Fundamentals of Conduction and Radiation
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Lecture - 18
Transient Heat Conduction – Part I

Hello everyone. So today we will learn Transient Heat Conduction. Till now we have studied the steady state heat conduction where temperature was not function of time. So now we will consider Transient Heat Conduction where temperature may vary with time as well as space. There are many applications where we can find the Transient Heat Conduction. You can see the turbine blades where suddenly the hot steam comes inside the turbine that time turbine blades get heat and its temperature increases.

So it is an example of Transient Heat Conduction. Similarly, you can see the earth surface get heated from the sunshine so in the morning to evening there will be change in the temperature so this is also one example of Transient Heat Conduction. In addition, you can see the motorbike engine where you can see that when combustion takes place the temperature rises very high and later it comes down. So there will be periodically change in temperature which is also one example of Transient Heat Conduction.

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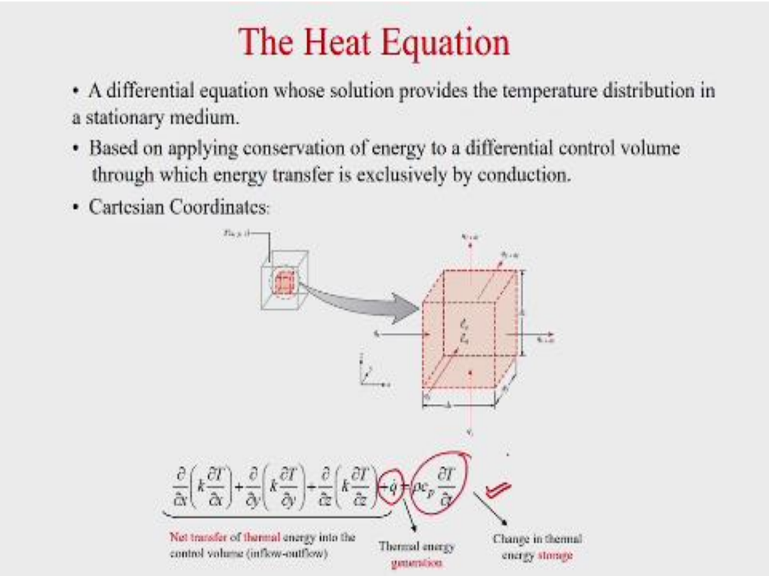

So let us see in earlier classes you have already derived this energy equation from the energy balance.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

So first three terms contains the net transfer of thermal energy into the control volume, essentially it is inflow minus outflow and 4th term is the thermal energy generation per unit volume and right hand side is the transient temperature okay. So you can see it is rate or change of temperature with respect to time.

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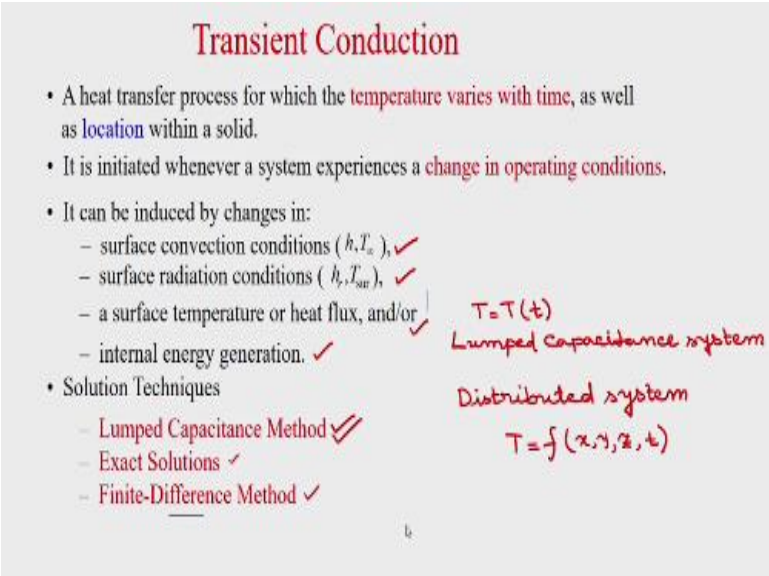

So a heat transfer process for which the temperature varies with time as well as location within a solid is known as Transient Conduction. So there may be two possibilities. One is temperature may vary only with time okay. So your T may vary only with time okay, but there will be no spatial variation of temperature okay, so that is known as Lumped Capacitance System; where everywhere inside the solid temperature will be same.

Another case may be distributed system so where temperature may vary with time as well as space. So another one is Distributed System, where temperature is function of space as well as time. So in this case spatial variation will be there as well as temperature will vary with time.

So what causes the transient heat transfer inside the solid? There may be different reasons, let us see. So it is initiated whenever a system experience a change in operating condition okay. So it can be induced by changes in surface convection condition; so ambient temperature changes or ambient fluid changes. So if you change the fluid that means your heat transfer coefficient of the ambient fluid will change.

So that means if you change with time this h or T_∞ then there may be transient heat conduction inside the solid. Next see surface radiation condition. So obviously your outside surface temperature if it changes or effective heat transfer coefficient changes then also there will be Transient Heat Conduction. And also a surface temperature or heat flux on the boundary if it changes with time then we will get the Transient Heat Conduction. Transient Heat Conduction will also appear if there is a internal energy generation.

So there are different solution techniques. One already we have told that lumped capacitance method where temperature will not vary in space only it will vary with time. So mostly we will consider in this course this lumped capacitance system. Distributed system just we will discuss, but it is not possible to find the solution in this course. We can also get exact solution and there are some numerical methods we can use like finite difference method so which we will do in this course in some module.

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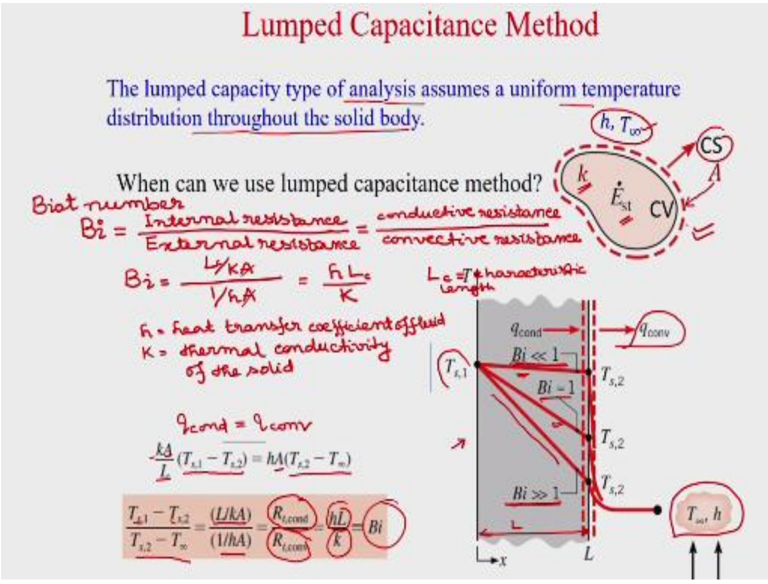

So we already told that the lumped capacity type of analysis assumes a uniform temperature distribution throughout the solid body okay. So when can we use lumped capacitance method? So when this lumped system analysis is valid? It is actually determined by one non dimensional number which is known as Biot number. So this Biot number actually determines whether you can use the lumped capacitance system method or not for the solution of the temperature in a solid.

So let us consider here one control volume okay with a control surface. And its thermal conductivity is k and there maybe with time some energy storage and ambient condition is your heat transfer coefficient is h and T_∞ is the ambient temperature and surface area this heat conducting surface area is A. So with this if we define the Biot number,

$$Bi = \frac{internal\ resistance}{external\ resistance} = \frac{conductive\ resistance}{convective\ resistance}$$

So here this heat conduction taking place inside the solid and heat loss or gain is happening due to the heat convection from the ambient. So in this case what is your internal resistance; internal resistance is nothing but the conductive resistance. And what is external; that is your convective resistance.

So Biot number is the ratio of conductive resistance to the convective resistance. So let us write it. So in this case if you consider this system okay, so what is the conductive resistance. So you have already derived in earlier classes that is nothing but L/kA.

And what is the external resistance, means convective resistance that is heat transfer coefficient is h and heat transfer area is A then you will get 1/hA as the convective resistance. So let us write it.

$$Bi = \frac{\frac{L}{kA}}{\frac{1}{hA}} = \frac{hL}{k} = \frac{hL_c}{k}$$

Instead of L you can write L_c where L_c is the characteristic length, and h is the heat transfer coefficient of the ambient fluid and k is the thermal conductivity of the solid.

So if it is a plane wall or sphere what will be the L_c; that we will define later. So Biot number is just ratio of conductive resistance to the convective resistance. So let us see one plane wall as you see this figure okay.

So this is one plane wall where your left wall temperature is maintain at $T_{s,1}$ okay and in the right wall the heat loss is there due to convection okay, and you have the heat transfer coefficient h and the ambient temperature is T_∞. So now depending on the Biot number you will see whether your internal resistance is more or less. So accordingly your temperature distribution will happen inside the solid okay.

So let's do the energy balance on right wall.

$$q_{cond} = q_{conv}$$

Using Fourier's law and newton's law of cooling

$$-\frac{kA}{L}\left(T_{s,1}-T_{s,2}\right)=hA\left(T_{s,2}-T_{\infty}\right)$$

Where, k is the thermal conductivity of the plane wall, A is the area of wall perpendicular ot the plane of paper, L the distance between right and left wall, h is the heat transfer coefficient of the ambient fluid and $T_{s,1}, T_{s,2}$ and T_{∞} area the temperatures of left wall right wall and ambient fluid respectively. So now if you rearrange it

$$\frac{\left(T_{s,1}-T_{s,2}\right)}{\left(T_{s,2}-T_{\infty}\right)}=\frac{\frac{L}{kA}}{\frac{1}{hA}}=\frac{R_{cond}}{R_{conv}}=\frac{hL}{k}=Bi$$

So now you have got the definition of Biot number. It is a non dimensional number which is the ratio of conductive resistance to the convective resistance and it is mathematically defined as $\frac{hL_c}{k}$ and which is a important parameter to decide whether lumped capacitance method you can use or not.

Tthat means whether your temperature will be function of only with time or it will be both space and time. So here you can see that if Biot number is very less than 1 okay, then your internal resistance is very less okay. Internal resistance is very less because your k is very high. So in this case there is not much temperature variation inside the solid okay. So temperature gradient is very, very less inside the solid.

So you can assume that your temperature is function of only time and not space, but if you can see Biot number is of the order of 1 then there is some significant temperature gradient and if Biot number >> 1 then also very large temperature gradient you will find. So you cannot use lumped capacitance system there. You have to use distributed system because you can see that here spatial variation of temperature is taking place.

Look here, so here you see your temperature gradient is like this; so your temperature variation is taking place spatially along the X, but in this case when Biot number <<1 then you can see is temperature gradient is very less. So we have defined the Biot number as the ratio of conductive resistance to the convective resistance.

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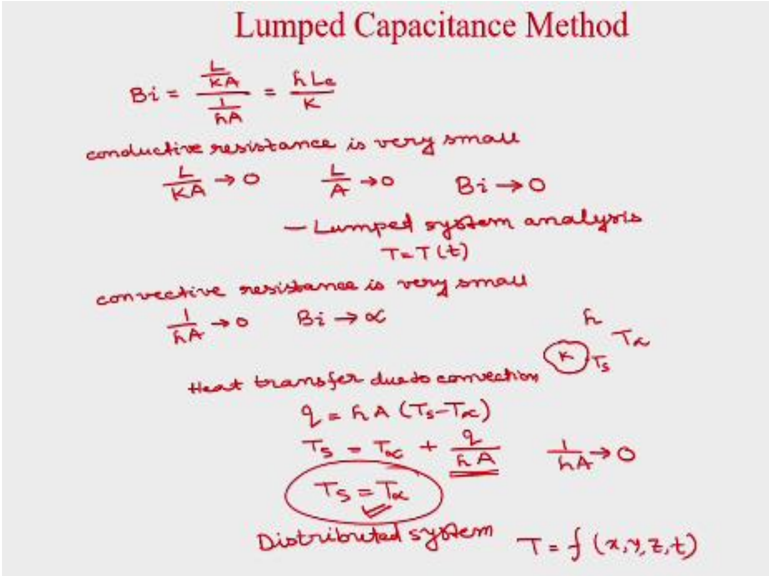

So now let us take two conditions where internal resistance is very small and another case where convective resistance is very small okay. So if you consider first case where conductive resistance is very small okay so what does it mean. So conductive resistance

$$\frac{L}{kA} \longrightarrow 0$$

So for a solid with finite thermal conductivity it will be

$$\frac{L}{A} \longrightarrow 0$$

So in this case what will be Biot number?

$$Bi \longrightarrow 0$$

So if you have a low Biot number then only you can see that your internal resistance is small. So that means there will be no spatial gradient of temperature inside the solid body and where you can use the lumped system analysis because in lumped system analysis we assume the temperature is function of time only.

So here we can use lumped system analysis okay where T is function of time only so no spatial gradient. Now the next condition let us consider where convective resistance is very small.

$$\frac{1}{hA} \longrightarrow 0 \ \Rightarrow Bi \rightarrow \infty$$

Let us see what will be your surface temperature okay. So if your temperature of the surface is T_s and outside is T_∞ and h is your heat transfer coefficient, k is the thermal conductivity of the solid then due to convection what will be the heat transfer? It will be

$$q = hA(T_s - T_\infty)$$

So what can we write for surface temperature?

$$T_s = T_\infty + \frac{q}{hA}$$

So now we have considered convective resistance is very small that means 1/hA is very small. So the second term tends to zero. So we get

$$T_s = T_\infty$$

So what does it means? So if you have a very low convective resistance then the surface temperature will be equal to the ambient temperature T_∞. That means there will be spatial variation inside the solid because your boundary condition is now T_∞. So obviously there will be a spatial variation of temperature inside the solid. So in this case when Biot number is very high you have to use distributed system where T is function of space as well as time okay.

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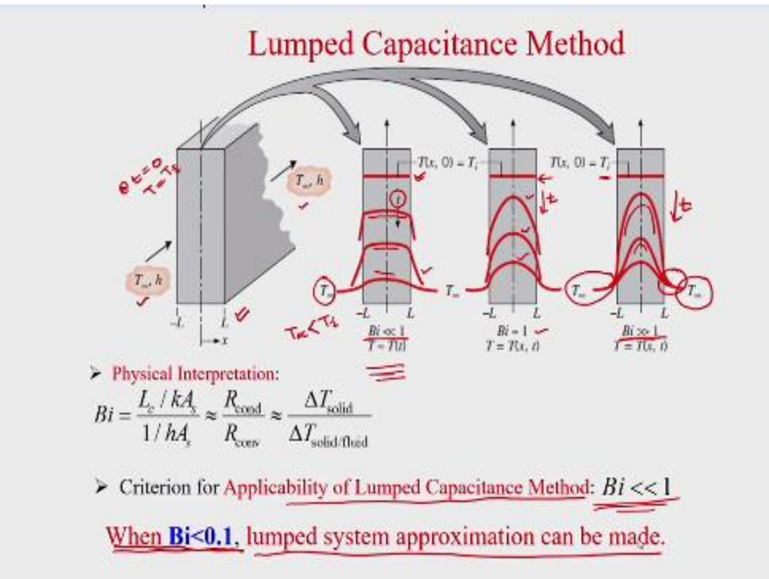

So we will go to the next slide. So now let us consider one plane wall and symmetrically your convection is taking place from both the sides and we will see for different Biot number what will be the temperature distribution inside the solid. So we have considered one plane wall both sides we have same condition, convection condition h and T_∞. So now let us consider where Biot number << 1 where internal resistance you can neglect.

So at at t = 0; initially it was temperature T_i okay and now T_∞ is the ambient temperature where $T_\infty < T_i$. So suddenly this hot plane wall is brought to a medium where you have a low temperature T_∞ and heat transfer coefficient h. So for the case of low Biot number where Bi<< 1; let us see how the temperature will vary inside the solid.

So here you see, so at t = 0 this is the temperature, it is constant. Now slowly with time your temperature will fall because of convection heat loss taking place so your body will cool or plane wall in this case will cool and your temperature will decrease. And how the temperature decreases inside the solid; you can see from this temperature distribution.

So as time decreases you can see that your temperature almost remains same inside the solid that means your internal resistance is very small. For very low Biot number already we have seen that internal resistance is low so it will have the same temperature inside the solid.

So this is the case where Biot number << 1. Now let us consider Biot number of the order of 1. So in this case obviously there will be internal resistance is compared to the convective resistance. So in this case obviously you will have a temperature gradient inside the solid as you can see from here. So here this is the temperature at t = 0 and in this way with time these are the distribution. You can see there is a significant temperature gradient inside the solid at different time okay.

So that means when your internal resistance is comparable to convective resistance then you will have spatial gradient of temperature okay and also it will vary with time that we can see. And for Biot number >> 1; that we have already considered. So in this case your surface temperature will remain same because it is almost T_∞ that we have already shown right.

So if you have very high Biot number that means your convective resistance is very, very small in that case we have told that

$$T_s = T_\infty$$

So in that case you see at time t= 0 this was the temperature distribution, but when time increases you can see that your surface temperature remaining constant because it is almost T_∞ right as we have shown earlier.

But there is a much, much gradient of temperature you can see inside the solid okay. So that means in this case also you have to use distributed system. So already we have discussed this Biot number is the conductive resistance to the convective resistance. And criteria for applicability of lumped system lumped capacitance method is Biot <<1. Wherever this is the case, we can neglect the spatial variation of temperature.

So in general we would say that when Biot number < 0.1 okay you remember it when Biot number < 0.1 then lumped system approximation can be made. So when we solve any problem using lumped system analysis first we have to check the Biot number. If Biot number < 0.1 then we can consider that temperature only varies with time and we can use lumped system analysis.

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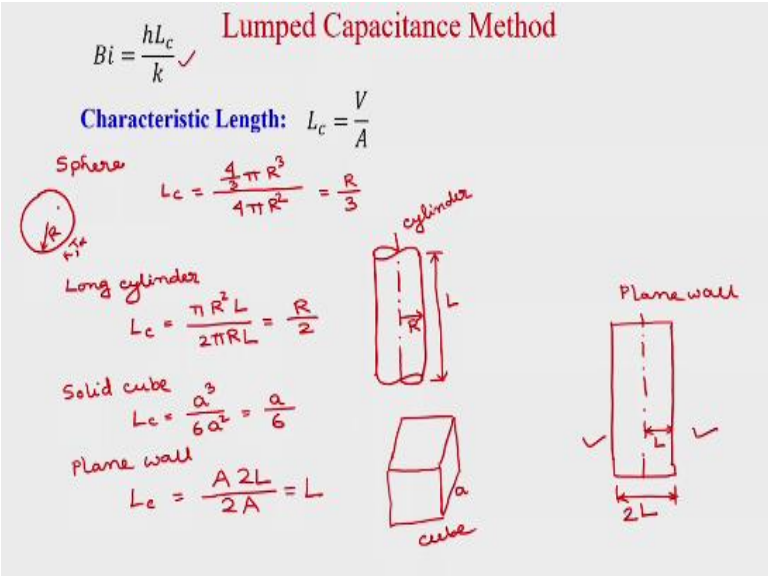

So now we already talked that

$$Bi = \frac{hL_c}{k}$$

Where L_c is the characteristic length and it is defined as

$$L_c = \frac{volume}{heat\ transfer\ area} = \frac{V}{A}$$

Or, heat conducting area in this case because only conduction is taking place. So now for different shape of the body your volume will be different okay, and surface area will be different so you will get characteristic length different okay.

So let us consider one sphere okay, some spherical body. So in this case let us consider what will be characteristics length. So for sphere with radius R

$$L_c = \frac{V}{A} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

So in the case of a solid sphere you will get the characteristics length as R/3. Now let us consider a long cylinder okay. So if it is a long cylinder you can neglect the heat transfer from the two ends and end surfaces will not be included in the heat transfer area. Now if L is length of the solid cylinder and R is the radius of the cylinder then

$$L_c = \frac{V}{A} = \frac{\pi R^2 L}{2\pi R L} = \frac{R}{2}$$

So for a long cylinder your characteristic length is R/2. Now you consider one solid cube of side a okay.

$$L_c = \frac{V}{A} = \frac{a^3}{6a^2} = \frac{a}{6}$$

So for solid cube your characteristic length is a/6. Now let us consider a plane wall of width 2L. So in the plane wall from the both side you will find the heat transfer is taking place. Let us say L is the distance to one side from the central line, so it is a 2L distance plane wall. So if the cross-sectional area of the plane wall is A then its volume is $A \times 2L$. And as from both sides heat transfer is taking place so heat transfer area will be 2A. So

$$L_c = \frac{V}{A} = \frac{A \times 2L}{2A} = L$$

So if you have a plane wall of 2L distance or 2L width then you will get the characteristic length as L okay. So when you use Biot number calculation so Biot number hL_c/k. So L_c you have to calculate the characteristic length in this way whatever way we have discussed.

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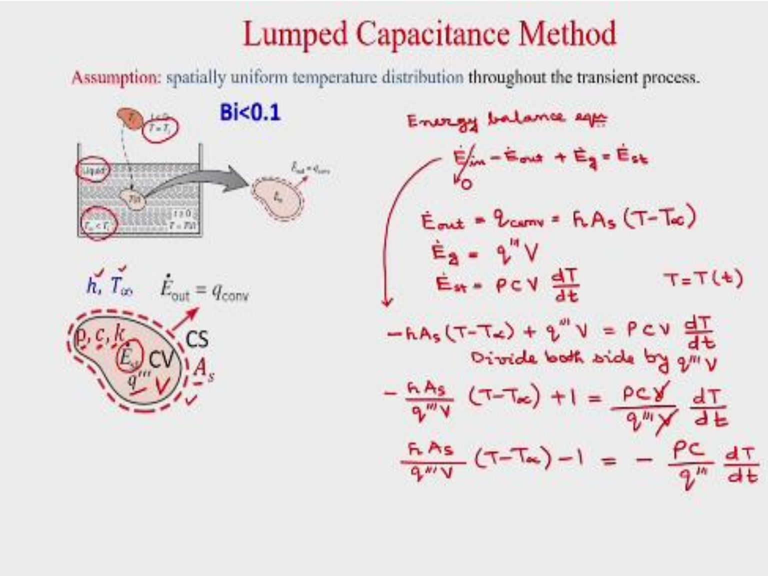

Now we will consider lumped capacitance method where temperature varies with time only. That means we will consider Biot number < 0.1 and in a generalized way we will derive the temperature distribution of the body. So actually how temperature varies with time that we will find in this case. So let us consider a solid body okay, or solid metal body solid metal forging which was initially temperature at T_i, initially it was very hot and its temperature is

T_i. And suddenly it is immersed in the liquid okay, where temperature is T_∞ which is less than T_i okay. So obviously initially it was T_i and when it is immersed inside this liquid and so it will be quenched, so it will lose the heat due to convection to the liquid and it will cool down. And now we want to find the temperature decay with time. So in this case we will consider a generalized case.

And we will consider that inside this solid metal we have some heat generation per unit volume q'''. So this is your solid body where surface area is A_s which is heat transfer area. V is the volume of the body okay. This is the control volume and this is the control surface and there will be energy storage with time $\dot{E}_{st}$. ρ is the density of the solid, heat capacity of the solid is c, k is the thermal conductivity and ambient temperature is T_∞ as we have seen here and h is the heat transfer coefficient okay. So in a generalized way this is we are considering and now we will apply the energy balance equation here. So if you write energy balance, so what you can write

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

So now it is a Transient Heat Conduction so it is a function of time. So as heat is lost in this case from the solid to the liquid so there is no heat in, so $\dot{E}_{in} = 0$ and what is $\dot{E}_{out}$; $\dot{E}_{out}$ is just the heat convection taking place and that is nothing but $hA_s(T_s - T_\infty)$ okay, and in this case T is same everywhere in the solid at the surface as well as inside the body.

Because we have considered Biot number less than 1 and we are doing the lumped system analysis. Now what is Energy generated? So it is $q'''V$, where V is the volume of this body What is $\dot{E}_{st}$? That is you know $\rho cV\frac{dT}{dt}$. So because T is function of t only so we can write dT/dt. So we know all these things. Now you put it in the energy equation

$$0 - hA_s(T - T_\infty) + q'''V = \rho cV\frac{dT}{dt}$$

Now divide both side by $-q'''V$.

$$\frac{hA_s}{q'''V}(T - T_\infty) - 1 = -\frac{\rho c}{q'''}\frac{dT}{dt}$$

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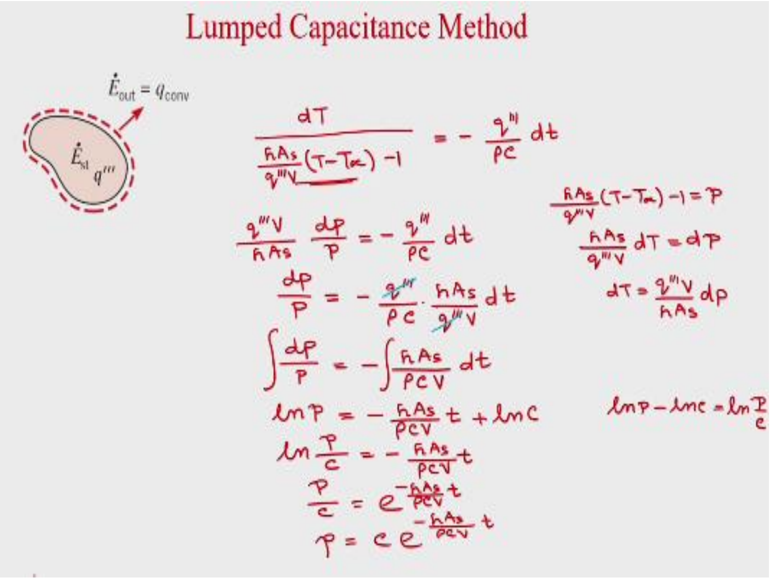

So next slide let us write, so we will write

$$\frac{dT}{\frac{hA_s}{q'''V}(T-T_\infty)-1} = -\frac{q'''}{\rho c}dt$$

So now integrate it. Before that lets write

$$\frac{hA_s}{q'''V}(T-T_\infty)-1 = P$$

$$\Rightarrow \frac{hA_s}{q'''V}dT = dP$$

$$\Rightarrow dT = \frac{q'''V}{hA_s}dP$$

Putting this in earlier equation

$$\frac{q'''V}{hA_s}\frac{dP}{P} = -\frac{q'''}{\rho c}dt$$

$$\Rightarrow \frac{dP}{P} = -\frac{hA_s}{\rho cV}dt$$

So now it is in easier form so you can integrate it.

$$\Rightarrow \int \frac{dP}{P} = -\int \frac{hA_s}{\rho cV}dt$$

$$\Rightarrow \ln P = -\left(\frac{hA_s}{\rho cV}\right)t + \ln C$$

$$\Rightarrow \ln\frac{P}{C} = -\left(\frac{hA_s}{\rho cV}\right)t$$

Everything in the bracket is constant so they can come out of the integration.

$$\Rightarrow P = Ce^{-\left(\frac{hA_s}{\rho cV}\right)t}$$

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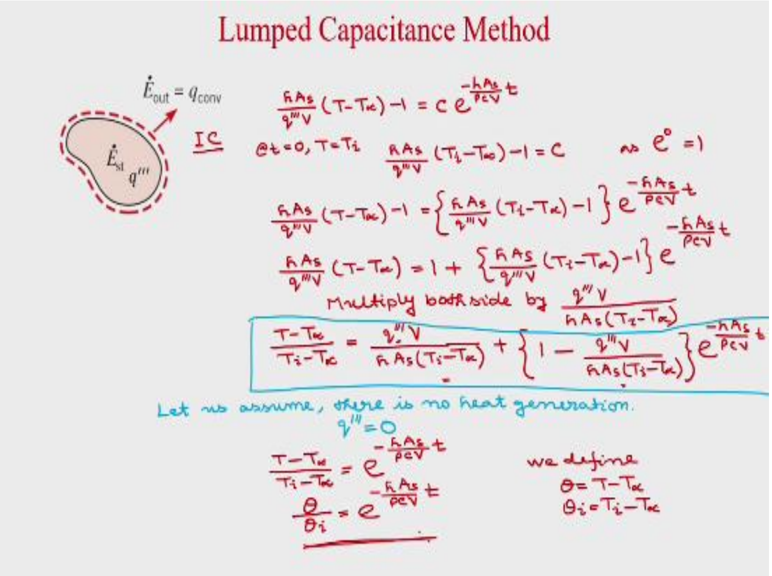

So writning the value of P

$$\frac{hA_s}{q'''V}(T - T_\infty) - 1 = Ce^{-\left(\frac{hA_s}{\rho cV}\right)t}$$

So this is the temperature variation with time. Now we need to calculate the integration constant C. So how you will calculate? You know the initial condition at t = 0; temperature is T_i. Putting it in equation

$$\frac{hA_s}{q'''V}(T_i - T_\infty) - 1 = Ce^{-\left(\frac{hA_s}{\rho cV}\right)\times 0} = C$$

Putting the value of C in the original eequation

$$\Rightarrow \frac{hA_s}{q'''V}(T - T_\infty) - 1 = \left\{\frac{hA_s}{q'''V}(T_i - T_\infty) - 1\right\}e^{-\left(\frac{hA_s}{\rho cV}\right)t}$$

So let us write in a simplified form.

$$\Rightarrow \frac{hA_s}{q'''V}(T - T_\infty) = 1 + \left\{\frac{hA_s}{q'''V}(T_i - T_\infty) - 1\right\}e^{-\left(\frac{hA_s}{\rho cV}\right)t}$$

Multiply both sides by $\frac{q'''V}{hA_s(T_i - T_\infty)}$

$$\Rightarrow \frac{(T - T_\infty)}{(T_i - T_\infty)} = \frac{q'''V}{hA_s(T_i - T_\infty)} + \left\{1 - \frac{q'''V}{hA_s(T_i - T_\infty)}\right\}e^{-\left(\frac{hA_s}{\rho cV}\right)t}$$

So now this is the temperature variation with time okay.

So we considered a generalized case where heat loss is taking place due to convection as well as you have heat generation inside the solid body. Now you, if you make it simpler and let us assume there is no heat generation. So $q''' = 0$. So the equation becomes

$$\frac{(T - T_\infty)}{(T_i - T_\infty)} = e^{-\left(\frac{hA_s}{\rho cV}\right)t}$$

So this is a simplified form where no heat generation we have considered as well as only heat loss is taking place due to convection. So it is a very simplified form and if we define

$$\theta = (T - T_\infty)\ and\ \theta_i = (T_i - T_\infty)$$

Then we can write

$$\frac{\theta}{\theta_i} = e^{-\left(\frac{hA_s}{\rho cV}\right)t}$$

Okay so how the temperature decay in this cases. So temperature decays exponentially okay. So with time you can see that exponential decay of temperature is happening inside the solid because it is a lumped system that means there is no spatial variation inside the solid. So only with time your temperature of the solid body is decaying exponentially okay. So this is the simplified case okay. So in the next class we will start from here. Thank you.