Fundamentals of Conduction and Radiation
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Lecture - 25
Fundamentals of Radiation Heat Transfer – Part 2

Hello friends. So how are you now? We are into the second lecture of module number 9 where we were discussing about the fundamentals of radiation heat transfer. In the previous lecture, we have talked about different kinds of radiation heat fluxes, we have talked about the concept of solid angle, which we need to use while calculating the radiative heat fluxes and also towards the end of the lecture, we were introduced to the concept of spectral intensity.

Now that is precisely the point from where I am going to start again. The spectral intensity of emission and also the spectral radiative heat flux, I have introduced in the previous lecture, but as I went through that a bit quickly, I would like to repeat that again to start this lecture. And I also feel that it is important to get that concept very clearly and that is why I am repeating the same definition again, a bit quickly.

And then I shall be using the definition to define different kinds of radiative heat fluxes. So how you have defined the spectral intensity of emission?

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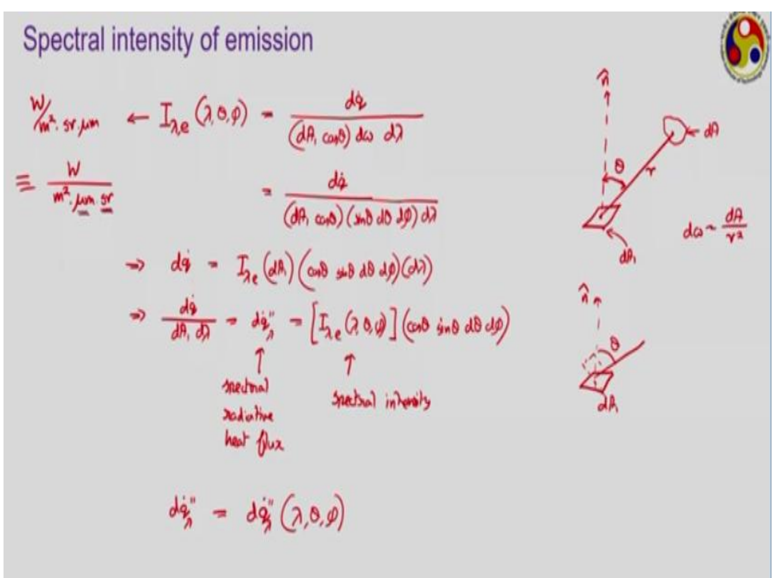

As I mentioned the spectral intensity of emission can also be called as spectral directional intensity of emission, for which we use the symbol I, which denotes the intensity and we are not using the term directional explicitly, because whenever you are talking about intensity, we talk about a direction only. Intensity can happen only in a particular direction. So we use only the subscript λ to denote it is spectral.

Because by default it is directional and also with a comma use a symbol e, to denote that we are talking about the intensity associated with the emission and not with irradiation or something else. It is a function of λ and θ and φ; for λ corresponds to the spectral dependence, θ, φ corresponds to the directional dependence. Now before I write the right hand side, what is the SI unit for this quantity? What do you feel?

It is spectral as well as directional, so what will be the unit? Okay, keep on thinking, let me write the right hand side, from there I think it will be much easier for you to identify. Here we are talking about an area, something like this, an infinitesimally small area say dA_1, which is having its normal in this direction $\hat{n}$ and we are talking about the intensity of emission in a certain direction, say θ, corresponding polar direction is θ.

Azimuthal direction also you can easily identify. So in a particular θ, φ direction, if we say there is another infinitesimally small area, say dA, each may you also, maybe it may be just your eye, where we are standing. So we are trying to identify the intensity in this particular direction. The distance is not coming into picture; rather the distance is already taken care of. The relation between this area dA and the distance, this r can be taken care of through the solid angle.

Because we know that the solid angle associated with this particular θ φ direction can be written as dA/r^2 and the relation of this one with the θ and φ that also we have derived in the previous lecture. So we have to identify the spectral intensities in this particular direction. Then, how can we associate, how can we write that? So it will be defined as the amount of radiant energy, let us say dq is the amount of radiant energy that has been received at that particular point at infinitesimally small area dA; this is the amount of radiant energy that is emitted in that particular direction θ, φ and of a particular wavelength λ. As we are talking about both spectral and directional dependence, so we are talking about the amount of radiation energy dq that is going only in a particular θ, φ direction, so that it is able to reach only that small area dA.

And also only a particular wavelength, not all possible wavelengths. So we are talking about a particular wavelength only. And only for a certain value of λ and a certain combination of θ, φ we are having this dq. Then this is the amount of energy. Now, that is going through that particular direction with a particular wavelength and the corresponding energy dq needs to be divided by the area, which is having the normal to this particular direction.

Now the area is dA_1, which is the source of this emission. But the θ, φ direction that we are talking about, particularly the θ direction in this context, that direction, we have to talk about the area normal to that particular direction. So if this is your dA and this is the normal, this is your dA_1, this is the normal as already drawn here, and this is the θ direction, then we have to talk about a projection of this area which is having its normal in this particular direction.

What will be that projection? That projection obviously will be equal to $dA_1 \cos\theta$; where, θ is the polar angle. So we are not taking the actual area dA_1, rather we are taking a projection of that area which is having normal to this particular direction. So it is per unit area normal to that direction, per unit solid angle around that θ, φ direction; so per unit solid angle is dω and per unit wavelength interval around that λ.

$$I_{\lambda,e}(\lambda,\theta,\phi) = \frac{d\dot{q}}{(dA_1 \cos\theta) d\omega d\lambda}$$

So though we are talking about a particular wavelength, but it is very difficult to talk about a precise wavelength, so we are actually selecting extremely small band of wavelengths. λ can be thought about the mean of this band dλ and again this dω is an infinitesimally small solid angle around the θ, φ direction. So we separated that out, but before that we can express that solid angle in terms of its component.

$$I_{\lambda,e}(\lambda,\theta,\phi) = \frac{d\dot{q}}{(dA_1 \cos\theta)(\sin\theta \, d\theta d\phi) d\lambda}$$

Before I write this term again, now just seeing the terms on the right hand side of this expression, can you tell me what is going to be the unit of this spectral intensity? We have $d\dot{q}$ in the numerator, because that is rate of energy transmitted that is power we are talking about. So $d\dot{q}$ is giving you W? We have an area in the denominator, so W/m^2. And then, we have solid angle plus we have wavelength interval. So we have steradian and we have micrometer as well or if we write properly, then its unit is going to be W/m^2.μm.Sr. Because here we are having several parameters coming in the denominator and as we are having this micron and steradian in the denominator, which indicates it has both spectral and directional dependence.

So dq dot that is the power or rate of radiative energy transmission, then can be written

$$d\dot{q} = I_{\lambda,e}\,(dA_1)(\cos\theta\sin\theta\, d\theta d\phi)\, d\lambda$$

So we generally divide this $d\dot{q}$ by both the source area and this wavelength dλ that gives us $d\dot{q}"_\lambda$, which is called the spectral radiative heat flux

$$d\dot{q}"_\lambda = \frac{d\dot{q}}{dA_1.\,d\lambda} = \left[I_{\lambda,e}(\lambda,\theta,\phi)\right](\cos\theta\sin\theta\, d\theta d\phi)$$

So first term in the right hand side is the spectral intensity of emission and the left hand side is the spectral radiative heat flux. So the spectral radiative heat flux talks about the power per unit area of the source surface, which is able to reach in the θ, φ direction with a particular wavelength λ.

So now we have to use this definition of this spectral radiative heat flux and the spectral intensity of emission to define the other radiative heat fluxes. Now before that, this $d\dot{q}"_\lambda$ that we have identified, this one is a function of what. This is definitely a function of λ and also a function of θ and φ.

$$d\dot{q}"_\lambda = d\dot{q}"_\lambda(\lambda,\theta,\phi)$$

So it is actually both spectral and directional. So it can also be called as spectral, directional radiative heat flux. While with intensity, we generally do not use the term directional, here you could have used the term directional also, we can easily call this spectral directional radiative heat flux as well, but we are not doing that because the definition will be too long, already there are four words in the name. Now you have to identify the relation of spectral intensity of emission and the emissive powers.

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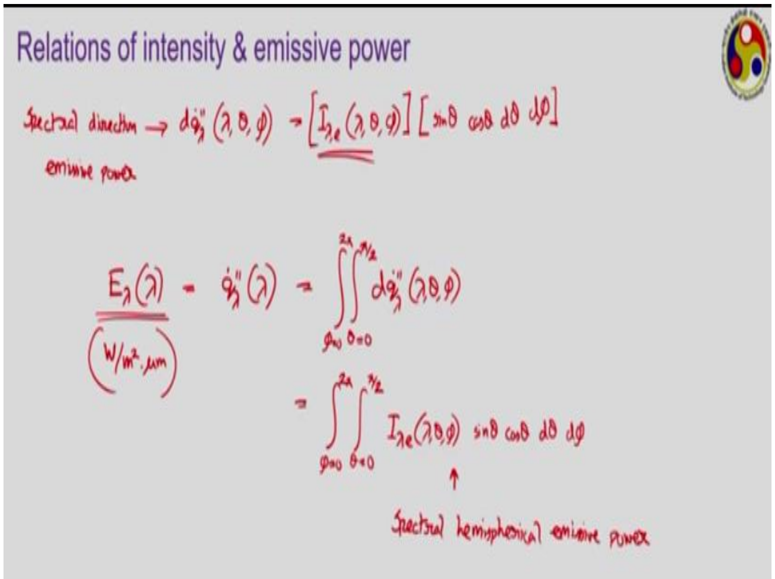

So we have just now identified that

$$d\dot{q}"_{\lambda}(\lambda,\theta,\phi)=\left[I_{\lambda,e}(\lambda,\theta,\phi)\right](\cos\theta\sin\theta\ d\theta d\phi)$$

So if we want to eliminate the spectral dependence, then what we have to do? We have to integrate this over the entire band of wavelengths, whereas if we want to eliminate the directional dependence, then what we have to do?

We have to eliminate the entire band of θ and φ. Accordingly, we can get two different kinds of emissive powers. But before that what was emissive power, how we defined emissive power? Emissive power was defined as the rate at which radiation energy is emitted by a surface per unit area. Now we are already into a heat flux that is per unit area of the source surface, but this is both spectral and directional dependence.

So this one can also be talked about as the spectral directional emissive power, though we do not use that name, but this one can easily be called spectral directional emissive power. Because it is talking about the amount of energy or rate at which energy is being emitted by a surface in a certain direction and for a certain wavelength. So this is nothing but emissive power with both spectral and directional dependency.

So the first thing that we do is to remove the directional dependency. If we remove the directional dependency, then what we get, that is defined as E_λ, which is going to be a function of λ only that is nothing but

$$E_\lambda(\lambda) = \dot{q}"_\lambda(\lambda) = \iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} d\dot{q}"_\lambda(\lambda,\theta,\phi)$$

What will be the limits of integration? Here we are integrating this over the entire range of θ and φ, over the entire range of direction over a hemisphere, because remember as we have discussed we are talking about radiation from a surface and therefore it is possible over all directions, all the rays of a hemisphere. So to get a hemisphere our limits of integration will be polar angle varying from 0 to $\frac{\pi}{2}$, and the azimuthal angle varying from 0 to 2π . So if we write in explicit way,

$$= \iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,e} \cos\theta \sin\theta \, d\theta d\phi$$

So this quantity is going to give you the emissive power for a certain wavelength over all possible directions of a hemisphere. Then what name we should give this?

This is emissive power, but what kind of emissive power, it is spectral because spectral dependence is still there, but there is no directional dependence, so we call it spectral hemispherical emissive power. We call it spectral hemispherical emissive power. The term spectral indicates that it still contains the spectral dependency or dependence on wavelength, so this definition that we have got that corresponds to one particular wavelength only.

But the term hemispherical indicates that it does not have any directional dependency or we have already integrated this one over all possible directions of a hemisphere and accordingly we are getting a definition of spectral hemispherical emissive power. What will be the dimensions of this quantity in SI unit? What do you feel? The dimension of intensity, if you probably remember that was W/m^2.μm.Sr. Now here we have integrated over the directions.

So the directional dependency is not there. It will be $W/m^2.\mu m$. Now you may question that why you are not coupling m^2 and micrometer, you can easily do that. But generally you do not do this and we keep the micrometer separately, so that we can get the idea that there is spectral dependence for this. So here this flux E_λ that you have identified is based upon the actual area, because already we have defined this one by this term dA_1, and so it is based upon the actual area whereas this I_λ that we have defined earlier, this one, that was based upon the projected area, because here this projected area was used to get the definition for this. So the intensity was defined in terms of the projected area, but this spectral radiative heat flux are the spectral directional emissive power and now the spectral hemispherical emissive power, both of them are defined in terms of the actual area for this. Now we have to remove the spectral dependence. To remove the spectral dependence then we have to integrate this E_λ that we have got over this $d\lambda$ or over this entire range.

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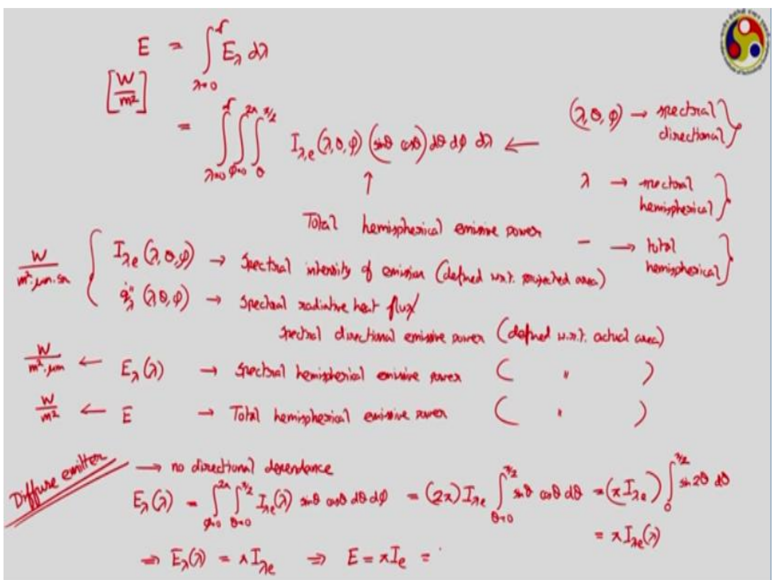

And theoretically λ can vary from 0 to ∞. So if we put this here, then

$$E = \int_{\lambda=0}^{\infty} E_\lambda d\lambda$$

$$= \iiint_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta d\phi d\lambda$$

Then what we should call this one?

This one does not have any spectral dependency, does not have any directional dependency also. It is still a hemispherical definition, it is still a hemispherical emissive power, but there is no spectral dependency, so we call it a total hemispherical emissive power. Hence, total hemispherical emissive power refers to the net value of radiation energy that can be emitted from a surface covering all possible wavelengths and all possible directions.

So just to repeat that, when we are talking about all variations λ, θ and φ, then use both the terms spectral and directional, like the spectral directional emissive power or which is also called as spectral radiative heat flux that we have defined. Now when it is only λ variation, that is it has already been integrated over the θ and φ, over a hemisphere, then it remains a spectral and hemispherical.

And when none of the variations are there, that is it is integrated over the wavelength as well as on direction, we call it total and hemispherical. So spectral directional refers to the dependency on both wavelength and direction, spectral hemispherical represents that dependency only on wavelength, but not on direction and total hemispherical indicates that it is independent of both direction as well as wavelength. Just repeat the definitions then.

The spectral direction or directional emissive power also called as spectral radiative heat flux is defined like this where we have the dependence on both λ and θ, φ, then we have the spectral hemispherical emissive power where we integrate over a hemisphere to remove the directional dependence, then we have this total hemispherical emissive power where we are integrating over the wavelength as well to remove any kind of spectral dependence as well.

So there are three definitions of emissive power that we have got, the spectral directional emissive power, spectral hemispherical emissive power and total hemispherical emissive power. The spectral directional emissive power is also defined as the spectral radiative heat flux. And we started by defining the spectral intensity of emission, which is again a spectral and directional quantity because that talks about the emission in a particular direction for a particular wavelength.

But what is the difference between then this quantity and this quantity? This particular quantity is defined as spectral directional heat flux only, but this is defined with respect to the actual area of the surface, whereas this is defined with respect to the projected area and accordingly we get this particular relation. And what will be the unit for this total hemispherical emissive power?

As there is no dependency on direction or wavelength, so it is nothing but just heat flux, W/m^2. So I am repeating the terms again and again because these definitions are a bit complicated or a bit confusing, so be very careful about what we are talking about. So the first term that we have defined, that is $I_{\lambda,e}(\lambda,\theta,\phi)$ which we have defined as spectral intensity of emission and here we are talking about a particular wavelength in a particular direction.

It is defined with respect to the projected area, whereas $\dot{q}"_{\lambda}(\lambda,\theta,\phi)$ that is called spectral radiative heat flux, but we can also call it spectral directional emissive power. It is defined with respect to actual area, area of the source that is I am talking about. Now both of them are spectral directional quantities, so their unit is, SI unit $W/m^2.\mu m.Sr$. Then we have defined $E_{\lambda}(\lambda)$.

This is the function of λ only, that is defined as spectral hemispherical emissive power that is also defined with respect to the actual area and its unit is, here the directional dependence goes off, so the steradian is not there. It is $W/m^2.\mu m$, and finally we have defined E, where the directional dependence has also been taken care of, so it is total hemispherical emissive power, again defined on the overall area.

This is just heat flux, no directional dependence, so this is just W/m^2. So this way we can define a spectral intensity based upon the projected area and based upon that we have got 3 kinds of heat fluxes, though the last 2 that is spectral hemispherical emissive power and the total hemispherical emissive powers are the ones that are commonly used, but the spectral radiative heat flux or the spectral directional emissive power also can be used.

Because that has the direct relation with the intensity and this intensity is the one that is easier to identify or that is easy to measure. So generally we have to start with the intensity and then gradually move to the spectral radiative heat flux, then the spectral hemispherical emissive power and finally the total hemispherical emissive power. And before I move on to the next topic, here another term that I am going to introduce, that is called diffuse emitter.

In relation with radiation, whenever you are introducing this term diffuse, diffuse may represent that there is no directional dependence. The term diffuse by default always represents no directional dependence. There may be spectral variation, but there is no variation with respect to direction. So when you are talking about a diffuse emitter, that means that the concerned surface is emitting with the same intensity in all possible direction for a given wavelength.

Of course, its intensity of emission can vary with the wavelength, but that is not going to vary with direction. So if we pick up any particular wavelength say, if we are talking about a wavelength of 1 micron, then in every possible direction the intensity of emission of 1 micron wavelength will be the same. It does not have any directional dependence. And if it does not have any directional dependence, then it becomes quite easy to calculate the corresponding heat fluxes or other corresponding emissive powers. Let us say if we want to calculate this spectral hemispherical emissive power, E_λ. So as per the definition,

$$E_\lambda(\lambda) = \iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,e}(\lambda)\cos\theta\sin\theta\, d\theta d\phi$$

So this intensity is a function of only λ. It is only spectral dependence. Then $I_{\lambda,e}$ can come out of this integration. Similarly, you can take this dφ out also. So if we take dφ out, that after integration becomes

$$= (2\pi)I_{\lambda,e}\int_{\theta=0}^{\theta=\frac{\pi}{2}}\cos\theta\sin\theta\, d\theta$$

Now what will be the result of this integration, can you tell me? How can you perform this? It is very easy to do this. You can easily make it sin2θ, because twice of sinθ cosθ is equal to sin2θ. So we can write this

$$= (\pi)I_{\lambda,e}\int_{\theta=0}^{\theta=\frac{\pi}{2}}\sin 2\theta\, d\theta = \pi I_{\lambda,e}(\lambda)$$

$I_{\lambda,e}$ is a function of λ alone. Therefore, for a diffuse emitter, we can easily calculate the spectral hemispherical emissive power just by multiplying the intensity with π. And from there straight away, we can also write the total hemispherical emissive power it is going to be

$$E = \pi I_e = \pi \int_0^\infty I_{\lambda,e}(\lambda)$$

So if we are talking about a diffuse emitter, then calculation becomes quite simple, we do not have to talk about the directional dependence, only the wavelength dependence is the one that we have to be bothered about.

Similar to diffuse emitter a surface can have diffuse behavior with respect to irradiation as well, as we shall be seeing later on.

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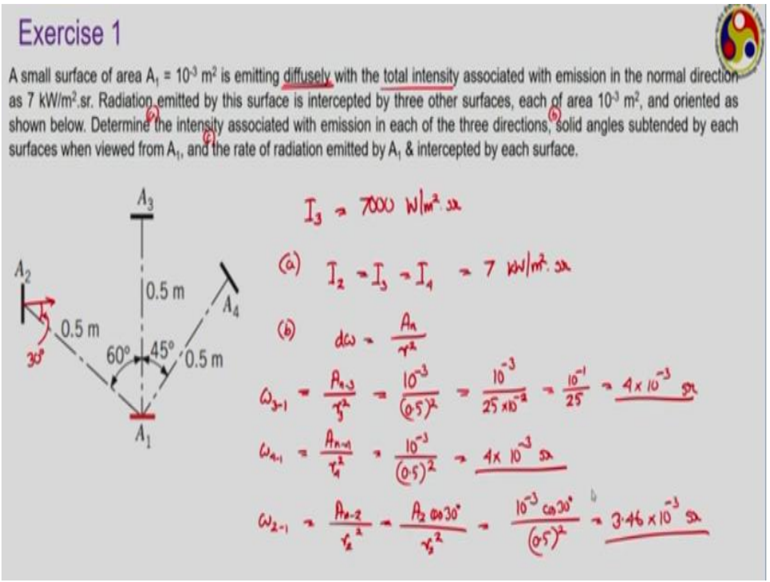

But before that, let us solve one exercise. Here we are talking about a small surface of area A_1 of 10^{-3} m^2; this is your area A_1. This is the source of radiation. It is emitting diffusely. It is important that it is emitting diffusely means, its emission intensity does not have or the spectral intensity of emission does not have any direction dependence. Now here the total intensity is given, that is it has been already integrated over all possible wavelengths.

So it is a total intensity that is mentioned. Associated with emission in the normal direction is 7 kW/m^2.Sr. So there are three directions shown here. So in this direction we call the intensity to be I_2. In this direction, we call it to be I_3. In this direction, we call it to be I_4. Then the intensity of emission in normal direction, which clearly is I_3 is equal to 7000 W/m^2.Sr.

Radiation emitted by the surface intersected by three other surfaces like shown here, each of them are having the same area 10^{-3} m^2 and oriented as shown. Clearly A_3 is just parallel to this A_1; A_4 is oriented at an angle 45 º or the normal to the surface A_4 is making an angle of 45 º with the normal to the surface A_1.

And the surface A_2 is oriented in an angle 60 º, but normal to the surface A_2, is actually normal to the normal of A_1 and therefore this direction of emission the normal to A_2 and the direction of emission is making an angle of 30 º. So we have to identify the intensity, part A, intensity associated with emission in each of the three direction, part B the solid angles subtended by each surfaces when viewed from A_1.

And part C, the rate of radiation emitted by A_1 and intersected by each surface. So for part A, we have to calculate the intensity associated with emission in each of the three directions. Now how can we get this? It is given that the surface is a diffuse emitter. Diffuse emitter means, it does not have direction dependency, so in all direction it is emitting with equal intensity and here the total intensity is given, we are not talking about any kind of spectral dependence as well.

So no spectral dependence we also have to consider and accordingly we can straight away write

$$I_2 = I_3 = I_4 = 7\frac{kW}{m^2.sr}$$

Here actually the total intensity term is given, which maybe a bit confusing, but the steradian is also there. Here we are talking about entire thing is happening for just one wavelength. So assume that we are talking about a single wavelength and accordingly we are doing all calculations.

So whatever values are given, they are all spectral values corresponds to a particular wavelength. So you got the first part straightaway. Now part B, solid angle subtended by each surfaces when viewed from A_1. We have to see the solid angles. So we know that the solid angle can be viewed as

$$d\omega = \frac{A_n}{r^2}$$

Where, A_n refers to the projected area normal to the direction of emission. Let's see the solid angle subtended by surface 3 to 1. Here the distance is given to be of 0.5 m. Now the surface A_3 is having its normal in the direction of emission only. So you do not have to calculate normal of this. Directly we can put the area is given as 10 to the power minus 3 meter square.

$$\omega_{3-1} = \frac{A_{n,3}}{r_3^2} = \frac{10^{-3}}{(0.5)^2} = 4 \times 10^{-3}\ sr$$

So that is done. Now here again the surface 4 is also oriented in such that its normal is in the direction of emission only. So here also we can just straightaway write

$$\omega_{4-1} = \frac{A_{n,4}}{r_4^2} = \frac{10^{-3}}{(0.5)^2} = 4 \times 10^{-3}\ sr$$

But for surface 2 the normal to the surface is not oriented to the direction of emission, rather there is an angle of 30 º, so what it will be? It will be equal to

$$\omega_{2-1} = \frac{A_{n,2}}{r_2^2} = \frac{A_2 \cos 30}{r_2^2} = \frac{10^{-3} \cos 30}{(0.5)^2} = 3.46 \times 10^{-3}\ sr$$

So you can clearly say based upon the orientation, how the result changes. While the surface A_4 is having normal in the same direction as the direction of emission A_2 is not. Accordingly, it can keep on varying. If you are interested you can also try to vary the angle made by A_2 like here it is making an angle of 30 º, you can make the angle to be 60 º or - 30 º and see how the definition of this $\omega_{2,\ 1}$ varies. Now I have to calculate the third part that is I have to calculate the rate of radiation emitted by A_1 intersected by each of the surfaces.

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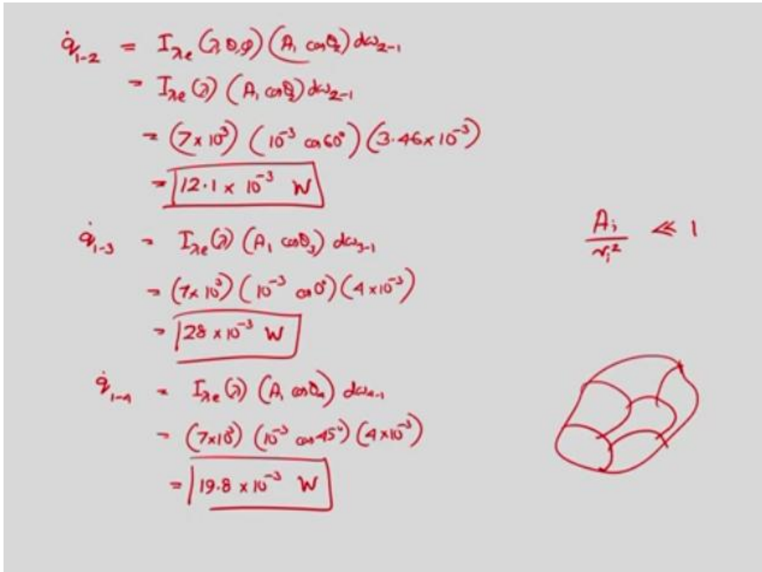

So we have to calculate this $\dot{q}_{1-2}$ or rate of emission. So how can we calculate this? We know that this is something like spectral directional emissive power. So what is the definition of this? As per the definition, it is

$$\dot{q}_{1-2} = I_{\lambda,e}(\lambda, \theta, \phi)(A_1 \cos\theta_2)d\omega_{2-1}$$

And we are talking about a particular wavelength range also. It is already spectral range, so you do not have to bother about this. Now in this case, surface is a diffuse emitter, so the θ, φ variation can be neglected. It just becomes

$$= I_{\lambda,e}(\lambda)(A_1 \cos\theta_2)d\omega_{2-1}$$
$$= (7 \times 10^3)(10^{-3} \cos 60\,)(3.46 \times 10^{-3})$$
$$= 12.1 \times 10^{-3} W$$

Area was $10^{-3} \cos 60$ because here this θ_2 that I am talking about that is the projected area of A_1, like when you are talking about emission on 2, then we have to take the area of A_1 or the projection of A_1 that is in the direction of normal to this. So it is making an angle of 60 º.

Similarly,

$$\dot{q}_{1-2} = I_{\lambda,e}(\lambda)(A_1 \cos\theta_3)d\omega_{3-1}$$
$$= (7 \times 10^3)(10^{-3} \cos 0\,)(4 \times 10^{-3})$$
$$= 28 \times 10^{-3} W$$

Here it is making an angle of 0 º. So $\theta_3 = 0$. And

$$\dot{q}_{1-4} = I_{\lambda,e}(\lambda)(A_1 \cos\theta_4)d\omega_{4-1}$$

This $A_1\cos\theta_4$ gives you the area of A_1 projected in the direction of emission towards 4.

$$= (7 \times 10^3)(10^{-3} \cos 45\,)(4 \times 10^{-3})$$

$$= 19.8 \times 10^{-3} W$$

So these are the corresponding energy emission or amount of energy intersected by each of these 3 surfaces 2, 3, and 4, which has been emitted by surface 1.

Here of course one important assumption we have considered. Here we have considered for each of the surfaces, the ratio of the area of surface A_i to r^2_i to be extremely small. That is compared to the distance squared, the area of the surface is extremely small. If that is small, then only you can treat them as infinitesimally small surface, otherwise the solid angle will keep on varying.

If your surface is a finite one, then the solid angle will keep on varying and then surface like suppose if this is your A_2, it is a large surface, then we may have to divide the surface into several small compartments and calculate the solid angle corresponding to each of them and then analyze each of them and add them together to get the final power diffused by this. So this way we can easily calculate.

It is a very simple example, but using the definition of emissive power and spectral intensity, we can easily calculate the energy exchange or energy received by different surfaces. And you can try to solve the same problem by varying this angle, varying the orientations as well and see how the result varies. Now here we have so far talked only about the emission, but along with emission all the surfaces are also subjected to irradiation.

Like in this example, we have calculated the amount of energy that is emitted by A_1 and received by A_2, A_3 and A_4, but while A_1 is emitting energy towards these 3 surfaces and also towards all other space, it is also receiving energy from these surfaces via irradiation; and not only from these surfaces, whatever it has on its neighbourhood, it is receiving energy from everything and that is why, the next we have to talk about is the incidence intensity and its relation with irradiation.

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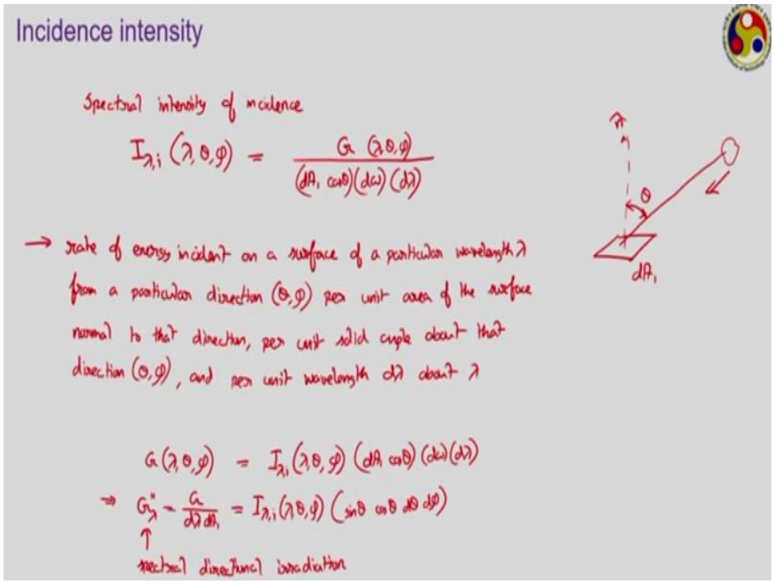

Incident intensity or intensity of incidence; like we have talked about the intensity of emission, similarly here also we are going to talk about the spectral intensity of incidence. We can also call it spectral directional, but again the term intensity by default indicates directional nature, so it will be indicated by $I_{\lambda i}$, i for incidence. There we used e for emission, here i for incidence and again it is a function of wavelength and the direction.

So it will be defined as the total amount of energy, say if G_{λ} is the amount of irradiation that has been received by this surface, then it will be defined as just try to remember the way we have defined the intensity of emission. Similarly, here also if we are talking about a surface A_1 and now this surface dA_1 that is the recipient and is receiving from some surface here. So this is a normal direction $\hat{n}$, this is coming from a direction θ here. And now energy is coming from this direction, so again we have to think about a particular wavelength λ and wavelength band around that dλ, a particular θ, φ direction and a solid angle around that direction θ, φ; and also the projection of this angle in that particular direction, projection of this area rather in that direction. So it will be again dA_1 cos θ; per unit solid angle d ω; per unit wavelength interval dλ. And this is a function of λ, θ and φ.

$$I_{\lambda,i}\,(\lambda,\theta,\phi) = \frac{dG(\lambda,\theta,\phi)}{(dA_1\cos\theta)d\omega d\lambda}$$

So spectral intensity of incidence $I_{\lambda i}$ can be defined as the rate at which energy of a particular wavelength λ coming from a particular direction θ, φ is incident on the surface per unit area of the surface having its normal in the direction of incidence, per unit solid angle around that direction and per unit wavelength interval around that λ.

I am not writing the full definition, but still trying to write something slightly briefly. So it is the rate of energy incident on a surface of a particular wavelength λ from a particular direction θ, φ; per unit area of the surface normal to that direction per unit solid angle about that direction θ, φ and per unit wavelength dλ about λ. So, the way we have defined the spectral intensity of emission, the same we are defining spectral intensity of incidence.

So from there we can define this G as

$$dG(\lambda, \theta, \phi) = I_{\lambda,i}\,(\lambda, \theta, \phi)(dA_1 \cos\theta)d\omega d\lambda$$

Let us write this as $G"_{\lambda}$.

$$dG"_{\lambda} = \frac{G}{d\lambda dA_1} = I_{\lambda,i}\,(\lambda, \theta, \phi)(\cos\theta \sin\theta \, d\theta d\phi)$$

So what we should call this quantity then? Like previous case when we got that $\dot{q}"_{\lambda}$, we call that the spectral radiative heat flux or spectral directional emissive power, similarly we can call this one as spectral directional irradiation. Because this is the irradiation energy that has been received by the surface coming from a particular direction and of a particular wavelength, so we are calling in the spectral directional irradiation.

So if we want to define the irradiation now, the spectral directional irradiation, which we have defined. Let me repeat that again.

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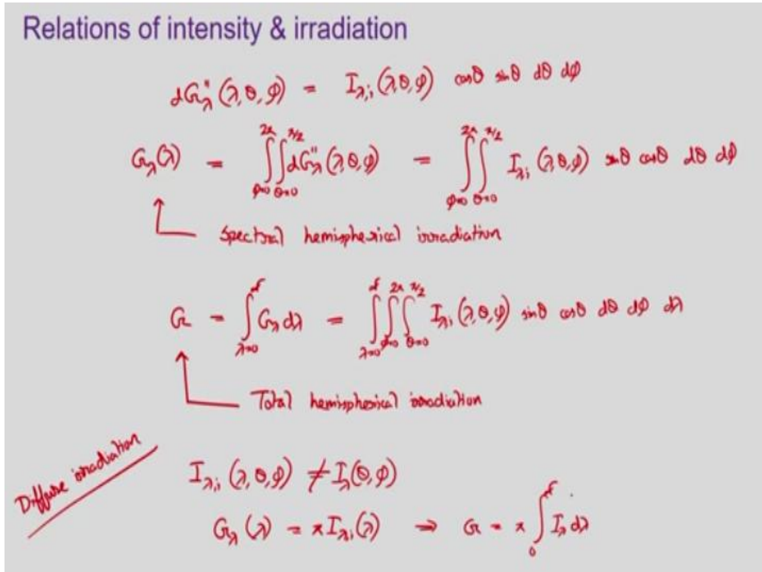

The spectral directional irradiation, is a function of λ, θ and φ, we have just now defined as

$$dG"_{\lambda}(\lambda,\theta,\phi) = I_{\lambda,i}\,(\lambda,\theta,\phi)(\cos\theta\sin\theta\,d\theta d\phi)$$

So once we integrate this over all possible directions, that is

$$\iint\limits_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} dG"_{\lambda}(\lambda,\theta,\phi) = \iint\limits_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,i}\,(\lambda,\theta,\phi)(\cos\theta\sin\theta\,d\theta d\phi) = G_{\lambda}(\lambda)$$

So what notation we should use now. θ, φ dependence goes off, so this is G(λ) only. And what name we should give this one now. Previous one was a spectral directional irradiation. So the spectral dependence is still there, so it is still a spectral one, but we have integrated this over all the direction, so it is spectral hemispherical irradiation and now if we integrate this G_{λ} over the entire range of λ, then what we should call this?

$$G = \int_{0}^{\infty} G_{\lambda} d\lambda$$

This is G, which has no spectral dependence, so this is total hemispherical irradiation. So combining this,

$$G = \int_{0}^{\infty} G_{\lambda} d\lambda = \iiint\limits_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta\,d\theta d\phi d\lambda$$

So we have got the definition of spectral hemispherical irradiation and total hemispherical irradiation. There is nothing like a diffuse emitter, a counterpart is quite difficult to get, but in certain cases we call it if surface is subjected to diffuse irradiation. If a subject is subjected to diffuse irradiation that means

$$I_{\lambda,i}(\lambda,\theta,\phi) \neq I_{\lambda,i}(\theta,\phi)$$

It remains a function of only λ. And in that case, just the way we have calculated earlier, it can be shown that the spectral hemispherical irradiation becomes

$$G_\lambda(\lambda) = \pi I_{\lambda,i}\,(\lambda)$$

And

$$G = \pi \int_0^\infty I_{\lambda,i}\, d\lambda$$

This is when a body is subjected to diffuse irradiation. Something like if you have a small particle or small ball located at the center of a big sphere. And the entire of the sphere is illuminated with equal intensity. So only in certain limited cases, we may have a diffuse irradiation kind of situation.

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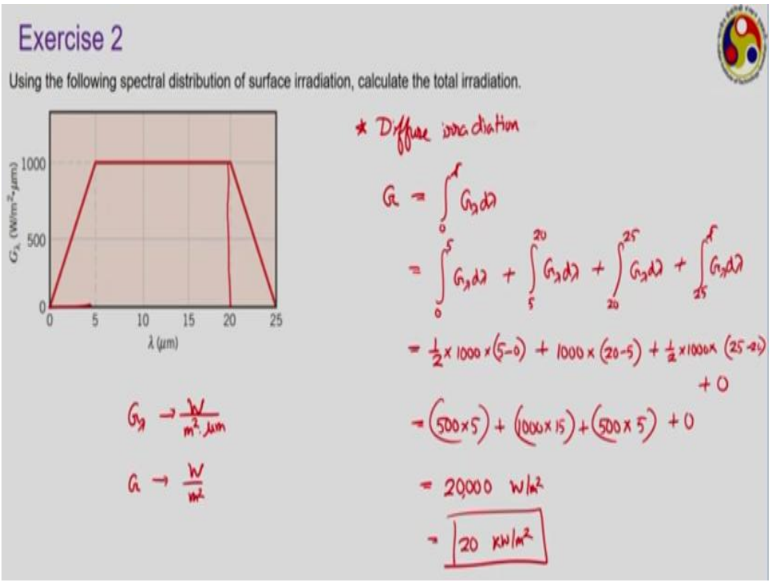

Let us quickly check another numerical example where we are trying to make use of this spectral distribution to get the total irradiation. A spectral distribution of surface irradiation is shown. You have to calculate the total irradiation. So here we are talking about only the spectral dependence. Let us assume it to be subjected to diffuse irradiation. So we are assuming diffuse irradiation. It is not mentioned in the problem, but for the moment we are assuming diffuse irradiation.

And then we have to calculate this total irradiation. So for a diffuse irradiation, we can easily say that

$$G = \int_0^\infty G_\lambda d\lambda$$

Now you can see there are 3 levels. Like in the first level up to 0 to 5, it is varying following a straight line, then 5 to 20 it is almost constant, and then 20 to 25 it is again decreasing linearly to 0 and after 25, which is not shown in the graph, we can assume that to be 0.

Then we can break this integration into 4 parts. That is

$$G = \int_0^5 G_\lambda d\lambda + \int_5^{20} G_\lambda d\lambda + \int_{20}^{25} G_\lambda d\lambda + \int_{25}^\infty G_\lambda d\lambda$$

As it is integration we basically have to calculate the area under each of the curves. So from 0 to 5, we can easily see there is a triangular kind of curve. For the second case we have rectangular nature only. For the third case, we are getting another triangle like shown here. Fourth level it is equal to 0. So, putting the values for all the regions

$$G = \frac{1}{2} \times 1000 \times (5 - 0) + 1000 \times (20 - 5) + \frac{1}{2} \times 1000 \times (25 - 20) + 0 = 20000$$

G_λ that is shown here has a unit of W/m^2.micron, because it is a spectral quantity. Here we have assumed a diffuse irradiation, so the directional dependence is not coming into picture; unit is W/m^2.micron.

Then G, here we have integrated over the wavelength, so its unit will be just W/m^2. So it is 20,000 W/m^2. We can write in a smaller number 20 kW/m^2, which is going to be the total irradiation for this particular case, the surface which is being subjected to diffuse irradiation following such kind of spectral distribution.

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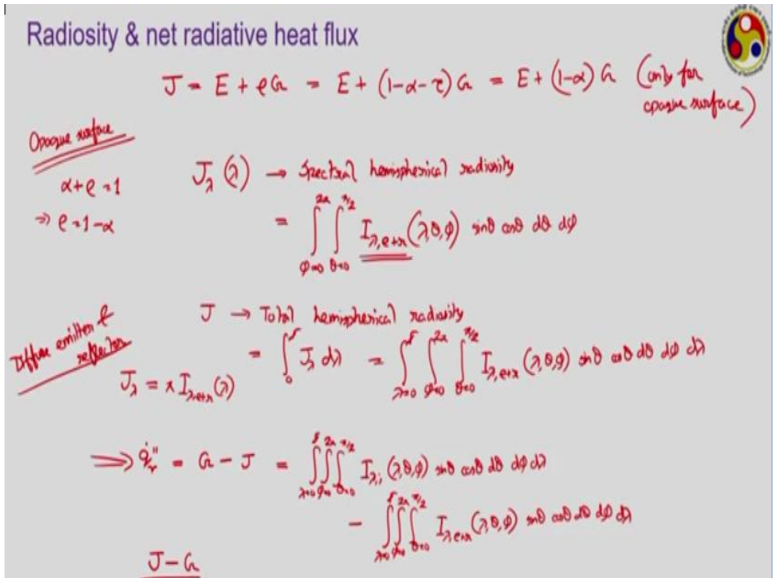

Now finally, I would like to finish up by defining the two other quantities, other type of heat fluxes that we have defined. Of course, emissive power and irradiation are the two fundamental ones and these are just their combination. So what was the definition of radiosity, do you remember? Radiosity was defined as the rate at which radiation energy is leaving a surface and radiation energy can leave a surface in the form of emission and also in the form of reflection.

So it is

$$J = E + \rho G$$

Now using the definition,

$$= E + (1 - \alpha - \tau)G$$

For an opaque surface

$$= E + (1 - \alpha)G$$

Now we are going to write the following expressions only assuming an opaque surface. If you assume an opaque surface that means

$$\alpha + \rho = 1 \Rightarrow \rho = 1 - \alpha$$

So what about the radiosity then? Similarly, here also we can have all the definitions of spectral directional, spectral hemispherical and total hemispherical. If we write, suppose J_λ (λ), then which quantity we are talking about, from this notation can you identify? Spectral dependence is there, but directional dependence is not there. So it is spectral hemispherical radiosity and what will be definition of this? This hemispherical definition will be

$$J_\lambda(\lambda) = \iint_{\theta=0,\phi=0}^{\theta=\frac{\pi}{2},\phi=2\pi} I_{\lambda,e+r}(\lambda,\theta,\phi)(\cos\theta\sin\theta\, d\theta d\phi)$$

For the intensity here we have to consider intensity of emission (e) plus reflection (r). So similarly the total hemispherical radiosity can be defined as

$$J = \int_0^\infty J_\lambda d\lambda = \iiint_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,e+r}(\lambda,\theta,\phi)(\cos\theta\sin\theta\, d\theta d\phi)d\lambda$$

This is how we can define the spectral hemispherical radiosity and total hemispherical radiosity. We could have defined a spectral directional radiosity also, but not required here. In most of the cases that will not be required. And if we are talking about a surface which is both diffuse emitter and diffuse reflector in that case,

$$J_\lambda = \pi I_{\lambda,e+r}(\lambda)$$

Here, there is no directional dependence. And finally the net radiation heat flux that is the difference in the total incoming radiation and outgoing radiation,

$$\dot{q}''_r = G - J$$

So if we want to define in terms of the total hemispherical net radiative heat flux, then what it will be? We shall be having total amount of energy that is being received by the surface. That is

$$\dot{q}''_r = \iiint_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi)(\cos\theta\sin\theta\, d\theta d\phi)d\lambda - \iiint_{\lambda=0,\phi=0,\theta=0}^{\lambda=\infty,\phi=2\pi,\theta=\frac{\pi}{2}} I_{\lambda,e+r}(\lambda,\theta,\phi)(\cos\theta\sin\theta\, d\theta d\phi)d\lambda$$

But the net radiative heat flux that we have defined here, here we have defined as the net radiative heat flux received by the surface.

But in certain books, it is defined the other way also, that is defined as the total outgoing minus total incoming. That definition is also true. That depends upon what way you can define. Here I have defined net radiative heat flux as the amount of net energy received by the surface in the form of radiation, but you can define the other way also. So this is the time where I would like to stop. Today I have defined all the radiative heat fluxes, all the four radiative heat fluxes using the definition of intensities and we have seen that each of them can have spectral directional, spectral hemispherical and total hemispherical definition.

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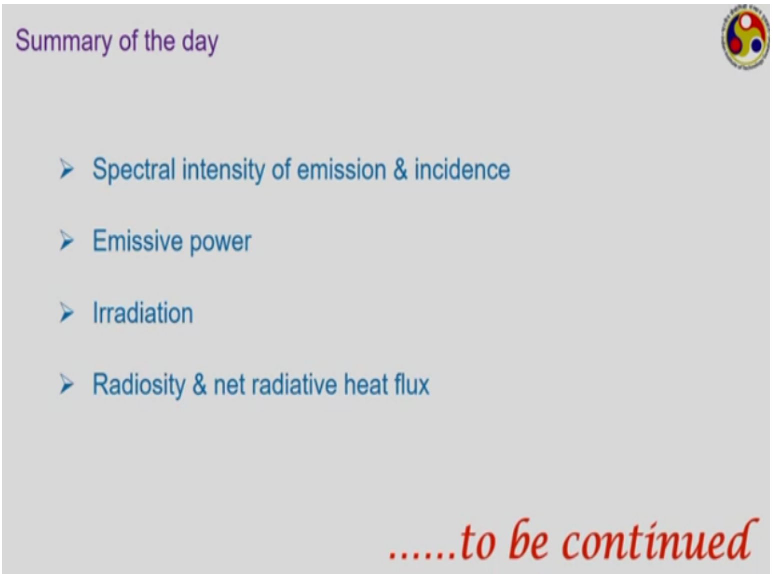

So we have discussed about the spectral intensity of emission and incidence both and from there we have tried to develop the spectral directional fluxes of spectral direction definition of emissive power in irradiation, then different definition of emissive power and irradiation was discussed and we have also briefly discussed about the definition of radiosity and net radiative heat flux.

So please try to review this definition, try to go through them repeatedly, so that you do not have any kind of confusion about the use of notation. Always try to note the subscript and also when you are writing, try to use the subscript properly so that from your subscript itself it will be clear whether you are talking about a spectral directional definition or spectral hemispherical definition or total hemispherical definition.

In the next class, I shall be talking about the black surfaces, which is the most ideal kind of emitter that we can have. Till then, please revise this lecture, go through the books and try to grasp the concept of this one. Thank you very much.