Applied Thermodynamics for Engineers

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Lecture - 04

Concept of Exergy and Exergy Destruction

Morning friends, so we are into the fourth lecture of our first week where we are using our basic fundamentals of thermodynamics and we have already covered three lectures where we have reviewed most of the basic topics like basic thermodynamic systems, different kind of interactions the system and surroundings can have in the form of heat, work and mass transfer.

Then, we have reviewed the laws of thermodynamics; the zeroth, first and second law. And from using the laws of thermodynamics particularly the first law, we have done the analysis of thermodynamic system. And there we have found that anyone of the laws alone is not sufficient rather for any kind of thermodynamic analysis, we basically have to consider both first and second law together.

Because first law of thermodynamics provides only a balance of energy or I should say it provides you a balance of the energy change or change in the stored energy of the system and the energy interaction it can have with the surrounding across its boundary. Now, it may seem that just knowing the first law of thermodynamics alone is sufficient for analyzing any energy associated device because that will give you the magnitude of all possible kind of energy interaction.

But if you are depending or relying only on the first law of thermodynamics, then you may lead to a situation where you have conceptually designed one PMM-II a perpetual motion machine of the second kind which satisfies the first law of thermodynamics. However, whatever may be the approach you follow to fabricate such a device, it will always lead to a failure in practice, because that actually violates the second law of thermodynamics.

Therefore, first law of thermodynamics alone is not sufficient. We have to consider the second law of thermodynamics also which provides a realistic dimension in which practical processes can go and also provides a limit of operation for any kind of heat engines or reversed heat engines. Now, in the previous lecture, you were introduced to the concept of entropy or I should say you already know the concept of entropy from your basic thermodynamics.

But we reviewed that, we have seen how to calculate the value of entropy for closed and open system, how to perform entropy based analysis and also we have derived couple of tedious relations which will be very useful in the next module that we are going to talk about in the next week but entropy based analysis sometimes can be quite confusing because entropy as a property itself is a bit confusing.

And therefore, there is another way of performing the second law-based analysis of thermodynamic systems, which is using the concept of exergy or work potential or sometimes called availability. In today's lecture we are going to review this very interesting concept of exergy. Just quickly we see the situation that we can have for a heat engine.

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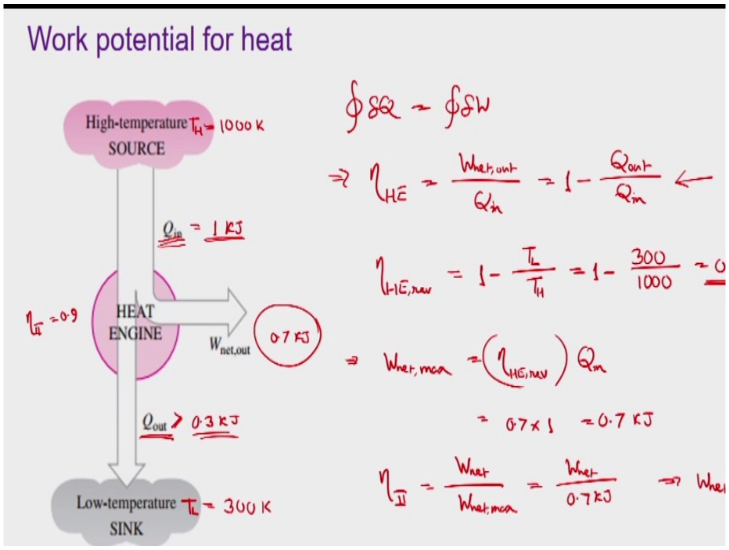

Work potential for heat

$$\oint \delta Q = \oint \delta W$$

$$\Rightarrow \eta_{HE} = \frac{W_{net,out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \leftarrow$$

$$\eta_{HE,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1000} = 0$$

$$\Rightarrow W_{net,max} = \left(\eta_{HE,rev}\right) Q_{in}$$

$$= 0.7 \times 1 = 0.7\ KJ$$

$$\eta_{II} = \frac{W_{net}}{W_{net,max}} = \frac{W_{net}}{0.7\ KJ} \Rightarrow W_{ne}$$

Here, we have a heat engine which is operating between a high temperature source, let us assume this is at a temperature T_H and low temperature sink which is operating at a temperature of T_L. It is receiving this amount of heat from this high temperature source converting a part of that to work and then rejecting a part of this into the low temperature sink. Now, using our concept of first law of thermodynamics, you know that we must have some balance between the heat and work transfer.

That is when this heat engine is operating over a cycle, we must have

$$\oint \delta Q = \oint \delta W$$

and using that concept we can define the efficiency of this heat engine as:

$$\eta_{HE} = \frac{W_{net,out}}{Q_{in}}$$

We are given to get that which eventually applying the first law will come out as:

$$\eta_{HE} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now, if your Q_{out} is 0, then your efficiency will be equal to 1.

That means if we can convert entire portion of this Q_{in} to output work then you can define a 100% efficient engine. However, as per the second law of thermodynamics that is never possible, we can never convert entire part of this input heat to work rather, there has to be some kind of heat reaction and only a part of this input energy can be converted to work output which we know now from the second law of thermodynamics.

And how much can we convert? The maximum efficiency that we can get between these two temperature reservoirs operating at T_H and T_L respectively is corresponding to our reversible engine and therefore for any reversible heat engine with Carnot cycle being an example, we know that the efficiency can be given as:

$$\eta_{HE,rev} = 1 - \frac{T_L}{T_H}$$

therefore, that is a maximum possible efficiency you can get.

Or in a way, the maximum possible work output that you can get from this is:

$$W_{net,max} = (\eta_{HE,rev})Q_{in}$$

where

$\eta_{HE,rev}$ is the efficiency for the reversible engine

Q_{in} is the amount of heat that you are giving

Let us take an example, suppose your high temperature reservoir is at a temperature of 1000 K and low temperature reservoir is at a temperature of 300 K. We are choosing values which will be easy to deal with. So, if this heat engine is a reversible one, then corresponding efficiency will be:

$$\eta_{HE,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1000}$$

Now, if your Q in is say 1 kJ, then the maximum work output you can get is:

$$W_{net,max} = (\eta_{HE,rev})Q_{in} = 0.7 \times 1 = 0.7\ kJ$$

That means whatever may be the effort you put in, there will be at least 0.3 kJ of heat loss to the surrounding. So, the maximum and this is a situation for a reversible engine.

And if the engine is an irreversible one, your efficiency will be even lower. So, this is always higher than the efficiency of any irreversible engine:

$$0.7 > \eta_{HE,irr}$$

0.7 or 70% being the maximum possible efficiency that we can get and this 0.3 kJ is the minimum possible heat rejection that we can have. So, Q_{out} will always be greater equal to 0.3 kJ and $W_{net,out}$ will always be less equal to 0.7 kJ.

So, the maximum possible output that you can get work output from this heat engine is 0.7 kJ or an efficiency of 70%. Whatever effort you may put in, you will never get an efficiency higher than this or any work output greater than this 0.7 kJ. Therefore this 1 kJ of heat in this particular combination of T_H and T_L has a maximum possible work potential of 0.7 kJ only.

You can never convert anything more than 70% of this input energy to work and that is why whenever we are analyzing the performance of this heat engine, it is not logical to define efficiency alone like this, rather we should always define the efficiency of a real heat engine in terms of the maximum possible value that we can achieve from this. And that leads to the concept of a second law efficiency which occasionally can be defined as there are several possible definitions.

And this can be defined as, for this particular heat engine that is:

$$\eta_{II} = \frac{W_{net}}{W_{net,max}}$$

i.e., in this particular situation you have:

$$\eta_{II} = \frac{W_{net}}{0.7\ kJ}$$

Then, you have a more realistic estimate of the efficiency of the engine.

So, if you are talking about a real heat engine which is said to have a second law efficiency of say 0.9, then you know that the actual work output that you are getting from this engine is:

$$W_{net} = 0.9 \times 0.7 = 0.63\ kJ$$

This is the output that you are getting from the engine and now this 0.63 kJ, you are comparing with the maximum possible output is 0.7 kJ and not with the ideal possible value of 1 kJ.

So, second law efficiency truly speaking provides a much more realistic way of defining the efficiency or performance of a system. And taking this concept of second law efficiency now, let us try to calculate what can be the maximum possible work potential. Like in this example, this 1 kJ of heat that we are providing that has a maximum possible work potential of this 0.7 kJ only.

So, you can never convert anything more than this 0.7 kJ to heat or to work and therefore this 1 kJ of energy heat energy has a work potential of only 0.7 kJ. This work potential is the one that we refer to as the exergy.

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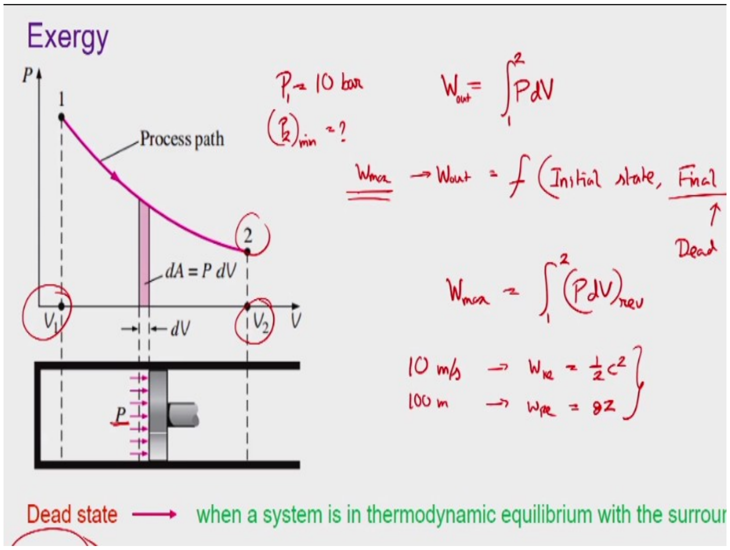

Exergy

$P_1 = 10$ bar

$(P_2)_{min} = ?$

$W_{out} = \int_1^2 PdV$

$\underline{\underline{W_{max}}} \rightarrow W_{out} = f$ (Initial state, $\underline{\text{Final}}$

$\uparrow$

Dead

$W_{max} = \int_1^2 (PdV)_{rev}$

10 m/s $\rightarrow W_{ke} = \frac{1}{2}c^2$

100 m $\rightarrow W_{pe} = gz$

Dead state ⟶ when a system is in thermodynamic equilibrium with the surrour

Let us consider one simple compressible system which is undergoing a process from point 1 to point 2. We have a piston cylinder arrangement just shown like this. Here the system is expanding from an initial volume of V_1 to the final volume of V_2 during when the pressure is changing. P is the pressure that is acting on an inner side of the cylinder. So, how much is the work output from this?

$$W = \int PdV$$

and if we know the relation between P and V, you can easily calculate this one. Now, this is the work output that you can get from this particular engine. Now, the value of this W that you are getting or let me know it say W_{out} that depends upon what I should put the integration limits also 1 to 2. Now, this W_{out} that we have written here that is depending on what?

That is a function of which quantities here? That definitely depends upon the two endpoints initial point 1 and the final point 2. So, it is a function of the initial state, it is a function of the final state that is these are the two limits of the integration 1 and 2 and it depends upon the PdV relation that has been followed while plotting this pink line or during the process. i.e., it depends upon the process path. So the output actually depends upon three quantities, your initial state point, the final state point and the real process that has been followed between this 1 and 2 and can be mathematically represented as:

$$W_{out} = f\,(Initial\ state, final\ state, process\ path)$$

Like between 1 and 2 if we follow a process something like this, your work output will be different, whereas if we follow a process somewhat like this, your work output will be different. Or say for example you follow a two-step process like during one step it remains a constant volume up to this. And during the second step, let say constant pressure process like this. So, your work output again will be different because on a PV plane we know that the area under the curve is a work output and each of these lines that I have drawn here, each of them has a different area under the curve, accordingly the work output also will be different and therefore the output definitely depends upon the process path.

Now, once we have fixed up the initial and final state initial, initial state and final step I have fixed, these are 1 and 2 respectively. Then, for which kind of process you are going to get the maximum work output? Yes, you know the answer, if the process is reversible one you are going to get the maximum possible work output. Therefore, if this process is a reversible one then you are going to get the maximum possible work output and that will be:

$$W_{max} = \int_1^2 (PdV)_{rev}$$

That is between these two state points, you can get the maximum output, only when a reversible path has been followed and that reversible path can be related to any kind of process or combination of any processes but as long as that is reversible, you are going to get the maximum possible work output between those two state points. Now, I remove the restriction on this final state also.

You are following a reversible path starting from point 1, then what is the ultimate value of point 2 or ultimate position of point 2 that you can go for? Like suppose, for example, in this piston cylinder arrangement, you have started with an initial pressure say, your initial value of P_1 is something like 10 bar, then what can be the smallest value of P_2 that you can realistically achieve or maybe this piston cylinder can achieve?

Of course, the smallest value of pressure at point 2 can be the atmospheric pressure because you cannot go beyond atmospheric. Because outside here, you have the atmospheric pressure working against which this work is getting done and your piston once the pressure inside reaches the value of atmospheric pressure, then it reaches some kind of thermal equilibrium or I should say the system and surrounding has reached thermal equilibrium. And so, it is not possible to have any further movement of this piston. So, the smallest possible position the piston can reach is the atmospheric pressure. And therefore this point 2 should be the one which allows the system to reach some kind of thermodynamic equilibrium to the system. Like the example that we are talking about here, we only the mechanical equilibrium is relevant but there can be several other potentials still left in the system.

Like say for example, when the system reaches the atmospheric pressure, still it is having some velocity of 10 m/s. Now, can you convert this velocity to work? Of course, we can convert and corresponding work output that we can get corresponding to the kinetic energy of this one is:

$$W_{KE} = \frac{1}{2}c^2$$

Or if you are talking per unit mass of course or if the system is some elevation of 100 meter from the datum. Then, corresponding potential energy can also be converted to work which magnitude will be:

$$W_{PE} = gz$$

Therefore, as long as the system is not in perfect thermal equilibrium with the surrounding, we can find newer ways of producing work output from the system. And the maximum possible work output we can get only when the system reaches in perfect thermal equilibrium to the surrounding, then, that state we refer to as the dead state. Dead state refers to the state when a system is in perfect thermal equilibrium with the surrounding, means it is in thermal equilibrium, it is in mechanical equilibrium and all possible source of equilibrium.

Like the example, we have talked about if the system is at a temperature higher than surrounding, then what will happen? You can easily run a heat engine between the system temperature and surrounding temperature and get some work output from this. If the system pressure is higher, you can allow the piston to move a bit more to get some additional work. If the system is having some kinetic energy or potential energy with respect to surrounding, this way you can extract some work output from this.

So, the system has to be in perfect thermal equilibrium for it to reach this dead state. And once the system reaches the dead state, then we have been able to harness maximum possible work output from the system starting from this initial point 1. Therefore, once I have given you one particular initial point, then what is the maximum possible work output that you can get? When will be this W_{out} maximum?

When the process path that has been followed is a reversible one and secondly the final state is the dead state or the state of the surrounding. Then, corresponding with the initial state, you have harnessed maximum possible work output from the system and that is what we define as the exergy. Exergy refers to the maximum possible work a system can deliver as it undergoes a reversible process from a specified initial state to a final state. Or we can say this is the exergy of the system at that specified initial state.

Corresponding to every state point, the system can have exergy. Exergy is always a positive quantity. And therefore whenever this initial state point is not the dead state, then we can find some way of producing some work output. Even when the system is at a temperature lower than the surrounding, then you can run a heat engine using the surrounding as a source and the system as the sink and get some work output from this.

So, even when the system is at a temperature lower than surrounding or at a pressure lower than surrounding you can get some work output from this. So, exergy is always a positive quantity and it refers to the maximum possible work output that you can get from a system at a given state point once we execute a reversible process from that initial state to a dead state. So, exergy gives you the work potential of a system at a given state point and for every state point we can calculate exergy differently.

Here, I should mention that in certain literatures this exergy is referred as available work or availability particularly some of the older books use the term availability but nowadays, the scientific community has given the consent to use this term exergy alone. And therefore, we are going to use this term exergy also but, in some books, if you find the title of any chapter to be availability do not get confused that refers to the same thing.

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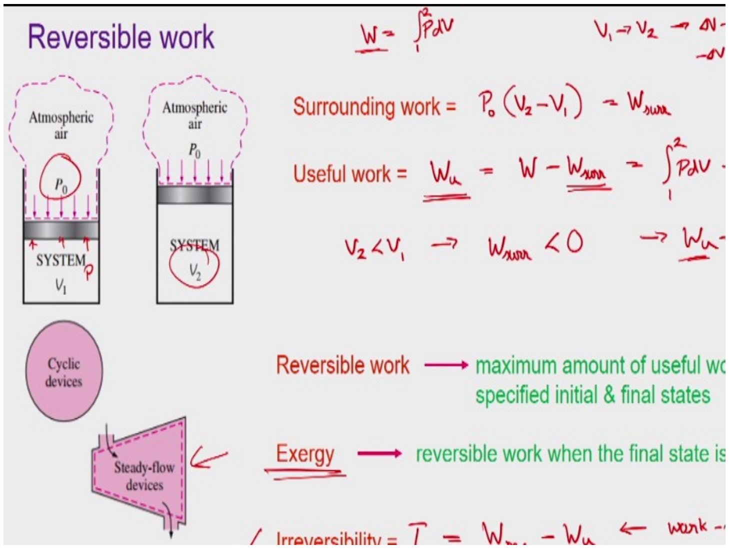

Reversible work

$W = \int_1^2 PdV$

$V_1 \rightarrow V_2 \rightarrow \Delta V$

$-\Delta V$

Surrounding work = $P_0 (V_2 - V_1) = W_{surr}$

Useful work = $W_u = W - W_{surr} = \int_1^2 PdV$

$V_2 < V_1 \rightarrow W_{surr} < 0 \rightarrow W_u$

Reversible work → maximum amount of useful w
specified initial & final states

Exergy → reversible work when the final state is

Irreversibility = $I = W_{rev} - W_u \leftarrow$ work

Now, there are several other terms that we generally have to define in conjunction with exergy, and let us quickly review them. Let us consider another piston cylinder arrangement which is initially at a volume V_1 and it is undergoing some process to reach a final state point of V_2. The pressure that is acting on the inner side of this cylinder is let us say is P. Now this P has to be greater than the surrounding pressure for the system to expand and to produce some work output.

Now, we know that the work output that the system is going to produce that is the W that we are going to get from the system is:

$$W = \int_1^2 PdV$$

However, this work actually is getting done against the atmospheric pressure and therefore some work will be some portion of this integral PdV actually we lost to overcome the pressure from the surrounding or you can visualize that separate way.

Instead of talking about system and surrounding, let us say we have two systems, the surrounding being the second one which is at a constant pressure of P_0. Now, when the system volume is changing from V_1 to V_2, the system is having actually a change in volume of δV that is:

$$\delta V = V_2 - V_1$$

Correspondingly, surroundings also having a change of $-\delta V$ amount in this volume.

And therefore, when the system is giving a positive work output because of this positive change in volume, the surrounding is actually having a work input because of this negative change in volume. Therefore a portion of the W produced by the system will be consumed by the surrounding in order to overcome the effect of the surrounding pressure P_0 and that is something refer as the surrounding work.

Surrounding work refers to this particular portion of the work that is being consumed by the surrounding and therefore it can be referred as surrounding pressure P_0. It can always be taken as constant, so it can be referred as:

$$\text{Surrounding work} = P_o\,(V_2 - V_1)$$

This is the amount of work which has been given to the surrounding because of the movement of the boundary.

And therefore, what is the actual work output or usable work output we are getting? That is, we refer as the useful work, for surrounding work let us use the symbol W_{surr}. So, useful work we shall be using the symbol W_u which is the actual work produced by the system-the surrounding work that is:

$$W_u = W - W_{surr} = \int_1^2 PdV - P_0\,(V_2 - V_1)$$

And hence whatever may be the magnitude of actual work your system is producing, actually you will always be losing this amount of work to overcome the surrounding pressure or atmospheric pressure, and you will be getting only this much of useful work. In situations your surrounding work may be extremely small or negligibly small but again under certain situations it can be quite significant.

In addition to that, surrounding work may not always be a loss, just think about the situation where the volume of the system is actually reducing that is:

$$V_2 < V_1$$

Then, what about your surrounding work? Your W_{surr} is actually negative, that is:

$$W_{surr} < 0$$

$$\begin{aligned} W_u &= W - W_{surr} \\ &= W + P_o\,(V_1 - V_2) \end{aligned}$$

and in this case, ($V_1 - V_2$) being a positive quantity where the system is undergoing a reduction in this volume, then actually the surrounding work will help. i.e., when you are going for a compression work, the added pressure of the atmosphere helps in reducing the actual work requirement for your system. Also your W will be then lesser than your useful work and you will be gaining from the surrounding work. Also, if your system does not involve moving boundary work, there is no surrounding work involved. Like if you are talking about a rigid system, rigid body or you are talking about steady flow device where there is no change in volume then there is no surrounding work.

Also, when you are talking about a cyclic device, in a cyclic process we start from one particular state and going through several states come back to that state point. So, what is the net change in volume of the system over a cycle that is 0, and therefore there is no surrounding work. You can visualize this way that during one process of the cycle, the system may be losing some surrounding work. Whereas it is gaining the same amount of surrounding or doing some other process in the same cycle. And, therefore over one full cycle, the surrounding work or net surrounding work is 0, so it does not need to be considered during analysis. However, when you are talking about a single process during which there is movement of the system boundary, we have to consider the surrounding work and therefore useful work will be different from the surrounding work.

Next term that we are concerned about is reversible work. Reversible work refers to the maximum value of the useful work when we are performing a reversible process between a specified initial and final state point i.e.,

$W_{rev} = W_u \mid_{1^{rev}\rightarrow 2}$
because a reversible process is going to give you the maximum possible work output between 2 given state points.

And W_{rev} or reversible work refers to this maximum value of useful work between two given state points. And when this final state is the dead state then what happens? Then, this one we are going to the dead state which commonly referred as zero, then we have harnessed the maximum possible work that we could have got from the system and that is what is exergy. So, exergy refers to the maximum value of reversible work that or the reversible work we can harness from the system when the final state is the dead state.

So, we are starting from our initial point and going to the dead state following a reversible process and whatever reversible work that we are getting that is the exergy. So, this is an alternate definition of the exergy and the difference between reversible work and useful work refers the irreversibility. Generally, we use the symbol I. In the previous lecture, I mentioned that I shall be coming back to the term irreversibility.

Now, here you have the definition, irreversibility is the definition between the reversible work and useful work. Because if your practical process is an irreversible one, then your useful work for a work producing device will be less than the reversible work, you will be losing something because of the presence of irreversibilities. So, for a work producing device your irreversibility will be:

$$I = W_{rev} - W_u \qquad \leftarrow \quad \textit{for work producing device}$$

Because for a work producing device, reversible process gives you the maximum possible output because of the presence of irreversibilities, we lose something and the difference between reversible work and useful work is the irreversibility. Whereas it can also for a work consuming device like a pump or compressor, what will be the implication because of the presence of irreversibilities?

Because of the presence of irreversibilities, the work requirement will be more, so the irreversibility is defined as:

$$I = W_{rev} - W_u \qquad \leftarrow \quad \textit{for work consuming device}$$

because a reversible process gives the minimum possible work input requirement for a work consuming device. So, depending upon whether we are talking about a work producing process or work consuming process, the definition of irreversibility has to be considered.

Sometimes, we can define a common definition which is just the magnitude of the difference between the reversible process and the useful process

$$I = |W_{rev} - W_u|$$

If we are talking about a work producing device, reversible work will be higher. If we are talking about a work producing device, reversible work will be higher, whereas if we are talking about work consuming process, the useful work will be higher. And their difference is the irreversibility is always a positive quantity. And this irreversibility can now be referred to the entropy generation which we have introduced in the previous lecture as:

$$I = T_o S_{gen}$$

where

T_0 is the surrounding temperature

S_{gen} is the magnitude of entropy generation

So, this way we can relate the irreversibility with the entropy generation. And we can also get a relationship between exergy and the entropy generation because when the final state is a dead state, this W reversible refers to the exergy of the system.

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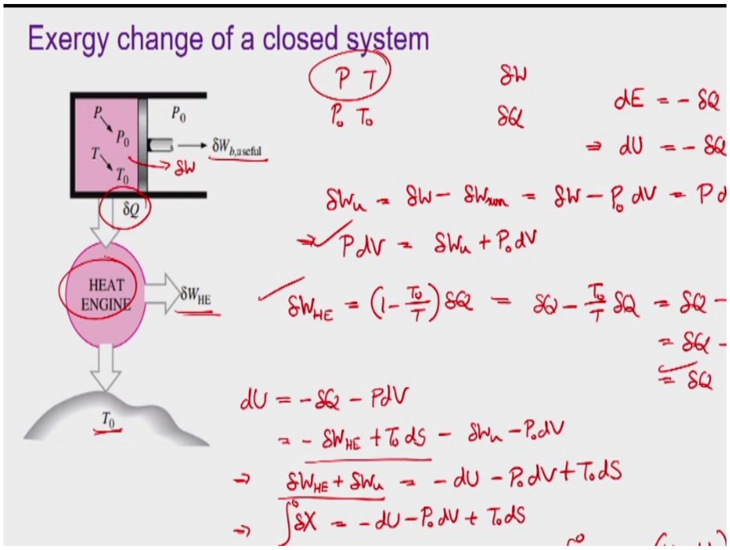

Exergy change of a closed system

$P\ T$

$P_0\ T_0$

δW

δQ

$dE = -\delta Q$

$\Rightarrow dU = -\delta Q$

$\delta W_u = \delta W - \delta W_{surr} = \delta W - P_0\, dV = P\, d$

$\Rightarrow P\, dV = \delta W_u + P_0\, dV$

$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right)\delta Q = \delta Q - \frac{T_0}{T}\delta Q = \delta Q -$

$= \delta Q -$

$= \delta Q$

$dU = -\delta Q - P\, dV$

$= -\delta W_{HE} + T_0\, dS - \delta W_u - P_0\, dV$

$\Rightarrow \delta W_{HE} + \delta W_u = -dU - P_0\, dV + T_0\, dS$

$\Rightarrow \int^{0} \delta X = -dU - P_0\, dV + T_0\, dS$

So, let us see how we can calculate the exergy change of a closed system. Your system is initially at a state point say 1, pressure is P, temperature is T etc whereas the surroundings at a condition of P_0 and T_0 and whatever may be the other parameters. This subscript '$_0$' refers to the dead state.

Now, let us consider a situation where the system undergoes an infinitesimally small process during which it gives you δW amount of work output and δQ amount of heat is rejected by the system. This is the amount of heat rejected by the system and this is the amount of work output, actually this is δW_{useful} is given. Let us say the system produce δW amount of work output.

If we subtract surrounding work, you are going to get this δW useful from this. Here, be careful we are taking both heat transfer and work transfer from system to the surrounding. So, if you write the first law of thermodynamics then we can write that dE the change in the stored energy of the system will be:

$$\mathrm{d}E = -\,\delta Q - \delta W$$

And now let us assume a simple compressible system i.e., the system does not have any kind of motion or gravitational effect. In that case, dE reduces to dU that is the change in the internal energy

$$\mathrm{d}U = -\,\delta Q - \delta W$$

And we have to calculate the exergy of this system at this initial situation of P, T etc. Now, your δW_u will be what, will be equal to:

$$\delta W_u = \delta W - \delta W_{surr} = \delta W - P_0\mathrm{d}V$$

where

dV refers to the infinitesimally small change in the volume of the system during this particular process

Now, δW for a simple compressible system can be written as:

$$\delta W_u = P\mathrm{d}V - P_0\mathrm{d}V$$

or we can write this PdV as:

$$P\mathrm{d}V = \delta W_u + P_0\mathrm{d}V$$

Let us now consider the heat transfer. The heat that has been transferred from the system to the surrounding is actually lost unless we supply this heat to a reversible heat engine to get some positive work output from the system.

So, we have to calculate the work potential for this heat by considering this heat engine to a reversible one. Then, how much work output that we can get from this heat engine or reversible heat engine? That is equal to:

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right) \delta Q$$

.

where

T_0 is the surrounding temperature

T is the temperature at which the heat has been supplied by the system to the heat engine

δQ is the heat received by the heat engine

So, this way we are not losing any energy, we are extracting or harnessing the maximum possible work potential of this heat δQ. So, we can write the above equation as:

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right) \delta Q = \delta Q - \frac{T_0}{T} \delta Q = \delta Q - T_0 \left(\frac{\delta Q}{T}\right)$$

Now, what is your $\delta Q/T$?

$\delta Q/T$ is the magnitude of the change in the entropy of the system during this particular process as for the definition of the entropy. So, the above can be written as:

$$= \delta Q - T_0 (-dS)$$

Here, this minus sign comes in because while δQ we are taking to be positive because we are writing this expression in terms of the heat engine but actually a system is losing heat. So, δQ is negative for the system and that is why this $-dS$ is coming into picture. That is equation can be in the following form:

$$= \delta Q + T_0 \, dS$$

Now, what is the maximum exergy of the system or I should say what is the maximum work potential or exergy of the system? So, if we add all this up, then we can get an expression of the maximum work potential of the system. So, let us go back to the first law expression, we had:

$$\mathrm{d}U = -\delta Q - \delta W$$

or instead of δW let us write this to be equal to $P\mathrm{d}V$, so that the equation becomes:

$$\mathrm{d}U = -\delta Q - P dV$$

So $-\delta Q$ taking from the above expression and taking $-P\mathrm{d}V$ *by rearranging the equation*

$$\delta W_u = P\mathrm{d}V - P_0 \mathrm{d}V$$

Maximum work potential of the system, $\mathrm{d}U$ can be written as:

$$\mathrm{d}U = \delta W_{HE} - T_0 \, dS - \delta W_u - P_0 \mathrm{d}V$$

Rearranging the above equation, we get the following expression:

$$\delta W_{HE} + \delta W_u = -\, dU - P_0 dV + T_0\, dS$$

So, we are left with this '$\delta W_{HE} + \delta W_u$' and what is this?

This is the total amount of work that you could have harness from the system. We are using the boundary work and also, we are using the heat to produce some reversible work from this heat engine and so this is the exergy of the system. So, often referred as the change in the exergy of the system, δX as:

$$\delta X = -\, dU - P_0 dV - T_0 dS.$$

Actually, this is a reversible work that we could have got from the system during this infinitesimally small process. And exergy will be once we integrate this from this initial point to the final point which is the atmospheric pressure. So the exergy of this system is often referred as X and is quite often used as a symbol for exergy. So, it will be:

$$X = \int_1^0 dU - P_0 \int_1^0 dV + T_0 \int_1^0 dS = (U_1 - U_0) + P_0(V_1 - V_0) - T_0(S_1 - S_0)$$

and that is the exergy of any system. Quite often, we write this one as per unit mass basis which is the symbol ϕ which is nothing but X/m,

So that can be written as:

$$\phi = \frac{X}{m} = (u_1 - u_0) + P_0(v_1 - v_0) - T_0(s_1 - s_0)$$

and this is the exergy of a closed system at a given state point 1. So, once you know the initial state point in terms of whatever properties say pressure and temperature, you can easily calculate the expression for the exergy of this closed system.

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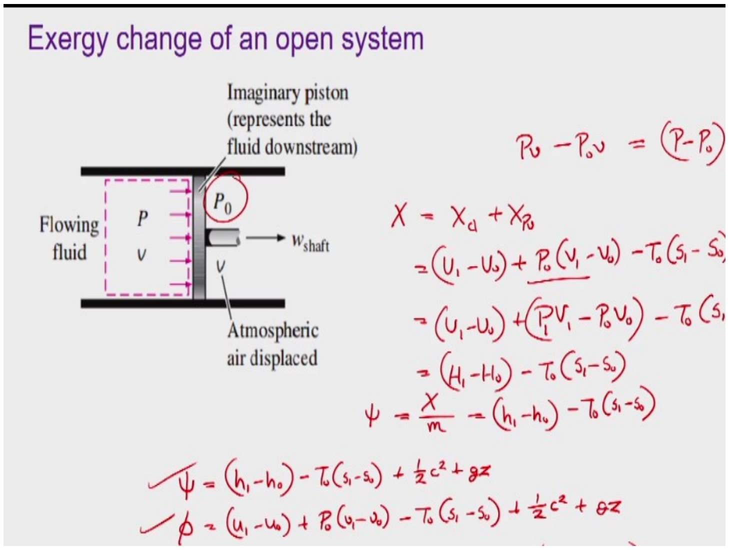

Exergy change of an open system

$$P v - P_0 v = (P - P_0)$$

$$X = X_d + X_{P_0}$$

$$= (U_1 - U_0) + \underline{P_0(V_1} - V_0) - T_0(S_1 - S_0$$

$$= (U_1 - U_0) + (P_1 V_1 - P_0 V_0) - T_0(S_1$$

$$= (H_1 - H_0) - T_0(S_1 - S_0)$$

$$\psi = \frac{X}{m} = (h_1 - h_0) - T_0(s_1 - s_0)$$

✓ $\psi = (h_1 - h_0) - T_0(s_1 - s_0) + \frac{1}{2}c^2 + gz$

✓ $\phi = (u_1 - u_0) + P_0(v_1 - v_0) - T_0(s_1 - s_0) + \frac{1}{2}c^2 + gz$

And if we now move to the open system, in open system only one additional effect comes into picture that is because of the flow work. So, how much is the work potential for a flow work? We know the magnitude of flow work is Pv and as this work is being done against the surrounding pressure P_0, so you are losing this P_0v, so

$$Pv - P_0v = (P - P_0)v$$

is the surrounding work.

So, if we want to calculate the exergy for an open system then what will it be? It will be:

$$X = X_{cl} + X_{Pv}$$

where

X_d is the exergy for the closed system that we have just developed

X_{Pv} is the exergy for the closed system

The expression of exergy for the closed system that we have developed previous page is:

$$X_{cl} = (U_1 - U_0) + P_0(V_1 - V_0) - T_0(S_1 - S_0)$$

so, if we combine these expressions, we get X as

$$X = (U_1 - U_0) + P_0(V_1 - V_0) - T_0(S_1 - S_0) + (P_1 - P_0)v_1$$

here we are using *'V'* because you are not writing in specific sense, so we have:

$$= (U_1 - U_0) + (P_1V_1 - P_0V_0) - T_0(S_1 - S_0)$$

and now if we combine is U and Pv part, we actually have enthalpy and the equation will become:

$$= (H_1 - H_0) - T_0(S_1 - S_0)$$

and generally, we use the symbol ψ to represent exergy of an open system per unit mass which is:

$$\psi = \frac{X}{m}$$

For this open system that is:

$$\psi = \frac{X}{m} = (h_1 - h_0) - T_0(s_1 - s_0)$$

and here we are talking about a simple compressible system which does not have any kinetic or potential energies. But if the system is having kinetic and potential energies, you have to add the kinetic and potential components to that one also, quite similar thing you have to do for the closed system as well. Like so the generalized expression for exergy per unit mass for an open system can be written as:

$$\psi = \frac{X}{m} = (h_1 - h_0) - T_0(s_1 - s_0) + \frac{1}{2}c^2 + gz$$

where

c is the velocity with respect to the surrounding

z is the elevation of the system with respect to the surrounding

Similarly, for a closed system, it will be equal to:

$$\phi = (u_1 - u_0) + P_0(v_1 - v_0) - T_0(s_1 - s_0) + \frac{1}{2}c^2 + gz$$

This way for any given state point, you can calculate the exergy of the system and when the system undergoes certain process say, from a process from state point 1 to state point 2, then, the total change in the exergy of the system, if it is a closed system then it will be just equal to initial exergy was ϕ_1 and it changes to ϕ_2 and if we multiply by mass then you have the total change in the exergy of the system during this process as given below:

$$m(\phi_1 - \phi_2)$$

So, this concept now you can use for any problem and this way you can calculate the total exergy that changes during a particular process.

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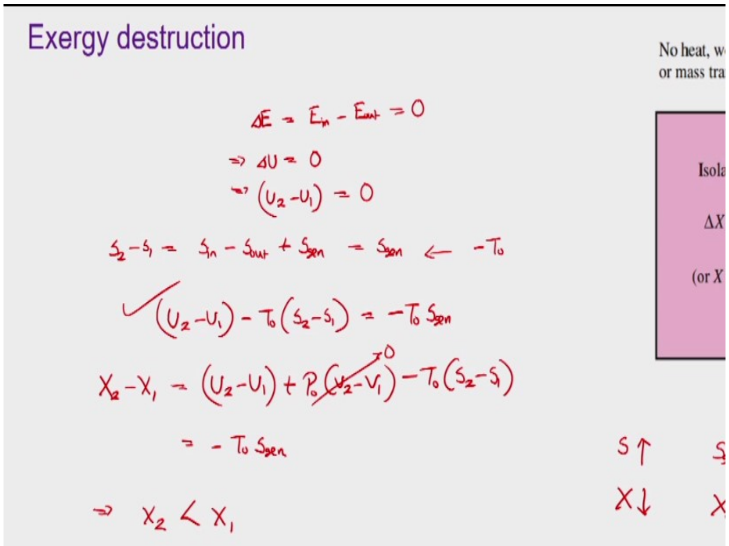

Exergy destruction

No heat, w
or mass tra

Isola

ΔX

(or X

$$\Delta E = E_{in} - E_{out} = 0$$

$$\Rightarrow \Delta U = 0$$

$$\Rightarrow (U_2 - U_1) = 0$$

$$S_2 - S_1 = S_{in} - S_{out} + S_{gen} = S_{gen} \leftarrow -T_0$$

$$✓ (U_2 - U_1) - T_0(S_2 - S_1) = -T_0 S_{gen}$$

$$X_2 - X_1 = (U_2 - U_1) + P_0\cancelto{0}{(V_2 - V_1)} - T_0(S_2 - S_1)$$

$$= -T_0 S_{gen}$$

$$\Rightarrow X_2 < X_1$$

$S \uparrow$

$X \downarrow$

S

X

Now, we added the concept of exergy destruction and for which we are considering an isolated system. So, an isolated system refers to a system which does not have any kind of interaction with the surrounding. Now, if we write the energy equation for an isolated system what you can write? The change in the energy content for the isolation system is:

$$\Delta E = E_{in} - E_{out}$$

Now, an isolated system does not have any kind of energy interaction, so that has to be equal to 0. Or we can say the total energy interaction in terms of either it does not have kinetic and potential energies, then we can write for this isolated system:

$$\Delta U=0$$

Now, if we talk about the entropy balance for the isolated system, we know that the entropy of an isolated system can only increase because of some irreversible process. So, writing that in formal terms, we have:

$$(U_2 - U_1) = 0$$

Similarly, for exergy balance says, change in exergy:

$$S_2 - S_1 = S_{in} - S_{out} + S_{gen}$$

Now, it is being an isolated system, there is no entropy in or out. So, only way its entropy can change is through the generation. Now, we multiply this equation with $-T_0$ add with the first equation. Then, what you have? You have:

$$(U_2 - U_1) - T_0(S_2 - S_1) = -T_0 S_{gen}$$

Then, isolated system cannot have any kind of interaction with the surrounding and therefore it cannot produce any moving boundary work also. That means is volume will also remain the same. So, if we write an expression for the change in the exergy of this isolated system:

$$X_2 - X_1 = (U_2 - U_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1)$$

neglecting any kinetic and potential energy changes.

It cannot have any kind of volume change, so $P_0(V_2 - V_1)$ is equal to 0. So, now using this expression, we have this change in the exergy equal to:

$$X_2 - X_1 = -T_0 S_{gen}$$

Now, here T_0 is the absolute temperature of the surrounding and entropy generation. So, absolute temperature is always a positive quantity. Similarly, entropy generation is also a positive quantity. So, what does that mean?

It means:

$$X_2 < X_1$$

that is entropy of an isolated system can only reduce or decrease and therefore this principle is known as the principle of exergy destruction. Exergy of an isolated system can only decrease. So, exergy of a system can definitely increase, but system surrounding combination should lead to a decrease in the exergy. So, this actually is an alternative statement of this principle of entropy generation.

While entropy of an isolated system can only increase, exergy of an isolated system can only decrease. And therefore in the real process can happen only in the direction in which either we have positive entropy generation or sometimes we call it to be exergy destruction. Exergy destruction is the change in the exergy or reduction in the exergy of the isolated system because of the presence of irreversibilities.

So, real process can proceed only in the direction in which you will be having a positive entropy generation or exergy destruction. So, this leads to the final concept that we had to discuss here which is the balance of exergy of a system.

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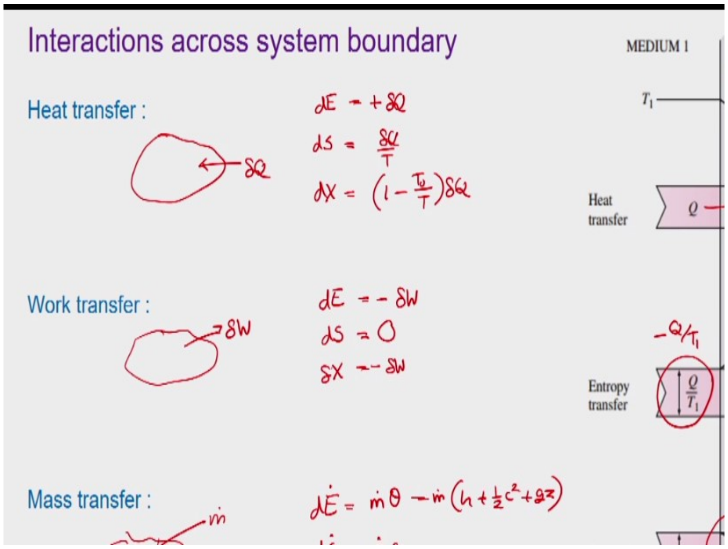

Interactions across system boundary

Heat transfer :

$$dE = +\delta Q$$
$$dS = \frac{\delta Q}{T}$$
$$dX = \left(1-\frac{T_0}{T}\right)\delta Q$$

Work transfer :

$$dE = -\delta W$$
$$dS = 0$$
$$\delta X = -\delta W$$

Mass transfer :

$$d\dot{E} = \dot{m}\theta - \dot{m}\left(h+\frac{1}{2}c^2+gz\right)$$

Before that let us quickly check what are the interactions a system can have across this boundary and the surrounding. We know we can have three kinds of interaction: heat, work or mass. So, first is heat transfer. Let us say this is your system and it is receiving δQ amount of heat from the surrounding. Then, how you can write the energy equation, so the change in the energy of the system, dE will be:

$$\mathrm{d}E = +\,\delta Q$$

where

δQ is the change in the energy because of this heat transfer

How much will be the change in the entropy of the system? You know the change in the entropy, dS will be:

$$dS = \frac{\delta Q}{T}$$

and if you are interested in the total change in entropy, then we have to integrate this over the change because whenever heat is being transferred to a system, it also is associated with some entropy transfer and now how much is the exergy that flows into the system with this heat? That is:

$$dX = \left(1 - \frac{T_0}{T}\right)\delta Q$$

that is the work potential of this heat. So, this way with heat we can have energy entropy and exergy transfer into the system. This is a way we can define like here across a wall you are having δQ amount of heat transfer, but inside in medium 1 we have a temperature T_1 and outside temperature is T_2. Now, energy remains conserved and accordingly it flows smoothly through the wall.

However, entropy is generated inside this, so inside your temperature will be, entropy transfer will be Q/T_1. So, here this medium 1 is losing Q/T_1 amount of heat but medium 2 is gaining Q/T_2 amount of heat, which actually will be higher than what value you got for 1 because T_2 is smaller than T_1 and the reason is this entropy generation inside the wall because of the change in temperature.

And what about exergy? Here, this amount of exergy is being transferred from medium 1 into the wall but outside we get only this much, there is this amount of loss, this amount of exergy destruction because of the temperature change inside this wall. That is how we calculate entropy and exergy transfer associated with heat. Now, what about work transfer? So, again you have a system like this and let us assume system is transferring δW amount of work to the surrounding.

Then, how much is the energy change of the system? This will be:

$$\mathrm{d}E = -\,\delta W$$

How much is the entropy transfer of the system? Work being organized form of energy that does not cause any kind of change in entropy and therefore it will be:

$$\mathrm{d}S=0$$

but how much will be your exergy transfer with this work? What is exergy? Exergy is work potential only, so your exergy loss will be:

$$\delta X = \delta W$$

The direction of δW can be opposite, in that case system will be gaining exergy, in this case it is losing exergy.

And finally, we have mass transfer. So, again we consider a system from where at a rate $\dot{m}$ mass is flowing into the system. So, how much is the change in the energy content of the system? That will be equal to:

$$d\dot{E} = \dot{m}\theta = \dot{m}\left(h + \frac{1}{2}c^2 + gz\right)$$

So, this is the rate at which energy of the system will increase.

What will be the rate at which entropy changes? That will be equal to:

$$d\dot{S} = \dot{m}s$$

very simple.

How much will be the rate of exergy change of the system? That will be equal to:

$$d\dot{X} = \dot{m\psi}$$

we have just calculated the exergy per unit mass. So, this way exergy rate or transfer of exergy into system can be calculated. So, using this now we know how to calculate the exergy of a system closed or open and also, we know how to calculate the exergy transfer and entropy transfer to a system or from a system associated with heat, work and mass transfer.

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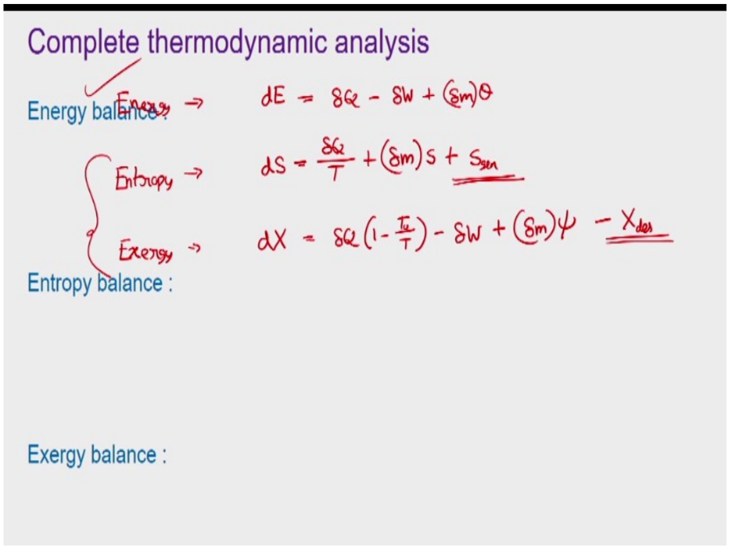

Complete thermodynamic analysis

Energy balance :

Energy → $dE = \delta Q - \delta W + (\delta m)\theta$

Entropy → $dS = \frac{\delta Q}{T} + (\delta m)s + \underline{\underline{S_{gen}}}$

Exergy → $dX = \delta Q\left(1 - \frac{T_0}{T}\right) - \delta W + (\delta m)\psi - \underline{\underline{X_{des}}}$

Entropy balance :

Exergy balance :

So, we finally combine all of them to get a complete thermodynamic analysis. To perform a thermodynamic analysis, we have to perform both first law and second law analysis. First law analysis means we have to write an energy conservation principle. Remember, energy is a conserved property. Of course, energy may reduce or increase because of the conversion of mass to energy following the theory of relativity. However, that happens only in very limited cases of nuclear reactions which we are neglecting here. So, the change in the energy content of a system will be:

$$dE = \delta Q - \delta W + (\delta m)\theta$$

This is the energy equation; we can write this a conservation equation.

What about entropy? Entropy is a non-conserved property because of the presence of entropy generation. So, we have change in entropy:

$$dS = \frac{\delta Q}{T} + (\delta m)S + S_{gen}$$

Here, we are writing in per unit mass basis and finally what about the exergy? If we write for a closed system suppose, change in exergy, we are writing everything in extensive sense. So, this is entropy generation and finally the change in exergy will be exergy transferred with heat which is:

$$dX = \delta Q\left(1 - \frac{T_0}{T}\right) - \delta W + (\delta m)\psi - X_{des}$$

where

$-\delta W$ is the exergy loss by the system because of work

$(\delta m)\theta$ is the exergy transfer with mass

X_{des} is the exergy destroyed during the process

So, you have to be very careful, these two are not conserved properties for entropy balance, you have to consider this entropy generation, for exergy as well you have to consider this exergy destruction and we can choose either of these two equations both should give you the same result. And therefore, for a complete thermodynamic analysis of system, we have to solve the energy equation and either of entropy or exergy balance equation thereby giving you a complete picture of the system.

So, that I forgot about the symbols that I have put in, but the idea is I hope is well conveyed to. So, that takes us to the end of our first module.

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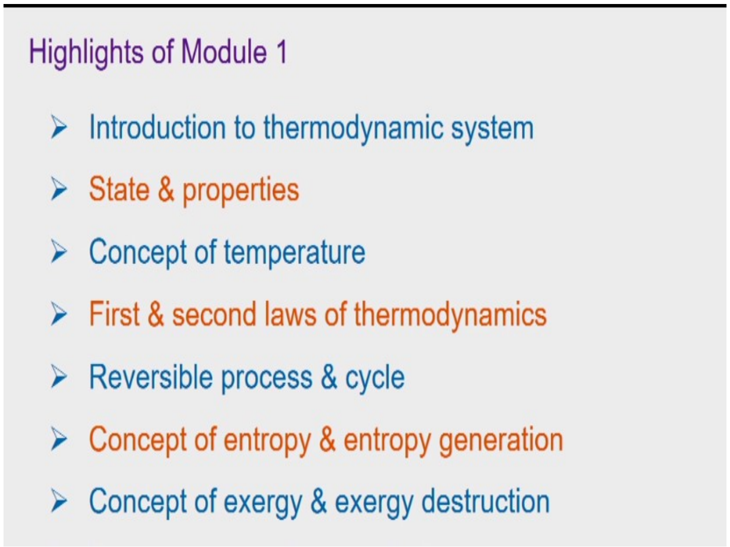

Highlights of Module 1

- Introduction to thermodynamic system
- State & properties
- Concept of temperature
- First & second laws of thermodynamics
- Reversible process & cycle
- Concept of entropy & entropy generation
- Concept of exergy & exergy destruction

In this module, we have reviewed the basic principle of thermodynamics by introducing you to the concept of thermodynamic system that talked about state, properties, equilibrium, etc. Then, the concept of temperature was introduced using the zeroth law of thermodynamics, then we reviewed the first and second law of thermodynamics. Using first law, we can get an energy balance of a system and can perform corresponding energy analysis, calculate the efficiency of heat engines or COP's of reversed heat engines.

Then, we talked about the reversible processes and reversible cycles from where we got the concept of Carnot cycle and Carnot efficiency. Then, in the previous lecture, we talked about the entropy and entropy generation. Today, we reviewed the concept of exergy and exergy destruction and finally combined all of them to get a complete thermodynamic analysis of a system.

So, we had a very quick review of basic thermodynamics in this particular week. You can solve some problems from the textbooks. I am not solving any problem here because you are expected to have already solved such problems but some of these will be given in the assignments.

Please try to solve those problems and also one suggestion whenever you are trying to performing second law analysis of any system, try to solve following both the approaches that is both entropy-based approach and exergy-based approach to check whether you can get the same result from them or not. So, that is it for this particular week. In the next week, we shall be starting the discussion on the calculation of thermodynamic properties.

And till then, you please review this particular module, review the lectures. If you have any doubt, please write to me immediately and so I can take some rest for this particular week. We shall be meeting again in the next week. Bye.