

Fundamentals of Conduction and Radiation
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Lecture - 24
Introduction to Radiative Heat Fluxes

Good morning friends. How are you all? So, I am back in this week number 9. Like you know that first 4 weeks I was your instructor and then the next four weeks Prof. Dalal was taking care of the course. Now, till this moment you have learnt the fundamentals of conduction heat transfer. Like initially I introduced you to the fundamental modes of heat transfer, 3 different modes, then developed the generalized heat diffusion equation.

Then, moved on to the discussion of 1D steady state heat conduction and also discussed about the special cases, particularly in relation with the concept of thermal resistance. Prof. Dalal has extended that to talk about the topic of fins or extended surfaces then moved on to the two-dimensional heat conduction, transient heat conduction and also he has extensively discussed about the numerical treatment of heat diffusion equation.

So, now as you have a gross overview of the fundamentals of conduction, so it's time for us to move to the topic of radiation that can be recognized as second most important mode of heat transfer. Like I have mentioned at the very beginning of the course that primarily from physics point of view conduction and radiation are the two primary modes of heat transfer whereas convection which of course is not within the purview of this course but generally included in any UG level heat transfer course.

Convection is just a combination of conduction plus advection or flow. And hence there are several classical physicists who would like to put convection not as a fundamental mode rather consider only two modes of heat transfer as conduction and radiation. And as we have already covered conduction, so now we are moving to the radiation heat transfer, which is probably the most interesting mode of heat transfer.

Now, the idea of conduction and also convection is the heat transmission in the presence of some medium. Like in case of conduction you know the primary mode of heat transmission is because of the molecular activities either because of the transmission or the random

movement of molecules within the liquids and gases, particularly in gases or it may be the lattice vibration of the molecules fixed in a lattice structure in case of solids.

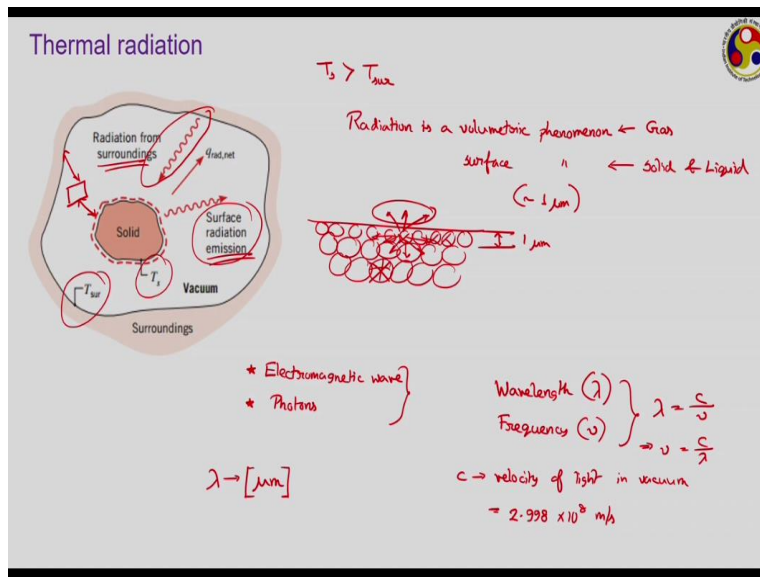
During the vibration it keeps on transmitting the energy from high temperature molecules to the low temperature molecules thereby leading to macroscopic heat transmission. But radiation interestingly can happen at the absence of any kind of medium in between. In fact, the radiative heat transfer between two surfaces can happen at the presence of some medium also. Like when the space interim to the two surfaces are occupied by some medium, the radiation heat transfer is still possible but it's the best when the medium is vacuum.

The medium which is present there may participate in this, that may not participate also. So, when the medium is not participating at all, then it's quite synonymous to the vacuum that is the two surfaces can see just each other and nothing in between. However, if the medium also participates in the radiation heat transfer like happens with certain gases present in a normal atmosphere like water vapour or carbon dioxide which can absorb a bit of radiation heat transfer of certain wavelengths.

Then, we need to have some idea about the nature or the radiative characteristic of the interim medium and that will be covered in the last week of this course that is the 12th module where we shall be talking about the radiation in presence of participating medium. But for the moment it's sufficient for us to assume that radiation can happen at the absence of any medium and that is what we are going to assume to start with.

That is for these 3 modules, module 9, 10 and 11 we are going to consider the radiation happening at the absence of any kind of medium. The role of medium will be discussed only in week number 12.

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Now, to discuss about radiation, say here a solid is shown here in this pink color, which is maintained at temperature T_s , or the surface of the solid is maintained at some temperature T_s and it is kept inside some enclosure and the temperature of the enclosure is this T_{sur} that is the surrounding temperature. Now, there are different ways heat transmission is possible from the solid to the surface or surrounding or from surrounding to the solid depending upon their temperature level.

Let us assume this T_s the surface temperature of the solid is greater than the surrounding temperature. Then, that means heat has to get transferred from the solid to the surrounding. If there is any medium say air is present in this chamber then heat transmission is possible by conduction and convection both. Via conduction heat will get transmitted through the air molecules. Similarly, if there is any bulk movement of the air present inside this enclosure, then that will also lead to convective heat transfer.

But if this medium is a vacuum, then conduction and convection neither of them are possible, but still you will find there is heat transmission happening from the solid to the surrounding and if the energy content of the solid is finite, then because of this heat transmission the temperature of the solid will reduce. The temperature of the surrounding may increase finally, and this heat transmission will keep on continuing till both of them reach certain kind of equilibrium. If the thermal inertia of the surrounding is too high, then the surrounding temperature may remain almost equal to T_{sur} but the solid temperature will fall back to its own level. Therefore in the presence of vacuum also, we can have radiation heat transfer.

Because conduction and convection is not possible whatever is the mode of heat transmission here that is purely radiation, which is the surface radiation emission coming from the solid surface shown by this dotted red line and going to the surrounding. But radiation interestingly is the mode of heat transmission which is possible from any surface maintained at a temperature higher than absolute zero.

That means a body which is maintained at a temperature higher than absolute zero will always lead to some radiation depending upon its own temperature level and a few other factors as well. As the temperature of both the solid and surrounding are higher than absolute zero or rather if we assume both of them to be higher than absolute zero, then heat transmission is possible or radiation emission is possible not only from the solid surface but also from the surrounding surface and the emission that is coming out of the surrounding surface, it's also possible that a part of that will fall back on the solid. Therefore, the amount of energy leaving the solid is actually a balance between the emission that is leaving the solid minus the emission which is received by the solid coming from the surrounding.

And if there is some other source like say if there is a third body present inside, then there will be a three way heat transfer, depending on the relative temperature level this body will have radiative heat exchange with the solid and also radiative heat exchange with the surrounding.

So, whenever you are talking about radiation heat exchange from a body or from a surface, then we have to consider both, the amount of energy leaving the surface because of emission and also the amount of energy received by the surface because of the emission from whatever it has on its own surrounding. So, whenever you are talking about radiation heat transfer or thermal energy balance for any surface by virtue of radiation we have to consider both this emission and radiation energy received by the surface.

Now, we are repeatedly talking about surfaces. Then, does radiation happens only from the surfaces? That is not true; by nature, radiation is definitely a volumetric phenomenon. When you are talking about gases, then each molecule will have their own radiation characteristics, each molecule will emit radiation.

And each molecule also will receive emission from the neighboring molecules and the other components of the surrounding. So, that is true somewhat about certain liquids also. But generally when you are talking about solids or most of the liquids, radiation goes down to a surface phenomenon. Why? This volumetric phenomenon is primarily applicable for gases. But let us talk about this is a solid surface and these are the molecules that we have.

Immediately below the surface, then we have another level of molecules just below this, then a third level of molecules something like this. So, these are several layers of molecules that are present below the surface of the solid. Now, the amount of energy emitted by say this particular molecule inside the solid, as it's packed from all around by other molecules it's very likely that whatever energy it's emitting by virtue of radiation will get absorbed by all these molecules which are around this and it will not be able to reach the surface.

The same is true for all these molecules which are far away from the surface. Only the molecules which are extremely close to the surface like this layer of molecules, it's likely that whatever energy they are emitting, a part of that of course will get absorbed by the neighboring molecules, like the energy which is going towards the other molecules that will get absorbed.

But as on one side it's exposed to a surrounding, so some amount of energy will leave the surface. And that is what if you are sitting somewhere here; you will be able to identify that fraction of the energy emitted by the molecules, which are very close to the surface. But energy actually is getting emitted by all molecules present inside the solid body and also whatever energy that is coming out that is not the total energy emitted by this molecule rather it's only a part of that.

And therefore for solids and also you can say about liquids, radiation comes down to be a surface phenomenon. And this is true for solids and liquids. That is because of their packed molecular structure. But, for gases the molecular structure is very irregular. Molecules are generally independent of each other and accordingly for gases it remains to be a volumetric phenomenon.

But for solids and liquids though radiation is happening in a volumetric way, we can easily treat that as a surface phenomenon and accordingly we shall be only talking about the

radiation leaving a surface and received by the surface. And also the radiative properties of the surface associated both with this emission and incidence.

But of course there are exceptions. Like the cases of a solid whose dimension is comparable with the diameter of the molecules like a nano crystalline structure or if we are talking about some extremely thin surface, extremely thin body, I should say. In those cases, the radiation from all the molecules may be able to reach the surface and come out of the surface. Hence in those cases, the radiation needs to be considered as a volumetric phenomenon or can be treated as a volumetric phenomenon.

But for most of the solids and liquids that we deal with, for them it's quite safe to assume radiation to be a surface phenomenon only. So, radiation is possible from all the surfaces of a body. Like if we take say this as the example, there are millions of molecules present inside this small cylindrical object but what will happen is that only the molecules which are present on the surface or on the circumferential surface that radiation will be able to come out.

And the role of the molecules present inside the body away from the surface that we don't need to consider while treating with radiation heat transfer. A common thumb rule says that only the molecules present within 1 micron from the surface only their contribution needs to be considered while talking about gross radiation from outside and therefore radiation we treat as a surface phenomenon.

Now, what is the nature of radiation? Radiation of course is energy emitted by the surface depending on its own temperature level. And that emission can happen based on two theories. One theory says that radiation is happening in the form of electromagnetic waves, other theory says that radiation is happening in the mode of transmission of photons. Now, both theories have their own advantage and disadvantage.

And depending upon exactly what kind of approach you would like to follow that is up to you only. But for our case, we are not going into the detailed of this. Any of them can be considered. We shall mostly be talking radiation as a form of electromagnetic wave. Now, whenever we are talking about electromagnetic wave, any wave can be characterized in terms of two parameters.

One is the wavelength which commonly is represented by λ , other is the frequency commonly characterized by ν . And what is their relation? Their relation you know that

$$\lambda = \frac{c}{\nu}$$

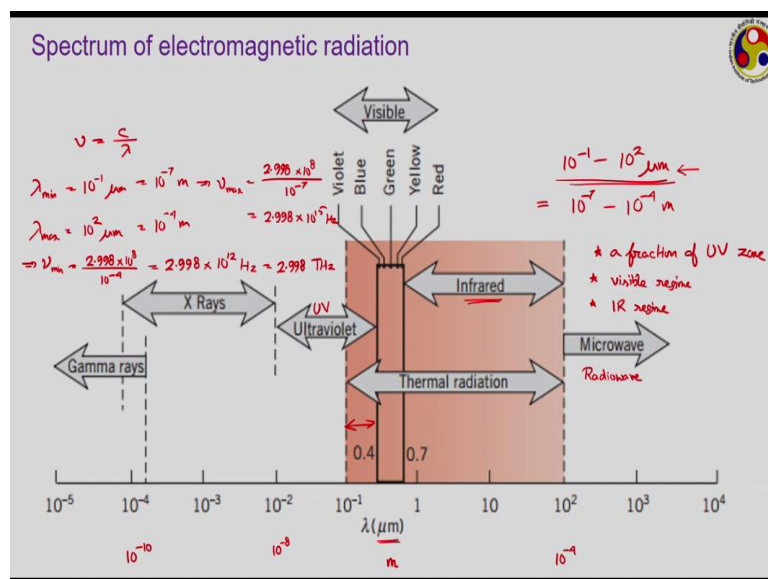
Where, c is the velocity of light in vacuum, which is generally given as 2.998×10^8 m/s. So, wavelength and frequency can easily be related in terms of this and generally the velocity of light being a constant we can use any one of them.

Now, in relation with the thermal radiation, the wavelengths that we primarily consider, they are always in the range of μm and therefore the unit for wavelength. Though m is the fundamental SI unit and I have repeatedly stressed on using the SI units but here I am going to make an exception as I am talking about wavelengths associated with thermal radiation. I shall be using this μm or micron as the unit for wavelength. Frequency, we are not going to talk too much about. We shall be talking everything in terms of wavelengths only. If you are interested to know the frequency of any particular emission, you can easily interchange the position of these two parameters, so that you have

$$\nu = \frac{c}{\lambda}$$

Now, what is the range of wavelength that we are concerned about? Electromagnetic wavelengths are available over an infinite spectrum right, from extremely small wavelengths to quite high wavelengths something like shown here.

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Like we know that when the wavelength is extremely small, something in the range of 10^{-4} micron or even less than that, that is in the range of 10^{-10} m, or in the order of that we call that gamma rays.

Whereas x-rays primarily goes in the range of 10^{-2} to 10^{-4} microns, that is 10^{-8} to 10^{-10} m in that range. Then, if we increase our wavelength bit further, we come to the UV rays or ultraviolet rays. So, this is ultraviolet or UV range which starts from 10^{-2} and goes somewhere around 0.5 microns or about 0.4 microns somewhere here. 0.4 to 0.7 micron is the visible range.

That we can see. The 0.4 corresponds to violet whereas 0.7 corresponds to red. Accordingly we get the visible light spectrum within the range of 0.4 to 0.7 micron. And once we increase it even further, we reach the infrared zone which spans from 0.7 micron to 10^2 micron which is nothing but 10^{-4} m and even higher than that is in the range of microwaves and also I should add radio waves as well.

Now, the gamma rays or x-rays, they are primarily relevant to nuclear physicists or persons who are working in the range of molecular spectroscopy etc. And the microwave and radio waves are primarily of interest to the electrical engineers. But here, being thermal engineers or mechanical engineers, our primary domain of interest is this thermal radiation zone which spans from 10^{-1} to 10^2 micron.

Or if I want to write in terms of m, then it spans from 10^{-7} to 10^{-4} m. As this approach we don't use too much, we shall be sticking to this micron approach only. Then, what are the things that come under the thermal radiation? Generally, the thermal radiation is most prevalent within this domain. It's possible to have some role of thermal radiation in the microwave zones and also in the zones of ultraviolet or x-rays but generally that does not need to be considered.

Whatever thermal radiation that we deal with in practice, they all fall within this wavelength range of 10^{-1} to 10^2 micron. So, what are the things that are coming within this? A fraction of UV zone or ultraviolet zone we are getting. Then, the entire spectrum of visible regime starting from 0.4 and going to 0.7 micron and also the entire spectrum of infrared regime; infrared starts from 0.7 micron and goes up to 10^2 micron or 10^{-4} m. And this entire zone

comes within the thermal radiation and therefore the thermal imaging or those kinds of technologies that we talked about they are primarily in the infrared zone, whereas whatever visible light source that we can see they all leads to the thermal radiation also, because the entire visible spectrum also falls within this thermal radiation zone and small contribution coming from the ultraviolet side. So, this entire discussion that we are going to consider here in relation with fundamentals of radiative heat transfer is restricted to this particular wavelength range and if your interest is to know the corresponding frequency, you can easily calculate as I have written.

$$\nu = \frac{c}{\lambda}$$

I don't have the calculated numbers but you can easily calculate. Like in our case the minimum value of λ that we have to consider is 10^{-1} micron that is 10^{-7} m and maximum value of λ that we are considering is 10^2 micron that is 10^{-4} m. so corresponding to λ_{\min} we shall be getting the maximum frequency as

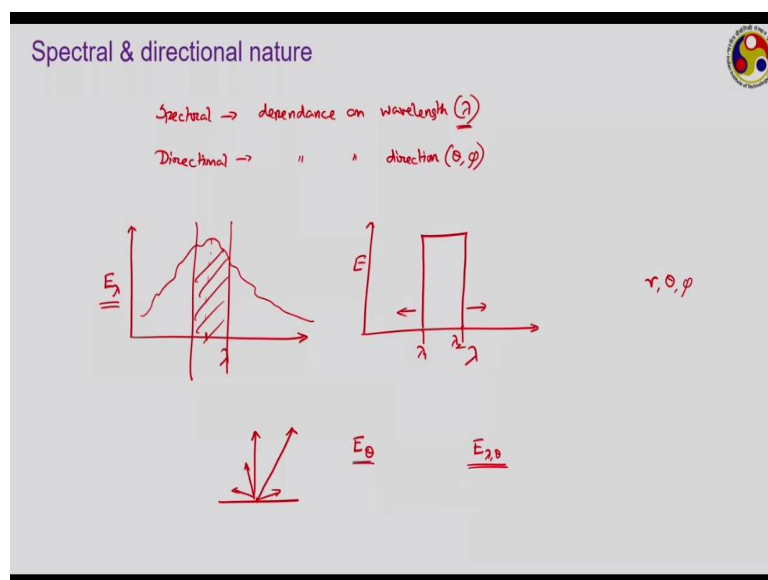
$$\nu_{\max} = \frac{2.998 \times 10^8}{10^{-7}} = 2.998 \times 10^{15} \text{ Hz}$$

So, it's an extremely high frequency that we are talking about. Similarly, corresponding to λ_{\max} we shall be getting the minimum frequency as

$$\nu_{\min} = \frac{2.998 \times 10^8}{10^{-4}} = 2.998 \times 10^{12} \text{ Hz} = 2.998 \text{ THz}$$

So, very high frequency we are talking about in relation with the thermal radiation because the wavelength is quite small.

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Now, whenever you are talking about radiation, there are several definitions that I am going to provide shortly in this lecture and also in the next lecture. Now, in each of these definitions there are two kinds of dependency that we have to consider. One is the spectral dependency, other is the directional dependency. What do we mean by spectral dependency? Spectral dependency talks about the dependence on wavelength, whereas the directional dependency talks about as the name suggests the direction from where it's coming. Say if we talk about the radiation properties of a surface, it can always be seen that if suppose this E is the total radiation emission that a surface is emitting, the emission heat flux and this is wavelength, then you may easily get a profile somewhat like this. That is at a certain wavelength the emission will be maximum whereas in other wavelengths the emission will be much lower. Emission maybe is 0 in certain other wavelengths also. It's very easy to identify elements, which has characteristics somewhat like a band with λ .

That is on this side, say λ_1 and this is some λ_2 . So, below λ_1 and greater than λ_2 or above λ_2 there is no emission. Whatever emission energy or radiation energy it's emitting that is restricted within this small band of λ_1 and λ_2 . So, this kind of wavelength dependence is called the spectral dependence and whenever you are talking about any spectral dependence we put this λ suffix to the corresponding quantity like E , that I shall be coming back to.

But if we are talking about E_λ we are considering the spectral dependence. And when we talk about directional dependence in association with radiation; we primarily use the spherical coordinate system. The reason I am going to show very shortly but in association, as we are talking about the spherical coordinate system, you know that for spherical coordinate our coordinate directions are r , θ and ϕ .

r being the position, θ is a polar angle and ϕ is the azimuthal angle. So, here the direction dependence actually talks about the dependence on this polar and azimuthal angle and if we are talking about direction dependency let us say this is a surface and this is a point on the surface from where we are talking about emission.

So, it's very much possible that in a particular angle, the amount of emission it's showing, in some other angle the emission is maybe quite small, means the surface is giving an emission profile something like this. Here, the length of each of those arrows represents the intensity of radiation in that particular direction. So, in certain direction the emission may be larger, I

mean the surfaces may have some kind of preferential direction in which it prefers that it will lead to higher radiation.

Whereas some other direction, you may have extremely small radiation level or may not be anything at all. So, whenever you are talking about such kind of directional dependence, if E is the quantity that we are going to use, we use the subscript θ to show that we are talking about the directional dependence and if we write something like λ and θ both then it means that we are talking about both spectral and directional dependence.


So, I repeat when we are writing only λ , we are talking about the spectral dependence but not directional dependence. When we are writing like this E_θ , then we are talking about only the directional dependence but not spectral dependence and when we are writing this way $E_{\lambda\theta}$, then you are talking about both directional and spectral dependence. Here, this E that I am using it's actually a dummy symbol, don't consider this now.

Because E and several other symbols I shall be introducing shortly. So, what we are getting from here is that every radiative surface or every emitting surface has its own directional preference that is it emits larger in certain direction, smaller in certain other direction and also the amount of energy it emits that keeps on varying with the wavelength, means it has certain preferential wavelength as well, it emits more in certain wavelength and its emission corresponding some other wavelengths will be less.

But theoretically, a surface will emit radiation over the entire spectrum starting from 0 to infinite wavelength values. But much larger in certain wavelength like the figure I have shown, maybe within this zone the wavelength is extremely high, in certain or rather within this zone of wavelengths the emission is are very high, in certain other zones emission may be extremely low.

So, both spectral and directional dependence that we have to consider whenever you are going to define any radiative property or any radiation related terminology.

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Radiative heat fluxes

Emissive power (E) \rightarrow rate at which radiation energy is emitted by a surface per unit area of that surface
(W/m^2)

$E_{\text{max}} = \sigma T^4$ $\sigma \rightarrow$ Stefan-Boltzmann constant
 (ideal surface)
 $E_{\text{real}} = \epsilon E_{\text{max}} = \epsilon \sigma T^4$ $\epsilon \rightarrow$ emissivity

Irradiation (G) \rightarrow rate at which radiation energy is incident on a surface per unit area of that surface
(W/m^2)

* absorption (α)	$E_{\text{ab}} = \alpha G$	$\Rightarrow \alpha = \frac{E_{\text{ab}}}{G}$	$\left. \begin{array}{l} \alpha + \rho + \tau \\ \Rightarrow \frac{1}{G} [E_{\text{ab}} + E_{\text{ref}} + E_{\text{tran}}] \\ = 1 \end{array} \right\}$
* reflection (ρ)	$E_{\text{ref}} = \rho G$	$\Rightarrow \rho = \frac{E_{\text{ref}}}{G}$	
* transmission (τ)	$E_{\text{tran}} = \tau G$	$\Rightarrow \tau = \frac{E_{\text{tran}}}{G}$	

$G = E_{\text{ab}} + E_{\text{ref}} + E_{\text{tran}}$

Now, you have to define the radiative heat fluxes. There are 4 different radiative heat fluxes that we need to know. The first one is emissive power that E that I am talking about. Emissive power refers to the total amount of energy leaving the surface per unit surface area. So, emissive power can be defined as the rate at which radiation energy is emitted by a surface per unit area of that surface.

So, emissive power is energy per unit area that is actually a flux, so its SI unit will be W/m^2 . What was the fundamental law of radiation that I have introduced in the very first lecture; the Stefan Boltzmann's law. So the maximum amount of emissive power that we can get from a surface is given by the Stefan Boltzmann law. For a given surface then

$$E_{\text{max}} = \sigma T^4$$

Where, T is the absolute temperature and this σ is the Stefan Boltzmann constant. That is the maximum value of emission or emissive power that you can get from a surface maintained at a temperature T . But this is actually true only for ideal surfaces.

For real surfaces, we don't get this one. So, for real surfaces we get this E_{max} multiplied by a fraction which is known as emissivity. So, it's

$$E_{\text{real}} = \epsilon E_{\text{max}} = \epsilon \sigma T^4$$

Where, this ϵ is a property of the surface which is called emissivity. More detail on this emissivity, we shall be learning in the next week, the different definitions of emissivity and the way to estimate that one.

But for the moment it's sufficient that if we know the temperature of the surface then we can calculate the maximum possible emission or emissive power that you can get from that surface and then if we have some idea about the emissivity, we can easily calculate the emissive power of the surface as $\epsilon\sigma T^4$. The next definition is irradiation.

Irradiation talks about the opposite that is the amount of energy incident on the surface per unit area. So, it can be defined as rate at which radiation energy is incident on a surface per unit area of that surface. Irradiation is denoted by a symbol G . Again irradiation is also heat flux because we are talking about rate of energy incident per unit area. So, here again the SI unit's W/m^2 .

And remember a surface can receive energy from all the sources. Like if we are talking about this palm or I should say the top of the palm, it's emitting energy to the surrounding. So, if we know the emissivity of the skin and also the temperature of the surface then we can calculate emissive power.

Because Stefan Boltzmann constant multiplied by T^4 , where T is absolute temperature, that is going to give you the maximum limit of emissive power and then once we multiply that with the emissivity of the skin of the palm, then that is going to be the total emissive power. But to identify the irradiation, we have to consider the entire surrounding means whatever is there in the surrounding everything is emitting some radiation and that radiation is going to all possible direction.

Now, from each of the components that I have in my surroundings like the wall, like this camera, like the lights, the screen of the tablet that I have in front of this, even the pen that I am using, from everything or the from the surface of each of these quantities or each of these items, radiation is getting emitted. And that radiation may reach the top of my palm, may not reach at all.

Like the pen that I am showing that pen is presently kept below my hand or below my palm and therefore its radiation energy may not be able to reach the top surface of the palm but it's reaching the bottom surface. And here again as you are talking about radiation being a surface phenomenon, this top surface and bottom surface are separate surfaces. So, when I

am talking about the top surface whatever energy is incident on the bottom surface that we can completely neglect.

Now, in order to calculate the irradiation on this top surface, then I have to identify the radiation energy or emissive power of all my neighbors, all the items that I have in my neighborhood and from there whatever amount of energy is able to reach the surface that is irradiation. Now, once I have some idea about irradiation, or once some amount of energy has reached the surface, then what will happen to that?

The amount of energy received by a surface by virtue of irradiation can have 3 kinds of results. So there are 3 kinds of fates. One is absorption, means the energy can get absorbed by the body itself by the surface itself, then it's possible to have reflection and thirdly it's possible to have transmission. Just think about any common radiation source that we can see in surrounding.

Probably the best source of radiation energy that we can visualize or you can think of is the sun. The amount of energy emitted by the sun is able to reach the atmosphere, earth atmosphere despite the entire space being vacuum. So, it is radiation energy transmission through the vacuum and once it reaches the surface of the earth, then once it falls on a particular surface there are 3 possible probabilities.

It can get reflected like some highly polished surface like a glass or some metallic flat plates. They will always reflect a significant fraction of the energy that is emitted on this but they can also absorb. Some of them can have significant amount of absorption and if the item is a transparent one, then some energy can get transmitted through this also. But if we are talking about an opaque surface, then the transmission will not come into picture.

Now, to characterize each of them, there are 3 parameters are defined for all. Like we have emissivity as a surface property, which is associated with the emissive power. Similarly associated with irradiation we can define 3 properties. One is absorptivity given by the symbol α , one is reflectivity given by the symbol ρ and one is transmissivity given by the symbol τ .

So, if E_{ab} is the amount of energy absorbed by the surface,

$$E_{ab} = \alpha G \text{ or } \alpha = \frac{E_{ab}}{G}$$

Similarly, the amount of energy reflected is given as

$$E_{ref} = \rho G \text{ or } \rho = \frac{E_{ref}}{G}$$

Where, ρ is defined as the fraction of energy reflected divided by total irradiation and then only for transparent or semitransparent bodies, we can have transmission as well which is given as

$$E_{trans} = \tau G \text{ or } \tau = \frac{E_{trans}}{G}$$

Accordingly, transmissivity is defined as amount of energy transmitted divided by G . Now, if we add all of them up,

$$\alpha + \rho + \tau = \frac{1}{G} [E_{ab} + E_{ref} + E_{trans}] = 1$$

Now, what is this term inside the bracket these 3 quantities together? That is nothing but G only because once the energy is incident on the surface by irradiation that can have only 3 kinds of fates. Some part can get absorbed, some part can get reflected and for transparent and semitransparent one is some part can get transmitted and therefore this becomes equal to 1. These are the 3 surface properties that are associated with irradiation.

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Radiosity (J) → rate at which radiation energy is leaving a surface per unit area of the same surface

$$J = E + \rho G = E + G_{ref}$$

For opaque, $\alpha + \rho + \tau = 1$
 $\Rightarrow \alpha + \rho = 1$

$$J = E + (1 - \alpha)G \leftarrow \text{only for opaque surface}$$

Net radiative flux (q_r'') → difference between incoming & outgoing energy fluxes for a surface

$$\begin{aligned} q_r'' &= G - J \\ &= G - (E + \rho G) \\ &= G(1 - \rho) + E \\ &= \epsilon \sigma T^4 + (1 - \epsilon)G \\ &= \epsilon \sigma T^4 + \alpha G \leftarrow \text{only for opaque surface} \end{aligned}$$

Next, we define a parameter called radiosity. Radiosity refers to total amount of energy leaving the surface. Here, look at this the emissive power is defined as the rate at which radiation energy is emitted by a surface. Whereas for radiosity, we define this one as rate at

which radiation energy is leaving a surface per unit area of the same surface. It's given by the symbol J .

Now, what is the difference of this radiosity and the emissive power? In case of emissive power, we talk about the amount of energy flux that is being emitted by the surface itself because of its own molecular activity, because of its own temperature level. Then what else we have in the radiosity? In the radiosity, definitely we have emissive power as one of the components.

As that is also the energy that is leaving the surface in the form of the radiation of the surface molecules. But also the amount of irradiation that is falling on a surface, a part of that is also coming back. What is that part; the part that gets reflected. So we have ρG also coming into picture. So, total radiosity is defined

$$J = E + \rho G = E + G_{ref}$$

Now, if we are talking about an opaque surface τ is not present that means

$$\alpha + \rho + \tau = 1 \Rightarrow \alpha + \rho = 1$$

In that case, the radiosity can also be written as

$$J = E + (1 - \alpha)G$$

This is true only for opaque surface. This is true only for opaque surfaces and why we are converting this to α that you will be learning later on.

Because this absorptivity, α and emissivity has a certain relation between them. So, radiosity refers to the rate at which radiation energy is leaving the surface per unit area of the same surface. And leaving the surface, it can leave in two ways. It can leave in the form of emission; it can also leave in the form of the reflection part of the irradiation. And finally we have the net radiative heat flux.

Net radiative heat flux \dot{q}''_r is defined as the difference between incoming and outgoing energy fluxes for a surface.

$$\dot{q}''_r = G - J$$

The suffix r denotes the radiation. So, what is the amount of energy the surface is receiving? It's receiving via irradiation. So, G is the amount it's receiving and how much is leaving the surface? That is the radiosity. So $G - J$ is referred as the net radiative flux. So, I repeat then;

there are 4 radiative fluxes that we have defined. First is emissive power, which refers to the flux which is associated with the emission by the surface on its own, the amount of energy that is emitted by the surface. Then is irradiation that refers to amount of energy that is received by the surface.

Then, radiosity is the total amount of flux that is leaving the surface and it can leave in the form of emission and also in the form of reflection. So, it has the entire emission part plus a part from the reflection or from the reflected part of irradiation and finally the net radiative flux which talks about the difference between the incoming and outgoing energy fluxes.

So, incoming flux is the irradiation and outgoing flux is the radiosity. So, $G - J$ is the net radiative flux. If we express them in terms of their components or their basic definitions then

$$\begin{aligned}\dot{q}''_r &= G - J \\ &= G - (E + \rho G) \\ &= G(1 - \rho) + E \\ &= \epsilon \sigma T^4 + (1 - \rho)G\end{aligned}$$

if you are talking about the special case of opaque surfaces, then this $1 - \rho$ can be replaced with α .

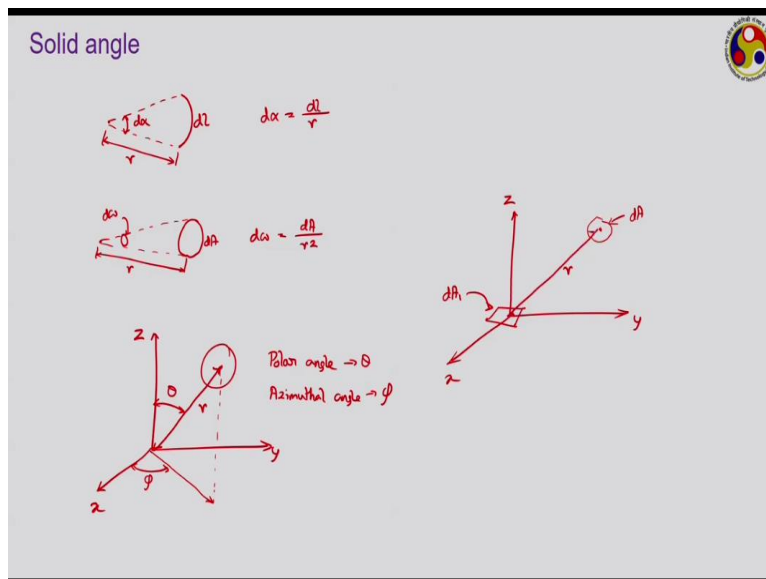
$$= \epsilon \sigma T^4 + \alpha G$$

This is true only for opaque surfaces. So, these are the 4 radiative fluxes for which we have to know because in any radiation calculation, all 4 of them can appear. Now, for all these radiative fluxes, again you have to consider the spectral and directional dependence like the emissive power.

We are talking about emissive power as the amount of radiant energy that is emitted by the surface. Now, that emission that is going from the surface that itself has its own spectral and directional dependence that is the emission of all wavelengths is not of equal intensity. In certain wavelengths, the intensity is high; in certain wavelengths, the intensity is low. Similarly, the directional dependence, in certain direction, there will be quite high rate of radiation emission; in certain other direction, the emissive power may be quite low.

So, that is the second thing or rather the next thing that we have to consider now. But before that let us quickly review the concept of solid angle, which I am sure all of you already know.

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If suppose we are talking about a 2D space and this is one arc and this is one point, then if we join the end of the arc by these lines to this and then if the length of this arc is dl and this angle is equal to $d\alpha$ and the radius of this arc is equal to r , then we know that we can relate this $d\alpha$ with this dl as

$$d\alpha = \frac{dl}{r}$$

as long as this $d\alpha$ is extremely small. Now, if we expand that to a three dimensional space where instead of an arc, we have an area. So, we have an area A , and again this is a point here. Then if we joined all the points located on the circumference of this surface to this point and if this distance continues to be r , then the three dimensional angle or rather the two-dimensional angle that we get here this angle is called the solid angle $d\omega$.

Now, we can very easily expand the logic that we used in case of relation between $d\alpha$ and dl . Similarly, we can get

$$d\omega = \frac{dA}{r^2}$$

So, this way we can define a solid angle. But why we are defining a solid angle? Because whenever we are talking about radiation from a single point, so this is one point, the tip of my finger is a point from where we are talking about radiation. The radiation can go in all possible directions or all possible angles and therefore the solid angle becomes very important as we are talking about three dimensional space. Now, to analyze this radiation, we generally consider spherical coordinate system. How that will look like? So, if our standard

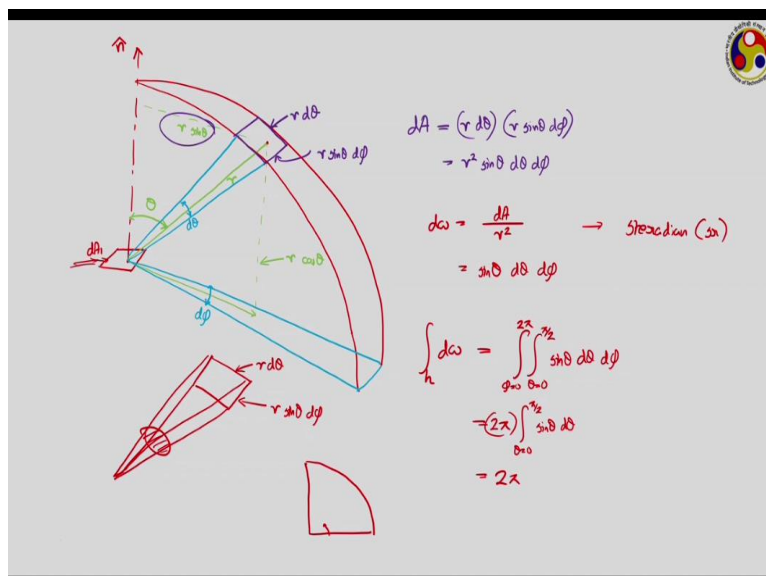
Cartesian coordinate system is like this x, y and z and we have a point somewhere here then we can easily identify the x, y, z coordinate.

But in spherical coordinate, we need to consider this radius r and then the azimuthal and polar angle. How to get the angles? Let us take the projection of this on the x-y plane. Let us say this is your projection. Then the angle this r is making with the z axis, θ is called the polar angle whereas the angle this projection is making with the x axis is called ϕ which is the azimuthal angle.

So, the location of this particular point we can define using x, y, z as per the Cartesian coordinate system or we can define in terms of r, θ and ϕ . Now, our interest here is to identify, let us say this point is a source of radiation. So, again I am drawing the Cartesian coordinate system. So this is y, this is x and this is z. Now, assume the origin here is located on an infinitesimally small surface and let us say the area of this surface is dA_1 .

And we want to know the emission that is leaving the surface dA_1 and what fraction of that is able to reach a point located somewhere here. Let us say there is an infinitesimally small area dA around this point then how much of the energy emitted by this area dA_1 is able to reach this dA that is something that we are trying to calculate. Let us say the location of this point is equal to r.

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So, how to identify this? Let us say this is our vertical axis and this is the small area that we have. Let us say this is the normal direction to the area. We have a small area say dA_1 , an

infinitesimally small area and its normal is this \hat{n} which is shown like this. And we are looking to identify the amount of energy received by the point here, this particular point.

To get this, let us first join this by a line. Let us say this line is having a length of r , which is a position vector of this point with respect to the coordinate framework. And this r is making an angle θ with this normal axis. So, this θ becomes the polar angle and to get the azimuthal angle, we know that we have to draw a projection on x y plane and then if this is the projection then what angle it makes with this x direction?

I have not shown the x , y , z direction but you can easily identify the azimuthal direction as well. But I am not showing this azimuthal angle ϕ because that is not required for our case but this θ is required. Now, what is the length of this dotted line? As we have taken the projection and if we take the other position basically this r is divided into two components.

One is parallel to the normal direction, other perpendicular to the normal direction. Then, this one will be having a length of $r\cos\theta$ and this one will be having a length of $r\sin\theta$. Now, around this point r , I am taking an infinitesimally small angle $d\theta$ by two lines. One line is something like this, another line is something like this. This small portion is $d\theta$, this infinitesimally small angle of deflection. Like say this is our origin and this is the point where we are looking to identify.

This is making an angle, this distance is r and it's making a polar angle θ and azimuthal angle ϕ . Now, if we swipe it by a very small amount $d\theta$ on either side, then this is what we are getting. So, if we swipe this slightly, then we may get this amount of distance. So, this is the amount of distance it has been traveled by the surface. Then, how much is the length of this portion? So, this was our original point and by traversing this small angle $d\theta$ about the polar angle θ , then probably we have got something like this. So, now if we draw one plane starting from here something like this and another somewhat like this, then around the azimuthal angle ϕ again if we traverse it. Like here now we have traversed this much by $d\theta$ right, traverse in this direction, by a small amount $d\phi$, then what we are going to get?

This line now we curtail up to this. So once we are drawing this, then maybe your $d\phi$ will be something like this much. So this angle is this $d\phi$. So, if we now complete the area that is of our concern. By traversing the angle $d\theta$, we are getting this line. By traversing the angle $d\phi$

maybe, we are getting something like this and if we complete the area, we are getting a small area something like this.

If I draw it using a separate color, let me pick up this purple color here, then probably this is the area that I am going to talk about. So, what are the dimensions of this area? The length of the arc, the distance or the position of the point was r and we have moved by distance $d\theta$ around this. Then, this arc will be having a distance of $rd\theta$ and how much is this? Here this $r\sin\theta$, this one has traversed an angle $d\phi$ around this.

So, this one will be having $r\sin\theta d\phi$. Then if this area that we are talking about, this area is equal to dA , then it will be equal to assuming it to be an infinitesimally small rectangle

$$\begin{aligned} dA &= (rd\theta)(r\sin\theta d\phi) \\ &= r^2 \sin\theta d\theta d\phi \end{aligned}$$

So, let's calculate the solid angle that has been extended or subtended by this. Now this becomes the area that we are talking about where this dimension is $r\sin\theta d\phi$. This dimension is $rd\theta$ and it will be probably something like a conical structure like this and in between we are having the solid angle that we are talking about. Difficult to visualize or difficult to draw rather, try to visualize this one exactly what I am talking about. Here, we are having a full surface. Like again I am talking about if this palm is the surface and some point I have somewhere here, then from this point I am joining all the points lying on the periphery of this palm and then we are getting a three dimensional volume.

Then, whatever angle it's creating that angle is this solid angle. So, how much will be the solid angle? If the solid angle is $d\omega$, then we know that it will be equal to

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

The solid angle has a unit of Steradian or in short is Sr. As we know that we use radian for angle, solid angles are generally given by steradian or Sr. Now, if we talk about the emission from a surface like this surface that I started with.

This is the surface from with where the emission is happening. Now, say if we have some point on this surface, then what are the maximum possible directions the emissions can go? The emissions can go in all the infinite possible directions; can go in this direction, this direction, can come to me, can go in this direction, in all possible direction. That is if we

draw a hemisphere on top of this surface, then emission can go in all possible directions of that hemisphere.

Of course, it's only a hemisphere because emission cannot go below the surface. It's a flat surface and emission can go only top of the surface, on the points available above the surface and there we are having a full hemisphere. That is why quite often we go for definitions in terms of hemisphere or hemispherical quantities. So, quite often we have to integrate the solid angle over a hemisphere.

So, if we integrate this solid angle $d\omega$ over a hemisphere

$$\int_h d\omega = \iint_{\phi=0, \theta=0}^{\phi=2\pi, \theta=\frac{\pi}{2}} \sin\theta d\theta d\phi$$

Putting this h refers to a hemisphere. Then, what does that mean? We are integrating this $\sin\theta d\theta d\phi$ over some limits of θ and ϕ . What will be the limits? Now, say if this is your ray lying at a position of $\theta=0$, $\phi=0$ and is a straight line. Using this straight line, you have to form a hemisphere. So, how we can do this?

Let us change the polar angle from 0 to $\frac{\pi}{2}$, so we are moving only in this direction so that the polar angle is changing from 0 to $\frac{\pi}{2}$. So, the possible directions of $\theta = 0$ to $\frac{\pi}{2}$. So, once we have done this like initially we are having a line like this as we move in this direction, then after completing you get a surface somewhat like this, which covers the entire polar angle of 0 to 90 degree.

Now, as we have the surface, a surface somewhat like this, now you keeping this point fixed, you rotate the surface like something like this way, then if this is the initial surface and once you rotate something like this, you get one quadrant of this hemisphere. So, if you make it full 360 degree rotation this way then you get the full hemisphere. That is the azimuthal angle you have to vary from 0 to 2π .

So, polar angle is varying from 0 to 90 degree and then the surface that has been created, this surface we are rotating by a full circle, full 360 degrees to form the hemisphere. So, if you integrate this,

$$= 2\pi \int_{\theta=0}^{\frac{\pi}{2}} \sin\theta d\theta = 2\pi$$

This surface I am rotating over the full circle, so full 360 degree. So to form the hemisphere and that is what the azimuthal angle is varying from 0 to 2π . So, we are getting 2π into integral of hemispherical integral of $\sin\theta d\theta$ which is equal to 1. So, this hemispherical integration of the solid angle becomes equal to 2π Steradian. So, that is something that we have to make use of very soon.

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Emission intensity

→ Spectral intensity $I_{\lambda e}(\lambda, \theta, \phi)$

Spectral directional intensity (λ, θ, ϕ)

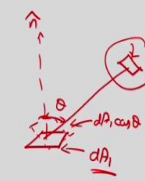
→ rate at which radiation energy is emitted at the wavelength λ in the (θ, ϕ) direction, per unit area of the emitted surface normal to this direction, per unit solid angle about this direction and per unit wavelength interval $d\lambda$ about λ .

$$I_{\lambda e}(\lambda, \theta, \phi) = \frac{dq_\lambda}{(dA_1 \cos\theta) d\omega d\lambda}$$

$$\Rightarrow \frac{dq_\lambda}{d\lambda} = dq_{\lambda} = I_{\lambda e}(\lambda, \theta, \phi) (dA_1 \cos\theta) (\sin\theta d\theta d\phi)$$

$$= I_{\lambda e}(\lambda, \theta, \phi) dA_1 \sin\theta \cos\theta d\theta d\phi$$

$$\Rightarrow dq_{\lambda} = \frac{dq_\lambda}{d\lambda} = I_{\lambda e}(\lambda, \theta, \phi) \sin\theta \cos\theta d\theta d\phi \leftarrow \text{spectral radiation flux}$$



So emission intensity is the final term that I am going to introduce today and using this several other definitions will be introduced in the next class. So, emission intensity is also called the spectral intensity. Though we are using that term spectral intensity but we truly should talk about spectral directional intensity, because here you are going to talk about both λ , θ , ϕ variation, both spectral and directional variation, though spectral intensities is the name which is commonly used.

Because whenever you are talking about intensity it by default indicates that we are talking about any particular direction. The symbol that is used for this is $I_{\lambda e}$. It's a function of λ , θ and ϕ . Here, the suffix λ indicates that it's spectral, as I have mentioned in we are not putting in θ , ϕ subscript to indicate.

Because the intensity term always talks about the direction and the λ , θ , ϕ dependence is also shown there. Now, what is the definition of this? The spectral intensity is defined as; let us say we talk about some area here, some area dA_1 , this is normal to this, the normal direction

and here is another surface, we are looking to identify which is making some angle θ with this vertical.

So, this surface or whatever projection that we are getting, this second surface it's actually not able to see this entirety of dA_1 rather it's able to see only a projected version of this which is having an area of $dA_1 \cos \theta$. Then, the spectral intensity is defined as the rate at which radiation energy is emitted at the wavelength λ in the θ, ϕ direction, per unit area of the emitted surface normal to this direction, per unit solid angle about this direction and per unit wavelength interval $d\lambda$ about λ .

If suppose dq is the amount of energy that is getting emitted at this wavelength λ in the θ, ϕ direction per unit area of the emitted surface normal to that direction. Now, though the area of the surface that we are talking about is dA_1 . But as this is the destination in the θ, ϕ direction, then the area that is projected in that direction is only $dA_1 \cos \theta$ and therefore this energy should be divided by this projected area of $dA_1 \cos \theta$ because here we are talking about per unit area of the emitted surface normal to this direction. So, we are not talking about dA_1 rather $dA_1 \cos \theta$. Per unit solid angle about this direction, so about the θ, ϕ direction whatever is the solid angle we have which is $d\omega$.

And also per unit wavelength interval $d\lambda$ around λ that is λ is a particular wavelength we are talking about an infinitesimally small band of wavelengths around that λ . This is going to be defined as this spectral intensity of emission is a function of λ and θ, ϕ .

$$I_{\lambda e}(\lambda, \theta, \phi) = \frac{dq}{(dA_1 \cos \theta) d\omega d\lambda}$$

Quite often, we generally separate out this $dq/d\lambda$ and denote that as dq_λ , called the spectral heat flux

$$\begin{aligned} \frac{dq}{d\lambda} &= dq_\lambda = I_{\lambda e}(\lambda, \theta, \phi) (dA_1 \cos \theta) \sin \theta d\theta d\phi \\ &= I_{\lambda e}(\lambda, \theta, \phi) dA_1 \sin \theta \cos \theta d\theta d\phi \end{aligned}$$

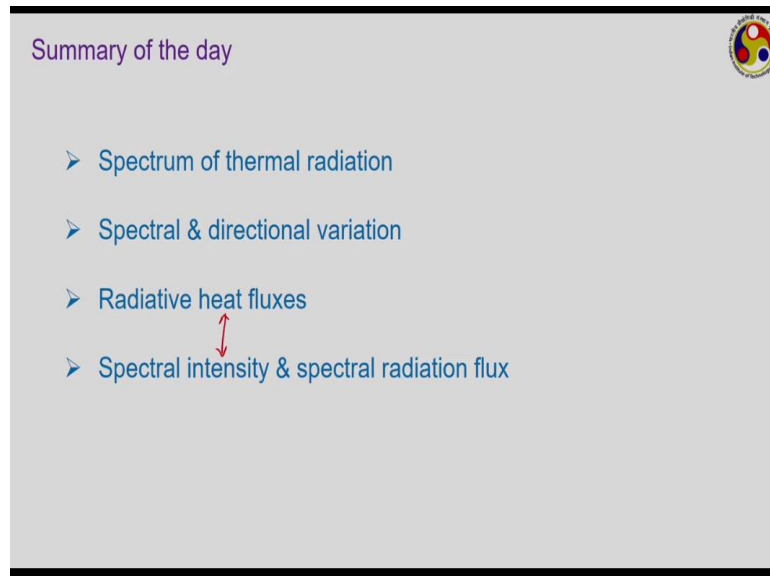
And the spectral heat flux or spectral radiation flux is

$$\frac{dq_\lambda}{dA_1} = I_{\lambda e}(\lambda, \theta, \phi) \sin \theta \cos \theta d\theta d\phi$$

So, this is called the spectral radiation flux. So, the emission intensity and spectral radiation flux I have defined. I will start again from this particular point from next lecture and we shall be using this definition of intensity to define different radiative fluxes.

So, please try to check this one out. I shall be repeating this exercise again, just for the purpose of your clarity.

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So, today we have learned about the spectrum of thermal radiation and we have seen that thermal radiation covers a fraction of ultraviolet radiation, the entire visible spectrum and the entire of the IR spectrum and the thermal radiation can have both spectral and directional dependence. And 4 different radiative heat fluxes that we have defined like emissive power, irradiation, radiosity and net radiative heat flux.

And finally the spectral intensity and spectral radiation flux was introduced which I shall be repeating again and trying to identify the relation between the spectral intensity and the heat fluxes. So, please revise this lecture and only once you have got the ideas that I have conveyed here. Then, only you try to move to the next lecture. So, thank you very much.