

Fundamentals of Conduction and Radiation
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Lecture - 2
Module 1: Introduction to Heat Transfer
Part 2

Hello friends, welcome for the second time on this week number 1 where we are just setting up the stepping stones for our discussion on conduction and radiation heat transfer. In the previous lecture, we have just done a bit of overview of thermodynamics, particularly the laws of thermodynamics and tried to establish the need of studying heat transfer and tried to identify the difference of heat transfer with thermodynamics.

Like we have seen that, among the three common laws of thermodynamics, the Zeroth law gives us the definition of a property known as temperature, which gives us an idea about the possible direction of heat transfer. The first law of thermodynamics gives us a way of measuring the magnitude of heat transfer involved with a process or associated with a cycle, whereas the second law of thermodynamics gives us the direction of that heat transfer, which I have just mentioned in terms of the temperature difference between the system and its surrounding.

However, thermodynamics always deals with a quantity of energy. It just identifies heat transfer as the energy which crosses the boundary of the system because of a difference in temperature between system and surrounding and that's all. It never talks anything else about heat transfer. Like, if you just think about your thermodynamic lessons, there you have identified or you have learned to calculate different types of work transfer.

The moving boundary work, the work associated with electrical energy, the flow work, etc. but you have never learned anything about how to calculate the magnitude of heat transfer because we always treated heat transfer just as a bulk quantity, where we are just concerned about the magnitude and may be the direction that's all. But we never considered how much time it is required to transfer a particular quantity of heat. And also the rate of heat transfer associated with a particular area of the system or area of the boundary, I should say. And

that's exactly where the heat transfer starts, where we are looking to identify the rate of heat transfer in form of power and also heat flux.

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Modes of heat transfer

Fourier's law of heat conduction $\dot{q}_x'' \propto -\frac{dT}{dx} \Rightarrow \dot{q}_{x,out}'' = -K\left(\frac{dT}{dx}\right)$

Stefan-Boltzmann law of radiation $\dot{q}_{rad}'' \propto (T_s^4 - T_{amb}^4) \Rightarrow \dot{q}_{rad}'' = \epsilon \sigma (T_s^4 - T_{amb}^4)$
 $\dot{q}_{rad}'' = \epsilon \sigma (T_s^4 - T_s^4)$ $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$

Newton's law of cooling $\dot{q}_{conv}'' \propto (T_s - T_f)$
 $\Rightarrow \dot{q}_{conv}'' = h(T_s - T_f) = h(\Delta T) \leftarrow$
 $\Rightarrow \dot{q}_{conv}'' = hA(T_s - T_f)$
 $h = h(\text{Velocity, Fluid properties, Solid properties, Nature of surface, ...})$

Process	h ($\text{W/m}^2 \cdot \text{K}$)
Free convection	
Gases	2-25
Liquids	50-1000
Forced convection	
Gases	25-250
Liquids	100-20,000
Convection with phase change	
Boiling or condensation	2500-100,000

You already know from your basic level physics discussion that there are three primary modes of heat transfer, conduction, convection and radiation. Conduction is associated with the molecular activities and the molecular structure particularly in case of solids, whereas radiation is an energy transmission in the form of electromagnetic waves.

And, a body which is at a temperature higher than absolute zero will lead to radiative energy transmission. Convection is more a combination of conduction and flow, where we basically talk about the energy transmitted from a solid surface by conduction to the nearest fluid layer and then that fluid layer will be advected away from the surface, thereby taking the energy away from the surface by the bulk motion of the fluid itself. And we have also introduced three basic laws of heat transfer.

The Fourier's law of heat conduction, which gives us the basics of conduction of heat transfer or allows us to calculate the magnitude of conduction heat transfer associated with any system. Here I have put one dot over the Q and also I have put two lines, two vertical lines or near vertical lines or two commas, two primes basically.

This dot refers to per unit time that is it is a rate whereas each of those lines represent per unit length scale. As there are 2 lines, it represents per unit length scale square that is area. So this is heat flux. As per Fourier's law of heat conduction, rate of conduction heat flux or I should

say conduction heat flux in a particular direction is proportional to the temperature gradient in that direction from where we can write the heat flux in a particular direction (x) is equal to

$$\dot{q}_{x,cond}'' = -k \frac{dT}{dx}$$

I should have added the minus sign here also, here K is the constant of proportionality which is actually property of the system that we are dealing with, or the property of the substance, which is known as the thermal conductivity and the minus sign indicates that as we increase x, T decreases. Or I should say, as per the second law of thermodynamics you already know that heat transfer always takes place on the direction of higher temperature to lower temperature.

So in the x direction, or to have heat conduction in the x direction, this $\frac{dT}{dx}$ quantity has to be negative. So, to have a positive magnitude for the heat transfer, we allow this negative sign. Then, the Stefan-Boltzmann law for radiation or law of radiation, which says that, the radiative heat transfer or radiative heat flux is proportional to, if Ts refers the temperature,

$$\dot{q}_{rad}'' \propto (T_s^4 - T_{surr}^4)$$

This leads to,

$$\dot{q}_{rad}'' = \sigma(T_s^4 - T_{surr}^4)$$

This is of course for ideal surface like the blackbodies. For real surface we have another parameter ϵ , which is the property of the surface known as emissivity.

$$\dot{q}_{rad}'' = \epsilon\sigma(T_s^4 - T_{surr}^4)$$

Its value varies from 0 to 1. 1 for ideal surfaces and here the sigma is known as the Stefan-Boltzmann constant.

So, the value of σ we shall again be calculating, and also we shall be developing this particular Stefan-Boltzmann law later on in module number 9. But for the moment you can take its value, because we may have to use them even when you are analyzing conduction, sometimes convection and radiation related knowledges may be important, where we have to make use of these relations. That's why I am just giving the basic formulas without derivation.

Later on, we shall be deriving them. We shall be discussing them in details. But for the moment, you just try to remember these relations. If we are talking about radiation between 2 points, say this is surface number 1, this is surface number 2, then the net radiative exchange between them can be written as or net radiative energy transmitted from 1 can be given as

$$\dot{q}_{rad}'' = \varepsilon\sigma(T_1^4 - T_2^4)$$

For the moment, you just remember this relation, but exact form of this one we shall be using later on.

Now the value of sigma, which I was talking about, is quite easy to remember. 5.67×10^{-8} is the magnitude of the Stefan-Boltzmann constant. This sigma is known as the Stefan-Boltzmann constant and what will be the unit for this? Its unit is $\frac{W}{m^2.K^4}$. From where this one comes? That we can see from the above equation, or I shall be coming back to this later on.

The third relation that is known as the Newton's law of cooling; we have seen for convective heat transfer. The heat flux is proportional to the temperature differences. T_s is the surface temperature or solid temperature from where the heat is being transferred to the fluid. It is surrounding by the fluid temperature or more commonly we write this as T_∞ , and the corresponding constant of proportionality is generally given the symbol h.

So, the convective heat transfer becomes

$$\dot{q}_{conv}'' = h(T_s - T_\infty)$$

Or, the product of h and the prevailing temperature difference, which is the reason for this heat transfer and h is known as the convective heat transfer coefficient. This is known as Newton's law of cooling. Now, we are not going to discuss any more about convective heat transfer, but we may have to make use of this relation quite a bit while studying conduction heat transfer, and also may be a bit in while studying radiation heat transfer.

So, please try to remember this formula, that is the convective heat flux is equal to the convective heat transfer coefficient into the corresponding temperature difference or if you are interested to calculate the corresponding heat rate or heat transfer rate, then this one will become

$$\dot{q}_{conv} = hA(T_s - T_\infty)$$

So, as flux refers to energy transfer rate per unit area, when you are calculating the energy transfer rate, this area (A) needs to be multiplied with this.

This heat transfer coefficient is a function of several quantities. It is a function of the flow velocity, it's a function of the fluid properties, and it's a function of the solid properties as well. Fluid properties play a more dominant role compared to the solid properties but solid properties are also important. Also there may be important consideration like the nature of the surface, like the heat transfer coefficient associated with a smooth surface and the heat transfer coefficient associated with a rough surface may be distinctly different.

So, the nature of the surface definitely plays a role and there may be quite a few others. In fact, the range of temperature that we are talking about that can also come into play. Generally the temperature dependence of h is very weak. The value of temperature can only dominate the value of the fluid properties, but otherwise h is not that much dependent on the fluid temperature as well as the solid temperature.

Also, there are different ways we can have convective heat transfer. Like convection can be classified into 2 categories. Primarily, one is the force convection; other is the natural or free convection. Free convection refers to where the velocity of the fluid is developed by natural processes just like buoyancy. And force convection refers to when the fluid is forced to flow, like say we are having a fan and by using the fan you are dragging some gas like air or maybe we are having a liquid pump and using that pump we are forcing some liquid to flow through a channel.

Then accordingly, convective heat transfer can be classified into force convection which I have just mentioned where it is being forced by a prime mover like pump or fan or may be compressor; whereas in natural convection because of the temperature difference there may be density gradient created inside the flow stream which leads to a buoyancy and accordingly the fluid may start flowing. That is called the natural convection.

Similarly, convection can also be of internal convection and external convection types. Internal convection refers to when the fluid flow is completely bounded by surfaces. That is fluid is flowing through some pipe or duct or channel. So, it is completely bounded by solid surfaces. And we call it an internal convection scenario; whereas when it is not completely

bounded like say flow of water through a river, or flow of the sewage through that sewage canal.

So these kinds of scenario are external convection because it is not completely bounded. And depending upon the scenario, the value of h can vary widely. Like this is a sample table which is showing some standard ranges. For free convection scenario, again we can see for gases and liquids, there are significant difference between the value of h . For gases the primary reason for having low heat transfer coefficient is low thermal conductivity as we shall be seeing in the next week. For liquid, it is generally much higher.

For force convection, if we just compare the values for gases for free convection and values for gases with force convection, that is at least one order higher, same for the liquids, there is at least one order higher. So, in force convection you get higher heat transfer coefficient. Therefore, most of the convective transfer application you may find in force convection mode. But there are its problem of its own when natural convection definitely has certain advantages

There is another scenario with convective heat transfer, where we have some phase change. In case of phase change, like when a boiling or condensation process happening, we can have even larger heat transfer coefficient, particularly values in these ranges are obtained with condensation.

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The diagram shows a horizontal surface at temperature T_s with area A . Above it is a gas at temperature T_g with heat transfer coefficient h . Red arrows indicate radiation E and convection q''_{conv} from the surface to the gas. A legend defines E as Emission and Q as Irradiation.

Handwritten equations on the right:

$$q''_{conv} = h (T_s - T_g) \leftarrow h \neq h(T_s, T_g)$$

$$q''_{rad} = \epsilon \sigma (T_s^4 - T_g^4) \leftarrow$$

$$= \epsilon \sigma (T_s^2 + T_g^2) (T_s + T_g)$$

$$= \left[\epsilon \sigma (T_s^2 + T_g^2) (T_s + T_g) \right] (T_s - T_g)$$

$$= \boxed{h_x (T_s - T_g)}$$

$$h_x = \epsilon \sigma (T_s^2 + T_g^2) (T_s + T_g) = h(T_s, T_g)$$

Handwritten equations at the bottom:

$$q'' = q''_{conv} + q''_{rad}$$

$$= (h + h_x) (T_s - T_g)$$

$$\rightarrow \dot{q} = A(h + h_x) (T_s - T_g)$$

Now, this picture I have shown in the last lecture while studying radiation where we know that if the straight line is representing some surface, then there are several kinds of convective and radiative heat transfer scenarios that we can get. This E refers to the radiant energy that is leaving the surface whereas G refers to irradiation. So, E is here the emission whereas G is irradiation.

Emission refers to the radiant energy emitted by the surface or leaving the surface, whereas irradiation is amount radiation energy that is reaching the surface or being incident on the surface from the surrounding. And this quantity refers to convection heat transfer caused by the flow of some gas having temperature T and heat transfer coefficient h. In that case, if we want to calculate the net heat transfer associated with this surface, then how can you calculate this?

Let us say the temperature for this surface is T_s , and just to be consistent with the early notation that we used in the previous slide, let us call this temperature of the gas to be T_∞ . Then, the amount of convective heat flux can be written as

$$\dot{q}_{conv}'' = h(T_s - T_\infty)$$

as the rate of convective heat flux is proportional to the temperature difference. However, the magnitude of radiative heat flux that will be given by ϵ , the emissivity for this surface.

$$\dot{q}_{rad}'' = \epsilon\sigma(T_s^4 - T_\infty^4)$$

So, it is depending upon the difference between the fourth power of the two temperatures. Now, quite often, it is easier to have a form similar to this. That is, a form where we can just represent this one directly proportional to the temperature difference.

To obtain that form, quite often we play around with this term a bit, it can be broken into

$$\dot{q}_{rad}'' = \epsilon\sigma(T_s^2 + T_\infty^2)(T_s^2 - T_\infty^2)$$

If we break this down even more,

$$\dot{q}_{rad}'' = [\epsilon\sigma(T_s^2 + T_\infty^2)(T_s + T_\infty)](T_s - T_\infty)$$

So the last term is the one that we actually want to have.

Then this portion in the square bracket gives us something like a radiative form of the heat transfer coefficient. But as h is a convective heat transfer coefficient, this h_r refers the radiative heat transfer coefficient and the radiative heat flux becomes

$$\dot{q}_{rad}'' = h_r(T_s - T_\infty)$$

quite similar to the Newton's law of cooling. But there is important difference between h and h_r . As you have seen h depends on several quantities or several quantities or several parameters during convection, but it depends very weakly on the temperature of the fluid and also the temperature of the surface (i.e. T_s and T_∞).

In most of the scenarios, we can consider the heat transfer coefficient to be independent of both the temperatures involved. Only in certain cases, where the properties of the fluid can vary strongly with the temperatures, we may have to consider the temperature dependence. But h is not directly dependent on T_s as such. What about h_r ? Here, the radiative heat transfer coefficient h_r that we have got, that is

$$h_r = \varepsilon\sigma(T_s^2 + T_\infty^2)(T_s + T_\infty)$$

It's a strong function of both T_s and T_∞ .

In fact, I should repeat that the radiative heat flux is proportional to the difference in the fourth power of the temperatures and not to $(T_s - T_\infty)$. This particular form that I have written, this is just a form that is written for convenience purpose, because once we have a form like this and calculate the value of h_r , then the total heat flux leaving the surface can be calculated as

$$\begin{aligned}\dot{q}'' &= \dot{q}_{conv}'' + \dot{q}_{rad}'' \\ &= (h + h_r)(T_s - T_\infty)\end{aligned}$$

And if we want to calculate the total heat transfer rate, then this should be multiplied by the area of the surface

$$\dot{q} = A(h + h_r)(T_s - T_\infty)$$

Where, this A refers to the area of this particular surface. So this way, we can express the radiative heat transfer equation also in terms of a radiative heat transfer coefficient, just for the convenience of certain calculations.

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Exercise 1

The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m.K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m × 1.2 m on a side?

The diagram shows a 3D perspective of a furnace wall with width $W = 1.2\text{ m}$ and height $H = 0.5\text{ m}$. A cross-section shows a brick wall of thickness $L = 0.15\text{ m}$ with thermal conductivity $k = 1.7\text{ W/m}\cdot\text{K}$. The inner surface is at $T_1 = 1400\text{ K}$ and the outer surface is at $T_2 = 1150\text{ K}$. Heat flux q_x'' is shown moving from left to right. The wall area is labeled A .

Assumptions

- steady-state operation
- constant thermal conductivity
- 1-D conduction heat transfer

Handwritten Calculations:

$$q_x'' = -k \frac{dT}{dx} \approx -k \left(\frac{T_1 - T_2}{L} \right)$$

$$\rightarrow q_x'' = -(1.7) \left(\frac{1400 - 1150}{0.15} \right)$$

$$= 2833\text{ W/m}^2$$

$$\rightarrow \dot{Q}_x = q_x'' A = (2833)(0.5 \times 1.2)$$

$$= 1700\text{ W}$$

$$= \boxed{1.7\text{ kW}}$$

Derivation of Fourier's Law:

$$T_1 = T|_{x=0} = T(x)$$

$$T_2 = T|_{x=L} = T(x+L)$$

$$\Rightarrow \frac{T(x+L) - T(x)}{L} = \frac{dT}{dx}|_x$$

Now, let us just quickly see couple of numerical examples of analyzing heat transfer scenario, very very simple heat transfer scenario. Though I have given you the 3 basic rules the Fourier's law of heat conduction, Newton's law of cooling and the Stefan-Boltzmann law radiation or radiative heat transfer, but you have not gone into the detail of heat transfer like the Fourier's law.

The form of the Fourier's law and it's further implication etc. we shall be learning only in the next lecture. And therefore, we are just trying to solve a few numerical problems where we can have a direct application of this relation without understanding their technical details. So just read this problem. The wall of an industrial furnace is constructed from 0.15 m thick fireclay brick having a thermal conductivity of 1.7 W/m.K. The unit of thermal conductivity is W/m.K. I am coming back to that after sometime.

Measurement made during steady state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces respectively. What is the rate of heat loss through a wall that is 0.5 m X 1.2 m on a side? So this is the scenario. We are having a wall. This height is 0.5 m. Say its width is 1.2 m. If we talk about this particular surface, then its total area is 1.2 X 0.5 m and across this surface it is a wall is of 0.15 m thick like shown here.

One side is maintained at a higher temperature of 1400 K. Other side is maintained at a lower temperature of 1150 K and the thermal conductivity of the corresponding material is also given to be 1.7 W/m.K. So, we have to calculate the rate of heat loss through the surface. It's

a very simple scenario. But before analyzing this, we have to identify what is the mode of heat transfer associated with this. Here, we are talking about heat transfer through a solid.

As we are talking about heat transfer through a solid, there is no flow involved. And whenever there is no moving fluid or flowing fluid involved, the convection never comes into picture. So, we are restricted only to conduction and radiation. Now, radiation is primarily concerned from a surface, though radiation as we shall be seeing later on is a volumetric phenomenon; but what happens, let's say this is one surface and there are several molecules just below the surface, then another layer of molecules here, another layer of molecules here.

So each of the molecules has their own radiative properties, but what happens, the amount of radiation leaving this particular molecule or the molecules which are present in this layer will get absorbed by the molecules which are present here or here. That is the molecules which are present around that particular molecule which is the source of energy. And therefore the energy emitted by molecules or layers which are a bit farther away from the surface; they are not able to reach the surface and the net radiation energy that leaves the surface is actually the result of the emission only from the layer which is extremely close to the surface, something like this. So, only this one, the radiation energy is able to come out, for these layers it is not. And therefore though radiation is a volumetric phenomenon, but in practical operation its implication is like a surface phenomenon only.

Only the molecules of the layers which are extremely close to the surface, they can participate in the net radiation energy leaving the surface and the very close molecules that we are referring to is of that of microns, and so we are really talking about some layers extremely close to the surface. Now here, we are talking about energy transmission through a solid and there is no free surface as such.

So, if we are talking about energy transmitted from this surface to this side or from this surface to this side, there radiation may come into picture and there will be convection also. If air or some other fluid is flowing over these 2 surfaces, surface 1 and 2. However, our interest is only from say this surface number is 1 and this surface number is 2. Our interest is not to calculate the heat transfer from 1 to neighboring fluid or 2 to neighboring air; rather it is only from 1 to 2.

So, we are talking about heat transfer within a surface or inside the solid body and therefore radiation also is not coming into picture. This is a scenario of pure conduction heat transfer. To solve this, we have to take a few assumptions like; first assumption that we are taking is steady-state operation, then we are assuming the thermal conductivity to be constant which is given as 1.7 W/m.K, and also we are assuming 1-D conduction heat transfer, that is heat is being transferred only from surface 1 to surface 2 in this particular direction.

The generalized conduction equation we shall be developing in the next module. There we shall be seeing that we are only considering a very specific form of heat conduction equation. So this is the x direction that is shown from 1 to 2. Then, we can write that the conduction heat flux in the x direction to be equal to

$$\dot{q}_x'' = -k \frac{dT}{dx}$$

which can be approximated as

$$\dot{q}_x'' = -k \left(\frac{T_1 - T_2}{\Delta x} \right)$$

where T1 and T2 are the temperatures of surface 1 and 2 respectively and Δx is the distance.

How can we write this? Let us say, we are talking about T₁ refers to T at 1, T₂ refers to T at surface 2, which is located at a position; say T₁ at x, this position T₂ at x + Δx. Now, if Δx is sufficiently small, this can be expanded following Taylor series.

$$T(x + \Delta x) = T(x) + \left. \frac{dT}{dx} \right|_x (\Delta x) + \left. \frac{d^2T}{dx^2} \right|_x (\Delta x)^2 / 2 + \dots$$

Now if Δx is sufficiently small, we can neglect all the terms from 2nd order onwards. In that case,

$$T(x + \Delta x) = T(x) + \left. \frac{dT}{dx} \right|_x (\Delta x)$$

So if Δx is sufficiently small, this temperature gradient can be approximated in terms of the corresponding temperature difference.

$$\frac{T(x + \Delta x) - T(x)}{\Delta x} = \left. \frac{dT}{dx} \right|_x$$

Here, the Δx is 0.15 m, which is not a very small length; but if we consider the other dimensions something like 1.2 m or 0.5 m, this may be quite small.

In that case, we can have this dT/dx to be quite significant in that x direction. So, let us calculate the magnitude of this. So, we can write the heat flux to be equal to

$$\begin{aligned}\dot{q}_x'' &= -1.7\left(\frac{1400 - 1150}{0 - 0.15}\right) \\ &= 2833\end{aligned}$$

as K is equal to 1.7 W/m.K , T_1 is 1400 K , T_2 is 1150 K , Δx is 0.15 m . Here actually we have written it this way; 1400 is the temperature at location 1 where the distance is 0 and here it is 0.15 , so this is the distance that has been transferred from 0 to 0.15 .

Accordingly, the minus sign cancels out and I have pre-calculated the value, it is coming to be equal to 2833 . Now what should be the unit of this quantity? This is heat flux. Here what is \dot{q}_x'' , q is heat or energy, so its unit is joule, dot refers to per unit time, so second; and double prime refers per unit length scale or per unit length scale squared that is area; so meter square. And joule per second we know is watt. So correspondingly the unit for this quantity becomes W/m^2 .

So this heat flux is having a unit of W/m^2 , and our interest is the total heat transfer in x direction. So that will become the heat flux multiplied by the area, so it is

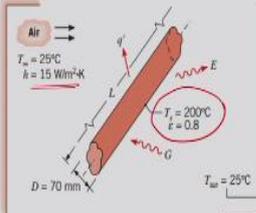
$$\begin{aligned}\dot{q}_x &= \dot{q}_x'' A = 2833 (0.15 \times 1.2) \\ &= 1700 \text{ W} \\ &= 1.7 \text{ kW}\end{aligned}$$

so it becomes 1700 W or 1.7 kW . So this is the total conduction heat transfer that we shall be getting under steady state using one dimensional approximation from surface 1 to surface 2 .

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Exercise 2

An uninsulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200°C and 0.8, respectively. What are the surface emissive power and irradiation? If the coefficient associated with free convection heat transfer from the surface to the air is 15 W/m²·K, what is the rate of heat loss from the surface per unit length of pipe?



Assumptions

- steady-state operation
- room much larger than pipe in relation with radiation exchange
- equal emissivity & absorptivity = 0.8

$T_s = 200^\circ\text{C} = 473.15\text{ K}$
 $T_\infty = 25^\circ\text{C} = 298.15\text{ K}$
 $\epsilon = \alpha = 0.8$
 $h = 15\text{ W/m}^2\cdot\text{K}$
 $\sigma = 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4$

$\rightarrow E = \epsilon \sigma T_s^4 = (0.8)(5.67 \times 10^{-8})(473.15)^4 = 2270\text{ W/m}^2$
 $\rightarrow G = \sigma T_\infty^4 = (5.67 \times 10^{-8})(298.15)^4 = 447\text{ W/m}^2$

$\dot{q} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}}$
 $= h(\pi D L)(T_s - T_\infty) + (\epsilon \sigma)(\pi D L)(T_s^4 - T_\infty^4)$

$\rightarrow \frac{\dot{q}}{L} = \dot{q}' = (\pi D) [h(T_s - T_\infty) + (\epsilon \sigma)(T_s^4 - T_\infty^4)]$
 $= 577 + 42$
 $= 998\text{ W/m}$

$\dot{q}'_{\text{rad}} = (\pi D) [E - \alpha G]$
 $= (\pi D)(\epsilon \sigma)(T_s^4 - T_\infty^4)$
 $= 42\text{ W/m}$

Similarly we can consider another numerical problem which involves convective and radiative heat transfer. Here, our problem is an uninsulated steam pipe which is passing through a room in which the air and walls are at 25 °C. So the surroundings are 25 °C. The outside diameter of the pipe is 70 mm. And its surface temperature and emissivity are 200 °C and 0.8 respectively. What are the surface emissive power and irradiation?

If the coefficient associated with free convection heat transfer from the surface is to be to the air is 15 W/m²·K, what is the rate of heat loss from the surface per unit length of the pipe? So there are 2 parts of the problem. This is the schematic representation. We have a pipe which is having a diameter of 70 mm. Its length is unknown and but we know that its surface temperature is 200 °C and its emissivity is 0.8 and it is kept inside the room which is maintained at a constant temperature of 25 °C everywhere and also air is flowing over this surface.

So convection will be there having magnitude of 15 W/m²·K, the convective heat transfer coefficient. So here, we definitely convection is in the picture because we are talking about heat transfer from the surface of the pipe to surrounding air and which air is flowing, so we have convection. Radiation is also there because from the surface radiation energy can go to the surrounding. Now, whether conduction will be there or not?

Conduction we don't need to consider in this case because conduction is generally prevalent inside a single medium whereas here we are generally talking about an interface from the solid to the fluid. So conduction is not coming into picture. Here we can assume again steady-

state operation. We are assuming the room to be much larger than the pipe in relation with the radiation exchange.

This gives us an advantage that whatever irradiation this surface of the pipe is receiving, that is coming only from the room. That is entire irradiation is coming from the room and the walls which are maintained at 25 °C. So let's assume the size of this surface or rather the pipe to be extremely small compared to the room. And we are also assuming emissivity and absorptivity to be equal, as no information is given about absorptivity. So we are assuming it to be equal to emissivity which is equal to 0.8.

Now, this absorptivity I have not mentioned earlier that will now come into picture in relation with the irradiation. Now, let us first try to calculate the emissive power and irradiation, but before that just note down the information. So, here your surface temperature is 200 °C and surrounding temperature is 25 °C. Now, it is a good practice to convert everything to absolute units or SI units. In conduction and convection analysis, we primarily deal with the temperature difference.

So whether it is in Celsius or Kelvin, it doesn't matter. But in radiation we have to deal with the absolute temperature, and therefore we have to mention or we have to do have to convert this temperature to the absolute counterpart and better to do it now itself. So it will become 473.15 K and this will become 298.15 K as absolute zero refers to minus 273.15 °C.

$$T_s = 200^{\circ}C = 473.15K$$

$$T_{\infty} = 25^{\circ}C = 298.15K$$

Now, we have

$$\varepsilon = \alpha = 0.8$$

Convective heat transfer coefficient is given as 15 W/m².K, and something not given here but we probably have to remember this one, please try to remember the value of this constant

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$$

that is the Stefan-Boltzmann constant. Please try to remember this value, it is quite easy 5 6 7 8 in order, just you have to identify the location of the decimal point. Now let us first calculate the emissive power.

The emissive power from the surface epsilon will be equal to if it is an ideal surface that is a black surface it will be equal to

$$E = \sigma T_s^4$$

but it is a real surface, so it has emissivity less than 1, so emissivity also comes into picture.

$$E = \epsilon \sigma T_s^4$$

So putting this,

$$E = \epsilon \sigma T_s^4 = 0.8 \times 5.67 \times 10^{-8} (473.15)^4 = 2270$$

What should be the unit for this?

Emissive power, though we are calling it a power, actually is a heat flux that is a radiant heat flux and therefore its unit again will be W/m^2 . Now irradiation G, refers to the total radiation energy received by the surface of the pipe and here this assumption comes into picture. Here, we have assumed the room to be much larger than the pipe and therefore whatever irradiation the pipe is receiving that is coming only from the room.

So, the total radiation energy that is falling on this surface will be equal to

$$G = \sigma T_\infty^4 = 5.67 \times 10^{-8} \times 298.15^4 = 447$$

as the room is much larger actually we can assume the room to be an ideal surface or black surface, you don't have to consider emissivity of the surface. We shall be learning more technical details about all these in module 9. So, that is coming in this particular case is 447. This is also a heat flux, so it is W/m^2 .

This heat flux refers to the total radiation energy leaving the surface. This one refers to total radiation energy received by the surface. So, now we have to calculate the heat loss from the surface per unit length of the pipe. Now, we know that total energy transmitted from the surface is a summation of the convective part and the radiative part. Now for the convective part, we can write this one to be convective heat transfer coefficient into the area involved with this, how much is the area.

$$\begin{aligned} \dot{q} &= \dot{q}_{conv} + \dot{q}_{rad} \\ &= h(\pi DL)(T_s - T_\infty) + \epsilon \sigma (\pi DL)(T_s^4 - T_\infty^4) \end{aligned}$$

It is a cylindrical pipe, so the area can be written as πDL where d is the diameter, L is the length of the tube. And the radiative heat transfer will be equal to $\epsilon \sigma (\pi DL)(T_s^4 - T_\infty^4)$. So

area remains the same. So, you can easily put all the numbers, the problem is don't know the length and actually we don't need to know the length also because the entire calculation you have to do as per unit length of the pipe.

So, if you divide this heat transfer by L, then we have the heat loss per unit length of the pipe, but what notation we should use? This is energy or heat that we are talking about, we are talking about the rate, so the dot comes into picture and it is per unit length, so only one line, one prime. So this is the heat loss per unit length of the pipe. It should be equal to

$$\frac{\dot{q}}{L} = \dot{q}' = (\pi D)[h(T_s - T_\infty) + \epsilon\sigma(T_s^4 - T_\infty^4)]$$

Look at here, here the T_s and T_∞ if you are putting them in Celsius, it doesn't matter for the convective heat transfer part because there we are dealing with the quantity $T_s - T_\infty$ and therefore whether you are putting the values in Celsius or Kelvin, it doesn't matter because 1 °C is equivalent to 1 °K. However, in case of radiative part, it does matter where we have to put it in SI units. That is in Kelvin and therefore it is a good practice always to convert everything to SI units.

So if you put the numbers, it will be coming to be equal to, this part will be coming something like 577 approximately and this will be coming to be something like 421. So total energy is total heat loss per unit length is 998.

$$\begin{aligned} &= 577 + 421 \\ &= 998 \frac{W}{m} \end{aligned}$$

What is the unit? W/m, because we are calculating the rate of heat loss per unit length of the pipe, so we have W/m. So, this is the emissive power, this is the irradiation, and now this is the rate of heat loss.

This radiative part we could have calculated in a different way also. Like if we want to calculate this radiative part per unit length, then that can be written as total energy leaving the surface minus the amount of energy absorbed by the surface; and how much energy is absorbed by the surface? G is the amount of irradiation that is falling on the surface, but entire part of G will not get absorbed by the surface. There are generally 3 possible phenomena, 3 possible scenarios on irradiation falling on a surface.

The surface can absorb a part of the irradiation, surface can reflect a part of the irradiation, and third possibility, if the surface is transparent some amount of energy can also get transmitted through the surface. So there are 3 possible scenarios, that total irradiation can be divided into 3 parts. One is absorption part, one is transmission part, and other is reflection part. Now the reflection means surface is coming back to the surrounding, transmission means it is just going straight to the surface without getting absorbed.

It is only the absorption part that leads to the energy gain on the surface, not the other two part, and that is why the absorptivity α comes into picture. Here α , absorptivity refers to the fraction of energy, irradiation rather, absorbed by the surface. So α is the fraction of irradiation absorbed by the surface; and here we have taken α to be equal to ϵ . So from there we can write this to be equal

$$\begin{aligned}\dot{q}_{rad} &= (\pi D)[E - \alpha G] \\ &= (\pi D)[\epsilon\sigma(T_s^4 - T_\infty^4)]\end{aligned}$$

which is the same expression which we have used here. If you calculate this number, this again will be equal to 421 W/m. So, this way we can perform simple calculation, simple energy related calculations involving convection and radiation as well.

(Refer Slide Time: 39:11)

Concept of thermal resistance

$\dot{q}_{conv} = h(T_s - T_f)$
 $\Rightarrow \dot{q}_{conv} = h(\Delta T)$
 $\Rightarrow \frac{\Delta T}{\dot{q}_{conv}} = \frac{1}{h}$

$\Delta T \rightarrow$ driving potential
 $\dot{q} \rightarrow$ effect

$E = IR$
 $\Rightarrow \frac{E}{I} = R$

$\frac{1}{h} \equiv R \leftarrow$ Thermal resistance

$\frac{\Delta T}{\dot{q}_{conv}} = \frac{1}{hA} \Rightarrow R_{conv} = \frac{1}{hA}$

$\dot{q}_{conv} = (hA)\Delta T$
 $= (hA)\Delta T \Rightarrow \frac{\Delta T}{\dot{q}_{conv}} = \frac{1}{hA} \Rightarrow R_{conv} = \frac{1}{hA}$

So, I shall be finishing this lecture by discussing a very small concept or but a very powerful concept particularly in relation with conduction heat transfer, which is the concept of thermal resistance. Now, just take a look at the convective heat transfer relation that we have used, the Newton's law of cooling. There we had the relation as heat flux to be equal

$$\dot{q}_{conv}'' = h(T_s - T_\infty)$$

$$\dot{q}_{conv}'' = h\Delta T$$

This is convection. Now, what is the left hand side? That is your convective heat flux.

On the right hand side we have ΔT . Now, ΔT is the temperature difference. Now, here the heat transfer that is happening by convection. Why that is happening, that is happening because of this ΔT , because of the temperature difference; because from the second law of thermodynamics we know that, the heat transfer is possible only when there is a temperature difference. If there is no temperature difference, there will be no heat transfer. Therefore, this ΔT can be identified as the cause of having this heat transfer.

Therefore, we can always identify this ΔT to be some kind of driving potential. In this case, the magnitude of this heat flux is also directly proportional to this temperature difference. So the ΔT is the cause or the driving potential, whereas the corresponding heat flux that we are getting that is the effect or outcome of this driving potential.

Or, now if we represent this one in a slightly alternate way

$$\frac{\Delta T}{\dot{q}_{conv}''} = \frac{1}{h}$$

Now, what we have on the left hand side. In the numerator we have the driving potential, on the denominator we have the effect of imposing the driving potential. Have you seen any other equation of similar form, you must have seen in your school level physics in relation with electricity.

Do you remember the Ohm's law of heat conduction, Ohm's law of electricity, there what you had? You had

$$E = IR$$

where E refers to a potential difference, I refers the current, R is the resistance or

$$\frac{E}{I} = R$$

So, here on the left hand side in the numerator you have the cause or potential, on the denominator you have the effect of that or the outcome of that. Because of the imposing this potential difference only we have the flow of electrons, which leads to this current. And these

2 equations, are quite analogous to each other because in both cases on the left hand side, you have the ratio of the cause to the effect of this and accordingly this $1/h$ quantity is equivalent to the resistance and we call it to be thermal resistance.

$$\frac{1}{h} \equiv R$$

This concept of thermal resistance is a very useful one in case of heat transfer analysis, where for any kind of heat transfer scenario conduction, convection or radiation, we can always calculate a corresponding thermal resistance and then perform the analysis following a way quite similar to the electrical circuits, because just the way we can keep on adding electrical resistances in series or parallel, the same way we can assume a thermal system to comprise of several thermal resistances connected in series and parallel. More on this one we shall be seeing in the next week, but before that what we are seeing that in case of convective heat transfer, the thermal resistance can be written as $1/h$ or more commonly we generally write this as

$$\frac{\Delta T}{\dot{q}_{conv}} = \frac{1}{hA}$$

Accordingly, we get the convective resistance to be equal to

$$R_{conv} = \frac{1}{hA}$$

If we just compare this one to the radiation situation that we have seen, can you do it? The radiation heat transfer if we use the idea of radiative heat transfer coefficient, we can write this on quite similar to the Newton's law of cooling that is

$$\dot{q}_{rad} = h_r A(T_s - T_\infty)$$

$$\dot{q}_{rad} = h_r A\Delta T$$

So from there, you can again write

$$\frac{\Delta T}{\dot{q}_{rad}} = \frac{1}{h_r A}$$

So the thermal resistance corresponding to radiative heat transfer equal to

$$R_{rad} = \frac{1}{h_r A}$$

Mind you, this h_r actually is the function of the temperature itself. So, it is more like an implicit relation. But still we can get a radiation related thermal resistance the same way. For conduction, we shall be discussing in detail in the next lecture.

(Refer Slide Time: 44:49)

A note on dimensions

$L \rightarrow m$
 $M \rightarrow kg$
 $T \rightarrow s$
 $\theta \rightarrow K$

$Q \rightarrow J$
 $\dot{Q} \rightarrow W$
 $\dot{Q}'' \rightarrow W/m^2$

$\dot{Q}_{cond}'' = -k \frac{dT}{dx} \Rightarrow k = -\frac{\dot{Q}''}{\frac{dT}{dx}} \equiv \frac{W}{m^2} \frac{m}{K} \equiv \frac{W}{m \cdot K} \equiv W/m \cdot K$
 $\dot{Q}_{conv}'' = h(T_s - T_f) \Rightarrow h = \frac{\dot{Q}_{conv}''}{T_s - T_f} \equiv \frac{W}{m^2} \frac{1}{K} \equiv \frac{W}{m^2 \cdot K} \equiv W/m^2 \cdot K$
 $\rightarrow \dot{Q}_{rad}'' = \epsilon \sigma (T_s^4 - T_f^4)$
 $\Rightarrow \epsilon = \frac{\dot{Q}_{rad}''}{\sigma (T_s^4 - T_f^4)} \equiv \frac{W}{m^2} \frac{m^2 \cdot K^4}{W} \frac{1}{K^4} \equiv \text{---}$

And Finally, I would like to finish with a note on the dimensions. In this course, we are strictly going to follow the SI units and we are going to restrict ourselves to the basic dimensions as length, mass, time and temperature. So, the basic unit for length is meter. Generally to denote the dimensions we use capital symbols, so this is mass, this is time, and for temperature we generally use the θ .

So we are going to follow the LMT θ system where mass is having the basic unit of kg, T is having the basic unit of second and θ is having the basic unit of Kelvin. So, these are the fundamental units that we are going to consider. Of course, you may have prefix added to this meter and kg or may be with second also, like kilometer or something similar.

Similarly, the quantities like q will be having a unit of joule whereas \dot{q} will be primarily having the unit of J/s or W. We can of course have kJ, kW or MW kind of quantities. Similarly as we have seen heat flux refers to W/m^2 . So the same way, can we calculate the units for the thermal conductivity? We have already seen the unit but let's try to derive the unit again here.

You have seen that for conduction using the Fourier's law of heat conduction, we have the conduction flux to be equal to

$$\dot{q}'' = -k \frac{dT}{dx}$$

That is,

$$k = -\dot{q}'' / \left(\frac{dT}{dx} \right)$$

From there, if we try to get its unit, then in the numerator you have the heat flux, which is having the unit of W/m^2 . In the denominator you have the temperature gradient, so its unit will be equal to $W/m.K$, this dot is important between meter and Kelvin, because if you don't put the dot, it will look more like a milli Kelvin. So, just to indicate that it is this m is not a part of K, rather it is a separate unit, we have to put the dot. Similarly if we do for Newton's law of cooling,

$$\dot{q}_{conv}'' = h(T_s - T_\infty)$$

That is

$$\begin{aligned} h &= \dot{q}_{conv}'' / (T_s - T_\infty) \\ &\equiv \frac{W}{m^2} \frac{1}{K} \equiv \frac{W}{m^2 \cdot K} \end{aligned}$$

So from there if we get the try to get the unit, in the numerator you have the heat flux which have the unit of W/m^2 again, in the denominator you have temperature difference you are having the unit of K.

So the unit becomes W/m^2K ; or more commonly written as $W/m^2.K$. So there is a length scale difference between the unit of this K and h. What about the unit for the emissivity that we have used earlier? For radiative heat flux, we have seen that it is equal to

$$\dot{q}_{rad}'' = \varepsilon \sigma (T_s^4 - T_\infty^4)$$

So, emissivity can be represented as,

$$\varepsilon = \dot{q}_{rad}'' / \sigma (T_s^4 - T_\infty^4)$$

So, from there if we try to get the unit, in the numerator we have the heat flux, so it is W/m^2 , Truly speaking, we should have got the dimension of the Stefan-Boltzmann's constant from here but as I have already given that, so I am trying to get the unit of emissivity from here. So for Stefan-Boltzmann constant we know unit is W/m^2K^4 , and also you have it here $(T_s^4 - T_\infty^4)$, which is K^4 . So, what is the dimension?

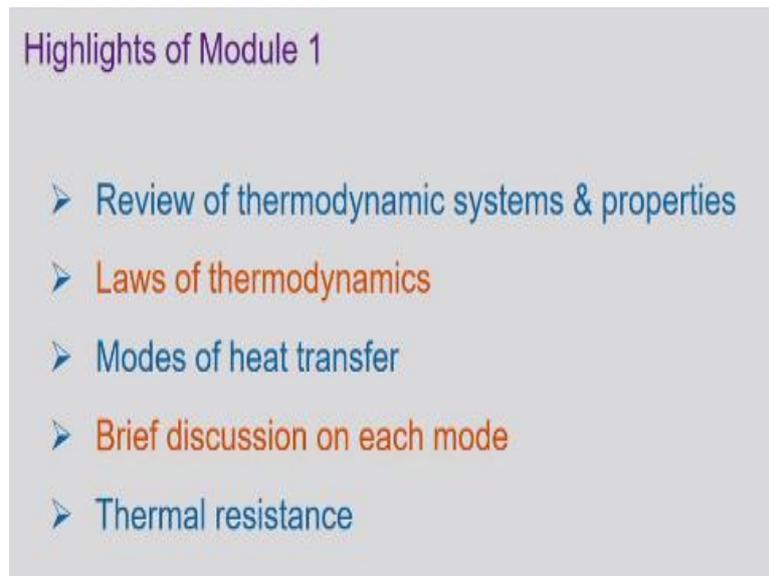
$$\equiv \frac{W}{m^2} \frac{m^2 \cdot K^4}{W} \frac{1}{K^4} \equiv -$$

Everything cancels out, there is nothing left out, and emissivity is a dimensionless quantity. Same is about reflectivity, and that is logical also because emissivity just compares the behavior of a real surface to the ideal surface and that is why it is only a ratio of heat fluxes.

But just be in a nutshell, always try to use SI units and ensure that whenever you are writing an equation, something like this, the right hand side and left hand side should always have the same dimension.

Any dimensional inconsistent equation has to be wrong and this dimension always gives us some idea to us whether our calculation is going the right direction or not at least. So, this is where I would like to stop for this week.

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In this module 1, we have just discussed about some initial concepts. We started with reviewing the concept of thermodynamic systems and properties, then briefly went through the Zeroth, first and second law of thermodynamics and tried to establish the relation and need of heat transfer from there. Then we have seen there are 3 primary modes of heat transfer and the difference between each of them has been explained.

Then we discussed the fundamental laws for each of them like Fourier's law of heat conduction, which gives the conduction heat flux to be proportional to the temperature gradient in a particular direction. The convective heat flux has been found to be proportional to the temperature difference which is given by the Newton's law of cooling, whereas the Stefan-Boltzmann's law of radiative heat transfer says that the radiative heat flux is proportional the difference of temperature raise to the power 4 and we have solved some simple numerical problem also using this relations.

And finally, the concept of thermal resistance was introduced which will come back in much greater detail in the next week. So that takes us to the end of module number 1. Please go through the corresponding chapter of your text books like Incropera and Dewitt. Please go through chapter number 1 of that text book. Even that 2 numerical problems that I have solved are taken directly from the solved problems of that book, but you can try to solve a few problems from the exercise from that book or any other standard text books like the ones mentioned in the course outline.

And if you have any query, please write back to me. The assignments have already been available online. So please try to solve the assignments also in the stipulated period. Thank you very much.