Applied Thermodynamics For Engineers
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Lecture – 09
Use of Property Tables

Hello everyone, welcome to the second lecture of our week number three where we are talking about the properties of pure substance. In the previous lecture, I have very quickly tried to review the thermodynamics involved in the phase change process with particular emphasis on the liquid to vapor or vapor to liquid phase change because that is the more prevalent situation in engineering applications.

Of course, we could have discussed the other possible phase changes as well like the one involving solid and liquid phase or solid and gaseous phase. But the general thermodynamics remains the same. And therefore, we generally like to discuss about the phase change process involving pure substances mostly in terms of the liquid vapor phase change. Now from that concept we have seen that generally during the liquid to vapor or vapor to liquid phase change commonly can have three different kinds of regions: the compressed liquid regime, the super-heated vapor regime and the mixture regime or saturated mixture regime. The saturated liquid state is the transition point between the compressed liquid state and the saturated mixture state where the saturated vapor is a transition point between the saturated mixture and the super-heated vapor stage.

And we have seen that so-far the saturated state generally corresponds to a situation where pressure and temperature are dependent on each other. Therefore, while for compressed liquid and superheated vapor state we can use pressure and temperature as the two independent intensive properties required to identify any other possible property. We cannot do that for the mixture zone rather we have to define a new property which we have identified as the quality.

I just briefly touched up on the name quality, but we shall be discussing a bit more on that. And normally during a typical thermodynamics course or undergraduate the thermodynamics course I would have prefer to spend two or even a two and half lectures on whatever we have summarized the previous one. But these being a second thermodynamics course or second level thermodynamics course where it is expected that you already know basic thermodynamics.

So, I have tried to go through very quickly. Still if you have any queries you can always write it back to me for any kind of clarification. Now there, couple of very important points that we have identified is the critical point and the triple point. And also, one thing we have learned that whenever you are talking about two phases of a substance in equilibrium, then that must be under saturation condition.

That is whenever we are having a liquid phase and the vapor phase of the same substance under equilibrium situation in an equilibrium state, then that must be under the saturated condition that is the corresponding pressure and temperature will be the saturation pressure and saturation temperature. Similarly, if we have three phases of the same substance in equilibrium then that must be moving somewhere along the triple line.

During which or along which the pressure and temperature are again depending on each other, but the specific volume may keep on varying depending upon the relative fraction of both the phases. The same applies for liquid-vapor phase-change as well like temperature and pressure remains constant but the vapor fraction that keeps on changing along with the increase in the vapor content or along with the change in the liquid content.

Now the critical point is something that I would like to spend a bit more time today to start the discussion. And after discussing the critical point I shall be moving on to the use of thermodynamic property tables. Remember in the previous week we talked about or we developed the generalized relation for thermodynamic properties. But as I mentioned during the previous lecture there are several situations where it is not easy to identify a straightforward *PvT* relationship which you can subject to some mathematical manipulation. And therefore, quite often we may have to depend on this thermodynamic tables. So today we shall be learning the use of those thermodynamic tables in relation with the liquid vapor phase change process or in relation with the three regimes that we have identified. But before that we have that critical point to talk about.

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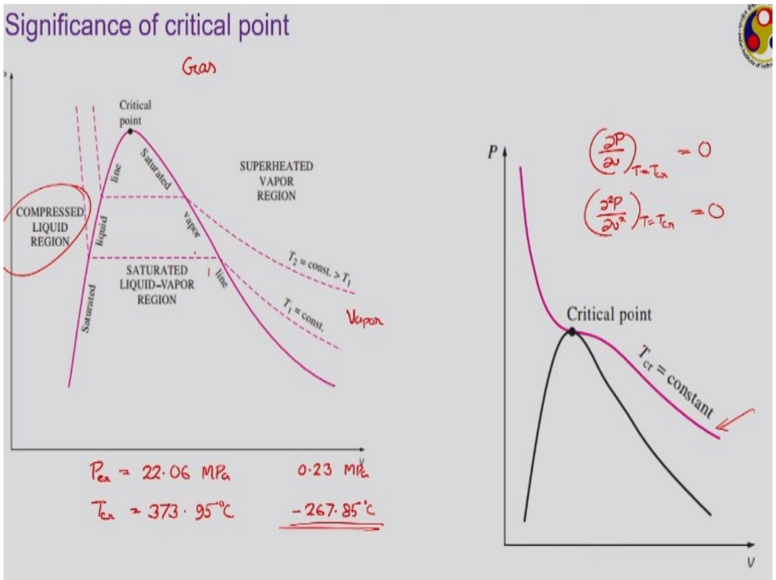

Significance of critical point

$$\left(\frac{\partial P}{\partial v}\right)_{T=T_{cr}} = 0$$

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_{cr}} = 0$$

$P_{cr} = 22.06$ MPa 0.23 MPa

$T_{cr} = 373.95$°C -267.85°C

Now this is the generalised *Pv* diagram that we have seen which is applicable for any substance. Remember here we are discussing mostly in context of water, but that is true for any kind of pure substance. So, we have seen the *Pv* diagram and constant temperature line will look like the couple of dotted lines shown here. That is as the pressure keeps on decreasing the volume increases. But in the compressed liquid zone, liquid being mostly incompressible, the change in volume or specific volume with pressure is extremely small. That is why if you follow any one dotted line, let us see if we start from this particular point and move up to the corresponding saturation point which is this, you can find the corresponding change in specific volume may be only this much. So only this much change in specific volume that we are talking about during that change in pressure from some level like this to, so the level like this the saturation pressure that is a large change in pressure is causing only small change in volume. That is mostly because of the near incompressible nature of the liquid. During the phase change process, pressure and temperature both remains constant, and then in the super-heated vapor regime again with change in pressure with reduction in pressure volume increases rapidly.

Now vapor is much more compressible there are also small change in pressure can cause a large change in volume. Like if we pick up two random pressure level let us say we pick up one pressure level like this and another pressure level like this, you can see corresponding change in volume is this much which is a very large amount in comparison with the liquid that is because of the compressible nature of the vapor phase.

Now, we have the saturated liquid line which is the locus of all the saturated liquid state for all possible pressures, all possible isotherms. Similarly, we have that saturated vapor line this one which is a locus of all the saturated vapor state corresponding to all possible isotherms. And as we keep on moving to higher pressure side the distance between saturated liquid state and saturated vapor state for a given pressure or temperature that keeps on reducing i.e., the change in the specific volume that keeps on reducing. Like if you see in the diagram, we have talked in the previous lecture also. For the temperature T_1, the saturated liquid volume and saturated vapor volume so this much is the difference in their specific volume. Whereas if we go to higher temperature T_2, then the specific volume corresponding to saturated liquid may be something like this and specifically corresponding to saturated vapor maybe something like this so corresponding change in specific volume is significantly smaller in the second situation. So, this is v_{12} for temperature T_1, this is your v_{12} corresponding to temperature T_2 which is definitely much smaller. And as we keep on moving to higher pressure or higher temperature this value keeps on reducing continuously till, we reach to the critical point where the saturated liquid state and saturated vapor state merge with each other and there is no distinction between liquid and vapor at this particular junction. So critical point is a property for any fluid and therefore any substance has a fixed value of pressure and temperature at the critical point. But apart from being the point which removes the distinction between liquid and vapor phase there are some further importance or significance of critical point as well.

Now, let us follow any particular temperature line. Let us follow the temperature line shown as T_1. So, let us start with a super-heated vapor state somewhere here. We are starting from the state point, and at constant pressure we are increasing the pressure of the system. So what will happen? As the pressure increases, we shall be moving in this line that is a volume keeps on decreasing and at some point or after a continuous increasing pressure at some point we shall be reaching the saturation pressure which is this much, so this is your P_{sat} corresponding to this temperature T_1. So, at this particular portion with further with any further effort to increase the pressure will not cause any change in pressure, here that that will cause a change in phase. That is the super-heated vapor does not start to condense to gets converted to the liquid phase.

And with continuous effort to increase the pressure, will keep on condensing of vapor till it reaches a corresponding saturated liquid state, and from that point onwards they can get the pressure keep on increasing. So that we all know now. Now let us do this for a temperature which is higher than the critical temperature. Then a constant temperature line where the temperature is higher than critical temperature, how that will look like?

The line may be somewhat like this. Okay let me draw in a better way the line may be somewhat like this. So here the temperature along this particular line temperature is higher than the critical temperature. So, if we start from say this particular point and maintaining this constant temperature, we are increasing the pressure. Then what will happen? As you keep on increasing the pressure it is not coming in contact with this saturation dome, therefore the phase change will never get initiated and you will see continuous increasing pressure and continuous reduction in the specific volume, but no distinct change in phase. Therefore, when we are operating at a temperature higher than the critical temperature, then just by simple pressurizing we cannot convert a vapor to the liquid phase or you cannot force a vapor to condense.

So, in a way when we are dealing with a temperature level higher than the critical temperature the vapor phase becomes non-condensable for during any constant temperature process. Of course, we can cause a change in temperature from this point onwards like moving through several different temperature levels and may reach at a saturation point like this. But that is not the focus here.

Here we are looking to move along a constant temperature line and the line such that temperature is higher than the critical temperature. Then we can see that we can never condense a super-heated vapor by operating at that level. Similarly, if we are operating at a pressure higher than the critical pressure. Let us say we pick up a line like this, here that definitely the pressure is higher than the critical pressure.

Now we start from a point like this. From this point keep maintaining the pressure constant, if we reduce the temperature of the vapor. If the vapor temperature reduces, then the specific volume will reduce so we are moving along this line remember the pressure is constant. So were moving along the horizontal line towards smaller volume side and you can again see here that the pressure remains constant, temperature decreases correspondingly, specific volume decreases but there is no change in phase because it is not at all coming in contact with the vapor dome. So, when we are operating at a condition, where we are above the critical point then we cannot convert the vapor to a liquid phase or we cannot initiate any condensation process seemed by any simple way. That is for any temperature level above critical temperature, we cannot do it by simply a simple compression.

Similarly, for any pressure level beyond the critical point we cannot do it by simply changing the temperature at a constant pressure. We have to change both pressure and temperature simultaneously. And that is why often the substances which belong at a state above the critical point we call them gas or non-condensable gases because if we cannot at a constant temperature ,we cannot condense these gaseous simply by increasing its pressure.

Whereas when you are dealing with a substance which is below its critical point, we call it a vapor. Like a substance or the super-heated vapor which is at temperature T_1 or T_2 they should be treated as vapor because maintaining that constant temperature simply by compressing we can convert them to liquid. But that we cannot do for any temperature level above the critical point.

For example, if we pick up say, water, for water the critical pressure is 22.06 MPa and critical temperature is 373.95 ^{0}C or 374 ^{0}C roughly. So, if we talk about the standard atmospheric condition which is something like 0.1 MPa and say 30 ^{0}C here both pressure and temperature for water is below the critical point values. Then what will happen? Then suppose if atmospheric temperature is 30 ^{0}C.

This temperature maybe this T_1 then maintaining that constant temperature simply by increasing its pressure you can make water vapor to condense to liquid. Similarly, if you are at the atmospheric pressure level which is 0.1 MPa or 1 bar then, at this constant pressure then you may be operating somewhere along this. At this constant pressure if you just reduce the temperature you can make this vapor to condense.

And therefore, the water under normal atmospheric conditions should be treated as vapor. However, if we are dealing with water in some industrial application water vapor, i.e., where the temperature is something like 1000 ^{0}C quite common possible in IC engine gas turbine applications. Here we are having a gas mixture with temperature is 1000 ^{0}C. Then the water vapor present at this temperature is at a temperature much higher than the critical temperature. And therefore at this temperature by simply pressurizing this vapor we cannot convert it to a liquid and so for this particular condition water should not be called a vapor, water should be called gas. Because it is a condition much beyond its critical point.

Let us talk about another substance. Let us say our second substance choosing is helium. For helium the critical point values are 0.23 MPa which is still higher than the atmospheric pressure but its critical temperature is -267.85 ^{0}C. So, this is extremely small that is the under normal atmospheric condition even in most of the cryogenic applications as well that the temperature of helium is well above its critical temperature.

So, we cannot convert helium to the liquid stage by simply compressing it we first have to reduce the temperature below this value and then only by simple compression you can convert helium to a liquid state. Therefore, helium under normal atmospheric condition should be called as gas and not vapor. The same is true for all common gases like oxygen, hydrogen, nitrogen.

They all have critical temperature well below the atmospheric temperature and that is why we should call them non-condensable gases. However, that is not true for water vapor or certain other substances like ammonia etc, even carbon dioxide you must have noticed during the previous lecture carbon dioxide as a critical temperature of only about a 31 ^{0}C about 304 K.

And therefore, under normal atmospheric condition it particularly in a hot climate of most of our prevailing in most part of India during summer the atmospheric temperature maybe above the critical temperature of carbon dioxide and therefore carbon dioxide vapor can be condensed to the liquid state. There is another significance of the critical point, mostly from a mathematical point of view which is this.

If we plot the Pv diagram and then plot the isotherm corresponding to critical temperature then you will find that this isotherm, which I am talking about this red line. This isotherm exhibits a point of inflection of the critical point, point of inflection means its slope becomes 0 at that point i.e., mathematically we can write as:

$$\left(\frac{\partial P}{\partial v}\right)_{T=T_{cr}} = 0$$

And not only this even it has been found that the second derivative of the same thing:

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_{cr}} = 0$$

These two are very important mathematical criteria which we shall be using in the next lecture while talking about the Van der Waal equation of state. So, this is the discussion that you need to have related to the critical point.

So, what we are going to discuss now is the way to calculate the properties in this compressed liquid regions, saturated liquid vapor region and super-heated vapor region as long as we are restricting ourselves to the critical point values.

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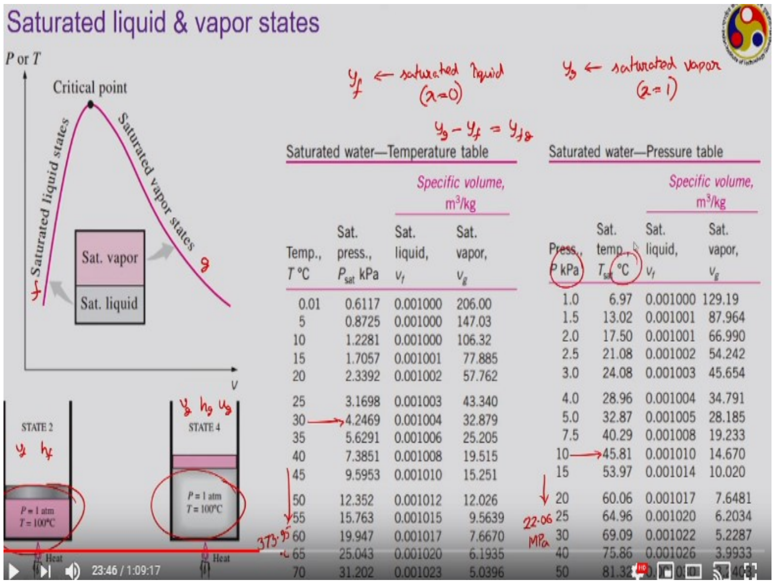

Saturated liquid & vapor states

$y_f \leftarrow$ saturated liquid $(x=0)$

$y_g - y_f = y_{fg}$

$y_g \leftarrow$ saturated vapor $(x=1)$

Saturated water—Temperature table

Temp., T °C	Sat. press., P_{sat} kPa	Specific volume, m³/kg Sat. liquid, v_f	Sat. vapor, v_g
0.01	0.6117	0.001000	206.00
5	0.8725	0.001000	147.03
10	1.2281	0.001000	106.32
15	1.7057	0.001001	77.885
20	2.3392	0.001002	57.762
25	3.1698	0.001003	43.340
30	4.2469	0.001004	32.879
35	5.6291	0.001006	25.205
40	7.3851	0.001008	19.515
45	9.5953	0.001010	15.251
50	12.352	0.001012	12.026
55	15.763	0.001015	9.5639
60	19.947	0.001017	7.6670
65	25.043	0.001020	6.1935
70	31.202	0.001023	5.0396

373.95

Saturated water—Pressure table

Press., P kPa	Sat. temp., T_{sat} °C	Specific volume, m³/kg Sat. liquid, v_f	Sat. vapor, v_g
1.0	6.97	0.001000	129.19
1.5	13.02	0.001001	87.964
2.0	17.50	0.001001	66.990
2.5	21.08	0.001002	54.242
3.0	24.08	0.001003	45.654
4.0	28.96	0.001004	34.791
5.0	32.87	0.001005	28.185
7.5	40.29	0.001008	19.233
10	45.81	0.001010	14.670
15	53.97	0.001014	10.020
20	60.06	0.001017	7.6481
25	64.96	0.001020	6.2034
30	69.09	0.001022	5.2287
40	75.86	0.001026	3.9933
50	81.32	[illegible]	[illegible]

22.06 MPa

So saturated liquid and vapor state as earlier mentioned refers to the two extreme points of the mixture region, i.e., for a given temperature or maybe given pressure the point where the liquid is just about to vaporize is referred to as a saturated liquid state. Where the vapor at a point where vapor is about to condense is referred to as saturated is vapor state. So, these are the two lines corresponding to this.

Now one important convention in thermodynamics. In thermodynamics, the saturated liquid state is generally presented by a subscript f or any property associated with the saturated liquid state is given the subscripts f whereas any property associated with a saturated vapor state is given the subscript g, i.e., when you are talking about something like specific volume and the saturated liquid state.

Let us say like the experiment of heating water at constant pressure that we talked about. Saturated liquid state refers to the state number 2 here. That is, under atmospheric condition at 100 ^{0}C it reaches a saturated liquid state. So, any property here like specific volume at this stage should be called a v_f like enthalpy at this stage should be called as h_f. Whereas if we talk about the state 4, there which is the saturated vapor state where 100% of the fluid has been converted to vapor then here we use the subscript g.

So, the specific volume of these vapor state they should be called v_g internal should be h_g internal should be called u_g just like this. So, if we are talking about any general intrinsic property y then y_f refers to the saturated liquid and y_g refers to the saturated vapor. So saturated liquid state corresponds response to a quality value of 0 where 100% of the mixture is liquid whereas saturated vapor with the g refers to $x = 1$ where 100% of the mixture is vapor. When we are talking about the changing certain property from the saturated liquid state to the saturated vapor state, then like say, $y_g - y_f$, the change in the value of this property y. Then from saturated liquid state to saturated vapor state is quite often denoted using subscript f and g like y_{12} we have written earlier is you have 1 and 2 we shall now be using f and g. So the change in specific volume, like say, we have started with this particular state where we have a 100% liquid under saturated condition and converted that on heating to 100% vapor under saturated conditions who have moved from state f to state g then the corresponding change in specific volume should be denoted as v_{fg} which is nothing but the difference between v_g and v_f. Corresponding change in the enthalpy of the mixture should be h_{fg} written as:

$$h_{fg} = h_g - h_f$$

This is often called the enthalpy of vaporization or more common the latent heat of vaporization. The change in enthaply from liquid state to vapor state or I should say saturated liquid state to saturated vapor state is called the latent heat of vaporisation. The property values quite often or at least the pressure and temperature and any other *f* and *g* values for any other property are commonly given in terms of standard tables. Here whatever tables that I am going to show are adopted from the book of Cengel and Boles, so I am thankful to that particular book for providing all this data. You just need to know you can refer to that book because of copy right issue I cannot provide you the table directly you can take it from this from the slides.

But also, this tables are not complete but for a detailed you can refer to any standard textbooks. So, look at this here for the saturated water temperature table we have temperature given on the left-hand side column and saturation pressure corresponding to the temperature in the second column. Like if we take of say 30 ^{0}C then you can directly see from the table that the pressure saturation pressure corresponding to 30 ^{0}C is 4.2469 kPa.

And then corresponding v_f and v_g values are also given in m^3/kg. Similarly, the temperature levels are also available corresponding to pressure where we have pressure at the independent variable in the first column. Then if you pick up say 10 kPa then you can clearly see the saturation temperature corresponding to 10 kPa is 45.81 that is if you want to convert a certain quantity of liquid maintain 10 kPa to vapor, then first you have to heat it up to the temperature of 45.81 and then you can easily see it to convert to vapor phase you do not have to go to 100 ^{0}C. Because 100 ^{0}C and saturation temperature corresponding to the atmospheric normal atmospheric pressure which is 101 kPa. Similarly, the v_f and v_g values are also there, as I mentioned these tables are incomplete.

So generally, the saturated water temperature table is available up to the critical point which I have just mentioned 373.95 ^{0}C. Whereas the saturated water pressure table is again available up to the critical pressure which is 22.06 MPa. You have to carefully note that unit here is kilo pascal and for temperature degree Celsius not their typical SI units. So, you have to be careful while taking any data from the table regarding this units.

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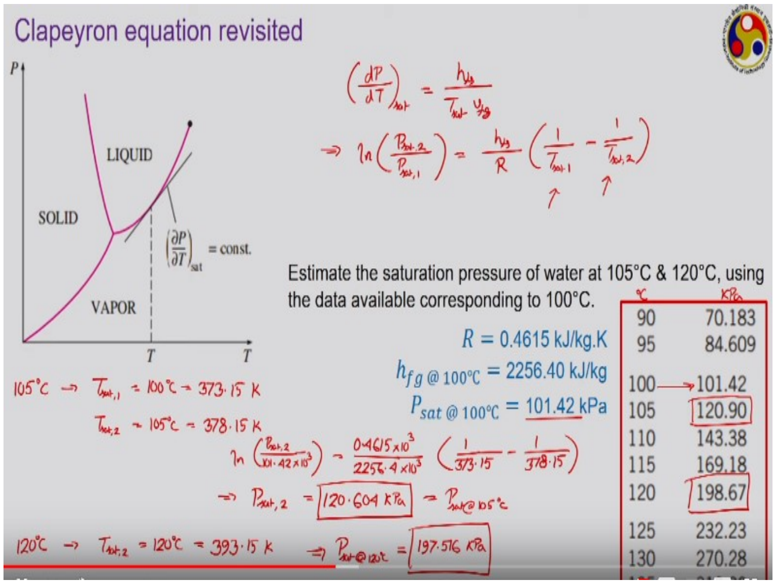

Clapeyron equation revisited

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{T_{sat}\, v_{fg}}$$

$$\Rightarrow \ln\left(\frac{P_{sat,2}}{P_{sat,1}}\right) = \frac{h_{fg}}{R}\left(\frac{1}{T_{sat,1}} - \frac{1}{T_{sat,2}}\right)$$

Estimate the saturation pressure of water at 105°C & 120°C, using the data available corresponding to 100°C.

$R = 0.4615$ kJ/kg.K

$h_{fg\,@\,100°C} = 2256.40$ kJ/kg

$P_{sat\,@\,100°C} = 101.42$ kPa

°C	kPa
90	70.183
95	84.609
100	→ 101.42
105	$\boxed{120.90}$
110	143.38
115	169.18
120	$\boxed{198.67}$
125	232.23
130	270.28

$105°C \rightarrow T_{sat,1} = 100°C = 373.15\ K$

$T_{sat,2} = 105°C = 378.15\ K$

$$\ln\left(\frac{P_{sat,2}}{101.42\times10^3}\right) = \frac{0.4615\times10^3}{2256.4\times10^3}\left(\frac{1}{373.15} - \frac{1}{378.15}\right)$$

$$\Rightarrow P_{sat,2} = \boxed{120.604\ kPa} = P_{sat@105°C}$$

$120°C \rightarrow T_{sat,2} = 120°C = 393.15\ K \Rightarrow P_{sat@120°C} = \boxed{197.516\ kPa}$

Now, I would quickly I could revisit the Clapeyron equation. In the Clapeyron equation that we have discussed during the previous week there we have to actually I showed you this diagram these are typical *PT* diagram involving water you can see this is the sublimation line involving liquid and vapor phase transition. This is the vaporisation line involving liquid vapor for phase change. And this is the melting line involving solid-liquid phase change and this is the triple point triple point for water corresponds to the temperature of 0.01 ^{0}C.

Now there corresponding to the liquid-vapor for phase-change we have seen that saturation pressure is a sole function of saturation temperature and accordingly we using the Maxwell's relation we derive this particular equation:

$$\left(\frac{\partial P}{\partial T}\right)_{sat} = \frac{h_{fg}}{T_{sat}\, v_{fg}}$$

They actually use the subscript 1 and 2, so here you have gone to the standards subscripts associated with liquid vapor phase change *f* and *g*. And then if we are assuming very low pressure then we can simplify this to the Clausius Clapeyron equation or Clapeyron Clausius equation where we can identify the saturation pressure corresponding to another temperature using knowledge for one, i.e.,

$$ln\left(\frac{P_{sat,2}}{P_{sat,1}}\right) = \frac{h_{fg}}{R}\left(\frac{1}{T_{sat,1}} - \frac{1}{T_{sat,2}}\right)$$

So, if we have knowledge about the saturation pressure corresponding temperature T_{sat1} then for T_{sat2} also we can calculate the saturation pressure using this equation as provided we are dealing with low pressure level where we can assume the vapor to be an ideal gas.

Let us see an application, we have seen the application of this one for R134a through numerical example. But now we have the exact values available for water from the table which I have shown in the previous slide. Like I am talking about this particular tables where we have the saturation pressure and saturation temperature data available. Let us try to see the validity of this relation in terms of this table for low temperature. So, we have to estimate the saturation pressure of water at 105 ^{0}C and 120 ^{0}C using the data available corresponding to 100 ^{0}C. And these are your data: the R for water is given, h_{fg} corresponding to 100 ^{0}C and saturation pressure corresponding to 100 ^{0}C these are also given.

R = 0.4615 kJ/K

$h_{fg\ @100^0C}$ = 2256.40 kJ/kg

$P_{sat\ @100^0C}$ = 101.42 kPa

So let us do it first for 105 ^{0}C, so here you have:

$T_{sat,1}$ = 100 ^{0}C = 373.15 K

Remember that data available in the table and also in the question are here in degrees Celsius. But these temperatures available in the Clausius equation or Clausius Clapeyron equation are in kelvin these are absolute temperature. So, you always have to do this conversion. So, we have:

$T_{sat,2}$ = 105 ^{0}C = 378.15 K

Here your P_{sat} is this one R and h_{fg} is also given. So, let us apply the Clapeyron equation. So:

$$ln\left(\frac{P_{sat,2}}{101.42 \times 10^3}\right) = \frac{0.4615 \times 10^3}{2256.4 \times 10^3}\left(\frac{1}{373.15} - \frac{1}{378.15}\right)$$

which actually is going to give you P_{sat2}, I have pre-calculated the value so it is going to come as:

P_{sat2} = 120.604 kPa = $P_{sat@105^0C}$

Now let us do the same calculation for 120 ^{0}C. Here your T_{sat2} is 120 ^{0}C i.e., 393.15 K. So if you do that calculation following the same procedure you can have precalculated numbers. You have: $P_{sat@120\,^0C}$ = 197.516 kPa.

Now how accurate are these numbers? Let us compare them with the table. This is the table. So first this is the data that is already given 101.42 corresponding to 100 ^{0}C is given here. I have not shown here the unit is in degree Celsius here the unit is in kilopascals for pressure. So, for 105 your table is giving 120.90 and you have got 120.60 kPa which are relatively very close to each other.

So, the value that you are getting for 105 ^{0}C is reasonably accurate following the Clapeyron Clausius equation. However, what about 120? For 120 the actual value which is 198.67 and you have got 197.51 so there is a difference of more than 1 kPa i.e., about 0.5 % error in the value. Now this much of error maybe okay in certain applications, but in precise application that may not be the most correct one.

Therefore, we can see that as we are moving to higher temperatures so we are talking about if further large distance between the reference which is 100 and the objective that is 120, then your error keeps on increasing. I would suggest to you try to calculate corresponding to 130 following the same procedure and see what value you are getting compared to the actual one which is 270.28. So, this is about does saturated liquid and vapor state and a saturation pressure temperature.

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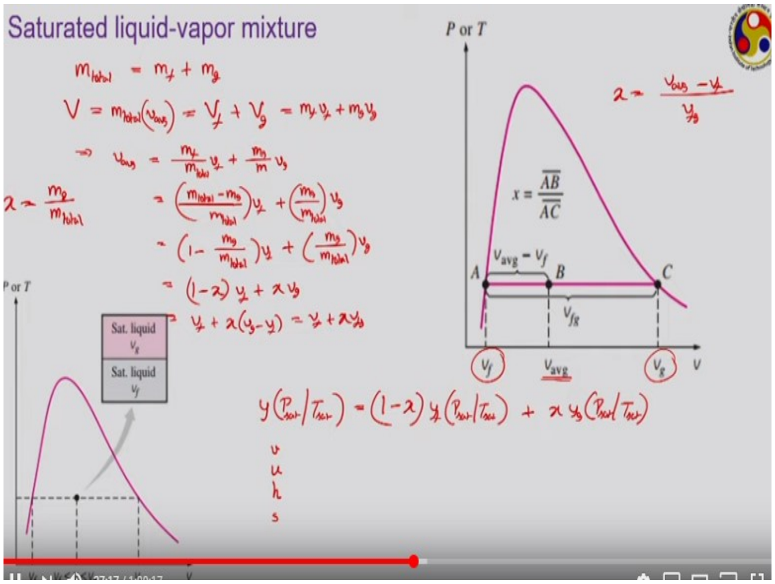

Saturated liquid-vapor mixture

$$m_{total} = m_f + m_g$$

$$V = m_{total}(v_{avg}) = V_f + V_g = m_f v_f + m_g v_g$$

$$\Rightarrow v_{avg} = \frac{m_f}{m_{total}} v_f + \frac{m_g}{m} v_g$$

$$x = \frac{m_g}{m_{total}}$$

$$= \left(\frac{m_{total} - m_g}{m_{total}}\right) v_f + \left(\frac{m_g}{m_{total}}\right) v_g$$

$$= \left(1 - \frac{m_g}{m_{total}}\right) v_f + \left(\frac{m_g}{m_{total}}\right) v_g$$

$$= (1 - x) v_f + x v_g$$

$$= v_f + x(v_g - v_f) = v_f + x v_{fg}$$

$$x = \frac{v_{avg} - v_f}{v_{fg}}$$

$$y(P_{sat}/T_{sat}) = (1 - x) y_f(P_{sat}/T_{sat}) + x y_g(P_{sat}/T_{sat})$$

v
u
h
s

Let us know move to the saturated liquid-vapor mixture. For a saturated liquid-vapor mixture we actually are dealing with a nonhomogeneous substance, truly speaking. But whereas the definition of pure substance involves the term uniform or homogeneous. But you must have always seen when water boils in some container you have a liquid in certain portion of the volume whereas vapor bubbles in certain other portions leading to a non-homogeneity.

However, in most of the thermodynamic analysis we neglect these non-homogeneities. Assume instead of having a liquid in some part and vapor in some other part we assume the entire volume to be filled up with some hypothetical fluid whose properties are average properties of the entire mixture. Something like this, where we are having properties v_f for the saturated liquid part v_g for that saturated vapor part.

And we assume the internal specific volume property v_{avg}. Now how to calculate this v_{avg} or any general property y_{avg}? Let us assume we are dealing with a mixture of having total mass m_{total} which is:

$$m_{total} = m_f + m_g$$

where

m_f is the part of mass occupied by liquid

m_g is the part of mass occupied by the vapor phase

Now one interesting characteristic for this mixture is that, the liquid part actually retains the saturation state properties and the vapor part retains the saturated vapor properties.

Let us assume, say, we have a mixture where 50% is liquid and 50% is vapor. If by some means if we can completely separate these two then you will find that the 50% of the mass which is liquid actually refers to the or the liquid state corresponding to that pressure of temperature. Whereas the vapor part that 50% or mass which is vapor corresponds to the saturation vapor state correspond to that pressure and temperature.

That is both liquid and we are per retains the saturation point properties or we can say the liquid part will always be corresponding to their *f* properties and vapor part will always be having their *g* properties. Then if we want to calculate the total volume of this mixture and then or specific volume of this mixture, let me say. Then if capital V refers to total volume then it will be:

$$V = m_{total}\, v_{avg} = V_f + V_g$$

where

v_{avg} is average specific volume

V_f is the volume occupied by the liquid phase

V_g is the volume occupied by the vapor phase.

Now how much volume occupied by the liquid phase? What we have just mentioned then we can say that the liquid part will always be having this v_f as a specific volume, so capital V_f should be small v_f. Similarly, the vapor part will always be having v_g as the specific volume.

$$= m_f v_f + m_g v_g$$

So, we can write then this v_{avg} as:

$$v_{avg} = \frac{m_f}{m_{total}} v_f + \frac{m_g}{m_{total}} v_g$$

Or we can write:

$$= \left(\frac{m_{total} - m_g}{m_{total}}\right) v_f + \left(\frac{m_g}{m_{total}}\right) v_g$$

Or we have:

$$= \left(1 - \frac{m_g}{m_{total}}\right) v_f + \left(\frac{m_g}{m_{total}}\right) v_g$$

Now what is m_g/m_{total} ? It represents the mass fraction of the vapor phase in the mixture. And what is our definition of quality x? Your quality definition was the same only which is a mass fraction of the vapor in a total mixture of liquid vapor. That means it becomes:

$$= (1 - x)v_f + (x)v_g$$

or occasionally we write this as:

$$= v_f + x(v_g - v_f) = v_f + xv_{fg}$$

and that is what is shown in this diagram. Here this AB portion, the line segment AB refers to the portion occupied by the vapor $v_{avg} - v_f$.

Similarly, other portion, this total distance AC is actually v_{fg}.

Now if you see from this relation, if we rearrange it then we can always write:

$$x = \frac{v_{avg} - v_f}{v_{fg}}$$

And therefore, x can be written as the ratio of these two-line segments AB and AC, that is what we have here. So, for any intermediate point i.e., during a mixture point we have v_f at 1 end which corresponds to $x = 0$, v_g at the other end corresponding to $x = 1$. And this instead of specific volume for any property y now for the mixture we can always write a mixture which is having a known pressure or temperature.

Say, P_{sat} or T_{sat} any one is sufficient should be having the property of:

$$y\left({}^{P_{sat}}/_{T_{sat}}\right) = (1 - x)y_f\left({}^{P_{sat}}/_{T_{sat}}\right) + xy_g\left({}^{P_{sat}}/_{T_{sat}}\right)$$

and this y can be any property is why we can have specific volume, we can have a specific internal energy, we can have specific enthalpy, we can have specific entropy or any other property which can always be presented by this mass ratios.

This way we can use the idea of this mass fraction or quality for a saturated liquid-vapor mixture to calculate the properties. We just need to know that f values and g values, or sometimes instead of writing this way we can also combine them to write as:

$$= y_f + x y_{fg}$$

which is the difference between g and f like this. So, this way for any saturated mixture we can always calculate the properties.

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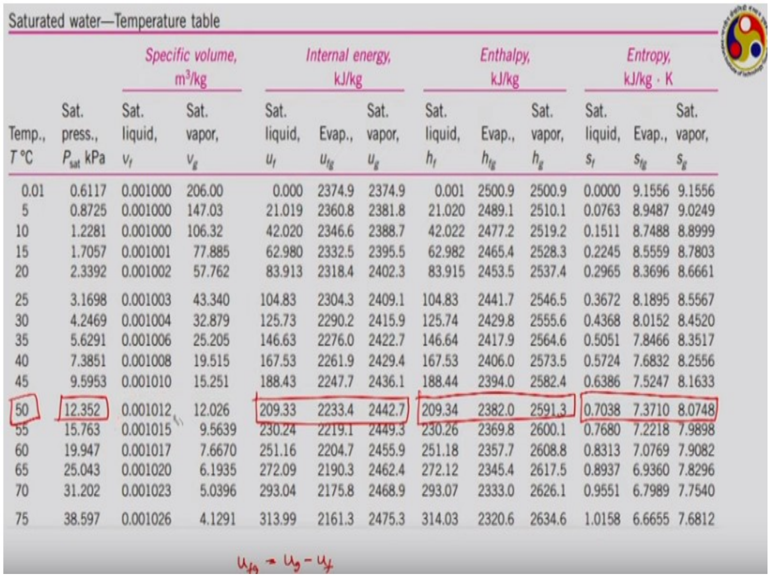

Saturated water—Temperature table

		Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4	0.2965	8.3696	8.6661
25	3.1698	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1895	8.5567
30	4.2469	0.001004	32.879	125.73	2290.2	2415.9	125.74	2429.8	2555.6	0.4368	8.0152	8.4520
35	5.6291	0.001006	25.205	146.63	2276.0	2422.7	146.64	2417.9	2564.6	0.5051	7.8466	8.3517
40	7.3851	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
45	9.5953	0.001010	15.251	188.43	2247.7	2436.1	188.44	2394.0	2582.4	0.6386	7.5247	8.1633
50	12.352	0.001012	12.026	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
55	15.763	0.001015	9.5639	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218	7.9898
60	19.947	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082
65	25.043	0.001020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9360	7.8296
70	31.202	0.001023	5.0396	293.04	2175.8	2468.9	293.07	2333.0	2626.1	0.9551	6.7989	7.7540
75	38.597	0.001026	4.1291	313.99	2161.3	2475.3	314.03	2320.6	2634.6	1.0158	6.6655	7.6812

$u_{fg} = u_g - u_f$

And these are the tables, actually I showed only a glimpse of that table the temperature, pressure and specific volume. Here it is a more complete table where we have internal energy, enthalpy and entropy or I should say specific internal energy, specific enthalpy and specific entropy all taken into consideration. You can follow any horizontal line these are temperature tables. So, temperature is the independent variable.

Let us say, we talk about 50 ^{0}C, so if we follow the horizontal line corresponding of 50 ^{0}C then this is the saturation pressure corresponding to this 12.352 kPa. These are the v_f and v_g values for this. These are the specific internal energies so you have u_f, u_{fg} and u_g. Truly speaking u_{fg} is not required because u_{fg} is:

$$u_{fg} = u_g - u_f$$

But are similar to these tables several tables also represent this fg separately. So, these are the h_f, h_{fg} and h_g and these are s_f, s_{fg} and s_g. So, for each of the cases if you are interested you can calculate this you will always find that that fg value is the difference between g and f. And therefore, g and f alone are sufficient. Of course, no one can remember these values you have to make use of this tables for calculating any mixture property. And another interesting thing to note here that this table is for water of course.

So for water the value of v_f for all this case is extremely small that is you can almost blindly, at least up to 75 ^{0}C and truly speaking even while beyond that you can blindly say this v_f is about 0.001 m^3/kg, and your v_g is much greater than v_f over a large range of temperature and pressure as well. This is the corresponding pressure table, again this table is incomplete this pressure level can go up to the critical pressure.

You can see, here again for any pressure level you can see the v_g value significantly larger than v_f. So, unless we are dealing with a very high-pressure situation, we generally can neglect the volume of v_f. So, this h_{fg} these values are the latent heat of vaporization whereas this is the entropy associated with the vaporization process representing the transition between f and g.

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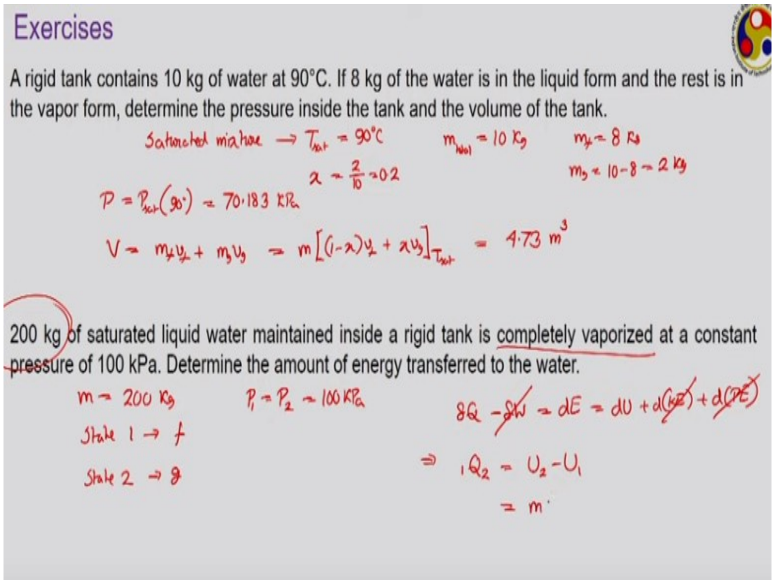

Exercises

A rigid tank contains 10 kg of water at 90°C. If 8 kg of the water is in the liquid form and the rest is in the vapor form, determine the pressure inside the tank and the volume of the tank.

Saturated mixture → $T_{sat} = 90°C$ $m_{total} = 10$ Kg $m_f = 8$ Kg

$x = \frac{2}{10} = 0.2$ $m_g = 10 - 8 = 2$ kg

$P = P_{sat}(90°) = 70.183$ kPa

$V = m_f v_f + m_g v_g = m\left[(1-x)v_f + x v_g\right]_{T_{sat}} = 4.73\ m^3$

200 kg of saturated liquid water maintained inside a rigid tank is completely vaporized at a constant pressure of 100 kPa. Determine the amount of energy transferred to the water.

$m = 200$ Kg $P_1 = P_2 = 100$ kPa

State 1 → f

State 2 → g

$\delta Q - \cancel{\delta W} = dE = dU + \cancel{d(KE)} + \cancel{d(PE)}$

$\Rightarrow {}_1Q_2 = U_2 - U_1$

$= m$

Let us know try to solve couple of numerical exercises quickly to understand the use of these tables. Here the table that I have shown here I am making use of the same table it is just copied from the book of Cengel and Boles. And the data values that I have here in this table is a other 1 that I am going to use to solve this problem. Look at the first one here you are talking about the rigid tank containing 10 kg of water at 90 ^{0}C if 8 kg of water is in liquid form and the rest is in the vapor form determine the pressure inside the tank and the volume of the tank. Now here we have a rigid tank containing a mixture of liquid and vapor phase of water. And whenever you are talking about the liquid and mixture of liquid and vapor phase under equilibrium, equilibrium term not specifically mentioned in the problem but we can assume it to be an equilibrium.

Then whenever you are having a mixture, that has to be under saturation condition. So actually, you are dealing with saturated that mixture so the state that we have state of the substance we have is that of saturated mixture. It is very important to understand nature of the state at the beginning itself. Now saturated mixture, what is the temperature or pressure anyone you need?

$T_{sat} = 90\ ^0\text{C}$

$m_{total} = 10$ kg

$m_f = 8$ kg

$m_g = 10 – 8 = 2$ kg

$x = 2/10 = 0.2$

So now you know the saturation temperature and you know the quality and therefore the state is completely defined. Because you know two independent intensive properties. Remember in the mixture, for a saturated mixture you need to know either of temperature and pressure and the quality value for this and using this now you can easily solve this problem. But before doing that I would just like to go back to the previous table to mention you something. If the history of quality you are giving it some other property value. Suppose it is given that you are dealing with a system where the pressure is 20 kPa and suppose the value of enthalpy given. Now you have to understand you are talking about what kind of state compressed liquid or mixture or the super-heated vapor. How can you do that? You have to check that two values corresponding this 20 kPa which are this h_f and h_g. Just check this hf and hg values now if your h is less than h_f then who state you are belonging to?

Your enthalpy of the fluid is less than the saturated liquid enthalpy then you are in the compressed liquid zone. When your h is greater than h_g then where you are? You are in there super-heated vapor zone. For compressed liquid and super-heated how to get the properties we will soon be discussing. And only when your h is restricted within h_f and h_g then you are in the mixture zone.

And the quality of the mixture you can calculate as:

$$x = \frac{h - h_f}{h_g - h_f}$$

thereby identifying the quality value or thereby making way to calculate all other properties. Therefore, even when the quality is not known but some other property value is known then you can easily calculate the state of the fluid. Now coming back to this particular problem. So, we know the state then how to solve this? You have to identify the pressure inside the tank. Now what will we the pressure? pressure has to be the saturation pressure corresponding to 90 ^{0}C. So, you can check your table the corresponding pressure will be coming to be 70.183 kPa. I am noting it from the table only the table that have shown from there I am noting these values.

$P = P_{sat}$ (90 ^{0}C) = 70.183 kPa

And then you have to calculate the volume inside the tank or the volume of the tank. Now total volume of the tank should be equal to:

$$V = m_f v_f + m_g v_g = m\left[(1 - x)v_f + x v_g\right]_{T_{sat}}$$

and you know the value of v_f and v_g corresponding to this 90 ^{0}C and then evaluate this value this final volume is going to be coming to be okay 4.73 m^3. You please do this calculation and check whether you are getting this number.

Another example, some more involved calculation here you are talking about 200 kg of saturated liquid water. So, it is clearly mentioned your initial state is of saturated liquid water. So

m= mass is 200 kg

state 1 is of *f* state, saturated liquid water.

Maintained inside the rigid tank is completely vaporized at a constant pressure of a 100 kPa, i.e., $P_1 = P_2 = 100$ kPa

and it is and the final state your state 2 is of, it is completely vaporized. So final state is that of a vapor which is *g*. No mention about super heating, super heating shall be taken in the saturated vapor state, you have to determine the amount of energy transferred to the water during the process.

So how can you calculate that? We are talking about some energy transfer and whenever we are talking about energy transfer, we have to make use of the first law of thermodynamics which gives you the energy balance. Now what kind of system we are talking about? We are talking about the rigid tank that means no more boundary work no movement of the boundary and we are talking about a fixed mass, mass is not changing so were talking about the closed system.

Then if we write the 1st law of thermodynamics for a closed system with rigid-wall, then we can write:

$$\delta Q - \delta W = dE = dU = d(KE) + d(PE)$$

Now here no mention is given of all the changes in kinetic and potential energy of the system so we are assuming to a stationary system. So these two cancels out and we have a rigid tank no moving boundary work so ∂W cancels out.

That is during that process that total heat transfer during the process 1 to 2 will be:

$$Q_1^2 = U_1 - U_2$$

the changes in the total internal energy of the substance. i.e.,

$$= m(u_1 - u_2)$$

Now what is your state 2? State 2 is u_g or g only and state 1 is if both corresponds to the same pressure of 100 kPa. i.e.,

$$= m(u_g - u_f)$$

So, you can use a table corresponding to a 100 kPa, get the uf and ug values, mass is given to it 200 kg and your finally calculated value is going to be coming as:

$$= 417.64 \text{ MJ}$$

So, use the table to check whether you are getting this number or not. This way we can make use of this tables for saturated mixtures. We shall be making several such use in a later modules as well.

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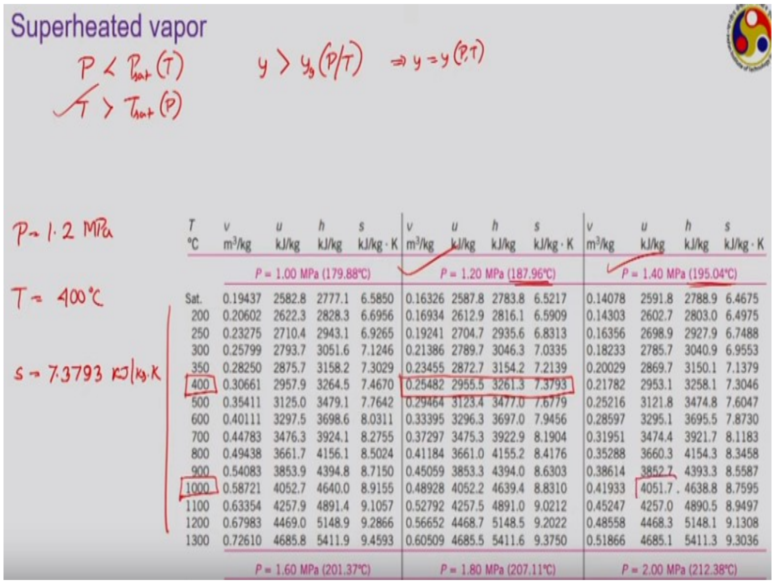

Superheated vapor

$P < P_{sat}(T)$

$T > T_{sat}(P)$

$y > y_g(P/T) \Rightarrow y = y(P,T)$

$P \to 1.2$ MPa

$T \to 400$°C

$s \to 7.3793$ kJ/kg.K

T	v	u	h	s	v	u	h	s	v	u	h	s
°C	m³/kg	kJ/kg	kJ/kg	kJ/kg · K	m³/kg	kJ/kg	kJ/kg	kJ/kg · K	m³/kg	kJ/kg	kJ/kg	kJ/kg · K
	P = 1.00 MPa (179.88°C)				P = 1.20 MPa (187.96°C)				P = 1.40 MPa (195.04°C)			
Sat.	0.19437	2582.8	2777.1	6.5850	0.16326	2587.8	2783.8	6.5217	0.14078	2591.8	2788.9	6.4675
200	0.20602	2622.3	2828.3	6.6956	0.16934	2612.9	2816.1	6.5909	0.14303	2602.7	2803.0	6.4975
250	0.23275	2710.4	2943.1	6.9265	0.19241	2704.7	2935.6	6.8313	0.16356	2698.9	2927.9	6.7488
300	0.25799	2793.7	3051.6	7.1246	0.21386	2789.7	3046.3	7.0335	0.18233	2785.7	3040.9	6.9553
350	0.28250	2875.7	3158.2	7.3029	0.23455	2872.7	3154.2	7.2139	0.20029	2869.7	3150.1	7.1379
400	0.30661	2957.9	3264.5	7.4670	0.25482	2955.5	3261.3	7.3793	0.21782	2953.1	3258.1	7.3046
500	0.35411	3125.0	3479.1	7.7642	0.29464	3123.4	3477.0	7.6779	0.25216	3121.8	3474.8	7.6047
600	0.40111	3297.5	3698.6	8.0311	0.33395	3296.3	3697.0	7.9456	0.28597	3295.1	3695.5	7.8730
700	0.44783	3476.3	3924.1	8.2755	0.37297	3475.3	3922.9	8.1904	0.31951	3474.4	3921.7	8.1183
800	0.49438	3661.7	4156.1	8.5024	0.41184	3661.0	4155.2	8.4176	0.35288	3660.3	4154.3	8.3458
900	0.54083	3853.9	4394.8	8.7150	0.45059	3853.3	4394.0	8.6303	0.38614	3852.7	4393.3	8.5587
1000	0.58721	4052.7	4640.0	8.9155	0.48928	4052.2	4639.4	8.8310	0.41933	4051.7	4638.8	8.7595
1100	0.63354	4257.9	4891.4	9.1057	0.52792	4257.5	4891.0	9.0212	0.45247	4257.0	4890.5	8.9497
1200	0.67983	4469.0	5148.9	9.2866	0.56652	4468.7	5148.5	9.2022	0.48558	4468.3	5148.1	9.1308
1300	0.72610	4685.8	5411.9	9.4593	0.60509	4685.5	5411.6	9.3750	0.51866	4685.1	5411.3	9.3036
	P = 1.60 MPa (201.37°C)				P = 1.80 MPa (207.11°C)				P = 2.00 MPa (212.38°C)			

Now let us quickly talk about super-heated vapor. As we have seen super-heated vapor state refers to the state beyond the saturated vapor condition. So super-heated vapor state can be identified when the pressure is less than the saturation pressure for a given temperature, say,

$$P < P_{sat}\,(T)$$

or, temperature is greater than the saturation temperature corresponding to given pressure, say,

$$T < T_{sat}\,(P)$$

And instead of pressure and temperature some other properties are given. Say property y is given specific value of the property then if the value of this y:

$$y\; > y_g\,(P/T)$$

corresponding to either of its pressure or temperature whichever is given, then it is super-heated vapor that were talking about. Yes for super-heated vapor here any property y:

$$y = y\,(P,\,T)$$

i.e., y is a function of both pressure and temperature we do not have any quality kind of thing and pressure and temperature independent of each other. And therefore, to have any kind of table you need to have separate table for each pressure and temperature. This is one typical table for super-heated vapor again adopted from the book of Cengel and Boles. Now here you can see there are several tables each corresponding to a different pressure. Like I am showing just a few of them here this is for 1 MPa. Corresponding to 1MPa that is 10 bar, saturation temperature is 179.88 ^{0}C the value given in bracket next to the pressure value is actually the saturation temperature. And you directly have this quantities *v, u, h* and *s* specific values in this. So, suppose you are giving it the condition that you are dealing with a system where the pressure is 1.2 MPa and the temperature is 400 ^{0}C. Then you have to identify the property values. How to do that? First, corresponding to this 1.2 MPa you check what is the saturation temperature corresponding to this. Now corresponding to 1.2 MPa saturation temperature is 187.96 ^{0}C. So, temperature is greater than saturation temperature corresponding to pressure so it is different in the super-heated condition. So, you identify the corresponding super-heated table and which is this particular table corresponding to 1.2 MPa.

And from this table you have to identify the entry for 400 ^{0}C. That entry is here on the extreme left, this is your 400 ^{0}C. So, corresponding to 400 ^{0}C i.e., you move along the horizontal line. So, the entry in 1.2 MPa, table is this or these are the values. So, whatever that you want to have you can directly take it from there. Like suppose if you are interest is entropy corresponding this condition, the entropy you can directly note as 7.3793 kJ/kgK, the specific entropy that is or any other value. Similarly, if you are suppose looking for the value of specific internal energy corresponding to 1.4 MPa and 1000 ^{0}C. Then where to find this? These are tables for 1.4 MPa, 1000 litres of saturation temperature 195 so 1000 ^{0}C if it is a super-heated one.

So, this is your 1000 ^{0}C correspondingly this is the specific volume. This is the specific internal energy and also units are also there, unit is kilo joule per kg, be very careful about choosing the unit and remember these values are in Celsius not in kelvin. So, this way we can take the values for super-heated vapor.

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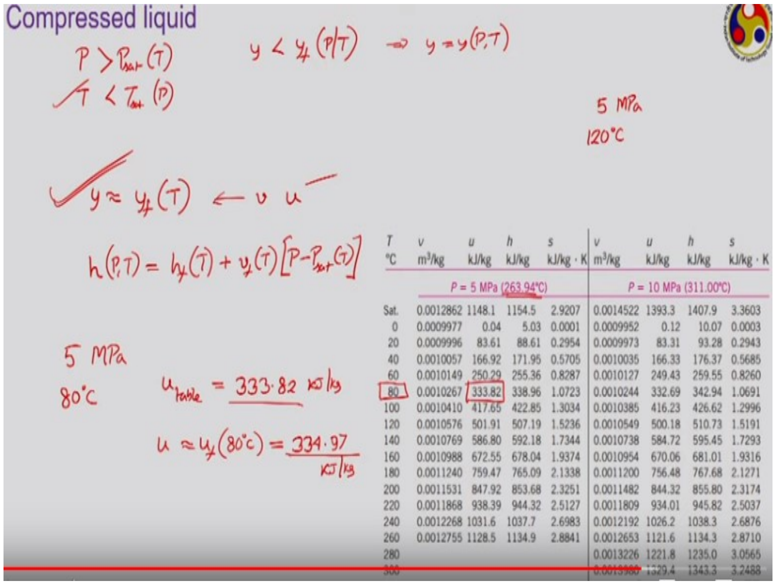

Compressed liquid

$P > P_{sat}(T)$

$T < T_{sat}(P)$

$y < y_f(P/T) \Rightarrow y = y(P,T)$

5 MPa

120°C

$y \approx y_f(T) \leftarrow v \; u$

$h(P,T) = h_f(T) + v_f(T)\left[P - P_{sat}(T)\right]$

5 MPa

80°C

$u_{table} = \underline{333.82}$ kJ/kg

$u \approx u_f(80^\circ C) = \underline{334.97}$ kJ/kg

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 5 MPa (263.94°C)				P = 10 MPa (311.00°C)			
Sat.	0.0012862	1148.1	1154.5	2.9207	0.0014522	1393.3	1407.9	3.3603
0	0.0009977	0.04	5.03	0.0001	0.0009952	0.12	10.07	0.0003
20	0.0009996	83.61	88.61	0.2954	0.0009973	83.31	93.28	0.2943
40	0.0010057	166.92	171.95	0.5705	0.0010035	166.33	176.37	0.5685
60	0.0010149	250.29	255.36	0.8287	0.0010127	249.43	259.55	0.8260
80	0.0010267	333.82	338.96	1.0723	0.0010244	332.69	342.94	1.0691
100	0.0010410	417.65	422.85	1.3034	0.0010385	416.23	426.62	1.2996
120	0.0010576	501.91	507.19	1.5236	0.0010549	500.18	510.73	1.5191
140	0.0010769	586.80	592.18	1.7344	0.0010738	584.72	595.45	1.7293
160	0.0010988	672.55	678.04	1.9374	0.0010954	670.06	681.01	1.9316
180	0.0011240	759.47	765.09	2.1338	0.0011200	756.48	767.68	2.1271
200	0.0011531	847.92	853.68	2.3251	0.0011482	844.32	855.80	2.3174
220	0.0011868	938.39	944.32	2.5127	0.0011809	934.01	945.82	2.5037
240	0.0012268	1031.6	1037.7	2.6983	0.0012192	1026.2	1038.3	2.6876
260	0.0012755	1128.5	1134.9	2.8841	0.0012653	1121.6	1134.3	2.8710
280					0.0013226	1221.8	1235.0	3.0565
300					0.0013980	1329.4	1343.3	3.2488

Similar tables or also available for compressed liquids. Compressed liquid means the general situation where the pressure is greater than saturation pressure corresponding to its temperature or temperature is less than the saturation temperature corresponding to its pressure, or if any general property y is given:

$$y < y_f\,(P/T)$$

So, from there you can easily identify similar to the super-heated vapor here also:

$$y = y\,(P,\,T)$$

and so, we need to have similar tables just like the one shown here. Like here my example is of pressure table 5 MPa and 10 MPa. So, suppose you want to identify the enthalpy of liquid water at 5 MPa and 120 ^{0}C. Say, our condition is 5 MPa and 120 ^{0}C.

So, you check the table corresponding to 5 MPa saturation temperature is 263 ^{0}C. So, this temperature is well beyond this so you are in the compressed liquid zone. So, in this 5 MPa now you identify 120, this is your 120 so this is the line that is of your interest from there you take the enthalpy which is nothing but 507.19 kJ/kg. So, this way we have similar to super-heated vapor you can also make use of that compressed liquid table.

But truly speaking, for compressed liquid often we do not have to use that table. Because it has been identified that the for the compressed liquid again, we are talking about compressed liquid which is very much incompressible or nearly incompressible. So that dependence of these properties on pressure is quite weak they are solely dependent on temperature or they are mostly dependent on temperature.

So therefore, quite often we take the properties y to be nearly or rather instead of taking that from the table we assume y to be:

$$y = y_f(T)$$

we just neglect the pressure dependence whatever temperature value you have we just take it corresponding to the y_f for this. Particularly properties like specific volume or specific internal energy these are fine.

Enthalpy is more dependent on pressure so for enthalpy for compressed liquid the function of pressure and temperature we use instead of using that table again use a slightly modified form where we take the hf value as:

$$h\,(P,T) = h_f(T) + v_f(T)[P - P_{sat}(T)]$$

the last term takes care of if there is any small amount of compressibility for this.

We can take one example to check let us suppose I give you a condition of 5 MPa and 80 ^{0}C. So, 5 MPa and 80 ^{0}C is a condition and you have to identify specific internal energy following the table and also using this approach and compare for this. So, 5 MPa and 80 ^{0}C firstly a saturation temperature is 263 degree.

So, this is compressed or subcooled liquid. So from the table the exact value is corresponding this 80 ^{0}C this is your exact value. So, u obtained from the table is 333.82 kJ/kg i.e.,

u_{table} = 333.82 kJ/kg

and if you follow the approximate approach then u will be:

$u = u_f$ (80^{0}C) = 334.97kJ/kg

And that will be coming you can check from the table again that will be coming as 334.97 and you can see both values are very close to each other therefore for properties particularly specific volume or specific internal energy and in most cases specific enthalpy also there is really hardly any error following this approximate approach and therefore compressed liquids will itself be less much used compare to the super-heated vapor table. So, these are the ways we should use the tables.

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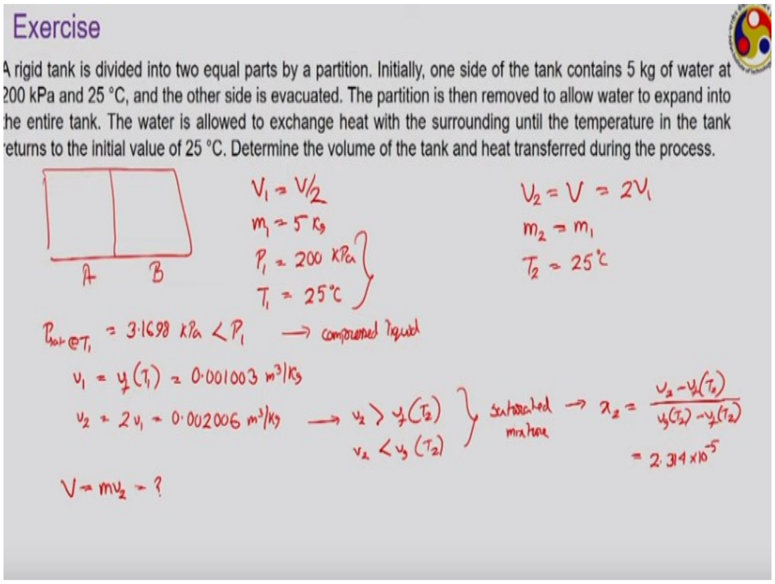

Exercise

A rigid tank is divided into two equal parts by a partition. Initially, one side of the tank contains 5 kg of water at 200 kPa and 25 °C, and the other side is evacuated. The partition is then removed to allow water to expand into he entire tank. The water is allowed to exchange heat with the surrounding until the temperature in the tank eturns to the initial value of 25 °C. Determine the volume of the tank and heat transferred during the process.

$V_1 = V/2$

$m_1 = 5\ kg$

$\left.\begin{array}{l} P_1 = 200\ kPa \\ T_1 = 25^\circ C \end{array}\right\}$

$V_2 = V = 2V_1$

$m_2 = m_1$

$T_2 = 25^\circ C$

$P_{sat@T_1} = 3.1698\ kPa < P_1 \longrightarrow$ compressed liquid

$v_1 = v_f(T_1) = 0.001003\ m^3/kg$

$v_2 = 2v_1 = 0.002006\ m^3/kg \longrightarrow \left.\begin{array}{l} v_2 > v_f(T_2) \\ v_2 < v_g(T_2) \end{array}\right\}$ saturated mixture $\rightarrow x_2 = \dfrac{v_2 - v_f(T_2)}{v_g(T_2) - v_f(T_2)}$

$= 2.314 \times 10^{-5}$

$V = mv_2 = ?$

I would like to wrap up this by solving couple of numerical problems quickly. I have a problem here please read it quickly. Here you are talking about the rigid tank which is divided into two equal parts by a partition. So, we have our tank which is divided into two equal parts so this is your part A and this is your part B which are having equal volume. Initially one side of the tank is contents 5 kg of water at 200 kPa and 25 ^{0}C.

So initially total volume is:

$V_1 = V/2$

where

V is the we are assuming to be the volume of the tank

$m_1 = 5$ kg

then we have

P_1=200 kPa

T_1=25 ^{0}C.

So, the other side is evacuated now then the partition has been removed to allow water to expand into the entire tank that water is allowed to exchange heat to the surrounding until temperature in the tank returns to the initial value of 25 ^{0}C.

So, in the final state your volume is:

$V_2 = V = 2\ V_1$

It is because this gas is now occupying the entire volume. what would be the mass? You say rigid tank no mass transfers. So,

$m_2 = m_1$

we are talking about the close system in terms of the gas. And the final temperature T_2 that is the same which is 25 ^{0}C, but pressure may not be same so you have to calculate the volume of the tank.

And the heat transfer associated with this process to get the final temperature back to 25 ^{0}C to how to approach it. Initial state is completely defined using these two values 200 kPa and 25 ^{0}C. So how you can get this 25 ^{0}C and 200 kPa? If I make use of the table now from that table I can see that:

$P_{sat@T1}$ at 25 ^{0}C = 3.1698 kPa

this is less than your pressure that means your state is of compressed liquid. So, we are taking the specific volume using the approximate approach:

$v_1 = v_f(T_1) = 0.001003\ \mathrm{m^3/kg}$

So, your v_2 will be:

$v_2 = 2\ v_1 = 0.002006\ \mathrm{m^3/kg}$

Now look at the second state temperature is 25 ^{0}C and now you know the specific volume.

From the specific volume and temperature data we have to identify the state this specific volume we have to now compare with the v_f and v_g corresponding to 25 ^{0}C. Now this:

$v_2 > v_f(T_2)$

However

$v_2 < v_g\,(T_2)$

that means we are dealing with a saturated mixture. So, corresponding to this we can calculate the quality your quality x_2 will be:

$$x_2 = \frac{v_2 - v_f(T_2)}{v_g(T_2) - v_f(T_2)}$$

you can do the calculation, again I have precalculated the number here x is coming to extremely small:

$x_2 = 2.314 \times 10^{-5}$

So, you know all the results or you know the quality of this. Now you have to calculate first the volume of the tank. How much would be the volume of the tank? You can easily calculate the volume of the tank to be:

$V = mv_2$

v_2 is known.

So your volume of the tank, actually I have not calculated this number so you please use your calculator mass is given to be 5 kg, v_2 is there you can get the volume of the tank. And secondly to calculate the heat transfer you have to go to the 1st law of thermodynamics. So, we are using:

$$\delta Q - \delta W = dU$$

I am directly writing dU as a need to measure my system is a rigid tank no moving boundary work.

$$Q_1^2 = m(u_1 - u_2) = m\left[\left(u_f - x_2 u_{fg}\right)|_{T2} + u_f|_{T1}\right]$$

So, this will become just:

$$= m x_2 u_{fg}|_{T2} = 0.2666\ kJ$$

So, this way we have made use of the compressed liquid data and mixture.

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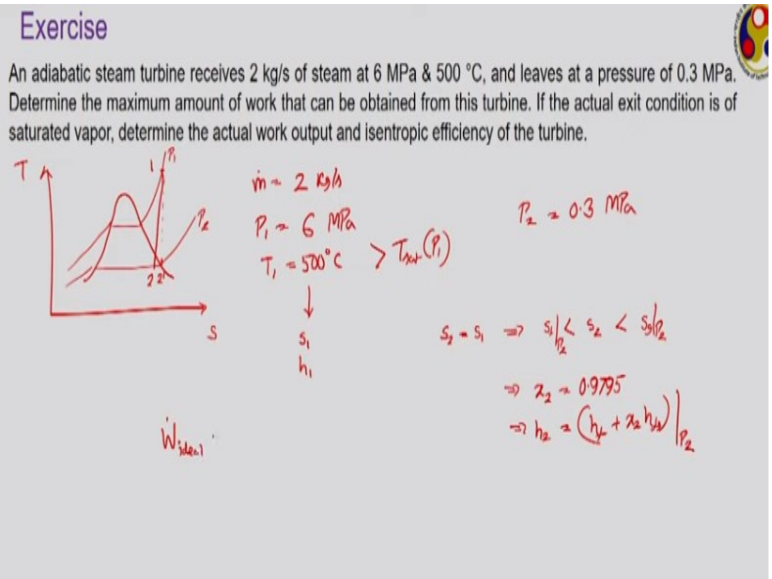

Exercise

An adiabatic steam turbine receives 2 kg/s of steam at 6 MPa & 500 °C, and leaves at a pressure of 0.3 MPa. Determine the maximum amount of work that can be obtained from this turbine. If the actual exit condition is of saturated vapor, determine the actual work output and isentropic efficiency of the turbine.

$\dot{m} = 2$ kg/s

$P_1 = 6$ MPa

$T_1 = 500°C > T_{sat}(P_1)$

$\downarrow$

s_1

h_1

$P_2 = 0.3$ MPa

$s_2 = s_1 \Rightarrow s_f|_{P_2} < s_2 < s_g|_{P_2}$

$\Rightarrow x_2 = 0.9795$

$\Rightarrow h_2 = (h_f + x_2 h_{fg})|_{P_2}$

$\dot{W}_{ideal}$

And now I would like to close up this by using the super-heated table. Here the question involves one of the adiabatic steam turbines. Let us try to do it very quickly, here let me draw that *TS* diagram of this. Here the initial state, here mass flow rate is given as:

$\dot{m} = 2$ kg/s,

$P_1 = 6$ MPa

$T1 = 500\ ^0C$

Now you can check the value of this T_1,

$$T_1 > T_{sat}\ (P_1)$$

we are talking about a super-heated condition. So on this TS diagram if we plot the constant pressure line let us say if I plot the 6 MPa line then your initial state maybe somewhere here the state 1. And we are talking about an isentropic process during which the pressure drops to 0.3 MPa so we draw the second pressure line this is your P_1 line this your P_2 line where:

$P_2 = 0.3$ MPa.

So, the ideal process will be somewhat like this and actual process is that of saturated vapor i.e., at the end of this process actually it is coming out at a condition like this. So 2 and 2', 2 is the ideal one to 2' is the actual one. So how can I approach this? P_1 and T_1 given so from this you can easily calculate whatever requires you can calculate as to s_1, h_1 etc. Now it is an isentropic process that is being followed in the turbine.

If it is isentropic process then

$$s_2 = s_1$$

and if you use the values properly, here I do not have the numbers, you please check our table. You will find that this is s_2 is coming to be greater than s_f corresponding to your P_2 but less than s_g corresponding to your P_2 indicating that the final state 2 is of saturated mixture. Correspondingly you can calculate the x_2, x_2 value I have it will be coming something around 0.9795.

So this is your x_2 value so from x_2 you can now calculate your h_2 which will be:

$$h_2 = h_f + x_1 h_{fg}$$

both h_f and h_{fg} will be corresponding to your P_2. And so, the ideal power output from this will be

$$W_{ideal} = \dot{m}\ (h_1 - h_2)$$

but the actual exit state because of the frictional losses because of the irreversible is that of that of saturated vapor. So, your h_2' prime is actually hg corresponding to P_2 which you can get again from the table. So, your

$$W_{actual} = \dot{m}\,(h_1 - h_2')$$

if you do the calculation correctly, if you are able to take the data properly you will find this will be coming less than the ideal one. And the isentropic efficiency for this turbine will be thus the ratio of this is:

$$\eta = \frac{\dot{W}_{actual}}{\dot{W}_{ideal}}$$

So this way we can make this a super-heated vapor table to use the concept of entropy. In the next lecture, I shall be trying to solve one problem involving exergy, where we shall be making better use of all these tables. Till then you please go through the lecture, go through the books try to see how you to use those tables hopefully you will be able to use those tables for solving any such kind of problems. And we shall be doing several such exercises in the future modules as well.

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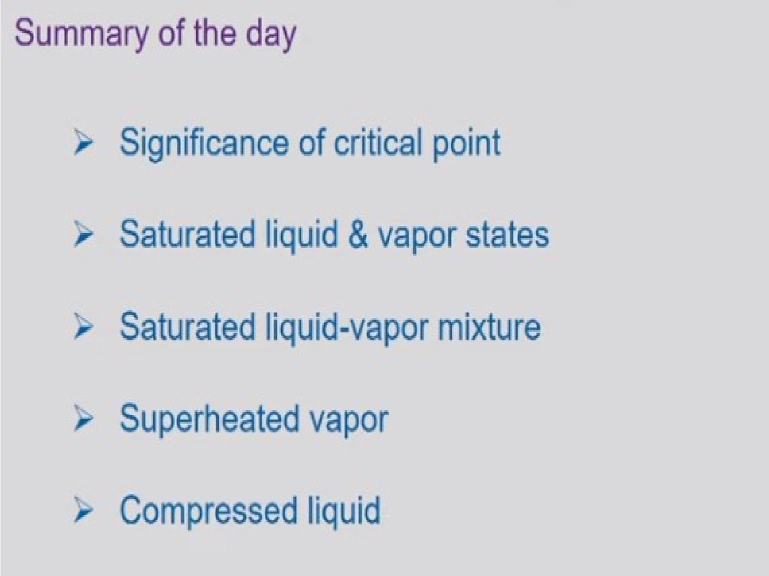

Summary of the day

- Significance of critical point
- Saturated liquid & vapor states
- Saturated liquid-vapor mixture
- Superheated vapor
- Compressed liquid

So, this is where I would like to stop today, we have briefly discussed the significance of critical point, then we have talked about the saturated liquid and vapor state, how to use a property tables to calculate properties of saturated liquid-vapor mixture, super-heated vapor and compressed liquid. And also, we have solved a few problems involving this. So, I hope you have enjoyed this lecture, we shall be coming back to the with the third lecture of this week where I shall be wrapping up with the equation of state for ideal and real gases. Till then, thank you.