

Applied Thermodynamics for Engineers
Dipankar N. Basu
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture – 28
Modifications in VCR Systems

Hello everyone, so today in this third lecture we are going to continue our discussion on the refrigeration cycles. Over the previous two lectures we have discussed primarily about the vapour compression refrigeration cycle where we have already seen different variations of that particularly starting with the Carnot cycle we have discussed about the SSS cycle which is a most common vapour compression type cycle and also you have discussed about its applications and refrigerants.

Now in the last lecture towards the end of the second lecture we have discussed about the desirable properties of the refrigerants. And then we were supposed to discuss about the naming convention of the synthetic refrigerants that we commonly use in refrigerators or in refrigeration systems, but we ran out of time.

(Refer Slide Time: 01:15)

Naming convention for refrigerants

1. Synthetic FC-based refrigerants \rightarrow Alkanes $\rightarrow C_n H_{2n+2}$ CH_4 C_2H_6

R_{xyz} $x \rightarrow$ no. of C-1 \rightarrow no. of C = $x+1$

$R134 \equiv R134$ $x=1$ \rightarrow no. of other atoms = $2(x+1)+2$

$C \rightarrow x+1 \rightarrow 1+1=2$ $y=3$ \rightarrow no. of H = $y-1$

$H \rightarrow y-1 \rightarrow 3-1=2$ $z=4$ \rightarrow no. of F = z

$F \rightarrow z=4$ \rightarrow no. of Cl = $[2(x+1)+2] - (y-1) - z$

$Cl \rightarrow [2(1+1)+2] - (3-1) - 4 = 6-2-4=0$

2. Pure fluids based on inorganic refrigerants $R7xy$ $xy \rightarrow$ molecular weight

$NH_3 \rightarrow 14+3 \times 1=17 \rightarrow R717$

$H_2O \rightarrow 2 \times 1+16=18 \rightarrow R718$

$CO_2 \rightarrow 12+2 \times 16=44 \rightarrow R744$

3. Pure fluids $\rightarrow HC_x \rightarrow R_{xyz}$ $R290$ $x=2 \rightarrow C=2+1=2+1=3$

$y=9 \rightarrow H=9-1=9-1=8$

$z=0 \rightarrow F=0$

C_3H_8

4. Mixtures Azeotropic $\rightarrow R5_ _$

Zeotropic $\rightarrow R4_ _$

So, today I shall be starting with the naming convention of refrigerants. Now there are several kinds of refrigerants that we commonly use in refrigeration system depending upon the application, depending upon the prevailing pressure and temperature ranges etc. And the most common type

of refrigerant that we have is the synthetic FC-based refrigerants and FC refers to fluoro chloro, that is fluorine and chlorine-based refrigerants.

So, synthetic fluorine chlorine-based refrigerants are the most common types. You probably remember that we have used the terms CFC, HCFC and HFC. In the previous lecture CFC refers to chlorofluorocarbon, HCFC is a hydrochlorofluorocarbon and HFC is the hydro fluorocarbon. These are different kinds of HC-based refrigerants where you generally have fluorine and chlorine in the molecular structure present.

Now, synthetic FC-based refrigerants can be derived from different kinds of hydrocarbon groups or saturated hydrocarbon groups. But we shall be restricting our focus on the saturated hydrocarbon that is, we shall be focusing only on the alkenes. Alkenes which have the structure generally C_nH_{2n+2} , like common example can be methane where $n = 1$. If you put $n = 1$ then H will you have in a number 4, or if we put $n = 2$ then what will be the number of hydrogen atoms? It will be 6 and we get C_2H_6 , ethane.

So, these are alkenes so we shall be talking about the synthetic FC-based refrigerants whose structures are based upon alkenes. The idea is the carbon that is there in the alkenes will remain the same but all the hydrogen atoms or maybe some of the hydrogen atoms will be replaced by fluorine and chlorine atoms. Then we shall be having some kind of FC-based refrigerant. And for this kind of refrigerant you will always find that the name is given with the convention Rxyz. R-11, R-12 and R134a, these are the different names that we have we keep on hearing about the refrigerants.

And this is the convention Rxyz where R refers to refrigerants and xyz refers or give some idea about the molecular structure. Here x is the number of carbon atoms (C) minus 1, x refers to the number of carbon atoms present in the molecular structure minus 1. Therefore,

$$\text{no. of C} = x + 1$$

and what is when we know that the number of carbon atoms is $x + 1$ what will be the number of other molecules or other atoms I should say that excluding carbon basically.

So, we know that has to be twice the number of carbon atoms plus 2, so it will be:

$$\text{no. of other atoms} = 2(x + 1) + 2$$

Now here y refers to the number of hydrogen atoms remaining in this +1. Therefore,

$$\text{no. of H} = y - 1$$

So x refers to the number of carbons - 1, whereas y refers to a number of hydrogen +1 and z this refers to number of fluorine that is present in this. Generally, we are too much bothered about the presence of chlorine in these synthetic refrigerants because chlorine is the one that leads to the ozone depletion potential as we have discussed in a previous lecture. Where this chlorine comes out of the molecular structure and in the presence of UV rays that generally breaks the ozone down to form oxygen thereby depleting the ozone layer. But unfortunately, in this naming convention chlorine does not appear directly, rather chlorine we have to calculate separately. Once we know the magnitude of x, y and z, we know the number of some carbon, hydrogen and fluorine present there. Then we now can write:

$$\text{no. of F} = z$$

then what will be the number of chlorine atoms? We know that apart from carbon, the number of other atoms is 2 multiplied by $(x + 1) + 2$. Out of this hydrogen is $y - 1$ number and fluorine is z number. So, from there we can easily found the number of total chlorine atoms that is present there using the expression below:

$$\text{no. of Cl} = [2(x+1) + 2] - (y - 1) - z$$

Let us take some examples. if we have R11 instead of writing here we have just 2 numbers present in step 3. So, you can also write this one as R011, so what will be the number of carbon atoms? Now here your $x = 0$, $y = 1$ and $z = 1$. Then what is the total number of carbon atoms?

$$C = x + 1 = 1$$

$$H = y - 1 = 0$$

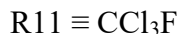
$$F = z = 1$$

Then how much will be the number of chlorine atoms? There number of chlorine atoms will be equal to:

$$Cl = [2(x+1) + 2] - (y - 1) - z$$

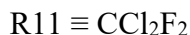
So, from there we can calculate directly as there is just one carbon atom. So, from the $2n + 2$ we can easily calculate the total number of other atoms will be equal to 4 out of this there is no

hydrogen there is one fluorine, so there will be 3 chlorine then what will be the molecular structure of R11? So



that is tri chlorofluorocarbon or I should say trichlorofluoromethane, that is the standard name or scientific name of R11. Instead of R11, if we talk about R12, so we are having R12 so it is 012, so $x = 0$, $y = 1$ and $z = 2$.

So, the number of carbon atoms will be $x + 1$ that is, 1 and let us remove all this. So, the number of carbon atoms will be $x + 1$ that is, 1, number of hydrogen atoms is $y - 1$ that remains 0 number of fluorine atoms in this case is 2. So how much will be the number of chlorine atoms? Number of chlorine atoms will be equal to $4 - 0 - 2$ that is equal to 2. So, how to represent this one R12 then



that is dichlorodifluoromethane. Where R11 is trichlorofluoromethane or trichloromonofluoromethane, R12 is dichlorodifluoro methane.

If we talk about something else, let us take another example of R22 another very popular refrigerant and generally used for large-scale commercial applications. Here even we can write R022, so

$$x = 0$$

$$y = 2$$

$$z = 2$$

Now how many carbon atoms are present here? That is 0 plus 1 that is equal to 1, so it is also methane-based. There is just one carbon atom now there is a hydrogen atom, 2 minus 1 that is 1 hydrogen atom is present and there is one fluorine atom also. Then the total number of chlorine atoms as there is only one carbon so total number of other atoms will be twice that number of carbon atoms plus 2 that is equal to 4 minus 1 hydrogen atom minus 2 fluorine atoms that will be equal to 1.

$$4 - 1 - 2 = 1$$

So,



So, these two are CFC, because in the molecular structure you have carbon, fluorine and chlorine whereas this one is an HCFC, hydrochlorofluorocarbons because hydrogen is also present there this HCFC.

We take another very common example R134a. So here we have let us write as R134. So,

$$x = 1$$

$$y = 3$$

$$z = 4$$

So, the number of chlorine atoms in this case? or okay first let us calculate the number of carbon atoms. So, there is $x = 1$, so it is $1 + 1 = 2$, so there are 2 carbons so that is actually ethane based. Previous three examples that we talked about R11, R12 or R22 they are all methane-based. Whereas R134a there it has 2 carbon atoms in its molecular structure. So, it is an ethane based 1.

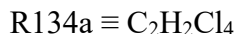
$$\text{Number of hydrogen atoms} = 3 - 1 = 2$$

$$\text{Number of fluorine atoms} = 4$$

Then total number of chlorine atoms how much it will be? So, as there are 2 carbons, so,

$$\text{Total number of other atoms} = 6 - 2 - 4 = 0$$

That is there is no chlorine present there. So,



Now we have two carbons and there is 2 hydrogen and 4 fluorine. So, it is an even structure and it is you can say tetrafluoroethane you can say the name of this one is tetra fluoro ethane right.

Now what is the type of this CFC, HCFC or HFC? This is an HFC because there is no chlorine present here and therefore its ozone depletion potential will be it is 0. As there is no chlorine present so, ozone depletion potential has to be 0. So, this is much more environment friendly one compared to the earlier three. Because earlier three: R11, R12 and R22 all are having chlorine present there. So they have ozone depletion potential which R134a does not have.

However global warming potential is there for all of them. So, this is the way we give the names to several common refrigerants or I should say the synthetic fluoro chloro based refrigerants. This is the naming convention that we commonly follow. Here of course I should mention that R134a,

the 'a' refers to is an isomer means it is also possible to have a few other molecular structures using the same combination. So, it is just one of several possible. I would request you to calculate a few others just following the same convention say R121 or something you can just put some numbers and try to see what can be the corresponding molecular structure.

Now second is, we have the pure fluid based inorganic refrigerants. Pure fluid based inorganic refrigerants refers to the natural refrigerants like ammonia, carbon dioxide, even water vapour though that is not very commonly used, but they all come under this category.

And here our naming convention is very straightforward. Naming convention is R7xy, the 7 is a fixed one and this xy, it refers to the molecular weight. So, it is a very simple convention this 7 actually corresponds to this inorganic refrigerants or natural refrigerants and xy is the molecular weight. Like suppose if we take ammonia, ammonia has a molecular weight of nitrogen is 14 + 3 multiplied 1 for hydrogen, so 17. Then its name will be R717. If we take H₂O, for H₂O it is 2 multiplied by 1 for hydrogen + 16 for water, so it is 18. So, it will be equal to R718 for water vapour. Another very popular refrigerants nowadays for large scale application particularly CO₂, how much it will be now carbon is 12 + 2 multiplied by 16 that is 44. So, 44 is the molecular weight for carbon dioxide. So, it will be R744 this way you can easily put something like sulphur dioxide or any other chemicals you can just put this xy or replace this xy by the molecular weight and you can get their name.

Third is the pure fluids pure fluids generally refers to hydrocarbons. Previous one was a pure fluid based on inorganic refrigerants here whereas here we are talking about pure fluids based on the organic refrigerants like hydrocarbons. Now here the naming convention is very similar to the synthetic FC-based refrigerants. That is we shall be following Rxyz and here the xyz convention is very similar to that number one.

Like let us take the example of say R290, so in this case we have

$$x = 2$$

$$y = 9$$

$$z = 0$$

They generally do not have fluorine present there so they always will become equal to zero. So, from there,

$$\text{Number of carbon atoms} = x + 1 = 2 + 1 = 3$$

$$\text{Number of hydrogen atoms} = y - 1 = 9 - 1 = 8$$

$$\text{Number of fluorine atoms} = 0$$

so chlorine or something else you can easily calculate actually there is no need to calculate as there are 3 carbons, so

$$\text{Total number of other atoms that can be present} = 3 \times 2 + 2 = 8.$$

So, this symbol is C_3H_8 , what is this? This is nothing but propane, so C_3H_8 is propane. Similarly we can calculate others also. You can easily calculate few others like say the ethanol, methane that we have written you can easily find what can be the corresponding symbol in terms of this convention.

The fourth possibility is the mixtures we shall only be talking about the mixtures but they are generally are two types of mixtures that we consider azeotropic and zeotropic. Now for an azeotropic one we write R5- - then something else where this F indicates to the azeotropic like for inorganic refrigerants or natural refrigerants we use that 7 and to indicate we are talking about natural refrigerants. Similarly, the 5 indicates that we are talking about azeotropic mixture and similarly R4- - talks about the zeotropic mixtures. And for this to fill up the two blank space we have to look to some reference tables.

So, these are the naming conventions that we commonly follow for refrigerants. Here this particular one and this one is the most common ones. The first category where you talk about the synthetic refrigerants which is most commonly used in for both domestic and industrial applications. So, you now know how we give the names this R11, R1,2 etc once we have the chemical composition known to us. Similarly, for pure fluids just like this way putting the 7 and then the molecular weight we can also form the symbols for any natural refrigerants and for similarly for any kinds of other refrigerants.

And one big problem that we have is when we have to deal with a very large temperature difference between the evaporator and condenser. Because we know that as the condenser temperature increases the *COP* decreases, similarly, as the evaporator temperature decreases the *COP* also decreases. Now the condenser temperature generally is fixed by the atmospheric consideration. But if we are looking to refrigerate something at very low temperatures that is your evaporator temperature is too low then if we are looking for just the normal cycle then your CO_2 will be extremely low. And therefore, we may have to go for several kind of modification just like this one.

Cascade refrigeration system

- ✓ reduction in compression work requirement
- ✓ increase in refrigeration capacity
- ✓ possible to reach very low evaporator temperature
- ✓ flexibility in choosing refrigerants

} COP ↑

$$\dot{m}_B (h_2 - h_3) = \dot{m}_A (h_5 - h_6)$$

$$\Rightarrow \frac{\dot{m}_A}{\dot{m}_B} = \frac{h_2 - h_3}{h_5 - h_6}$$

$$w_c = w_{c,A} + w_{c,B}$$

$$= \dot{m}_A (h_c - h_s) + \dot{m}_B (h_2 - h_1)$$

$$\text{COP} = \frac{Q_c}{w_c}$$

$$= \frac{\dot{m}_B (h_1 - h_4)}{\dot{m}_A (h_c - h_s) + \dot{m}_B (h_2 - h_1)}$$

$$= \frac{h_1 - h_4}{\frac{\dot{m}_A}{\dot{m}_B} (h_c - h_s) + (h_2 - h_1)}$$

$A \rightarrow 1$

$A \rightarrow 1$

$A \rightarrow 1$

$A \rightarrow 1$

The cascade refrigeration system, where not one but rather we have two refrigeration cycles. quite similar to the combined cycle that we have discussed earlier in conjunction in the power cycles. It is also something like combined cycle used for refrigeration purposes. Here just look at the bottom cycle given by this B, this bottom cycle is picking up the heat from the cold refrigerated space Q_{in}

amount of it has been picked up in the evaporator then it has the compressor the condenser and the expansion valve.

But its condenser is not rejecting heat to the surrounding rather the condenser is rejecting heat to the evaporator of the topping cycle which is A. So, the topping cycle is not receiving any heat from the refrigerated space whether it is receiving heat from the condenser of the bottoming cycle. But it is rejecting Q_H amount of heat to the warm environment or to the upper atmosphere. So, here this Q_L is equal to your evaporator load this Q_H is equal to your condenser load.

And whatever we have in between this Q_L does not seem to be correct let us strike this one off. So whatever amount of heat transfer is taking place in this heat exchanger from the condenser of the bottoming cycle to the evaporator of the topping cycle, that is only interior to the system. And therefore, when we shall be calculating the performance of this we only have to consider this evaporator load and this condenser load and of course the compressor work and here we have two compressors.

So, this is a compressor where we are giving W_{CB} amount of work input and this is the compressor where we were giving W_{CA} amount of work input so total work input definitely will correspond to these two compressors work together. Now what is the gain that we have here? Just look at the corresponding Ts diagram. Here this Ts diagram has been drawn assuming both the reference to be the same and that is why you are having the same Ts diagram.

But it is very much possible that we can use different refrigerants also. In that case the saturation dome also, we have to draw two saturation domes for two refrigerants. Now if it is the same refrigerant then look at the bottoming cycle. This is a standard say standard diagram for any VCRS that is 4 to 1 is the evaporation process then 1 to 2 is the compression process 2 to 3 is a condensation and 3 to 4 is the expansion in the valve.

Now how much is the total amount of heat this refrigerant reject in the condenser? The total amount of heat rejected will be equal to a rate of heat rejection in the condenser by the bottoming cycle

will be equal to the mass flow rate multiplied by the change in its enthalpy and the amount of energy that has been picked up by the topping cycle when it goes to this process from 8 to 5.

$$\dot{m}_B(h_2 - h_3) = \dot{m}_A(h_5 - h_8)$$

Or if we want to get the ratio of these two flow rates:

$$\frac{\dot{m}_A}{\dot{m}_B} = \frac{(h_2 - h_3)}{(h_5 - h_8)}$$

So, once we know all these state points then we can easily calculate the mass flow rates. And now, this 5 to 6 is the compression process in the topping cycle and then we have the rest of the process. 6 to 7 is the process during which it is finally getting rejected to the surroundings. Now if we want to reach from this, let us say talk about this temperature to be equal to your evaporator temperature of the combined system and this corresponds to the saturation temperature corresponding to your final condenser pressure. Then if we want to reach this final condenser pressure from the initial evaporator pressure. Then you would have ended up at after the compression at this particular point 2', that is 1 to 2' would have represented the single stage compression.

Rather what we are having here, here the compression is being achieved by two steps and as the temperature at point 5 is lower than the temperature point 2, so the work input required for the second stage of compression that is represented by this process 5 to 6 is definitely much lower than the work that would have been required during the compression 2 to 2' and therefore the shaded area, the area shaded in pink that is there is a decrease in the total compression work.

So, we are gaining from the compression work point of view and that is the first big advantage we are having significant reduction in the compression work requirement. Now how much is the total compression work:

$$\begin{aligned} W_C &= W_{C,A} + W_{C,B} \\ &= \dot{m}_A(h_6 - h_5) + \dot{m}_B(h_2 - h_1) \end{aligned}$$

So, *COP* of this combined cycle will be equal to:

$$COP = \frac{\dot{Q}_E}{\dot{W}_C}$$

$\dot{Q}_{dot E}$ is the evaporator load or actually the load picked up by the bottoming cycle and dividing it by total compressor load, as follows:

$$COP = \frac{\dot{m}_B(h_1 - h_4)}{\dot{m}_A(h_6 - h_5) + \dot{m}_B(h_2 - h_1)}$$

and if we divide everything by \dot{m}_B , then this becomes:

$$= \frac{(h_1 - h_4)}{\frac{\dot{m}_A}{\dot{m}_B}(h_6 - h_5) + (h_2 - h_1)}$$

So that is the *COP* of this combined cycle.

But there is another thing to look for. There is definitely a decrease in the compression work but there is also an increase in the refrigeration capacity. How is that possible? Instead of this combined cycle or cascaded cycle, if we are using just the single stage cycle, then the state at the end of compression is this point number 7. And as we are following an isenthalpic expansion so it should have ended up at this particular point say 4'.

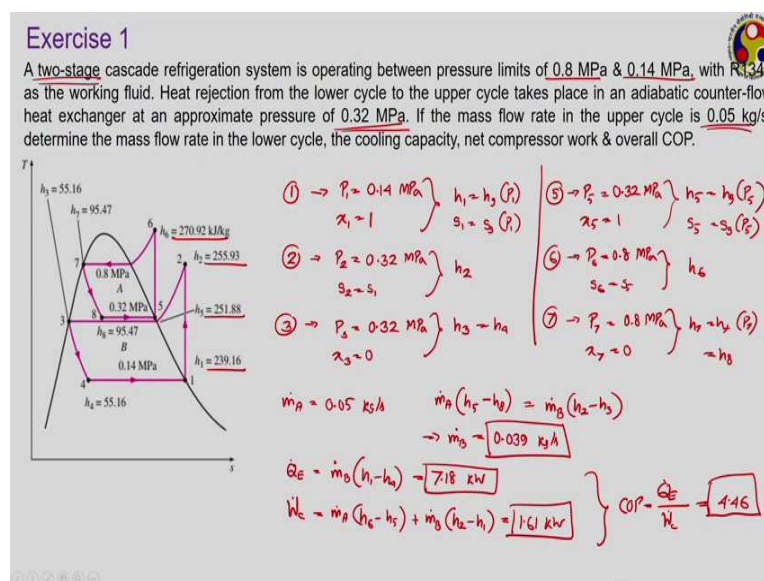
So, your evaporation process would have started from point 4' only and therefore this particular shaded area represents the amount of refrigeration load. This shaded area represents the amount of refrigeration capacity that we are actually losing or we would have lost. So, as you are going for the cascaded cycle, the state at the beginning of evaporation is 4 instead of 4'. So instead of having the evaporation going process going from 4'to 1, we are actually going from 4 to 1. So, we are able to put or able to have a much larger amount of cooling load being absorbed in the evaporator. So, this cascaded cycle also giving us an increase in the refrigeration capacity. Therefore, when we are going for a very large change in between the evaporation and condensation temperatures, then use of a cascade refrigeration system gives us advantage in twofold. It reduces the compression work requirement and also it increases the refrigeration capacity and therefore this whole thing gives an increase in the overall *COP* of the cascaded system compared to the equivalent single-phase system.

Also, it is possible to reach very low evaporator temperatures as I have mentioned earlier, each of the cycles has their own *COP*'s. And therefore, each of the cycle, none of the cycles are operating over a very large temperature change rather the temperature range correspond each of the cycles are much smaller compared to an equivalent single-phase cycle.

So, we can easily go to much lower evaporator temperature level despite getting a reasonably high *COP*. And also, as I have mentioned it is very much possible that we can choose two different refrigerants. That is, it is not mandatory to have the same refrigerant. If you are choosing two different refrigerants then we have to plot two different saturation domes to have the final *Ts* diagram. But it is very much possible like suppose your bottoming cycle is working within a temperature range of -50 to -10 °C whereas the top one topping cycle is working at -10 °C to 30 Celsius

Now you may have the refrigerant B which is more suitable for that sub-zero operation whereas refrigerant A is more suitable for the high temperature range. Therefore, you can easily go for two refrigerants, like suppose if your topping cycle has the lowest temperature of 5 °C and the highest temperature of 40 °C, then you can even use water as the refrigerant. Whereas for the bottoming cycle which is going to sub-zero temperature levels we definitely cannot use water because the water will become solidified.

(Refer Slide Time: 27:42)



Let us now take a numerical example just to see how we can deal with this cascaded refrigeration system. And I should mention that here though we have talked about two-stage but it is possible that we can have more than two stages also, three or four stages are also very much possible, particularly when we are looking to go for very low like cryogenic level temperatures. Now in this

problem we are talking about a two-stage cascade refrigeration system operating between pressure limits of 0.8 MPa and point 0.14 MPa with R134a as the working medium.

Heat rejection from the lower cycle to the upper cycle takes place in an adiabatic counter flow heat exchanger at an approximate pressure level of 0.32 MPa. Truly speaking here, we have assumed that the condenser temperature for the bottoming cycle and evaporator temperature for the topping cycles are the same and their pressures are also the same. But that is not true in practice, because the condenser temperature has to be slightly higher than the corresponding evaporator temperature otherwise there will be no heat transfer.

So, the condenser for the bottoming cycle will work at a higher-pressure level compared to the evaporator of the topping cycle. At least as long as we are going for the same refrigerant. But still here we are approximating the pressures to be the same that is 0.32 MPa. Now the mass flow rate for the upper cycle is given, so you have to determine the mass flow rate in a lower cycle, cooling capacity, net compression work and overall *COP*.

So, this is the *Ts* diagram for the cycle. State points are shown, once I provide you the property chart for R134a then you can easily find the properties for this. Just to show you some examples, say point number 1. We know that point number 1, corresponds to a pressure of 0.14 MPa and a quality of 1.

$$P_1 = 0.14 \text{ MPa}$$

$$x_1 = 1$$

Correspondingly h_1 will be equal to a h_g at that particular pressure i.e.

$$h_1 = h_g(P_1)$$

and s_1 will be equal to s_g at that particular pressure, i.e.,

$$s_1 = s_g(P_1)$$

so, point 1 is defined.

Point 2, the pressure at point 2 will be equal to the intermediate pressure which is 0.32 MPa,

$$P_2 = 0.32 \text{ MPa}$$

and we know that

$$s_2 = s_1$$

because this is isentropic expansion, nothing is mentioned about isentropic efficiencies we can assume that to be 1. So, using this we can easily get the value of h_2 also, so these are value of h_1 this is a value of h_2 taken from the tables. What other points do we know?

Point number 3, we know this is the this is the condenser for the topping cycle. So, here this is the condenser for the bottoming cycle rather pressure is 0.32 MPa and quality is equal to 0, i.e.

$$P_3 = 0.32 \text{ MPa}$$

$$x_3 = 0$$

so, you can easily calculate value of h_3 and we know

$$h_3 = h_4$$

so, you know the value of h_4 as well because 3 to 4 is an isenthalpic process. Similarly, for the topping cycle, for point number 5, we know that pressure of the at this particular point is an intermediate pressure that is 0.32 MPa. And quality is equal to 1 because a saturated vapour, i.e.,

$$x_5 = 1$$

So, combining this we can get:

$$h_5 = h_g(P_5)$$

$$s_5 = s_g(P_5)$$

Point number 6, we know

$$P_6 = 0.8 \text{ MPa}$$

which is the highest pressure of the cycle and

$$s_6 = s_5$$

from there we can get at h_6 . So, this is your h_6 and h_5 is marked here. Point number 7 that is the end of the condensation in the topping cycle. So here pressure is the highest pressure point 0.8 MPa, but here

$$x_7 = 0$$

so, we know that

$$h_7 = h_g(P_7)$$

and it is also equal to h_8 because that is also an isenthalpic process, so we know all the enthalpies. And it is also given that

$$\dot{m}_A = 0.005 \text{ kg/s for the topping cycle or upper cycle.}$$

So, you can easily go for the energy balance. First let us start with the energy balance for the heat exchanger so that we can get the mass flow rate in the bottoming cycle. So, amount of energy gained by the topping cycle is equal to the amount of energy lost in the bottoming cycle, i.e.,

$$\dot{m}_A (h_5 - h_8) = \dot{m}_B (h_2 - h_3)$$

as we know all the enthalpies and also the mass flow rate for the topping cycle, putting this \dot{m}_B , I have pre calculated the number will be equal to 0.039 kg/s. So, we have the mass flow rate for the bottoming cycle or for the lower cycle. Now the cooling capacity, cooling capacity will be equal to, we have to talk about the evaporator of the lower cycle so it is:

$$\dot{Q}_{dot E} = \dot{m}_B (h_1 - h_4) = 7.18 \text{ kW}$$

Total compression work input requirement you have to consider both the compressors. So, for the topping cycle it is $\dot{m}_A (h_6 - h_5)$ and for the bottoming cycle it is $\dot{m}_B (h_2 - h_1)$.

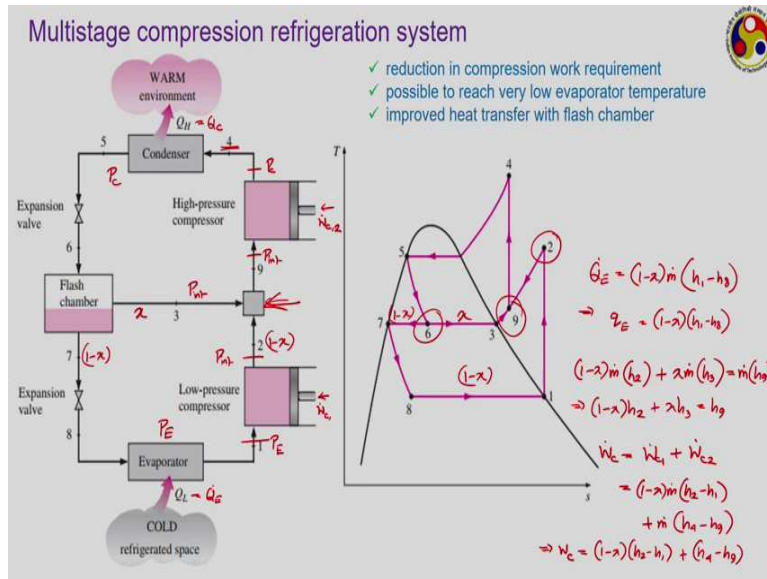
$$\dot{W}_{dot C} = \dot{m}_A (h_6 - h_5) + \dot{m}_B (h_2 - h_1) = 1.61 \text{ kW}$$

So, this is the net compression work input requirement of course assuming both compression process to be isentropic. And finally, the COP for this system will be equal to:

$$COP = \frac{\dot{Q}_E}{\dot{W}_C} = \frac{7.18}{1.612} = 4.46$$

So, this is the final result that we are looking for. So, we can see this is a reasonably high COP that we are talking about. If you remember yesterday's class, we have solved the same problem we were assuming a single stage cycle, where we assumed one R134a based vapour compression cycle is working between pressure range of 0.8 MPa and 0.14 MPa and accordingly to the calculation and the COP was 3.97 there so we are definitely having an improve in the COP because of this cascading.

(Refer Slide Time: 34:53)



Another way of getting the same effectiveness is the use of multistage compression refrigeration cycle. Here we are not having cascading or you are not having two different cycles rather we are having just the same cycle but we are having two compressors. Just look at how the cycle is operating. I am showing here both the schematic representation and also a Ts diagram. If we start from the exit of the higher compressor which is point number 4 from there into the entire refrigerant is crossing from point 4 to point 5 where the refrigeration is rejecting this heat to the surrounding which is your Q_C .

Then it is getting expanded, not to the evaporator pressure rather to some intermediate pressure. So, we are having some pressure here P_C here we having some evaporator pressure which are P_E , but here at this portion we have some kind of intermediate pressure. So, it is expanded to some intermediate pressure following this expansion valve and there we have a flash chamber. The purpose of the flash chamber is to separate out the liquid and vapour part.

Here the liquid and vapour parts are separated and x fraction of the vapour and $(1 - x)$ fraction that is liquid they are separated from each other. The liquid part is now further reduced or further expanded to the lowest pressure that is the P_E and it picks up the evaporator load which is your $\dot{Q}_{dot E}$ and that gets compressed in the lower level of compression. So, we are giving \dot{W}_{C1} here. And

now we have a mixing chamber where this $(1 - x)$ fraction of vapour at state point 2 is mixing with the x fraction of vapour that is coming from the flash chamber with state point 3.

And because of this, both are at the same pressure, this lower side compressor compresses the vapour from the P_E pressure to this intermediate pressure. And so, they mix here we have $(1 - x)$ fraction of superheated vapour where as we are having extraction of saturated vapour coming from the flash chamber they mix with each other here. And then the total quantity of mixture with this intermediate pressure or total quantity of this vapour with this intermediate pressure goes to the higher level of compressor where it gets compressed up to the final pressure of P_C . And we need to get W_{C2} to amount of work input there.

So, look at the Ts diagram here 1 to 2 here the lower cycle that is drawn that actually operates over $(1 - x)$ fraction of mass. Here with $(1 - x)$ fraction energy is being picked up at 1 and it is also compressed and at the end of compression the state is 2. Similarly, the point 6 is the flash chamber where we are separating this $(1 - x)$ fraction of liquid and extraction of vapour this x fraction of vapour and $(1 - x)$ fraction of superheated vapour they are mixing and giving this final mixture of point at state point 9 which is getting compressed to final maximum temperature of point 4.

So, how can we calculate the cooling load in this case? Here you have $\dot{Q}_{dot E}$, you have to remember that we have different mass prevailing during different operations. So, here it will be

$$\dot{Q}_E = (1 - x)\dot{m}(h_1 - h_2)$$

where

\dot{m} is total refrigerant flow rate

or if we are looking for specific effect it is:

$$q_E = (1 - x)(h_1 - h_2)$$

and similarly, at point number 9, that is in the mixing chamber here that is I am talking about we are having some kind of energy balance going on. How can you write that? The amount of energy coming from the compression side plus the amount coming from the flash chamber, i.e.,

$$(1 - x)\dot{m}(h_2) + x\dot{m}(h_3)$$

They are giving the final mass \dot{m} , they are mixing with each other to get the final mass \dot{m} and coming as straight point h_9 . So,

$$(1 - x)\dot{m}(h_2) + x\dot{m}(h_3) = \dot{m}(h_9)$$

So, if you remove \dot{m} from there it is,

$$(1 - x)(h_2) + x(h_3) = (h_9)$$

So, from there we can identify the location of this point 9 if we know 2 and 3. And how much will be the compression work? $\dot{W}_{dot C}$ for the lower side 1 which is $\dot{W}_{dot C1}$ and for the higher side one $\dot{W}_{dot C2}$. So, the sum will give:

$$\begin{aligned}\dot{W}_C &= \dot{W}_{C1} + \dot{W}_{C2} \\ &= (1 - x)\dot{m}(h_2 - h_1) + \dot{m}(h_4 - h_9)\end{aligned}$$

or dividing both sides by \dot{m} , the amount of compression work requirement per unit mass flow rate will be equal to:

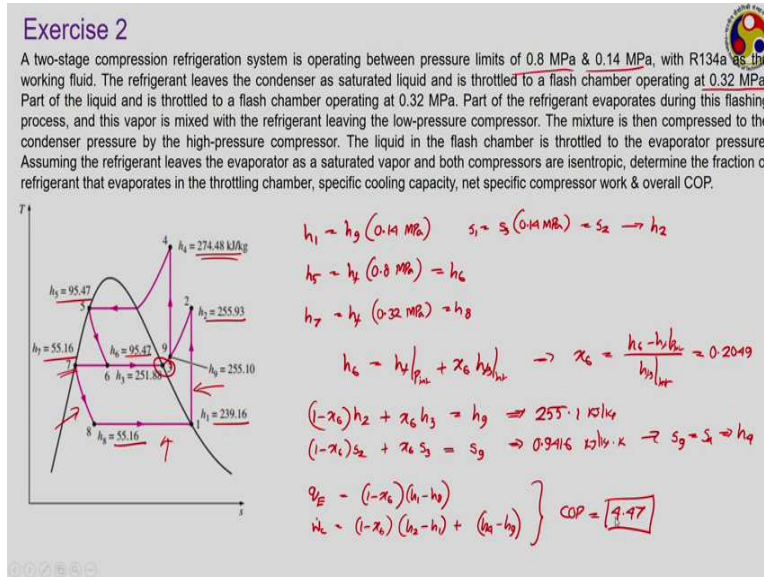
$$W_C = (1 - x)(h_2 - h_1) + (h_4 - h_9)$$

So, it is quite similar to the previous case but we are not having any heat exchanger rather we are having a flash chamber where we are having a direct mixing and we have phase separation.

So, the advantage of this multistage compression is like similarly a reduction in the compression work because had you been using the single stage compression without any flash chamber, then this point would have been somewhere here. Just by expanding this we would have got the final point 4' maybe somewhere here. So, a significant reduction in the compression work that we are getting from here, then it is possible to get a very low evaporator temperature. Even though I have not mentioned, but quite similar to the previous case we are also having a gain in the evaporator load.

And finally, the heat transfer is much improved in the flash chamber because here we do not need to bother about designing a heat exchanger with very high effectiveness. Rather we are just having a direct mixing in the mixing chamber and phase separation in the flash chamber.

(Refer Slide Time: 41:21)



Here I have a numerical example for such a configuration, it is a very long problem just read it very carefully let me go through it quickly. So, here we are working between the same pressure levels 0.8 MPa and 0.14 MPa and the flash chamber is working at this intermediate pressure of 0.32 MPa. So, we have to calculate the performance of this. So, just the way I explained in the previous case we can identify the point like h_1 is saturated vapour and h_2 we can get using the entropy balance. Then point 7 and point 5 corresponds to saturated liquid so from there we can identify point 6 and point 8 also. So, we can get:

$$h_1 = h_g(0.14 \text{ MPa})$$

and

$$s_1 = s_g(0.14 \text{ MPa})$$

which is equal to s_2

$$s_1 = s_g(0.14 \text{ MPa}) = s_2$$

So, from there we are getting h_2 by identifying the location of this particular point. And then we have,

$$h_5 = h_g(0.8 \text{ MPa}) = h_6$$

because that is an expansion process.

Similarly, you can get,

$$h_7 = h_f(0.32 \text{ MPa}) = h_8$$

because that is another isenthalpic process. So, these are the enthalpy values h_1 , h_2 , h_5 , h_7 and h_8 and similarly. But point 6 is the one that is most important from the flash chamber point of view because from here. Here, we are having a mixture from where we have to separate out the liquid and vapour part.

Then we have to identify the quality at this point. Now h_6 can be written as:

$$h_6 = h_f|_{P_{int}} + x_6 h_{fg}|_{P_{int}}$$

So, from there we know that,

$$x_6 = \frac{h_6 - h_f|_{P_{int}}}{h_{fg}|_{P_{int}}} = 0.2049$$

Now we have to perform the energy balance across the mixture to get point 9. So, there we know that $(1 - x_6)$ because x_6 is the fraction that goes to the bottoming cycle that gets expanded in this second expansion process, then in the evaporation process and this lower stage of compression. So it is:

$$(1 - x_6)(h_2) + x_6(h_3) = h_9$$

from there we are getting,

$$h_9 = 255.1 \text{ kJ/kg}$$

Similarly, we can also perform an entropy balance as:

$$(1 - x_6)(s_2) + x_6(s_3) = s_9$$

We have to remember here that we are picking up this point number 3 because once you have separated liquid and vapour phases, the vapour phase will assume the property of the corresponding saturated vapour and liquid phase will assume the property of the corresponding saturated liquid. So, we are using point 3 and not enthalpy of point 6 and same for entropy. So, similarly

$$s_9 = 0.9416 \text{ kJ/kgK}$$

and we know that

$$s_9 = s_4$$

from there we can identify the value of h_4 .

So, once we know the value of h_4 which is shown here then we can easily do the other calculation like,

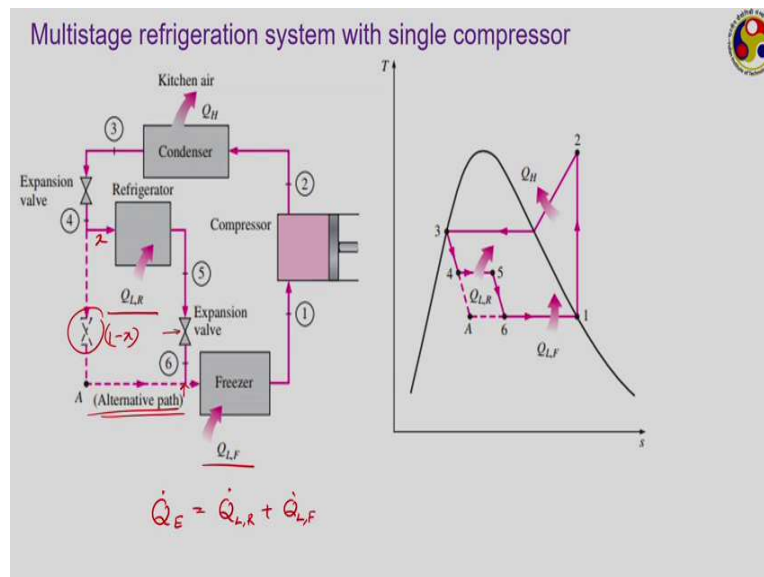
$$q_E = (1 - x_6)(h_1 - h_8)$$

total compression work:

$$\dot{W}_C = (1 - x_6)(h_2 - h_1) + (h_4 - h_9)$$

And once we combine this two, we shall be having a *COP* of 4.47 which is very similar to the one that we got in the previous problem where you used a heat exchanger instead of the flash chamber. So, flash chamber is a much easier configuration, much easier to control than the heat exchanger and cheaper as well. And therefore, quite often if you are designing a heat exchanger or going for a cascaded system, we go in for a flash chamber system just like this.

(Refer Slide Time: 46:37)



We can have a few other designs also. Like this is a multi-stage refrigeration system with single compressor. This is something that we have actually in our domestic refrigerators. Remember in your domestic refrigerators, we are actually in need of two different temperature levels. Like the common food stuffs, fruits, vegetables that we store in a normal refrigeration compartment, there

we do not need to go for very low temperatures, there we generally have lots of moisture content and therefore we generally maintain the temperature above 0 °C to avoid any kind of freezing.

But in the freezer compartment or in the chillers very quiet often we store the fish or meat kind of products, perishable products or maybe ice creams or similar kind of items there we have to go to much lower temperature quite often a temperature may be in the range of – 15 to –18 °C. Now if your freezer is at a temperature of – 18 °C then the refrigerant needs to be at a temperature of say – 25 °C whereas in the normal component or normal compartment rather your refrigeration and maybe just around 0 °C.

So, how can we have these two temperature levels using a single compressor, that is what is shown here. Here after the condensation process, while passing to the expansion valve we expand to the intermediate pressure level which gives us the required temperature for the normal refrigerator. And now once it has picked up the heat from this refrigerator compartment then it is allowed to expand further to the lowest possible pressure level, so that it can reach the freezer temperature.

And finally, this thing comes out and goes to the single compressor. So, here the cooling load comprises of two parts. Here your total cooling load \dot{Q}_E is the sum of the cooling load picked up from the refrigeration compartment and the cooling load picked up from the freezer compartment, i.e.,

$$\dot{Q}_E = \dot{Q}_{L,R} + \dot{Q}_{L,F}$$

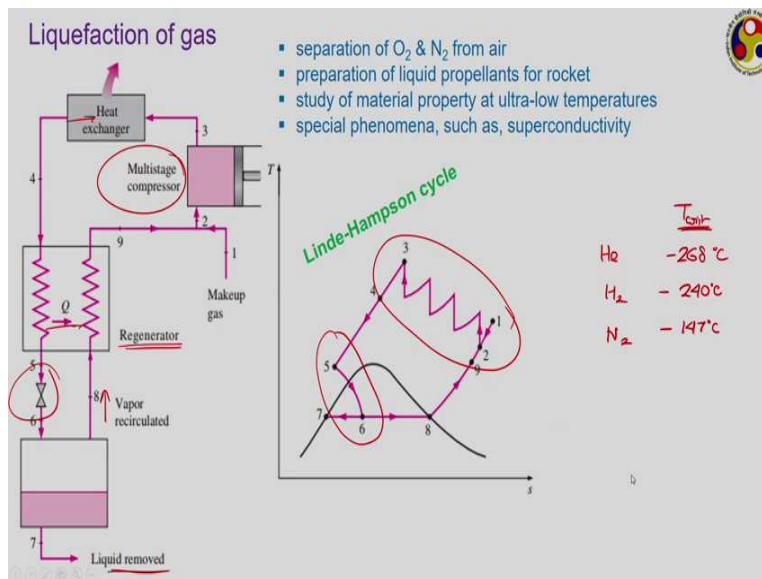
But we have a single compressor which is dealing with the entire mass. Sometimes we may have this alternative path as well. That is here a part of the mixture or rather a part of the refrigerant say x fraction goes to the refrigerator and remaining $(1 - x)$ fraction is allowed to expand further to be supplied to the freezer compartment.

And whatever is supplied to the refrigerator that comes back via the second expansion valve by this one and mixes at this particular point. That is what is shown on this Ts diagram. And also the advantage of having this alternative path is that we can easily run the refrigerator without running the freezer or maybe we can run the freezer without running the refrigerator as well. So, we have

lots of flexibilities. You must have heard different advertisements where your system has the flexibility of converting the refrigerator to a freezer or a freezer to a refrigerator or maybe run the entire system just as a refrigerator.

So, this is just by placing this expansion valve suitably and by their suitable combination that is all.

(Refer Slide Time: 49:41)



And a final application that you are going to talk about the VCR is the liquefaction of gases. Now in several situations we may have to go to extremely low temperatures. Like when you want to separate out the oxygen and nitrogen from air just by liquefying them. Under normal atmospheric condition they there is no way you can separate them out. But as as their boiling point are separate so if we keep on lowering the temperature, then beyond a particular temperature level or once you go below a particular temperature level, oxygen becomes liquid and thereby gets separated from the gas. Whereas nitrogen has a much lower boiling point so nitrogen still remains as gas. You know to separate nitrogen we have to lower the temperature even further.

There can be another application or preparation of liquid propellants for rockets, where we may have to go to extremely low cryogenic level temperatures, study of material property at ultra-low temperature, again something related to cryogenics. Special phenomena such as superconductivity

which appears only at extremely low temperatures. In these cases we go for multistage compression.

We have multi stage compressor where by several stages of compression we can move from extremely low-pressure levels to significantly high-pressure levels as well. And also, we have a regenerator as the refrigerant is coming from the heat exchanger which is nothing but a condenser rejecting heat to the surrounding. Where it is coming back it still has is at a higher temperature compared to the returning refrigerant.

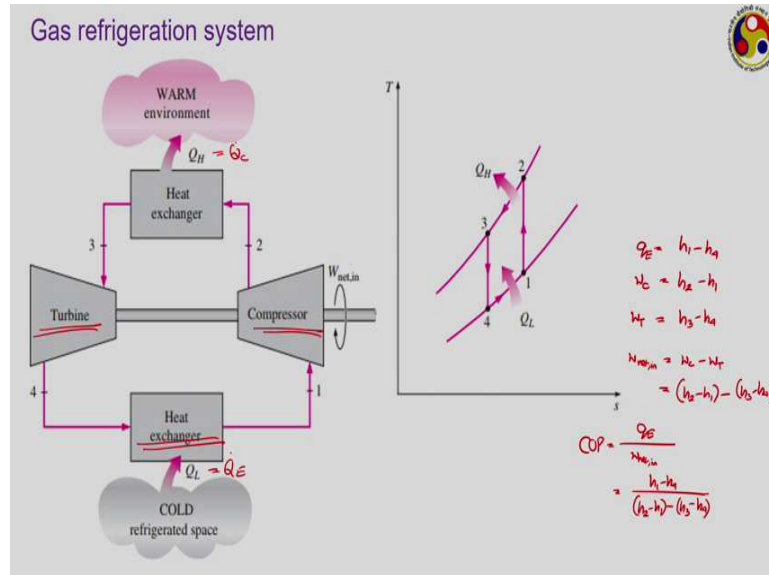
And therefore, it rejects heat to this then it there is expansion valve which takes it to the lowest possible temperature. And from there once it gets condensed, we move the liquid part like if you are talking about the liquid oxygen separation then this is the liquid oxygen that is separated and the vapour which is has very less amount of oxygen present that is recalculated back. And it is being at a lower temperature so we heat it up in the regenerator and then it goes to multi-stage compression process.

This is a multi-stage compression process shown here. As you are using multi-stage compression so with proper intercooling, we can have this compression being done nearly at constant temperature. Now, here this entire compression process and also the regeneration process is happening when the entire working medium is at the superheated condition. But only during this expansion process from this 5 to 6, it is expanded from some superheated pressure level to some supercritical pressure level to some sub-critical pressure level so that we can have this phase change and we have a mixture formation.

Of course, we have to go to very low temperature levels like, if we compare some of the critical temperatures for common gases like helium has a critical temperature of -268°C . And hydrogen has something like -240°C , nitrogen it is much higher about -147°C . So, we at least to get liquid nitrogen we have to lower the temperature below this -147°C . And in single stage of compression that is not possible, we can go for multi-stage compression with regeneration just like shown here.

Or we can go for a cascaded system, several stages of cascading so that we can reach such low temperature levels. So, these are all different varieties of vapour compression-based refrigeration system. This particular cycle is known as the Linde-Hampson cycle, the one that I have shown here.

(Refer Slide Time: 53:20)



So, these are all different varieties of vapour compression-based cycle where we are working with a vapour and the one that is getting compressed that is generally a superheated vapour. But during some stages of the cycle we are having phase change going on liquid to vapour conversion and also vapour to liquid conversion. But it is also possible that we may perform the inter-cycle above the critical point of the working medium, thereby maintaining the gas phase throughout.

And therefore, we can also have the gas refrigeration system, I should like to talk very quickly about this. We have now seen that once we reverse the direction of the arrows in a heat engine, we basically get a reverse heat engine. So, any power cycle if we reverse the direction, we get a refrigeration cycle. Like the vapour compression refrigeration cycle that we are talking about it is the reverse version of what? It is a reverse version of the Rankine cycle nothing else.

Just reverse the direction of the arrows in a Rankine cycle and you have the SSS cycle. Similarly, the all those air standard cycles that we have learned if we reverse the direction of them, we are going to get air refrigeration cycle. Like the Stirling cycle reversing that we get the reverse Stirling

cycle which is the Stirling refrigeration cycle which is gaining improved popularity. But the one that we are going to talk about here is the reverse Brayton cycle also known as the Bell-Coleman cycle occasionally.

This is a cycle just the Brayton cycle with reversed directions. So, you see the gas turbine we have the compressor here and where we have the compressor there, we have the turbine. And we have two heat exchangers here this Q_E amount of heat is being picked up from the cold refrigerated space and Q_C amount of it has been rejected to the warm surroundings. These are corresponding Ts diagram, so there are two constant pressure lines two constant pressure processes and two isothermal processes.

So, if we want to calculate the performance of this in terms of specific refrigeration:

$$q_E = (h_1 - h_4)$$

specific compression work requirement will be:

$$w_C = (h_2 - h_1)$$

and specific turbine work output will be:

$$w_T = (h_3 - h_4)$$

so, network input given is the difference between the compressor input and the turbine output that is,

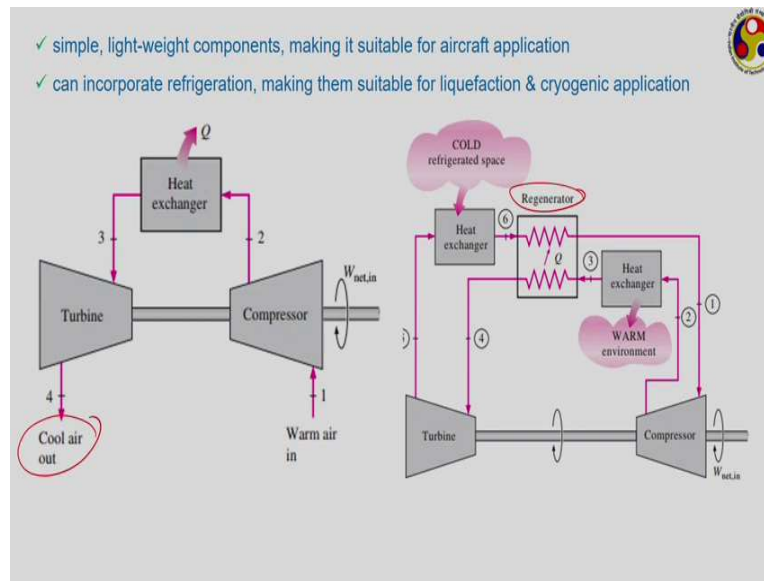
$$\begin{aligned} w_{net,in} &= w_C - w_T \\ &= (h_2 - h_1) - (h_3 - h_4) \end{aligned}$$

So, COP of this cycle is a specific refrigeration effect divided by specific work input network input requirement which is:

$$COP = \frac{q_E}{w_{net,in}} = \frac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)}$$

This is the way we can go for a gas-based refrigeration cycle. Of course in domestic application and even in large-scale industrial application this gas-based refrigeration cycle is not very common. One problem is that this heat exchanger, it is very difficult to maintain isothermal condition. There will be significant amount of temperature variation and when there is temperature variation, then there will be lot of more entropy generation they are very reducing the overall COP of the cycle.

(Refer Slide Time: 56:52)

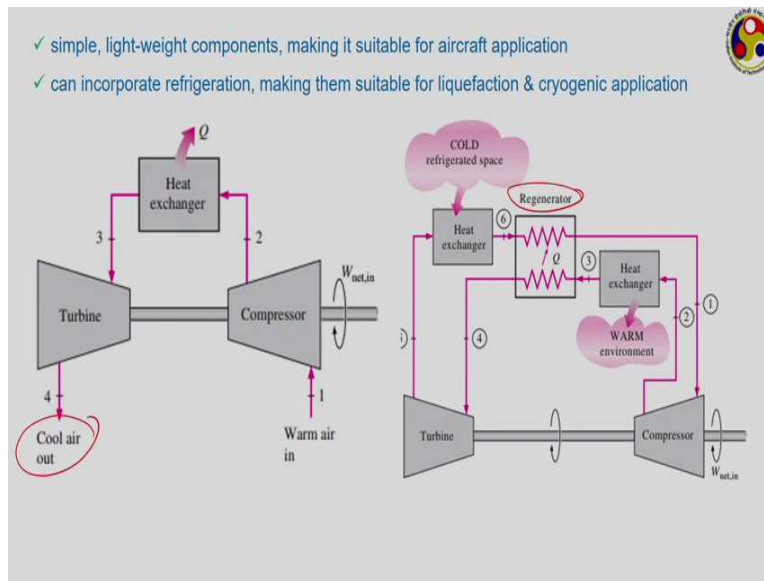


But one big advantage of this cycle is that, the components are simple and must light weight, we do not have to go for a bulk evaporator where we have the phase change going on. We are talking about simple gas-based compressors and turbines which are much simple and lightweight components there by this gas-based refrigeration system is very suitable for aircraft application, just like the one shown here. We have a compressor, we have the heat exchanger and we have the turbine and the cold air that is coming out of this, that cold air is circulated through the aircraft compartment to the cockpit.

And thereby picking up the heat and finally it is either rejected to the surrounding or it may come back to the compressor as well. Another very popular application is, if we can go for the liquification and cryogenic application through the multi-stage compression. If we are not looking to go below the critical temperature of the corresponding refrigerant then we can easily go for the gas-based cycle, something like this. Where you have the turbine, we have the compressor and we have the regenerator also.

So, these are quite popular area of application of gas-based refrigeration cycle. If you are interested about the gas phase refrigeration, you can read the books a bit more I would like to finish this one by discussing a simple problem.

(Refer Slide Time: 58:11)



So, here you are talking about an air-cooling system for a jet aircraft where air is bled from the engine compressor at 3 bar. Here we are not having a separate compressor, that is the gas turbine compressor which is running your aircraft from there at 3 bar pressure some air has been taken out and in a heat exchanger it has been cooled to 105 °C. Now we are expanding this to 0.69 bar in an air turbine having isentropic efficiency of 85%, the air is been delivered to the cockpit and finally leaves the aircraft 27 °C.

We want a refrigerating effect of 4 kW. So, this is the cycle, here isentropic efficiencies are given. So, we have air coming at 3 bar and 105 °C and going to the turbine. Then which state point you are talking about? This is the particular state point. Here

$$P_3 = 3 \text{ bar}$$

$$T_3 = 105 \text{ }^{\circ}\text{C} = 378 \text{ K}$$

And also, isentropic efficiency for the turbine,

$$\eta_T = 0.85$$

So, you know the pressures, at point number 4 the pressure is given in the turbine it is expanded to a pressure of 0.69 bar.

$$P_4 = 0.69 \text{ bar}$$

Now it is air that we are talking about so we can assume the properties of air such as:

$$k = 1.4$$

$$C_p = 1.0005 \text{ kJ/kgK}$$

So, we can write,

$$\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{(k-1)}{k}}$$

From there putting the numbers T_{4s} , I have the calculated value with me your

$$T_{4s} = 248.4 \text{ Kelvin}$$

which is well below the 0°C , but because of the isentropic efficiencies you will not be able to at such low temperatures.

So, η_T for the turbine is the maximum possible work output by actual work output which is:

$$\eta_T = \frac{T_3 - T_{4s}}{T_3 - T_4}$$

putting this we are getting

$$T_4 = 267.8 \text{ K}$$

So, just about -5°C , so, then how much is your \dot{Q}_E ? It is given as:

$$\dot{Q}_E = 4 \text{ kW} = \dot{m} c_p (T_{final} - T_4)$$

T_{final} is the final temperature with which this air is leaving and is given as 27°C

T_4 is the supply temperature the final temperature is that we have just calculated from there we can get the \dot{m} :

$$\dot{m} = 0.124 \text{ kg/s}$$

And the other values also you can calculate the same way. The mass flow rate we have got, the final temperature at which it is delivered to the cockpit is T_4 , this is the mass flow rate and power delivered by the turbine, so:

$$\dot{W}_C = \dot{m} c_p (T_3 - T_4)$$

which is coming as

$$= 13.7 \text{ kW}$$

So, 13.7 kW of power is available from this turbine which can be used for auxiliary uses as well.

So, that takes us to the end of today's discussion where we have talked about the naming convention of the refrigerants, we have talked about different kinds of VCRS system and finally the gas refrigeration system. So, we have covered the most part of our refrigeration cycles but there is one very interesting cycle that is still left which I shall be talking in the next lecture which is vapour absorption-based refrigeration cycles. So, till then you rehearse these lectures and if you have any query please write back to me. Thank you.