

Applied Thermodynamics for Engineers
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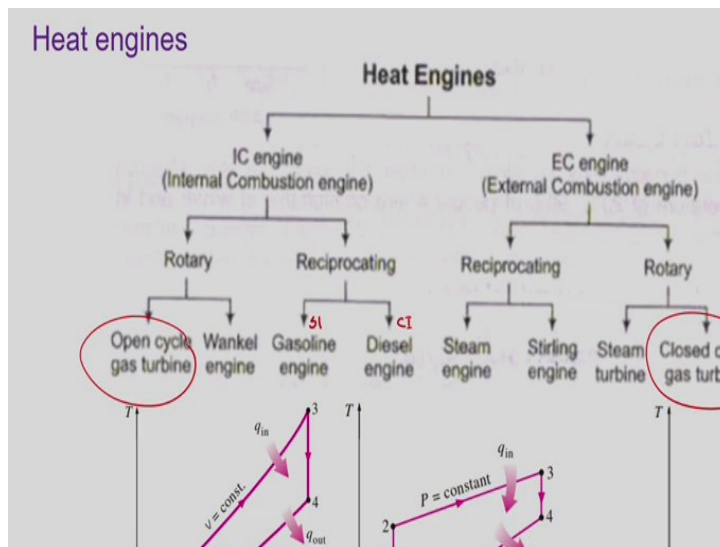
Lecture – 20
Vapour Power Cycle

Hello everyone, welcome to week number 7 where we are going to talk about the vapour power cycle. Over last few weeks we have talked about the power cycles only but where the working medium is some kind of non condensable gases. Like in week number 4 we talked about the ideal cycles for reciprocating type internal combustion engines or I should say which are the generally working cycles for all the automobile's cycles like the Otto cycle or Diesel cycle or Dual cycle. And also, in conjunction to that we have discussed about the Stirling and Ericsson cycles also.

In week number 5 we focussed more on some practical aspects related to those ideal cycles. And in the previous week we have talked about another gas power cycle, which was associated not with reciprocating engines was more with rotary type of engines i.e., the gas turbine. So, you talked about the Brayton cycle there. Now in both these two kinds of cycles i.e., the Otto or Diesel group of cycles and the Brayton cycle, the common working factor was we worked with one non condensable gas. That is the working temperature limit for the working medium over the entire range of operation over the entire span of the cycle remains well above the critical point.

In no way can come anywhere close the critical point, thereby completely eliminating the possibility of any kind of phase change. And hence we did only with the gaseous medium. However, in practical situations quite often we have to go for power cycles which involve some kind of phase change, commonly liquid to vapor and vapour to liquid phase change and corresponding cycles are the ones that you are going to discuss in this particular week under the title of vapour power cycles.

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Now, this is something that I have shown earlier also in last week only where we have classified the heat engines into two broad categories the IC or internal combustion engines and the external combustion engines. And there under IC engines we can have both rotary and reciprocating version same in case of EC engines as well. Now, under the reciprocating category, the gasoline engine also called the spark ignition engine or the SI engines and diesel engine also called CI engines are the ones which we have already discussed.

We have already discussed about this particular cycle from the diagram, can you identify? I am showing the Ts diagram only but in clearly see here we have two processes which have two processes which are constant volume in nature and two processes which are isentropic in nature. In which cycle is this? You got it correctly. This is the Otto cycle where we have constant volume heat addition and constant volume heat rejection, and isentropic expansion and isentropic compression processes.

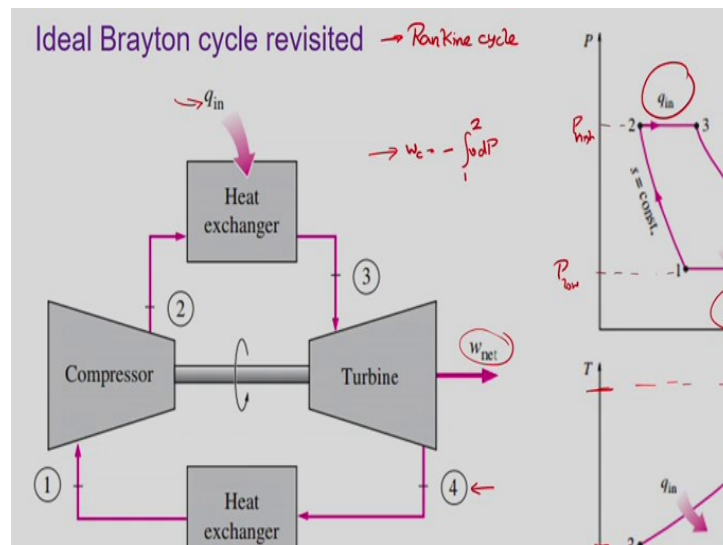
So, this is the Otto cycle which is ideal cycle for gasoline based reciprocating IC engines or the SI engines, whereas which cycle is this? Here we have heat addition constant pressure and heat rejection at constant volume and two isentropic processes. Definitely this is the engine of cycle corresponding to the diesel cycle or diesel engines, we call it a diesel cycle ideal cycle for diesel engines or the CI engines. We have already discussed about that. In the previous week we have discussed about this particular cycle.

Just look at it, I am sure you will be able to identify it. Here we have two constant pressure processes and two isentropic processes, both heat rejection and heat addition at constant

pressure and therefore this is Brayton cycle, which is the ideal cycle for both open cycle type gas turbine and closed cycle type gas turbine. In open cycle type gas turbine, we work with the IC engine mode only where the combustor remains inside engine cylinder.

Whereas in case of a closed cycle type gas turbine you work more than external combustion mode where instead of having a real combustion actually, where the gas receives energy from some external source. Both open and closed cycle type gas turbines are practical applications. In this week we are going to talk about this particular one. Steam engines are obsolete nowadays, so there is no point discussing about that. The steam turbine is you can see the prime mover of the modern industry because most part of the commercial power that we use for various purposes throughout the world at least 50 to 60% of that come from steam-based power stations. And if we include the nuclear stations which actually produces a power again in the form of a steam-based power station, so it can well go into the range of 75%. So, we have to discuss about the ideal cycle for steam turbine in this particular week.

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But before that we are quickly going to revisit the Brayton cycle, and this is the Brayton cycle. We have shown in the diagram the closed cycle mode. So, what we have here? We have a compressor where the working medium is a gas that gets compressed from some low pressure at P_1 to high pressure at P_2 . Then we have a heat exchanger where energy supplied from some external source. Remember this closed cycle is an external combustion typing. So here we do not have any combustion rather heat is getting supplied from some external source.

This q_{in} is coming from some external source and that gets added to the gas working in the heat exchanger thereby increasing temperature from T_2 to T_3 . Then in the turbine and goes for the expansion process, isentropic expansion from where we get the real work output. So, it comes back to a lower pressure level at point 4 and then it is exhausted in the form of q_{out} in the second heat exchanger and so that it can go back to stage number 1. This is the corresponding Pv diagram.

And this is the Ts representation which I also shown in the figure. So here we have 1-2 as a constant entropy or I should say isentropic type compression process during which the pressure increases from some say we can talk about say low pressure and high-pressure level. So, it works or the pressure increase in the compressor from this low-pressure level at state 1 to high pressure level at state 2. Then the ideal heat exchanger where there is no pressure drop and its temperature increases from 2 to 3.

We can see from the Ts diagram this is the temperature at the end of compression process. Where it increases upto this particular level is the temperature at the end heat addition process with significant increasing the temperature at constant pressure. Then we have the expansion process from 3 to 4 during which entropy remains constant because it is a frictionless adiabatic process, in ideal situation so it falls back to the low-pressure level. Then we have the heat rejection.

Now, we have already discussed about the ways to improve the performance of this in the form of intercooling or reheating or regeneration. And there is no point going for that just taking to the basic ideal Brayton cycle. We know how to identify the work output from the cycle, how to estimate the work efficiency etc. But one big concern that we have with Brayton cycle is that the temperature of the end of the expansion process, at the end of the turbine exposure that is. at this point number 4 that is still significantly higher compared to the atmospheric temperature.

And therefore, even when you are working in the ideal cycle mode there is no point, we should let it go to the atmosphere. Definitely we can go for regeneration to improve the cycle performance and then only we can reduce the amount of heat rejection. But another problem that we cannot avoid even when you are using all of those intercooling reheating and

regeneration, is the back work. Here the back work corresponds the work required by the compressor.

And as you are compressing a gas in the compressor, we already know the compression work required here is:

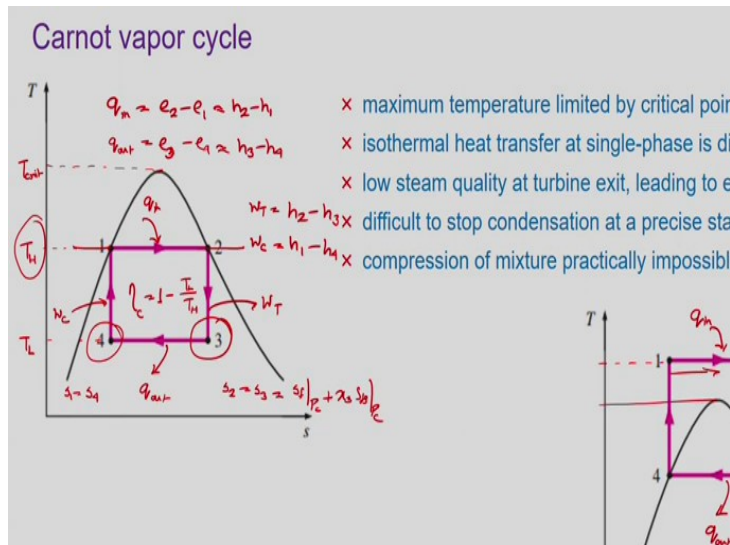
$$W_c = \int v dP$$

because this is an open cycle rotary kind of device, from point 1 to 2. And as you are working with the gaseous medium whose specific volume is very high and therefore the compression work is also significantly higher. So, such that even as much as 30, 40 or 50% of the work produced by the turbine which cannot be done by the compressor. Even when you are using the intercooling the back work ratio can be in the range of 30 to 40% which is a very, very significant number.

Then what is the way we can reduce compression work? Only possibility is that for a given pressure range of course only possibility is that if we can reduce the specific volume of the working medium. That is precisely what is done by virtue of the intercooling where will try to cool the gas to slightly what temperature so that its specific volume reduces. So instead of working with the gas, if we can work with the liquid, then we can get a drastic reduction in specific volume, thereby causing a significant reduction in the value of this compression work and hence even if the turbine work remains the same there will be a substantial increase in the net output that you should get from the cycle.

And that is precisely what motivates us to go for the vapour power cycle, where though the expansion is done with the gaseous working medium the compression is done with liquid working medium. And in between the heat addition and heat rejection stages that we get the phase change process, and that leads us to the ideal cycles for vapour based system.

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Before going to the detail of the cycle just look what we have. Here the heat addition and heat rejection processes are where we have some kind of phase change. If I just stick back to the Brayton cycle diagram just in context with that I would like to explain again. Here the process 3-4, the expansion process that is shown that will be continued to be done with a vapour because that turbine work that is also is equal to:

$$W_T = - \int_1^2 v dP$$

and here we want the specific volume to be high to get higher work output. So, we shall be using a gaseous medium and that also we want this gaseous medium to be at a temperature as high as possible. But this compression for compression part with doing the liquid that we shall be having a smaller specific volume and we can reduce the compression work. So, we may have similar level of turbine work but we can have a significant reduction in the compression work therefore, the net work output of the cycle:

$$W_{net} = W_T - W_C$$

will have a substantial increase. And these two processes: the heat addition process 2-3 and the heat rejection process 4-1 this is where we shall be going for some kind of phase change, so that we can have gaseous phase during expansion and liquid phase during compression. And correspondingly the Brayton cycle leads to the ideal cycle for vapour-based power generation which is called the Rankine cycle, based on the name of William Rankine. Rankine cycle can be viewed to be just the phase change version of the Brayton cycle, where we still have two isentropic processes and two isobaric processes. However instead of working always with gaseous medium we shall be working with the phase change in medium so that the expansion part is done with the gaseous substance and the compression part is

done with the liquid phase of the same substance. And both heat addition and heat rejection at constant pressure will be associated with phase change. But before we move on to the Rankine cycle, we should talk about the most ideal possible cycle, which is the Carnot cycle. This is the Carnot version of the vapour power cycle. Here 1-2 is the heat addition process. You can see here the black line; this particular line represents the vapour dome. we have talked in week number 3 about the properties of pure substances and there we were introduced to this phase change process.

We know that this particular line is known as saturated liquid line, this particular line as vapour line and this is a point where the two meet and is called as critical point. On the left side of the saturated liquid line we call the state to be compressed liquid or sub-cooled liquid, on the right side we call it superheated vapour and inside the dome we have a saturated liquid vapour mixture. So, the state 1 is lying on the saturated liquid line, so, here state 1 is of saturated liquid.

We are adding heat and heat addition is done such that at the end of the process with state 2, which is of saturated vapour. So, during this particular process we are having a change of phase from purely liquid to purely vapour. And during this entire process both pressure and temperature remains same. Remember during the phase change process pressure and temperature are functions of each other if one remains constant the other also has to remain constant. So, though the Carnot cycle corresponds to two isothermal and two isentropic processes, here the isothermal processes are also isobaric in nature.

Then the total amount of heat that you have added q_{in} corresponds to the change in phase from state 1 to state 2. We can therefore write this to be equal to:

$$q_{in} = e_2 - e_1$$

assuming there is no work transfer associated with this. And if the changes in kinetic and potential energy is neglected then:

$$e_2 - e_1 = h_2 - h_1$$

the changes in enthalpy because you know, that e is the summation of enthalpy plus kinetic energy plus potential energy for a flowing fluid.

And if the changes in kinetic and potential energies are unavailable, it is very good assumption to consider then it leads to:

$$h_2 - h_1$$

Similarly process 3-4, this is a heat rejection process again, we are having phase change going on. During 2-3, we are having isentropic expansion in the turbine. So, the system starts from point 2 at saturated vapour state and as it expands in the turbine it gets converted to liquid vapour mixture ending at which point 3 which is having significantly high vapour content.

Then we have heat rejection, between 3-4. So, we are having q_{in} during this process, we are having q_{out} during this process. This is the process during which we are going to get the work production in the turbine and this is a process of compression, where we are pressurizing, we are giving work input to this. I should put the arrows properly as, there work is given as input to the system. So, the q_{out} in this case can be written as:

$$q_{out} = e_3 - e_4$$

and again, if you neglect the changes in kinetic and potential energies, then this reduces to:

$$= h_3 - h_4$$

How much will be the turbine work? Very simple, the turbine work will be equal to:

$$W_T = h_2 - h_3$$

We are assuming this process to frictionless and adiabatic, so no heat transfer and no irreversible losses. So, the work output that we can get is the simple difference in enthalpies. And the compression work requirements similarly will be equal to:

$$W_C = h_1 - h_4$$

and the point number 3 and 4 also can be easily identified as these processes are isentropic in nature. So, we should have

$$s_2 = s_3$$

and what should be your s_3 ? We have done this in one of the earlier modules, I am just repeating the same thing.

$$s_2 = s_3 = s_f|_{p_c} + x_3 s_{fg}|_{p_c}$$

where

s_{fg} is the entropy corresponding to the condensation pressure

The same way we can calculate s_4 I should say because we know that as the compression process is isentropic, so s_1 should be equal to s_4 . This is the ideal cycle for a vapour power cycle in the form of Carnot cycle. We can easily calculate all the associated work interaction and heat interaction. And what will be the efficiency for this cycle? Efficiency for Carnot cycle, you know, this is the highest temperature of the cycle which can be written as T_H and these are the lowest temperature of the cycle is T_L so the efficiency for this Carnot cycle will be equal to:

$$\eta_C = 1 - \frac{T_L}{T_H}$$

Depending upon the level of the pressure at boiler condenser that is the temperature T_L and T_H we can easily get any value of efficiency. So, it is very simple and then why should not go we for this? Because this cycle is practically impossible to achieve, there are several reasons. As we are going for the phase change process, so we are restricted to the maximum temperature. In order to ensure this liquid to vapour and vapour to liquid phase change, the maximum temperature this T_H has to be lower than the critical temperature, has to be lower than this particular limit.

And for a Carnot cycle, you know higher the value of T_H , higher is the efficiency of the cycle. As here we are restricted by this critical temperature of the substance then we are also restricted by the maximum possible level of efficiency that we can achieve, that is a serious concern. It is not only for Carnot cycle but also for any kind of vapour power cycles, isothermal heat transfer at single phase is difficult. As long as we are doing within the vapour dome that is as long as you are having a phase change based on a transfer both pressure and temperature remains constant and it is quite easy to achieve in practice.

But as soon as we enter this side of the dome or the side of the dome into the single-phase side. While it is quite easy to have constant pressure heat addition of heat rejection, it is nearly impossible in practice to have isothermal heat transfer. And that is another big concerned about having this Carnot cycle in practice. Low steam quality at the turbine exit: As we starting with a saturated vapour at the beginning of the turbine expansion process, then this point number 3 will have a significantly high amount of vapour content.

This quality x_3 at this point can easily be in the range of 70 to 80%, which is significantly low compared to the practical levels. Infact as the quality at point number 3 keeps on dropping

there will be more and more liquid droplets that keeps on appearing in the mixture of liquid and vapour, at point number 3 or towards the latter stages of the turbine expansion. And as the mixture passes at a very very high velocity to turbine, these droplets can strike the turbine blades with very high velocity causing erosion to turbine blades which is quite significant.

And therefore, there is some kind of practical limit. In practice you never want this quality at this point x_3 to be less than 0.88 or 88 %. We never want this or I should say we always want this x_3 to remain greater than 0.88, that is some kind of practical operation-based limits, and that is quite difficult to have in the Carnot cycle-based operations. One way of doing this if we can identify some fluids for which the saturated vapour line is quite steep. That is instead of having a line which is shown in the line is somewhat like this then maybe we can have this expansion process with significantly higher value of x_3 .

But finding such a liquid may not also be so easy. Another problem is the difficulty to stop the condensation precisely at the state point 4. As we have the phase change going on stopping at a particular quality value again as required significant control operations and still may not be very much possible. And another big problem is with the compression from 4 to 1. At point number 4 we are having a mixture, a low-quality liquid vapour mixture. At point number 1 we have saturated liquid. As we are compressing this mixture, the liquid content keeps on increasing, i.e., the phase change is going on, that is very difficult.

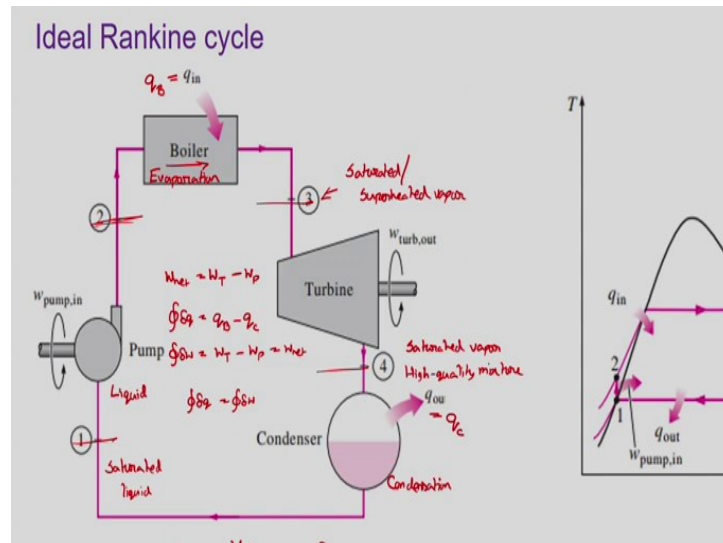
Compression of the mixture is practically impossible to control and impossible to achieve. And that is the reason that this Carnot cycle is not the most practical one to have a for large-scale power generation practice. Another problem that we shall be demonstrating through a numerical example shortly that Carnot cycle do is, it gives a very high efficiency. Corresponding work output will also be significantly low. We shall be seeing the shortly.

One way we could have achieved the cycle is by shifting the entire thing to higher temperature level. Something like this, where we are having phase change going on for the condensation, but there is no phase change in the boiling part. In the heat addition part, here we are having heat addition in the super critical stage. This particular temperature level is above the critical temperature, so we are somehow taking care of this particular limit, and the heat addition is taking place at supercritical level though heat rejection is still in this phase change dome. That is, still phase change is involved with heat rejection.

But one problem with this is that we have to compress to an extremely high pressure, the pressure at point number 1 can be much, much larger than pressure at point number 4. So, designing such a compressor is very difficult. And isothermal heat transfer that is happening from 1-2 that is happening now at variable pressure. Because your moving from this direction to this direction, the pressure also keeps on changing. Now designing a heat exchanger where temperature will remain constant, but pressure will keep on varying, here that causes loss of practical difficulties.

So, the Carnot cycle with vapour power cycles though since it is very easy to have from theoretical considerations are in no-kind of situation in practice. Then what we should do for? We just modify this Carnot cycle slightly to attain the Rankine cycle that I mentioned. We go more like the Brayton cycle mode, i.e., we shall be retaining the isentropic expansion in the turbine and isentropic compression. But, instead of having isothermal heat addition and heat rejection, we shall be having isobaric heat addition and heat rejection, i.e., this is the arrangement where you do this.

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Here we have a pump; in the pump the system pressure is increased from some level P_1 to P_2 and that is done using some kind of liquid as the working medium. So, in this process 1-2, the working medium is a liquid. At point number 2, we are having high pressure liquid then it passes to the boiler and within the boiler it goes through the phase change process. As it is passes through the boiler, we are having a liquid to vapour phase change which

conventionally is called evaporation. So instead of the term boiler, the term evaporator is also commonly used.

Then at point number 3 we can get several possible states. It may be saturated vapour or it may be a superheated vapour. So, point number 3, our state maybe saturated vapour or it can be superheated vapour. Then it passes to the turbine. It is already at high pressure level because of the action on the pump, so entire boiler is at the high-pressure level, i.e., from this point 2 to point 3 entire things is at high pressure. In ideal situation we assume no pressure loss i.e., P_2 and P_3 equal to each other. Then in the turbine, we have the pressure loss which leads to the work production P_1 and P_4 again equal to each other.

And this state of the system or working substance at state number 4 can be of saturated vapour or it may be a high-quality mixture. So, at state number 4 we are having ideal saturated vapour having high-quality mixture. And then it passes to the condenser where it condenses and heat is rejected. During this process, here we have the condensation, so at the end of the condensation process that saturated vapour or high-quality mixture that we had at the end of expansion get converted to generally saturated liquid. So, we have saturated liquid which goes back to the pump.

Now look here the work input is given to the pump but here work is done on a liquid, we can expect this work input to be extremely small. However, the expansion is done in the turbine where we are using saturated and superheated vapour. So we can expect the work output to be quite large and therefore the net output that is turbine work by 1 minus compression pump work can also be significantly small or at least we can expect that.

The net work output should be:

$$W_{net} = W_T - W_P$$

and q_{in} is the amount of heat that is added to the boiler. So, we shall be using the symbol q_B from now. To indicate the boiler heat addition q_{out} is the amount of heat rejecting the condenser. So, we shall be calling it q_C . So, the net heat interaction for the cycle, following the convention of first law of thermodynamics is:

$$\oint \delta q = q_B - q_C$$

because heat added to the system is positive, so q_B is positive and q_C is negative.

And net work interaction is:

$$\oint \delta W = W_T - W_P = W_{net}$$

because as per our convention the work done by the system is positive, so the turbine work is positive and pump work is negative. And as we are talking about the full cycle so:

$$\oint \delta q = \oint \delta W$$

so if you want to calculate the efficiency for this cycle, then the thermal efficiency for this cycle should be equal to:

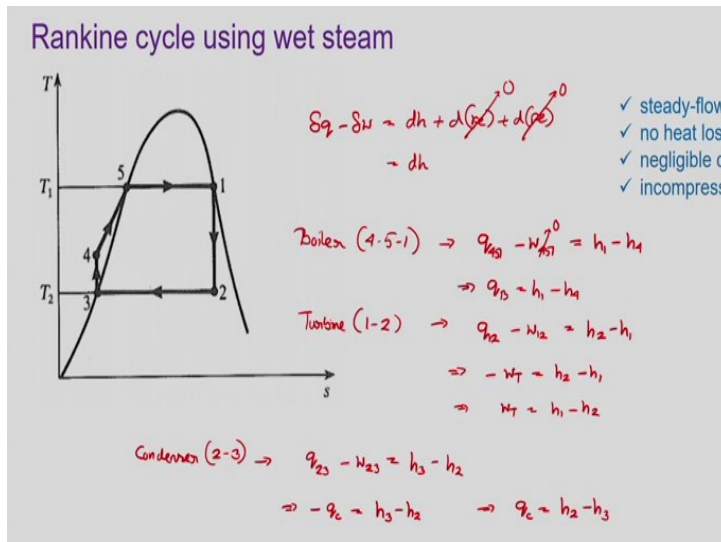
$$\eta_{th} = \frac{W_{net}}{q_B} = 1 - \frac{q_C}{q_B} = \frac{W_T - W_P}{q_B}$$

whichever is comfortable to us.

This is the Ts diagram of whatever we have discussed process 1-2 is done in the pump which is isentropic in nature and it corresponds to a liquid. Generally point number 1 corresponds to saturated liquid state this particular one and here heat is given. Point number 2 is subcooled liquid, but at higher pressure then here work input is given between this 1-2, then 2-3 is the heat addition process in the boiler. It can pass through several states the systems like 2 to this particular point it is liquid.

Then here to this, this is where we have the phase change going on. And then upto point 3 we have the superheated vapour heating. Then 3-4 is expansion isentropic expansion for ideal turbine and 4-1 is the compression. Now depending upon the exit state from the boiler where we can have either superheated or saturated vapour, we can have two kinds of Rankine cycle.

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First, we start with the basic one which is the Rankine cycle using wet steam. Here, the point number 1 corresponds to a saturated vapour, that is why we are calling it as wet steam. Because the system is not allowed to go to the superheated level. We are always staying within the saturated vapour limit. We are putting a few assumptions. First we are assuming it to be steady flow process. And we are neglecting any kind of possible heat loss during the boiling and condensation processes. We are assuming the changes in kinetic and potential energies to be negligible and the liquid phase is assumed to be incompressible. The liquid phases that we are dealing with in the pump are assumed to be incompressible. So if we talk about the first law of thermodynamics for an open system, then what do we know? We know that for an infinitesimal open process related to open system we can always write:

$$\delta q - \delta W = dh + d(ke) + d(pe)$$

that is the total net of heat and work interaction should be equal to the summation of changes in enthalpy, kinetic and potential energies.

As per our assumption these two are negligible and so it becomes equal to dh . Now let us go from one process to another or go to each of the process separately. Process 4-5-1 is done inside the boiler. So first we have the boiler which corresponds to the process 4-5-1. If we apply heat on this then:

$$q_{451} - W_{451} = h_1 - h_4$$

Now as this process is done inside heat exchanger, which you are calling a boiler or evaporator, here the working direction can be neglected. Or we can write this:

$$W_{451} = 0$$

and

$$q_{451} = q_B$$

So, we have:

$$q_B = h_1 - h_4$$

Now the turbine, the isentropic of expansion that is happening inside the turbine via process 1-2 so,

$$q_{12} - W_{12} = h_2 - h_1$$

Now it is an isentropic expansion, so the heat introduction during process 1-2,

$$q_{12} = 0$$

and the work interaction is turbine work,

$$W_{12} = W_T$$

it is positive because they work is given by the system. So, we can write:

$$-W_T = h_2 - h_1$$

that is,

$$W_T = h_1 - h_2$$

Now, we go to the condensation process, the condenser, process 2-3. So, following the same pattern we are writing:

$$q_{23} - W_{23} = h_3 - h_2$$

again heat is rejected during this process and we can assume the work interaction to be negligible or no work interaction. So,

$$W_{23} = 0$$

what about q_{23} ? Condensation heat is being rejected so it is:

$$q_{23} = -q_C$$

because heat is being rejected by the system it is negative. So, putting this,

$$-q_C = h_3 - h_2$$

which gives:

$$q_C = h_2 - h_3$$

Finally, we have the pump, process 3-4 so,

$$q_{34} - W_{34} = h_4 - h_3$$

we are assuming isentropic process in the pump. So, we have:

$$q_{34} = 0$$

and what about w_{34} ? There is work interaction the magnitude of which is the pump work. And is it positive or negative? there work is being done on the system, so it is negative. So, we have:

$$W_P = h_4 - h_3$$

where we have the plus sign. So, we have obtained all the four interactions that are required.

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$q_{net} = \oint \delta q = q_B - q_C = (h_1 - h_4) - (h_2 - h_3)$
 $W_{net} = \oint \delta W = W_T - W_P = (h_1 - h_2) - (h_4 - h_3) = (h_1 - h_2) - (h_4 - h_3)$
 $\eta_{th} = \frac{W_{net}}{q_B} = 1 - \frac{q_C}{q_B} = \frac{W_T - W_P}{q_B}$
 $= \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$
 $= \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_3)} = \frac{h_1 - h_2}{h_1 - h_3}$
 $W_P = h_4 - h_3$
 $= (u_4 + p_4 v_4) - (u_3 + p_3 v_3)$
 $\approx d(pv)$
 $\approx v dp = v(p_4 - p_3) \ll W_T$
 $\tau_{bu} = \frac{W_P}{W_T} \quad \tau_w = \frac{W_{net}}{W_T}$
 $\text{Specific steam consumption} = \frac{\dot{m}}{\dot{m} W_{net}} = \frac{1}{W_{net}} \text{ KJ/KWh}$
 $SSC = \frac{3600}{W_{net}} \text{ (KJ/KWh)}$
 $\eta_{th} = \frac{h_1 - h_2}{h_1 - h_3}$

Then the net heat interaction,

$$q_{net} = \oint \delta q = q_B - q_C = (h_1 - h_4) - (h_2 - h_3)$$

this is the net heat interaction. The net work interaction should be equal to:

$$W_{net} = \oint \delta W = W_T - W_P = (h_1 - h_2) - (h_4 - h_3)$$

and if you just re-orient these two then we can easily write this as:

$$= (h_1 - h_4) - (h_2 - h_3)$$

which is

$$= \oint \delta q$$

Of course, it is satisfying the first law of thermodynamics, which is definitely very logical.

Now how to get the thermal efficiency? The thermal efficiency for this cycle should be equal to:

$$\eta_{th} = \frac{W_{net}}{q_B} = 1 - \frac{q_C}{q_B} = \frac{W_T - W_P}{q_B}$$

If you keep on following the latter convention then, the above expression is equal to:

$$= \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Occasionally we can re-orient the denominator term, we get something like this:

$$= \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_3) - (h_4 - h_3)}$$

What is $h_4 - h_3$? It is pump work, how to get the pump work? Of course, the pump work is:

$$W_p = h_4 - h_3$$

that you have seen earlier, but there is a different way of calculating this one also. We know that it can be written as:

$$= (u_4 + P_4 v_4) - (u_3 + P_3 v_3)$$

Now as you are talking about a liquid during this particular process, its change in internal energy as we are talking about an isentropic process during which ok, I would like to request you to refer back to one of our earlier discussion probably in the very first module I had mentioned that if you are talking about an isentropic process for an incompressible substance that will also in isothermal.

Can you remember that you have proved that for an incompressible substance an isentropic process is also isothermal. And if that is isothermal, then that actually will lead to no change in the internal energy. So, this thing basically refers to the changes in the lower part. Now as you are talking about the liquid which is having extremely small changes in its specific volume then this can be written as:

$$\begin{aligned} &= d(Pv) \\ &= v dP \end{aligned}$$

or change in P. In this particular situation it is:

$$= v(P_B - P_C)$$

where

v is the specific volume

P_B is the boiler pressure

P_C is the condenser pressure

This is generally the way though we can easily calculate the pump work as $h_4 - h_3$. But quite often we would like to go for, these are:

$$= v(P_B - P_C)$$

in specific volume as long as the changes in the specific volume between state 3 and 4 are

negligible, then we can just blindly go for the change in pressure multiplied by the specific volume as the pump work. And this pump work magnitude of an is extremely small compared to the turbine work. So, if we neglect the pump work then this efficiency expression comes out to be:

$$= \frac{(h_1 - h_2)}{(h_1 - h_3)}$$

which of course is neglecting the pump work.

So, this is the way we can calculate the thermal efficiency. There are a couple of other terms that are quite often used in conjunction with any power cycle, particularly in conjunction with the vapour power cycles. One term that you have used in the previous week also, which is the back work ratio. Back work ratio is nothing but the pump work divided by the turbine work.

$$r_{bw} = \frac{W_p}{W_T}$$

This back work was quite significant in case of gas turbine, but it is expected to be much smaller in this case. They also can define work ratio, which is the net work output divided by the gross production which is the turbine work:

$$= \frac{W_{net}}{W_T}$$

Again, this one also has you seen the previous week. Another term quite commonly used for giving the specifications of power stations that is called the specific steam consumption or SSC. It refers to the amount of steam consumed produce unit power output that is where $\dot{m}$ is the mass flow rate and the amount of mass flow rate of steam required to produce unit per output that is $\dot{m}$ divided by $\dot{m}$ multiplied by W_{net} :

$$\text{Specific steam consumption (SSC)} = \frac{\dot{m}}{\dot{m}W_{net}}$$

which is of course the power output that we can get and therefore it is just nothing but:

$$= \frac{1}{W_{net}}$$

So, what will be the dimension of SI unit of this quantity? It is:

$$= \frac{1}{W_{net}}$$

so its unit will be kg/kJ. Quite often it is extremely small. So, what we do is we represent SSC as:

$$SSC = \frac{3600}{W_{net}} (kg/kWh)$$

by multiplying with 1 hour that is 3600 seconds. Now here whatever analysis that you have done that is assuming an ideal cycle that is no irreversibilities, but practically as you have seen for gas turbine we can have irreversibilities during expansion and compression processes just shown here.

If we have isentropic expansion, then we should have reached point 2s but because of the presence of the irreversibilities actually reached point number 2. Similarly, in the pump in there is no irreversibilities you should have reached 4s but because of the presence of irreversibilities reached point number 4. So here also we can define isentropic efficiency. What is isentropic efficiency for the turbine? That is actual work output divided by ideal possible work output so it will be:

$$\eta_T = \frac{\text{actual work output}}{\text{ideal possible work output}} = \frac{(h_1 - h_2)}{(h_1 - h_{2s})}$$

Similarly, the isentropic efficiency for the pump will be ideal work input requirement by actual work input requirement because it is a work consuming device and least work consumption will correspond to the ideal process, i.e., the fully reversible process. So, because of the presence of irreversibility more work input will be required so the isentropic efficiency for the pump will be:

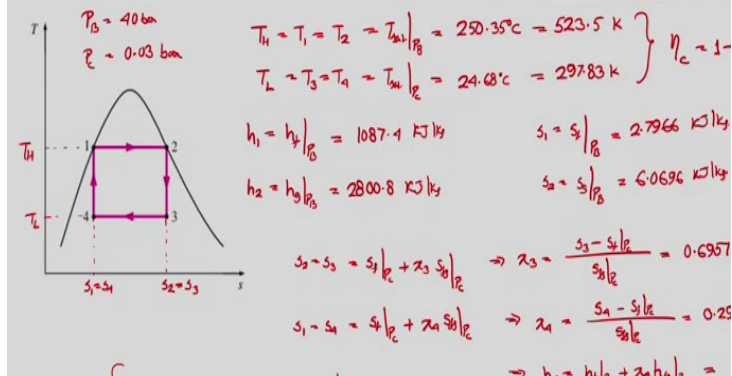
$$\eta_P = \frac{\text{ideal input requirement}}{\text{actual work requirement}} = \frac{(h_{4s} - h_3)}{(h_4 - h_3)}$$

The effects of both these isentropic efficiencies are to cause a reduction in the final output from the system.

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Exercise 1

A steam power plant operates between a boiler pressure of 40 bar and a condenser pressure of 0.03 bar. Calculate the cycle efficiency and specific steam consumption for (a) Carnot cycle using wet steam, (b) Rankine cycle with saturated steam at entry to turbine and (c) the same Rankine cycle with 80% isentropic efficiency.



Let us solve numerical example now to test whatever you have learnt so far. A steam power plant operates between the boiler pressure of 40 bar and condenser pressure of 0.03 bar. It is not mentioned specifically but we are assuming a wet cycle. That is, the state of steam at the exit of boiler or inlet to the turbine is that of saturated vapour not super-heated. So, we have to calculate the cycle efficiency of specific steam consumption for three different conditions.

First for a Carnot cycle using wet steam, then a Rankine cycle with saturated steam at the entry to turbines with wet steam and third the same Rankine cycle with 80% isentropic efficiency for the turbine. Nothing is given for the pumps so you can assume to identical ideal but turbine is an isentropic efficiency of 80%. Let us solve the first part. So, this is for the Carnot cycle. This is the diagram here your pressures are 40 bar and 0.03 bar. Then what will be that these two temperatures? The highest temperature T_H and the lowest temperature T_L how can you identify this?

Remember as due to the phase change process 1 to 2 which corresponds to 40 bar pressure and temperature T_H . This is a phase change so pressure and temperature both should remain constant and they should correspond to each other. So, the temperature T_H which is in this case is equal to:

$$T_H = T_1 = T_2 = T_{sat}|_{P_B}$$

where P_B is the boiler pressure which is 40 bar and I have pre-calculated the number or actually I have used the data table which we have visited earlier also from the table writing this to be:

$$= 250.35^{\circ}\text{C}$$

and if required we can convert the absolute Temperature also 523.5 K.

Similarly, the lowest temperature is T_L to be equal to:

$$T_L = T_3 = T_4 = T_{sat}|_{P_C}$$

where P_C is the condensor pressure at which T_4 is the saturation temperature corresponding the condenser pressure 0.03 bar. Here the boiler pressure is 40 bar and condenser pressure is 0.03 bar. Whatever data table you are using be careful about the units. Quite often the tables give the data in terms of kPa or MPa or maybe in bar. The table that you are using which unit is followed be very careful about that. So, from the table that I am using this is equal to:

$$= 24.68^{\circ}\text{C}$$

which gives you:

$$= 297.83\text{ K}$$

We know the highest and lowest temperature; can we calculate the efficiency? Surely, we can, the Carnot efficiency in this case is:

$$\eta_C = 1 - \frac{T_L}{T_H}$$

So, putting the number it will become something like 43.11%. So this is the first answer that you are looking for. This is the way we can always calculate efficiency but we shall be calculating it again using the heat and work interactions. So, let us try to locate the points.

h_1 what it will be in what will be h_1 is located on the saturated liquid line. So, it should be:

$$h_1 = h_f|_{P_B} = 1087.4\text{ kJ/kg}$$

Similarly, for s_1 :

$$s_1 = s_f|_{P_B} = 2.7966\text{ kJ/kgK}$$

Whatever data table that we are using please keep on doing at parallel to me and use own values. What about h_2 ? Point 2 is located on the saturated vapour line. So, h_2 should be equal to:

$$h_2 = h_g|_{P_B} = 2800.8\text{ kJ/kg}$$

h_g corresponding to the boiler pressure which is in my table this is the value.

Similarly, for s_2 :

$$s_2 = s_g|_{P_B} = 6.0696\text{ kJ/kgK}$$

So, we got the information for point 1 and 2 and that we are noted. If we expand these

vertical lines, then this is your s_2 which should be equal to s_3 and this is equal s_1 should be equal to s_4 because to 2-3 and 4-1 are isentropic process.

And this is the information that will allow us to locate the points 3 and 4. We know their pressure and temperature and now we know their entropies also. Then let us try to calculate s_3 ,

$$s_2 = s_3$$

then it should be what? s_3 is located on the condenser pressure. So, the value of s_3 should be equal to:

$$s_3 = s_f|_{P_c} + x_3 s_{fg}|_{P_c}$$

that is your x_3 should now be equal to:

$$x_3 = \frac{s_3 - s_f|_{P_c}}{s_{fg}|_{P_c}}$$

where

$$s_{fg} = s_g - s_f$$

So, for corresponding the condenser pressure you check the table, get the values of s_f and s_g or s_{fg} and your x_3 will be coming to be 0.6951. Look at that such low value of quality at the exit of the turbine, only in the range of 70%, which is extremely low and will lead to large amount of corrosion or erosion I should say. Corrosion is associated with chemicals and erosion is associated with steam. So, significant amount of erosion can take place here as I have mentioned earlier.

Now look at point 4, we know that:

$$s_1 = s_4 = s_f|_{P_c} + x_4 s_{fg}|_{P_c}$$

from there your x_4 should be equal to:

$$x_4 = \frac{s_4 - s_f|_{P_c}}{s_{fg}|_{P_c}}$$

and putting the values here, your x_4 is coming to be 0.2971. You please recheck all these calculations, because I may have done some calculation mistake it is possible. I have done quite hurriedly and I have not rechecked the numbers. So, you please do them again.

So, once we have this x_3 and x_4 then you can calculate h_3 and h_4 your h_3 will be equal to:

$$h_3 = h_f|_{P_c} + x_3 h_{fg}|_{P_c} = 1799.76 \text{ kJ/kg}$$

you have to get the values of h_f and h_g on the condenser pressure from your table.

Similarly, h_4 will be equal to:

$$h_4 = h_f|_{P_c} + x_4 h_{fg}|_{P_c} = ?$$

and this value of h_4 I have not noted actually. Ok you please calculate the number whatever you are getting that I am not noting the value of h_4 . Now using these values what information you can get?

q_B is the heat added to the boiler. What would that one be equal to as per this diagram? It should be equal to:

$$q_B = h_2 - h_1 = 1713.40 \text{ kJ/kg}$$

Here all calculations you are doing for per unit mass flow rate. So, we are having this kJ/kg as the unit. Then, how can you calculate q_C ? For h_4 you can easily calculate, what is another way you can calculate q_C also what is that? You know x_3 and x_4 then what will be your q_C ?

$$q_C = h_3 - h_4 = (x_3 - x_4) h_{fg}|_{P_c} = 972.67 \text{ kJ/kg}$$

The turbine work, W_T is:

$$W_T = h_2 - h_3 = 1001.07 \text{ kJ/kg}$$

so calculated the turbine work. And similarly, the pump work we can easily calculate. The pump work will be equal to:

$$W_p = h_1 - h_4 = 260.34 \text{ kJ/kg}$$

So, you have got all the four calculations, all the four parameters that we need. So, can you recalculate efficiency again? Your thermal efficiency will be equal to:

$$\eta_{th} = 1 - \frac{q_C}{q_B} = \frac{W_{net}}{q_B} = 43.11 \%$$

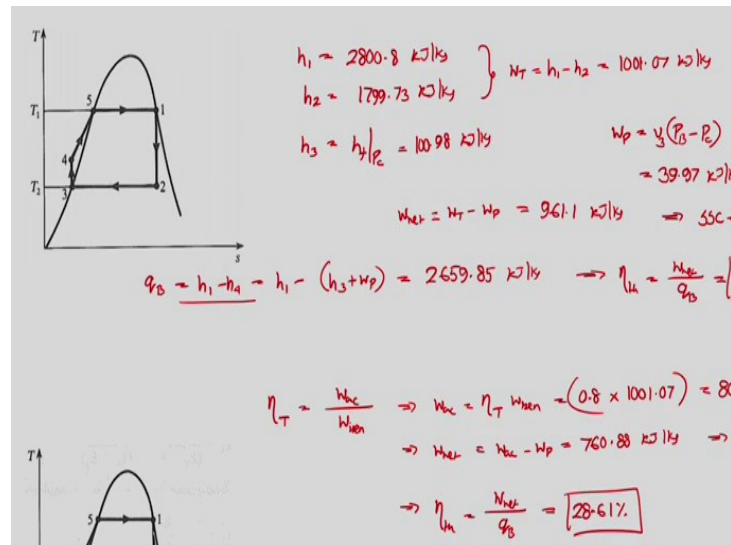
Whatever do you calculate that will come as the same 43.11% or maybe just small change in the fraction. And the specific steam consumption is:

$$SSC = \frac{1}{W_{net}}$$

now what is your W_{net} in this case?

$$W_{net} = W_T - W_c$$

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So, if we calculate your W_{net} will be coming as 740.73 kJ/kg that gives a specific steam consumption of 4.86 kg/kWh. So, this the other answer that you are looking for. So, note these values, now shall be comparing them with the Rankine cycle, an ideal Rankine cycle. This is a Rankine cycle for which we have to be the calculation. Look at this state number 1 and 2 are the same state 5 that is shown here that is similar to this particular state. So, the state 1 i.e., state the entry and exit of turbine are same but the change is at the end of condensation process. Here the condition is not stopping somewhere in between i.e., being extended up to the saturated liquid point.

Of course, I have got a different diagram, so the indexing will be a bit different. Just I am borrowing the values from the previous problem accordingly I can write:

$$h_1 = 2800.8 \text{ kJ/kg}$$

$$h_2 = 1799.73 \text{ kJ/kg}$$

and h_3 that is a new point coming in. Now what will the enthalpy there? h_3 definitely a saturated liquid so that it is h_f corresponding your P_C . So, very easy to get this it is:

$$h_3 = h_f|_{P_C} = 100.98 \text{ kJ/kg}$$

The difference of h_1 and h_2 is going to be the turbine work. That is difference of h_1 and h_2 which remains the same as in the previous case 1001.07 kJ/kg. So, the turbine work remains the same on adoption of Rankine cycle. But what about the pump work? The pump work definitely will be changing. So, let us try to calculate the pump work. The pump work in this case will be equal to:

$$W_p = v_3(P_B - P_C)$$

Now to check the value of this specific volume, let us put this v_3 , v_3 will be equal to v_4 which is equal to:

$$v_3 = v_4 = v_f|_{P_C}$$

This should be very nearly equal to each other. And if you put the values then we will get this value may be something like $0.001 \text{ m}^3/\text{kg}$ extremely small number that we get. And P_3 and P_4 are virtually the same and putting that we get pump work to be equal to 39.97 kJ/kg . The pump work is coming as 39.97 kJ/kg . Just go back to the previous case, turbine work is the same. Pump work was 260 kJ/kg . How much reduction that you are getting? Simply by expanding the condensation of the saturated liquid line this pump work has reduced to such level.

And so, your net work output that is:

$$W_{net} = W_T - W_p = 961.1 \text{ kJ/kg}$$

So, in the previous case it was in the range of 740 kJ/kg , now such a drastic increase. So, the total work output that is definitely increasing and from there we can calculate the specific steam consumption that is:

$$SSC = \frac{1}{W_{net}} = 3.74 \text{ kg/kWh}$$

So, the specific steam consumption is decreased because we are getting large amount of work output. But what about the efficiency? Work output is increasing efficiency will increase, but again we know that Carnot cycle gives us the highest possible efficiency.

Then you can calculate the efficiency we check the requirement also heat input requirement also. Heat input requirement here,

$$q_B = h_1 - h_4 = h_1 - h_3 + W_p = 2659.85 \text{ kJ/kg}$$

Now compare this with the previous case here it was significantly lower. So, the net work output was lower, but the heat input requirement was also much lower. Where we are getting high work output but at the cost of increased heat load. So, the thermal efficiency in this case will be equal to:

$$\eta_{th} = \frac{W_{net}}{q_B} = 36.13 \%$$

In the previous case Carnot cycle efficiency of about 43% . So, 7% reduction in the thermal efficiency because of the conversion from Carnot cycle to the Rankine cycle.

Now the third part of the problem where we are having an isentropic efficiency so irreversibilities in the turbine, so, some further loss, which is the situation something like this. We do not have irreversibilities in the pump side, but in the condenser side. So we have to calculate the pump work. Now we know that turbine efficiency will be equal to:

$$\eta_T = \frac{W_{actual}}{W_{isentropic}}$$

which is the W_T that you using so far. So,

$$W_{actual} = \eta_T W_{isentropic} = (0.8 \times 1001.07) = 800.86 \text{ kJ/kg}$$

Rest of things will remain the same. There will be no change in the pump size. So accordingly, net work output will be:

$$W_{net} = W_{actual} - W_P = 760.88 \text{ kJ/kg}$$

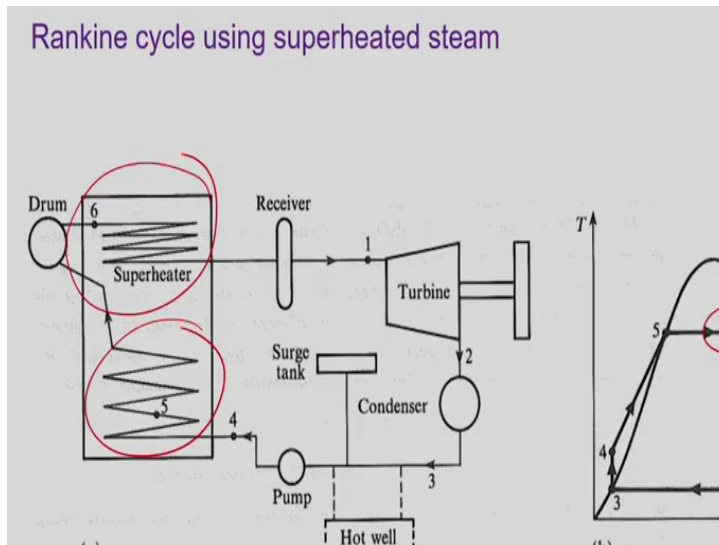
Point 4 is the same location of point 1 and is also the same. So

$$q_B = h_1 - h_4$$

that will also remain the same. So, accordingly your thermal efficiency will be equal to the new value of net work output by the earlier existing value of the boiler work or boiler heat input requirement and the efficiency will come in the range of 28.61%. And the specific steam consumption will be the reciprocal of this network output. So that is coming to be equal to 4.73 kg/kWh. So increase in the specific steam consumption as well.

I request you to repeat the same problem by using an 80% isentropic efficiency on both boiler and turbine side and see what were the final calculation. But this I am sure has demonstrated to you amply about how to do the calculation.

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Next we move to the more realistic Rankine cycle where this wet steam we go to the superheated steam. Just look at the diagram, here the heating in the boiler is not stopped at 0.6 whether we have extended this to point number 1, which is on the superheated side. And then once we are expanding this then the end of expansion either, as shown in the diagram we can go to the saturated liquid state or even when we are entering the mixture, we can maintain a very high quality mixture. So, that we do not have to bother too much about the erosion related problems.

Look at the diagram here in the high-pressure liquid that is coming out of the pump that goes through this series of heat exchangers where it is heated upto saturated mixture state i.e., state number 6. Then we supply to a boiler drum in the drum the liquid and vapour phases are separated and it is only the vapour phase that passes to another series of heat exchanger, which is called the superheater where it is heated upto the final temperature. So, this is the way that general Rankine cycle used in practice.

Instead of stopping with the wet steam or saturated steam we go to the superheated steam, in an effort to extract some more work output. Just look at this Ts diagrams. You can clearly see that the total area enclosed by the diagram is larger. Total heat interaction will be more total work interaction also will be more. So, the total output that we are going to get from the superheated steam or Rankine cycle with superheated steam definitely be more compared to the previous case but your heat interaction or heat input requirement that is also more.

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Exercise 2

A steam power plant operates between a boiler pressure of 40 bar and a condenser pressure of 0.03 bar. Calculate the cycle efficiency and specific steam consumption for a Rankine cycle with saturated steam superheated to 500°C.

$$\begin{aligned}
 h_1 \Big|_{40 \text{ bar}, 500^\circ\text{C}} &= 3485.5 \text{ kJ/kg} & s_1 &= 8.1933 \text{ kJ/kg}\cdot\text{K} \\
 s_2 \Big|_{0.03 \text{ bar}} &< s_2 = s_1 < s_g \Big|_{0.03 \text{ bar}} \rightarrow x_2 = 0.29 \\
 w_T &= h_1 - h_2 = 1059.75 \text{ kJ/kg} & \Rightarrow h_2 &= h_f \Big|_{0.03 \text{ bar}} \\
 w_P &= 39.97 \text{ kJ/kg} & &= 29.97 \\
 \hline
 w_{\text{net}} &= w_T - w_P = 1019.78 \text{ kJ/kg} \rightarrow h_3 \rightarrow q_g = h_1 - h_3 \\
 & & &\rightarrow \eta_k = \frac{w_{\text{net}}}{q_g}
 \end{aligned}$$

Just try this numerical problem the same cycle. We have the same boiler pressure of 40 bar and condenser pressure of 0.03 bar. But here there is a typing error you have to calculate a cycle efficiency and specific steam consumption for Rankine cycle, where the steam is superheated to 500 °C. Let the steam be superheated to 500 °C. So, your h_1 corresponds to a situation where your pressure is 40 bar and temperature is 500 °C, it is superheated. This temperature is well above the critical temperature. So, once you go for the superheating one of the earlier problems that have mentioned, once you are restricted with the critical temperature limit that is no more applicable. Because we are doing the phase change at a temperature lower than critical, then you are taking the vapour phase and then heating up to a higher temperature, so that we can go too much high temperature limit. So, to solve this you have to use a superheated vapour table. This value will be coming to be:

$$h_1 \Big|_{(40 \text{ bar}, 500^\circ\text{C})} = 3485.5 \text{ kJ/kg}$$

and your s_1 entropy for the same state is:

$$s_1 \Big|_{(40 \text{ bar}, 500^\circ\text{C})} = 8.1933 \text{ kJ/kgK}$$

Now, using this s_1 and ideal cycle to compare the existence of turbine, in ideal scenario we know that:

$$s_2 = s_1$$

and this is s_2 we have to check now s_2 corresponds to a pressure 0.03 bar. If this s_2 is greater than s_f corresponding to P_C , we are in the mixture part. And if this is greater than s_g , then we are still in the saturated part. But in this situation if you do the calculation or if you said that you will never find the s_2 is restricted between s_f corresponds to P_C and s_g corresponding to P_C which shows that is s_g is on the mixture side.

So, we can easily calculate x_2 , which will be equal to:

$$x_2 = \frac{s_2 - s_f|_{P_c}}{s_{fg}|_{P_c}} \\ = 0.9533$$

See such high value of quality we are getting a 95%, we do not have to bother about the erosion problem.

$$h_2 = h_f|_{P_c} + x_2 h_{fg}|_{P_c} \\ = 2430.75 \text{ kJ/kg}$$

So, the turbine work is equal to:

$$W_T = h_1 - h_2 = 1054.75 \text{ kJ/kg}$$

which is definitely higher than the previous case. The pump work remains the same that we had, the same pump side we have not changed. The pump work was:

$$W_p = 39.97 \text{ kJ/kg}$$

from where we get the net work output to be equal to:

$$W_{net} = W_T - W_p = 1014.78 \text{ kJ/kg}$$

and the q_B also will remain same. We have to identify the location of h_4 which is the exit of the pump. From there we have to identify the heat required in a boiler as:

$$q_B = h_1 - h_4$$

and from there we can calculate the thermal efficiency. I am leaving the calculations to you, just check the final answer which is:

$$\eta_{th} = \frac{W_{net}}{q_B}$$

in this case, it will be coming to be:

$$= 30.34 \%$$

And if you are able to locate the points properly your q_B will be equal to:

$$q_B = h_1 - h_4 = 3349.03 \text{ kJ/kg}$$

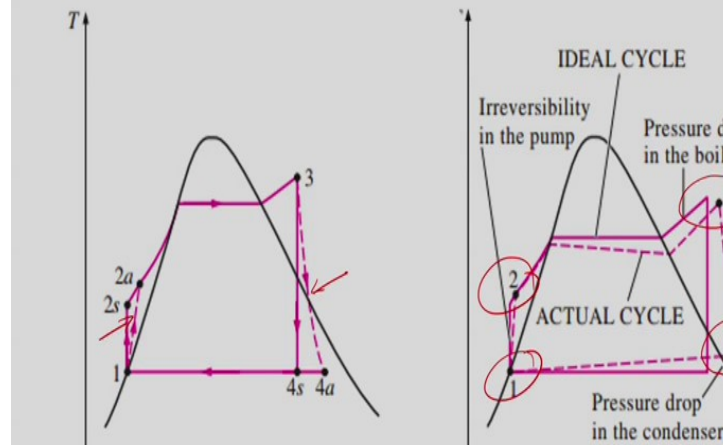
and your specific steam consumption will be coming as:

$$SSC = 3.547 \text{ kg/kWh}$$

So, this problem has demonstrated you the use of Rankine cycle with superheated steam at the boiler exit. So, that's it for today.

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Deviations in actual cycle



The deviations in actual cycle have already talked about as you have mentioned, we can have isentropic efficiency for turbine and also for the pump and we can also have pressure losses in the boiler and condenser side as shown in the diagram. Because of the pressure losses the pressure at the exit of the boiler and entry of the boiler may not be the same. Similarly the pressure at the exit of the condenser maybe much lower than at the entry to the condenser. So, in real cycle we have to be careful about these deviations also.

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Summary of the day

- Carnot vapor cycle
- Rankine cycle with wet steam
- Rankine cycle with superheated steam

So today we talked about the vapour power cycle. First, we talked about the Carnot cycle and explained why the Carnot cycle is not possible to achieve in practical operation. Then we mentioned about the Rankine cycle, first we analysed using wet steam that is saturated vapour at the entry to the turbine and then superheated at the entry to the turbine. So that is it for the day, next class we shall we talking about the factors on which the efficiency of

Rankine cycle depends and then the options of improving the efficiency of Rankine cycle till then, go through this lecture, try to solve a few more problems and rehearse the concept. Thank you.