T^4}$$

Now, with small manipulation we can change this to a form like this

$$= \int_0^{\lambda T} \frac{E_{\lambda,b}(\lambda,T)}{\sigma T^5} d(\lambda T) = f(\lambda T)$$

And now, it becomes a function of λT, from where we can calculate. We can see that this σT^4 that we have in a denominator, σ is a constant but we are getting this T^4 inside this integration, so that it became a function of λT.

So, once we know the λT product, then we can easily calculate the value of this $F_{0\to\lambda}$ and what this is giving? This is giving you the fraction of energy emitted within the wavelengths interval 0 to λ and this is how it will look like. This is $\lambda T X10^{-3}$ plotted in the horizontal axis. And this is the energy band. As the λT product keeps on increasing, it goes from 0 to 1.

This way we can easily calculate the fraction of energy emitted within a certain wavelength band as well. Like suppose, if our interest is to know the fraction emitted from a wavelength λ_1 to another larger wavelength of λ_2, then what we have to do? We just have to calculate the difference of energy emitted in range 0 to λ_2 to the energy in range 0 to λ_1. And take its ratio with the total amount of energy emitted over all wavelengths.

$$F_{\lambda_1\to\lambda_2} = \frac{\int_{\lambda=0}^{\lambda_2} E_{\lambda,b}(\lambda,T)d\lambda - \int_{\lambda=0}^{\lambda_1} E_{\lambda,b}(\lambda,T)d\lambda}{\int_{\lambda=0}^{\infty} E_{\lambda,b}(\lambda,T)d\lambda}$$

.

$$= \frac{\int_{\lambda=0}^{\lambda_2} E_{\lambda,b}(\lambda,T)d\lambda}{\int_{\lambda=0}^{\infty} E_{\lambda,b}(\lambda,T)d\lambda} - \frac{\int_{\lambda=0}^{\lambda_1} E_{\lambda,b}(\lambda,T)d\lambda}{\int_{\lambda=0}^{\infty} E_{\lambda,b}(\lambda,T)d\lambda}$$

$$= F_{0\to\lambda_2} - F_{0\to\lambda_1}$$

$$= f(\lambda_2 T) - f(\lambda_1 T)$$

So, if we know the value of this F, then we can easily calculate the other quantities. You can easily calculate the energy band because the denominator is always given by σT^4. Like somehow say if we know the value of this particular quantity or maybe if we know the value of this particular quantity, then on the right hand side, we know that the denominator is σT^4, this one; so we can easily get the numerator which is going to give you the band that you are looking for. And the values of this F have already been calculated using digital computers and standard tables are available.

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λT ($\mu m \cdot K$)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000

And I have taken a sample table from the book of Incropera, DeWitt, just to show you as an example. Here, you can see the values of λT are given on this side. I have curtailed this one at this 2898, but it keeps on going to very high values of this λT, generally up to 10^5. Then this is the F, so if we pick up any value say if we take λT =1000, then the corresponding value of F is 0.000321.

And additional quantities are also given, like this quantity gives $I_{\lambda,b}\ /\sigma T^5$ and on the fourth column, we have this $\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$. See here at the bottom this corresponds to 1, because λ_{max} you have to correspond to the value where the emissive power is the maximum. So here it is equal to 1 only.

This fraction continuously keeps on increasing, till it become 1 about a value of λ T equal to 10^5, this becomes almost is equal to 1, whereas this value increases becomes a maxima here, maxima of 1, then again it decreases and goes to 0. So this table generally is very useful in solving any problem associated with the radiation particularly, the spectral and directional dependence when you want to take care of.

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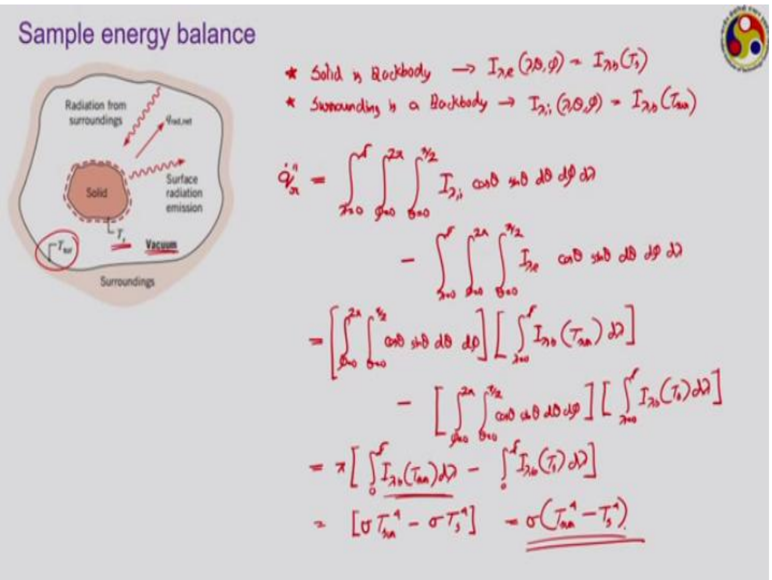

Let us finish this chapter by performing a simple energy balance and then solving one simple problem. This figure, I have shown you earlier also, here we have a very large cavity; inner surface is maintained at temperature T_{sur} and we have a solid maintained at temperature T_s. Let us assume the solid to be black body.

And this enclosure or the surrounding being it is very, very large compared to the solid, so we can treat this surrounding also to be a black body. So whatever energy is being emitted by the solid that is being received by the surrounding and also the solid is receiving energy only from the surrounding. And the surrounding is receiving energy from the solid and giving a part to the solid as well.

So, let’s perform a simple energy balance following radiation only because you are having a vacuum inside, so there is only energy transmission mode is radiation. So if we write an energy balance for the solid, then net radiative heat flux from the solid can be written as

$$\dot{q}"_r = \iiint_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,i}\cos\theta\sin\theta\, d\theta d\phi d\lambda - \iiint_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,e}\cos\theta\sin\theta\, d\theta d\phi d\lambda$$

If this solid is not assumed as a black surface, then instead of $I_{\lambda,e}$ we should have radiosity. Then it should be $I_{\lambda,e+r}$. But as we are talking about a black surface there is no reflection. So, this reflection part goes off. And remember here I have told earlier also the net radiative heat flux we are defining as the net energy gained by the solid in this case but you can write the other way also. In that case the emission will come first and incidence will come second. Now, we know that we have assumed the solid to be a black body.

As the solid is a black body, then for the solid

$$I_{\lambda,e}(\lambda,\theta,\phi) = I_{\lambda,b}(T_s)$$

Where, the solid temperature is T_s. Surrounding is a black surface and as the solid is receiving energy that is irradiation only from the surrounding, so

$$I_{\lambda,i}(\lambda,\theta,\phi) = I_{\lambda,b}(T_{sur})$$

And both of them are independent of θ and φ. And accordingly, it we can separate the components out, that is

$$= \left[\iint_{\phi=0,\theta=0}^{\phi=2\pi,\theta=\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta d\phi\right]\left[\int_{\lambda=0}^{\infty} I_{\lambda,b}(T_{sur}) d\lambda\right]$$

$$- \left[\iint_{\phi=0,\theta=0}^{\phi=2\pi,\theta=\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta d\phi\right]\left[\int_{\lambda=0}^{\infty} I_{\lambda,b}(T_s) d\lambda\right]$$

So, you can easily integrate. These space integrations will become equal to π. So

$$= \pi\left[\left[\int_{\lambda=0}^{\infty} I_{\lambda,b}(T_{sur}) d\lambda\right] - \left[\int_{\lambda=0}^{\infty} I_{\lambda,b}(T_s) d\lambda\right]\right]$$

So the terms inside integration are the total hemispherical emissive power for a black surface maintained at temperature T_{sur} and T_s. So, this becomes

$$= [\sigma T_{sur}^4 - \sigma T_s^4] = \sigma[T_{sur}^4 - T_s^4]$$

The π you remember, it has to be considered or to be multiplied with I to get a final value of $E_{\lambda,b}$.

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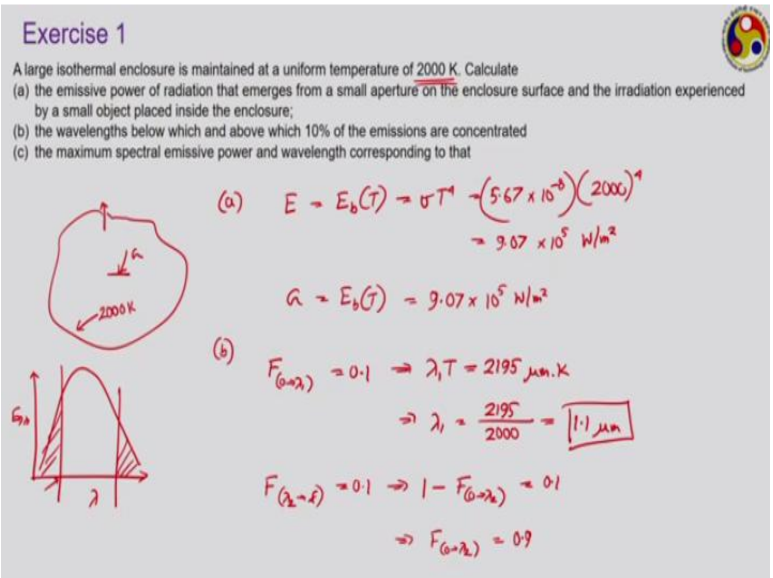

And that is what we are getting finally as the simple form in terms of Stefan Boltzmann law. So, I shall be rounding off quickly by solving 2 numerical problems. First one is corresponding to a large isothermal enclosure maintained at a uniform temperature of 2000K, so we have a large enclosure maintained at a uniform temperature of 2000K. Several things we have to calculate, first the emissive power of radiation that emerges from a small aperture.

There is a small aperture we have kept, we have to calculate the emissive power that comes out through this. Now, inner surface is maintained at a temperature of 2000K. So this being a very large enclosure we can easily treat this one as a black body and therefore, the emissive power that is coming out of this for part A, can easily be calculated as E_b, the total hemispherical emissive power corresponding to this temperature that is

$$E = E_b(T) = \sigma T^4 = (5.67 \times 10^{-8})(2000)^4 = 9.07 \times 10^5 W/m^2$$

Remember this temperature has to be in K if it is given in Celsius then please do the conversion. There are several small calculations, please follow them carefully. And also we have to calculate the irradiation experienced by a small object place inside the enclosure. If you have a small object kept here, we have to calculate G falling on this, and the G again will be

$$G = E_b(T) = 9.07 \times 10^5 W/m^2$$

Because it is kept inside a black surface and only radiation it is receiving is actually the emission from the black surface. Part B; the wavelengths below which and above which 10% of the emissions are concentrated. So first part is we have to calculate the wavelength say, if this is your $E_{\lambda,b}$, this is your λ we know that the curve will be something like this.

For the first part you have to calculate the wavelength below which 10% of the emissions are concentrated. So we have to calculate this one, so that this area is 10%. Then exactly what we are looking to identify? We have to get $F_{0\to\lambda_1}$ to be is equal to 0.1. So let us go back to the table. $F_{0\to\lambda}$ equal to 0.1, where we are getting that? 2200 is giving you 0.100888.

So, this value from the table then,

$$\lambda_1 T = 2195\ \mu m$$

To have a very precise value we could have taken 2200 also. Accordingly

$$\lambda_1 = \frac{2195}{2000} = 1.1\ \mu m$$

So, this way we can calculate. We can calculate that below 1.1 μm, only 10% of the emission will be there.

Second part you have to calculate the wavelength above which 10% of the emissions are concentrated. So, this is the other range that means, if that wavelength is λ_2; $F_{\lambda_2\to\infty}$ we have 0.1. So from there we can write that

$$F_{\lambda_2\to\infty} = 1 - F_{0\to\lambda_2} = 0.1$$

$$\Rightarrow F_{0\to\lambda_2} = 0.9$$

That is below λ_2, 90 % of the energy will be restricted.

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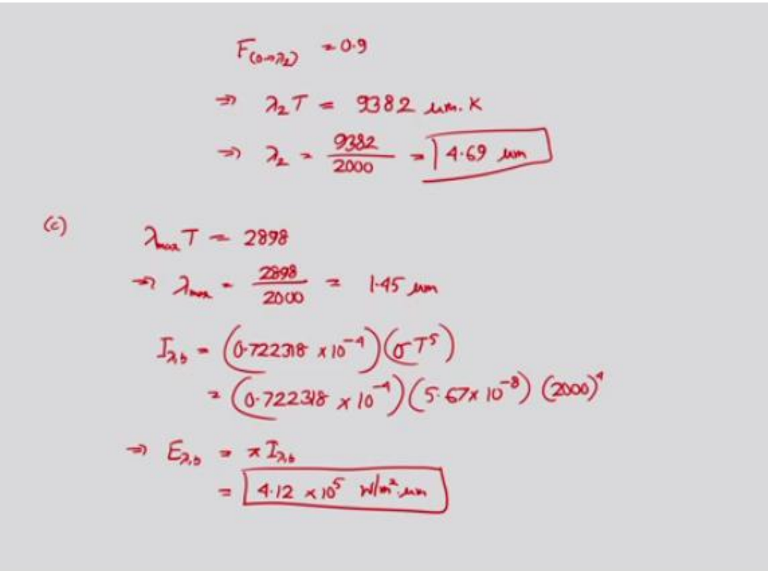

Now, the table that I have shown there it is restricted only up to 0.25. So I just got the numbers, you please refer to the books to get the value of λT for $F_{0\to\lambda_2} = 0.9$

$$\Rightarrow \lambda_2 T = 9382\ \mu K$$

$$\Rightarrow \lambda_2 = 4.69\ \mu m$$

That means we can see that between this small wavelength range of 1.1 micron to 4.69 micron; 80% of the emission will be restricted. And this is on which side? This is 1.1 micron, so this is actually on the IR side. 2000K it starts with and the 80% of the emission is restricted between 1.1 and 4.69 micron. Let's quickly solve part C of this problem.

The maximum spectral emissive power and wavelength corresponding to that so, we know that the maxima will always correspond to using the Wien's displacement law,

$$\lambda_{max} T = 2898\ \mu m.K$$

So

$$\lambda_{max} = \frac{2898}{2000} = 1.45\ \mu m$$

So using this we can calculate the value of $E_{\lambda,b}$. But there is another thing that we can do. Look at the table that I have shown. There is something shown here as $I_{\lambda,b}/\sigma T^5$. We can make use of this. We know that for 2898, this is a value of this quantity. Then as you know the temperature you can calculate $I_{\lambda,b}$ from there. So,

$$I_{\lambda,b} = 0.722318 \times 10^{-4}(\sigma T^5)$$

$$= (0.722318 \times 10^{-4})(5.67 \times 10^{-8})(2000)^5$$

And

$$E_{\lambda,b} = \pi I_{\lambda,b}$$

$$= 4.12 \times 10^5 \frac{W}{m^2.\mu m}$$

This is the maximum value of spectral hemispherical emissive power that we can get from this black surface. So this way we can make use of the other columns of this one as well.

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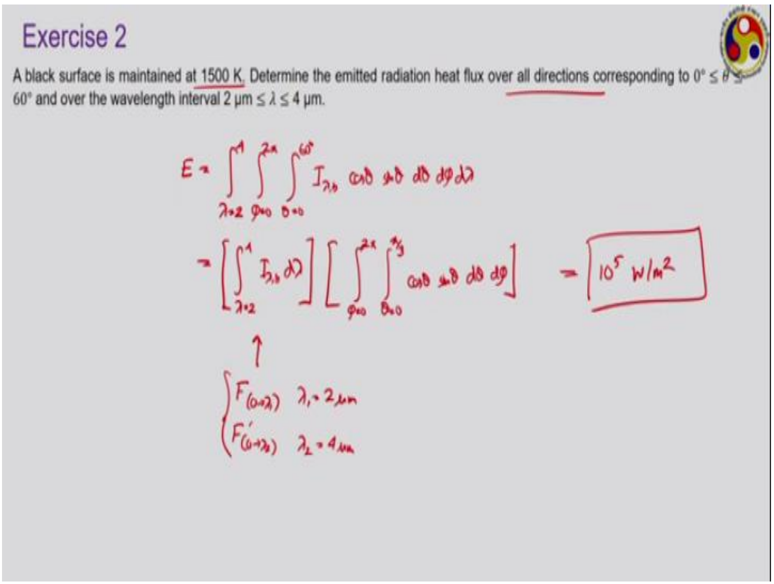

And another problem let us very quickly solve it to wrap up the chapter. We have a black surface maintained at this temperature. We have to determine the emitted radiation heat flux over all directions corresponding to θ equal to 0 to 60 and λ equal to 2 to 4 micron okay. Here all directions actually refer to all directions of φ. But θ variation is there. So you have to calculate the radiation heat flux.

So, how to do this? Here we can make use of values of F again because λ is given from 2 to 4 micron. And the θ integration we have to perform between 0 and 60. So how can we do this? Do we at all need to perform the θ integration? We are talking about a black surface which is a diffuse surface. So it does not matter what θ values that is given to us.

So, the energy that we are trying to identify is actually within this range of λ equal to 2 to 4 micron; φ equal to 0 to 2π; θ equal to 0 to 60º. So,

$$E = \iiint_{\lambda=2,\phi=0,\theta=0}^{\lambda=4,\phi=2\pi,\theta=60^{\circ}} I_{\lambda,b} \cos\theta \sin\theta \, d\theta d\phi d\lambda$$

Now, from there $I_{\lambda,b}$ being independent of θ and φ we can separate that out.

$$= \left[\int_{\lambda=2}^{4} I_{\lambda,b} d\lambda\right] \iint_{\phi=0,\theta=0}^{\phi=2\pi,\theta=\frac{\pi}{3}} \cos\theta \sin\theta \, d\theta d\phi = 105 \frac{\mathrm{W}}{\mathrm{m}^2}$$

First part we can make use of that F. Just give λ_1 =2 micron, λ_2 =4 micron. From there, you can get the first part. Second part you can perform the integration and I am just going to give you the final number, which is going to come as 10^5 W/m^2, please try to do the calculation using the table and get the final solution on your own, that will give you a good exercise of using these integrations.

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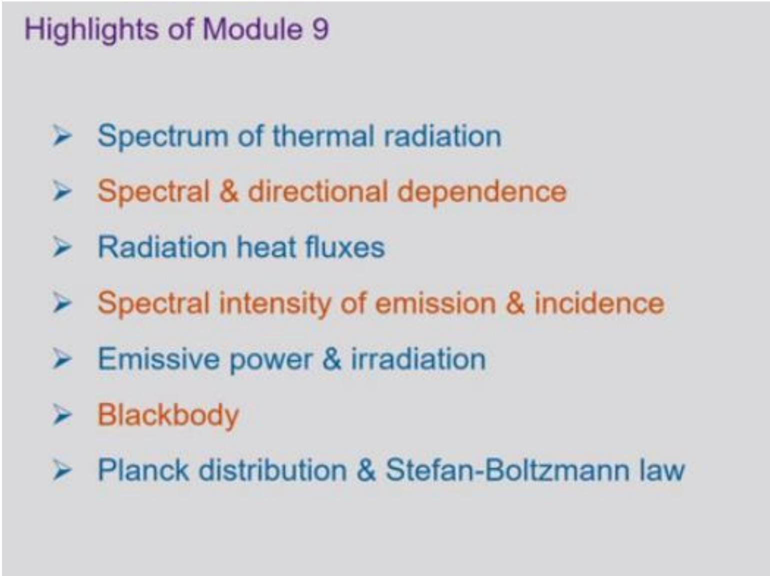

So that takes us to the end of our discussion of module 9, where we have learned about the fundamentals of radiation heat transfer. We started talking about the importance of radiation heat transfer and the spectrum of thermal radiation. Then the spectral and directional dependences were talked in detail. We developed the expressions for intensities of emission and incidence and correspondingly we got different kind of radiation heat fluxes.

Then, using the definition of spectral intensities, we developed the relations of emissive power irradiation, radiosity and net radiative heat flux. And today you have been introduced to a concept of blackbody or black surface where the Planck distribution is a very important one because that gives us the spectral hemispherical emissive power for a black surface. Using which we got the idea about the Wien's displacement law and the Stefan Boltzmann law; to calculate the total hemispherical emissive power from a black surface.

So that is it for module number 9. Please repeat the lectures go through the corresponding chapter in the books and also try to solve the assignments, so that you do not have any doubt. In the next week we shall be moving to the real surface where whatever you have discussed in this module, particularly about the black surface that will be used to calculate the properties and parameters for real surfaces.

And to go through that module 10, you need to have clear idea about all the definition that we have discussed here. So please revise the lectures and if you have any query, write back to me, thank you very much you.