Applied Thermodynamics for Engineers
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Lecture – 19
Regeneration in Brayton cycle

Hello everyone so welcome back for the third time this week. We are talking about the gas turbine cycles. So over previous two lectures we have learnt about the ideal cycle for the gas turbine, which is the Brayton cycle. We have discussed about different aspects of the Brayton cycle particularly, the role of the pressure ratio in determining the thermal efficiency and the work output for network output from the Brayton cycle.

And then in the second lecture we discussed two very common methods of enhancing the performance of a Brayton cycle with gas turbine which are the use of intercooler with multistage compression. I should say use of multistage compression with intercooling in between and the use of multistage expansion in the turbine with reheating in between.

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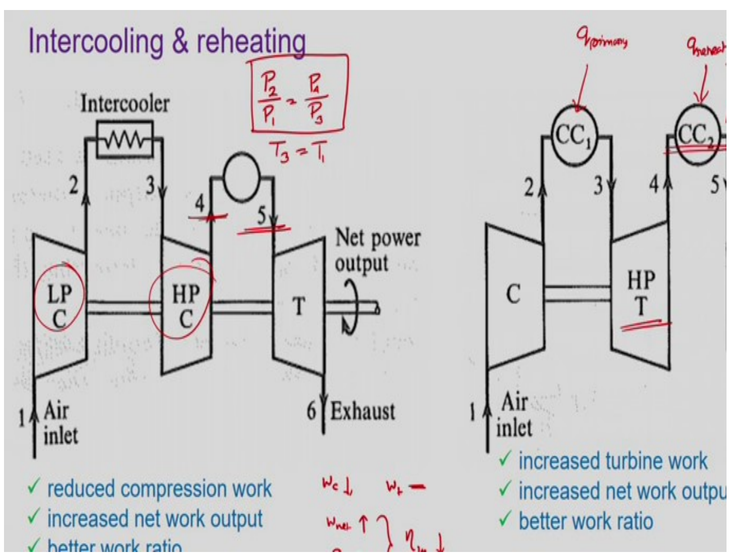

So, just to have a quick recap of that these are the two schemes that have discussed in one of them the compression stage is divided into two stages or more than two stages. The first one is as shown in the diagram, have just two stages of compression. First one is a low-pressure compressor and second one is high-pressure compressor. So, the air which is coming out of the low-pressure compressor that is supplied is allowed to pass through an intercooler, where is temperature is reduced ideally back to the initial temperature.

Then it goes to the second stage of compression to reach the desired pressure at this point 4. We have also discussed in detail about the optimum value of the pressure ratio and we have seen that pressure ratio for each of the compression stage should be equal to each other in order to have the minimum possible work requirement for the compressor. That is in this case we should have:

$$\frac{P_2}{P_1} = \frac{P_4}{P_3}$$

And if you are talking about, more number of stages, in the same way every should have the same pressure ratio due to have the maximum effect of the intercooling. And similarly, if we can separate out the expansion in multiple turbines then we can have two stages of a turbine as shown in the diagram here. We have an HP turbine or high-pressure turbine. So, the combustion products are coming from the combustion chamber allowed to pass through the high pressure turbine where they are expanding to some intermediate pressure level.

Then the gas or the exhaust coming out of the high-pressure turbine is subjected to a stage of reheating in the second combustion chamber, where ideally it is heated back to its initial temperature and then it goes to the second stage of expansion to reach the final pressure level P_6. So, like in case of intercooling in for ideal scenario or ideal intercooling, we should have:

$$T_3 = T_1$$

Similar in case of reheating, in order to have the maximum possible effects i.e., maximum possible gain in the work output we should have:

$$T_3 = T_5$$

i.e., the temperature of the gas entering every stages of expansion should be equal to each other. And also, to have the maximum possible work output from multistage compressor expansion, we should have equal pressure ratios in all the stages. Like in this case, if you are looking for the HP turbine stage the pressure ratio is:

$$\frac{P_3}{P_4}$$

And for the LP turbine stage the pressure ratio is:

$$\frac{P_5}{P_6}$$

So system will have maximum effect, these two should be equal to each other. The second one is of course we have not derived, but that is a direct consequence of this. I request you to try to do like the same relation as:

$$T_3 = T_5$$

But now what are the effects of intercooling and reheating? For intercooling of course, we are getting a reduction in the compression work, while the turbine work may remain the same, if there is no reheating or no change in the turbine side. Therefore, there is an increase in the net work output and improvement in the work ratio or reduction in the back work ratio.

However, because of the presence of intercooling there is an increase in the energy output requirement. Because, had there been no intercooling whatever would have been the temperature of the gas at the end of compression, because of the presence of intercooling the temperature is much lower. And therefore, to heat the gas back to the desired temperature T_5, we need to spend in larger amount of energy, which despite the reduction in the combustion work requirement generally leads to reduction in the overall thermal efficiency.

So because of the effect of intercooling is a reduction in the compression work while the turbine work remains the same and therefore the net work requirement that increases because net work requirement is turbine work minus compressor work. However, the heat input requirement that also increases and the combined effect of these two generally is the reduction in the thermal efficiency. And also the bulk of the system increases because you are going for multiple stages of compression plus one additional heat exchanger in the form of a intercooler in between.

So we need more space to house the same thing and also more complicated piping arrangement. What about the reheating then? For reheating again, we are having an increasing the turbine work. So, in this case the turbine work is increasing and the compression of may be remain in the same there is no change in the compression side. So, the effect will be the net work output, which is the turbine work minus compression work that is going to increase. However here again we are having additional energy requirement because here we have to supply energy into combustion chamber.

We are adding energy in this combustion chamber 1 and also adding the energy in the combustion chamber 2 because of the reheating requirement. Quite often we call this to be $q_{primary}$, because even if there is no reheating this amount of energy you always have to supply. Whereas this is called the q_{reheat} which is the additional energy body that is coming because of the presence of this reheater.

So, the total energy requirement which is the combination of this primary energy requirement or primary heat requirement and the reheat side requirement that is having an increase leading to reduction in the thermal efficiency as well. So, we are having increased turbine work output and therefore increased net work output. And similar to the intercooling better work ratio or reduction in the back work ratio but there is increased energy input requirement leading to increase or rather reduction in the thermal efficiency.

And another big implication of this reheating is that the exhaust temperature, this T_6 is much higher. If there is no reheating present then what will be the exhaust temperature? This exhaust temperature in presence of the reheater is significantly higher. There maybe a 300 to 400 K order of temperature difference which is one big issue to be resolved with reheating. So, this higher exhaust temperature is something that we have to be careful about, with the use of reheating. And of course, we have additional space requirement because of additional combustion chamber and additional stages of turbine and more complicated piping arrangement. But compared to the intercooling reheating generally requires less place because we may use the same combustion chamber i.e., CC_1 and CC_2 be the same temperature on the additional piping will be required for this. Whereas in case of intercooling we need to have a separate heat exchanger. Now we can see that while the multistage compression with intercooling and multistage expansion with reheating, both are leading to increase in the net work output and improved work ratio, but both of them are causing a reduction in the thermal efficiency.

So we also have to devise some option which gives us both things. That is the additional work output that we are getting that must be written but also the energy requirement energy input requirement that needs to reduce so that we can have a increasing the thermal efficiency as well and that option is regeneration.

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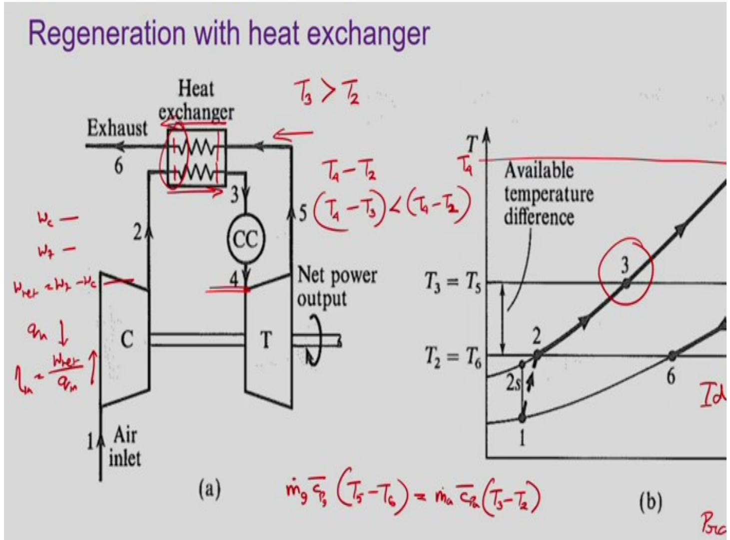

Regeneration plus another heat exchanger, which is called the regenerator. The idea is that the energy of the exhaust gas coming out of the turbine will be utilised to preheat the air before it is allowed to go to the combustion chamber. Like we have seen with the use of reheater, the exhaust gas temperature can be very, very high. Even though there is no reheater, size of temperature will be quite significantly higher compared to the air temperature coming out of the compressor.

So, if you can devise a heat exchanger to just like whatever is shown in this situation the exhaust coming out of the turbine is allowed to pass through the heat exchanger, where it is rejecting its additional or excess heat to the compressed air coming out the compressor. Thereby we are having an increase in the air temperature because this air temperature T_3 will be greater than T_2 and so it is entering the combustion chamber at higher temperature. So, the total energy requirement in the combustion chamber will be lower because ultimately, we want to gain the same temperature T_4.

If there is no heat exchanger, no regenerator, then the temperature rise that the air of the gas must experience in the combustion chamber should be equal to:

$$T_4 - T_2$$

but because of the presence of this regenerator, the temperature change that you want to have is:

$$T_4 - T_3$$

and T_3 can be significant T_2 causing a significant change in this energy requirement or the temperature is requirement:

$$T_4 - T_3$$

because that can be significantly lower than this

$$T_4 - T_2$$

So total energy requirement for the combustion chamber reduces. Whereas it does not have any effect on the work output because the work output depends on the turbine work output and the compressor work input requirement.

So, your compression work remains the same and the turbine work also remains the same. So, the net work output:

$$W_{net} = W_t - W_c$$

also remains the same. But the heat input requirements q_{in} is reducing because some part of the heat is being recovered from the exhaust gas. Therefore, the thermal efficiency which can be written as:

$$\eta_{th} = \frac{W_{net}}{q_{in}}$$

that is having a rise in this. So, regeneration is an option of increasing the thermal efficiency without affecting work output from the system.

Because its effect is not to change the work output rather to increase the heat input requirement. The simple diagram is shown just be careful here you are not talking about intercooling or reheating, we are just talking about regenerator as an option. So, what is the point at which air is entering the compressor? And had there been ideal compression, i.e., isentropic compression it should have reached 2s because of presence of irreversibilities in the compressor, it is reaching the point 2. Then, let us say T_4 is this temperature, which is our desirable temperature at the inlet to the turbine.

Now, if you want to have this entire temperature rise being done in the combustion chamber the amount of $T_4 - T_2$ or amount of energy that you have to supply to the air that will be equal to:

$$q_{in} = m_a c_{pa} (T_4 - T_2)$$

But here we are having regeneration done because of which this T_2 to T_3 amount of temperature rise has taken place. So, the amount of energy increase that has to be taken by the combustion chamber is now is not the same

$$m_a c_{pa} (T_4 - T_2)$$

But it is

$$= m_a c_{pa} (T_4 - T_3)$$

and from where is energy is coming? As shown in the diagram the same thing can get from the *Ts* diagram as well.

So, 4-5 is the expansion in the turbine. Had there been isentropic expansion we should have used this T_{5s}, but we reaching the T_5. Now instead of allowing this gas to go out on its own we are using the energy from the gas. So, the gas temperature in the region reduces from T_5 to T_6 and then only it is allowed to go out to the surrounding. So, in the regenerator then we are actually making use of the waste heat of the exhaust gas thereby is always a very profitable option. We generally go for a counter flow kind of heat exchanger, so that the air as it moves through like if you think about a counter flow heat exchanger arrangement: let me show from this diagram only. A counter flow kind of heat exchanger is shown. The air is flowing in this direction and the gas is flowing in this direction. Air is always encountering gas at higher temperature because at the inlet, the gas temperature is low. Similarly, at the inlet air temperature is low and similarly at the exit the air temperature also is low. So they are in close proximity of each other gas temperature being higher it will be able to add energy to the air.

Whereas towards the exit side where the air temperature is high, the gas temperature is also high. So this counter flow heat exchanger arrangement gives us quite high effectiveness and therefore we can record significant amount of energy from the exhaust gas. If you are talking about an ideal regenerator, then ideally you would like to have the air temperature at the end of regeneration, i.e.,

$$T_3 = T_5$$

this is the ideal scenario.

The ideal scenario is one the air temperature in the end of regenerator to be equal to the gas temperature at the beginning of regeneration. And similarly, the gas temperature at the end of regeneration T_6 that should be equal to the air temperature at the beginning of the regeneration process or the exit of the compressor that is equal to T_2 in the ideal scenario. But practically may not be able to get this because there maybe heat losses present in the system and also, we need to have a certain amount of temperature difference between the air and gas so that we can facilitate the heat transfer. Therefore, practically this T_3 will always be less than T_5 and T_2 will always be less than T_6. If we neglect any kind of heat loss from the system, then the total amount of energy rejected by the gas can be represented as:

$$\dot{m}_g \overline{c_{pg}}(T_5 - T_6)$$

This should be equal to the energy gained by the air, which is represented as:

$$\dot{m}_a \overline{c_{pa}}(T_3 - T_2)$$

As we are talking about heat exchanger we generally prefer to define one effectiveness to characterise the; or to compare the actual performance and the ideal performance.

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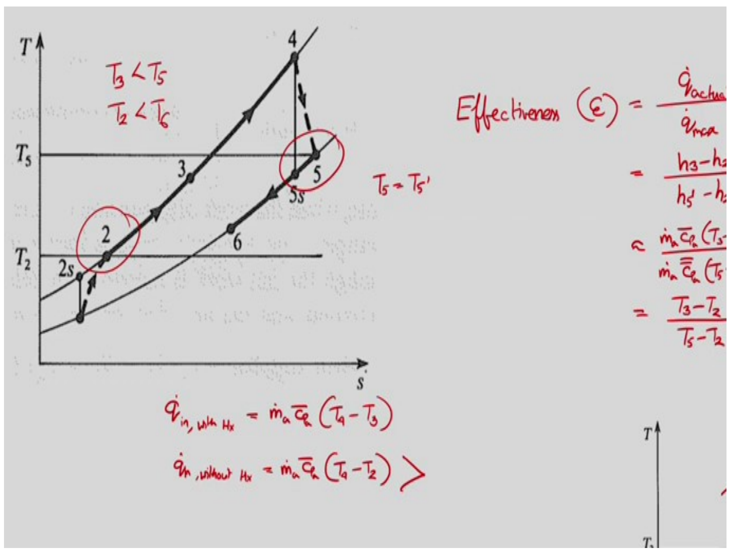

If you look at the diagram for an ideal scenario here look at what we have. Here 2 is the point at the end of compression and 5 is the point for the gas at the end of expansion. So, while in an ideal regenerator, the air temperature at the end of regeneration should be equal to T_5 but practically it remains at T_3 which is lower than T_5. Similarly, the gas at the end of regeneration that is 6 is not able to reach up to 2, so we have T_2 less than T_6 again. We generally prefer defining effectiveness as I have just mentioned. Effectiveness for a regenerator is similar to the way we define effectiveness for any heat exchanger.

Let us use ε it is given as the actual heat transfer by maximum possible transfer that can take place or actual rate of heat transfer by the maximum rate of heat transfer. The expression is as follows:

$$Effectiveness\ (\varepsilon) = \frac{\dot{q}_{actual}}{\dot{q}_{max}}$$

If you think from the air point of view, then what is the actual energy gained by the air? We can write that to be equal to actual enthalpy change of the air that is h3 - h2 but had it been we are talking about an ideal process

Then it should be equal to $h_5 - h_2$ or I should say h_5' where h_5' refers to this particular point, where the air temperature required becomes equal to the temperature at the beginning of regeneration, i.e., T_5 is equal to T_5'. So

$$= \frac{h_3 - h_2}{h_{5'} - h_2}$$

is the maximum possible energy gain that the air can have. Now, if neglect any heat loss and changes in kinetic energy and potential energy, then this is the way we can represent this.

And if we now assume the specific heat for the air to be constant, then we can write it as:

$$= \frac{\dot{m}_a \overline{c_{pa}} (T_3 - T_2)}{\dot{m}_a \overline{\overline{c_{pa}}} (T_5 - T_2)}$$

In the denominator, c_p for air double bar is double bar average value of specific heat for air between the point T_5 and T_2. So, it should be equal to $T_5 - T_2$ which gives us, if you use a cold air standard assumption i.e., all the specific heats and properties are assumed to correspond to the atmospheric temperature, then c_p bar and c_p double bar they are equal to each other. So, we get effectiveness equal to:

$$= \frac{(T_3 - T_2)}{(T_5 - T_2)}$$

as it is becoming ratio of temperature definition only this is often called the thermal ratio. So, in case of gas turbine, to characterise the regeneration process, we use effectiveness or thermal ratio either of the one can be used.

Now before move any further to perform any mathematical analysis of this gas turbine cycle with regeneration. We have to look into the practical sense. When can we go for a regeneration process? Can you say when? When can we go for regeneration process? What is the idea of regeneration? The idea of regeneration is to make use of the heat content by the exhaust gas to preheat the air before entering the combustion chamber. Now in order to have the heat transfer from gas to the air, the gas temperature has to be higher than the air temperature i.e., I should say gas temperature at the end of expansion has to be higher than the air temperature at the end of the combustion. So T_5 has to be greater than T_2 and then only it is possible to design a regenerator.

If we have a scenario something like this with the temperature at the end of the expansion process i.e., T_4 that is it is less than the temperature of air at the end of compression process. Like in this case T_2 is greater than T_4 and so regeneration is not possible in this case. Even when T_4 is greater than T_2 unless it is a significantly lot temperature difference we should not go for regenerator. Because the incorporation of the regenerator gets into picture several things. Like we have additional capital cost requirement, we have maintenance requirement for the heat exchanger and also large surface area requirement if we are talking to very small temperature change T_4 and T_2.

So, the heat exchanger or regenerator can be included in the design only when the gas exit temperature is significantly larger than the air temperature at the end of compression. Now how much is the total amount of heat input requirement with the heat exchanger, it is the total heat input requirement in the combustion chamber that will be equal to:

$$\dot{q}_{in,with\ Hx} = \dot{m}_a \overline{c_{pa}} (T_4 - T_3)$$

Whereas how much is the amount of heat input requirement without heat exchanger? That will be equal to:

$$\dot{q}_{in,without\ Hx} = \dot{m}_a \overline{c_{pa}} (T_4 - T_2)$$

T_3 is significantly greater than T_2 we can definitely have this term to be greater than the other term. And therefore, the effect of regeneration is to cause a reduction in the total energy requirement in the combustion chamber and accordingly we have an increase in thermal efficiency.

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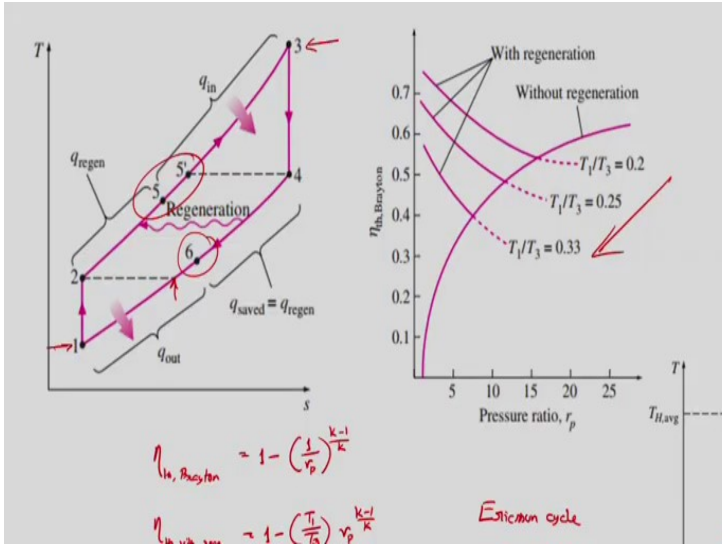

So, this is how a practical regenerator works where, we have 1-2 here of course, we are not considering the irreversibilities inside the turbine and compressor. That is, we are taking isentropic efficiency of both compressor and turbine to be equal to 1. Then 1-2 is a compression process 2-5 is the regeneration process, regeneration for air heating then 5-3 refers to the combustion process.

So, q_{regen} as we have seen will be equal to:

$$q_{regen} = m_a c_{pa}(T_5 - T_2)$$

and q_{in} is the amount of heat required in the combustion chamber will be equal to:

$$q_{in} = m_a c_{pa}(T_3 - T_5)$$

If you talking of an ideal regeneration, then the temperature of air at the end of regeneration should have been 5' which is equal to T_4 that is gas temperature the end of expansion. However because of the practical issues T_5 we are never able to reach to this temperature of 5', generally we are restricted to point number 5.

Similarly, the gas temperature is has dropped to T_6 ideally should have drop to this particular point. But again, because the practical issues may not be able to lower the temperature upto to level of T_2 it is restricted to T_6. And the amount of energy which is:

$$m_g c_{pg}(T_4 - T_6)$$

is amount of energy saving because of the regeneration otherwise you would have lost to the surrounding. Now, if I talk about from the thermal efficiency point of view, we already know that the thermal efficiency for a standard Brayton cycle without any reheat, regeneration etc is given as:

$$\eta_{th,Brayton} = 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

However, in regeneration is there we also have to consider the maximum and minimum temperature of the cycle. Now in the cycle shown here, the minimum temperature is T_1 and maximum temperature is T_3. It can be shown that I am not doing the derivation the thermal efficiency with regeneration, that will be coming to be:

$$\eta_{th,with\ regen} = 1 - \left(\frac{T_1}{T_3}\right) r_p^{\frac{k-1}{k}}$$

so it depends on the pressure ratio and also depends on the temperature ratio, maximum to minimum temperature of the cycle on minimum to maximum temperature of the cycle. As you can see the ratio of minimum to maximum temperature that keeps on increasing efficiency also keeps on reducing.

As the number of stages of compression and expansion that keeps on increasing, then we have seen scenario something like this. Let us look at here. So, how many stages of compression you can see here? There is, so many stages and each of them having is the identical pressure ratio and perfect intercooling in between. Similarly, on the turbine side we are having multiple reheaters each of them having the same pressure ratio. And accordingly, we are having ideal reheat for this. In that case how this curve was looking like or this total cycle that have been plotted? Definitely the cycle is looking quite similar to the Ericsson cycle is not it?

Because in case of Ericsson cycle also have two constant temperature process and two constant pressure processes, exactly what the Brayton cycle reduces to an infinite number of compression and expansion stages. However practically speaking we never go for more than two or three stages of compression and expansion two or at most three are sufficient. But if theoretically we can visualise infinite number of stages then the Brayton cycle reduce to the Ericsson cycle. Where of course the Brayton cycle needs to have all its improvement; that is a rigid regeneration and also intercooling.

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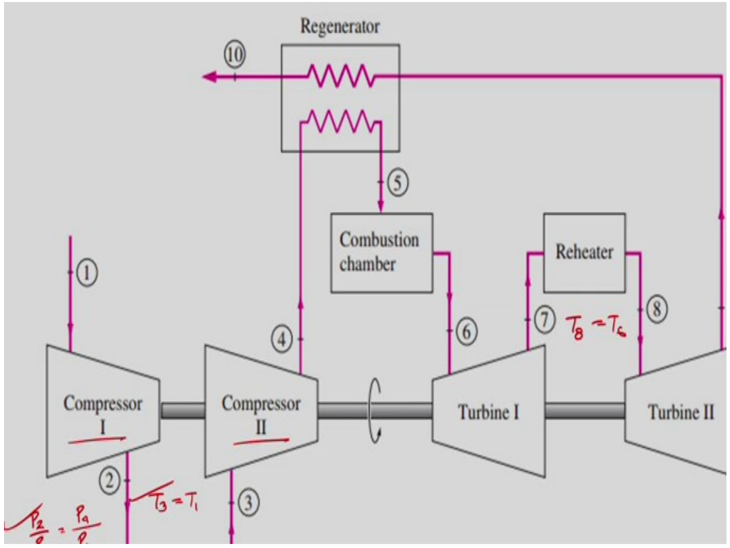

So this is how a practical gas storage system look like, where we can see the two stages of compression: compressor 1 and compressor 2 and in between we have an intercooler. We want ideal intercooling for which we have:

$$T_3 = T_1$$

and we have the pressure ratio to be equal for both stages i.e., pressure ratio for stage 1 should be equal to pressure ratio at stage 2, i.e.,

$$\frac{P_2}{P_1} = \frac{P_4}{P_3}$$

and also, in case of ideal scenario there is another factor that we neglect which is no pressure drop.

As the air is allowed to pass through the two stages of compression and also intercooler, there maybe some loss in a pressure of air, but you are neglecting any kind of pressure drop and we are assuming ideal intercooling. Now the air which is coming out of the compressor with temperature T_4 that goes to the regenerator where it gains energy from the exhaust gas and enters a combustion chamber with temperature T_5. The exhaust from the combustion chambers first goes to the turbine number 1 and the exit from turbine 1 is supplied through the reheater and it is energized again.

And again, quite similar to the intercooler in ideal scenario, we want T_8 to be equal to T_6 and we want the pressure ratio to be equal to each other. Now exhaust from the second turbine T_9 that goes to the regenerator thereby increasing the thermal efficiency of the cycle.

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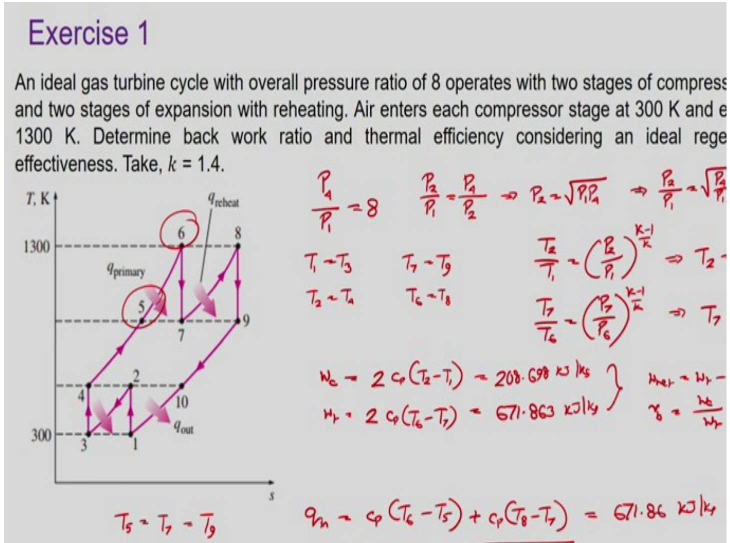

We shall now be doing a couple of numerical examples just to see the numerical effect of regeneration. The first problem; this is actually the litigation of one problem that you have solved in the previous lecture. Just read the problem statement carefully. We are talking about an ideal gas turbine cycle with overall pressure ratio of 8, it is operating with two stages of compression with intercooling and two stages of expansion with reheating. Air enters this compression stage, each compressor stage at 300 K and each turbines stages at 1300 K. And we want to work out the back work ratio and thermal efficiency.

We have solved this problem in the previous lecture, but now I have a change. Here we have to consider an ideal regenerator with 100% effectiveness. The value of k is given, you may consider the value of specific heat also. So, the same problem is solved in the previous lecture, but without regeneration. Now, this is a scenario where 1-2 and 3-4 are the two stages of compression 6-7 and 8-9 are the two stages of expansion, 2-3 refers to the intercooling process, similarly 7-8 refers to the reheating process. So, how to approach this particular problem now. We have solved this problem the previous lecture, but without the regeneration, so the similar procedure is what we are going to follow here also. Here, overall pressure ratio is given as 8, overall pressure ratio refers to what? It refers to the pressure at the end of compression, which is P_4 and pressure at the beginning of compression, which is P_1 that is equal to 8.

$$\frac{P_4}{P_1} = 8$$

And here we are talking about an ideal cycle that has no information given. So you can assume the multistage compression process to be an ideal one that is with perfect intercooling. If it is having a perfect intercooling then we have the pressure ratio to be equal, i.e.,

$$\frac{P_2}{P_1} = \frac{P_4}{P_3}$$

and as we are neglecting any kind of pressure loss, so we can write that P_2 and P_3 to be equal. So, let us replace P_2 and P_3 with P_2 which gives:

$$P_2 = \sqrt{P_1 P_4}$$

Now if you divide both sides of the equation by P_1 then what we have? We have:

$$\frac{P_2}{P_1} = \sqrt{\frac{P_4}{P_1}} = \sqrt{8}$$

So, actually I have derived this one as a part of the multistage compression exercise, but the same thing we have proved here. That is each stage is having the same number of same magnitude of pressure ratio and that can be directly calculated as the root of the overall pressure ratio.

If you are talking about k^{th} stage of compression, then the pressure ratio for each stage should be equal to:

$$\left(\frac{P_{final}}{P_{initial}}\right)^{1/k}$$

So, now we have the numbers for us:

$$\sqrt{8} = 2.83$$

this is for both the stages. Now, it is ideal intercooling so we have:

$$T_1 = T_3$$

and

$$T_2 = T_4$$

Similarly, we have ideal reheating so that we can have:

$$T_7 = T_9$$

and

$$T_6 = T_8$$

We have to calculate the temperature now there is no isentropic efficiency. So it is quite straight forward then we have information given as:

$$T_3 = T_1 = 300\ K$$

$$T_6 = T_8 = 1300\ K$$

So,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

from there we can get:

$$T_2 = T_4 = 403.83\ K$$

the same number that we calculated the previous lecture. And similarly, we can do the calculation for the reheating part or multiple stages of turbine. So, there we can write:

$$\frac{T_7}{T_6} = \left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}}$$

Now, though we have not done, but this being an ideal scenario in case of turbine also, so there also pressure ratio should be equal to $\sqrt{8}$ for each of the stages.

So, from here we can calculate:

$$T_7 = T_9 = 965.74\ K$$

in this case. So, how can you calculate the work output now? For the total work input requirement for the compressor for each stage it should be equal to:

$$c_p(T_2 - T_1)$$

and should be equal for both stages so it is:

$$W_c = 2\ c_p(T_2 - T_1)$$

and that will be coming to be equal to 208.698 kJ/kg. Similarly, for turbine it will be:

$$W_t = 2\ c_p(T_6 - T_7) = 671.863\ kJ/kg$$

Combining these two, we have net work output to be equal:

$$W_{net} = W_t - W_c = 463.165\ kJ/kg$$

So, we have got the work output and now the back work ratio r_b should be equal to:

$$r_b = \frac{W_c}{W_t} = 0.311$$

the back work ratio remains the same and work output also remains the same, despite the presence of the regenerator. Now we have to calculate the thermal efficiency and then the role of regeneration comes to the picture. Because had there been no regeneration then the total temperature rise for the air inside the combustion chamber would be from T_6 to T_4.

But because of the presence of regenerator, it is happening only from T_6 to T_5. Suppose you have to identify point number 5, and as we are talking about an ideal regenerator as well, so:

$$T_5 = T_7 = T_9$$

so, it is very straightforward. Now the total amount of thermal energy requirement, then in case of the combustion chamber it will be equal to:

$$q_{in} = c_p(T_6 - T_5) + c_p(T_8 - T_7)$$

Had there been no regeneration, the temperature difference in the first part would have been T_6 to T_4, but it is not and the energy required in the reheater which is the second part of the equation, this of course it is unaffected because of the regeneration.

So, total amount is coming as 671.86 kJ/kg. And then the total thermal efficiency that should be equal to:

$$\eta_{th} = \frac{W_{net}}{q_{in}} = 68.93\ \%$$

this is thermal efficiency. If you compare this one with the problem that we solved yesterday where there is no regeneration, thereby efficiency value that we got was something like 44.8%. You can clearly see there is a significant improvement in the efficiency simply by introducing an ideal regenerator.

So, regeneration is very useful process though there is no change in a net work output of work ratio or back work ratio, but there is significant change in the heat input requirement and hence the thermal efficiency of the cycle. Another interesting alteration you can get it from there, if instead of having two, if we put multiple stages of compression and multiple stages of expansion then, under ideal scenario there would be infinite number of expansions and compressions and it should resemble Ericsson cycle. Then what would have been the efficiency of the Ericsson cycle?

The thermal efficiency for that Ericsson cycle should be called the thermal efficiency of the corresponding Carnot cycle working between the same two temperatures because Ericsson cycle is also a totally reversible cycle. So, it should be equal to:

$$\eta_{Ericsson} = \eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1300} = 1 - \frac{3}{13} \approx 76.92\ \%$$

So, while introducing a simple regenerative in just two stage of compression and two stages of expansion, we are having an efficiency of 68.9%, which is a 24% increase compare to no regeneration scenario. With increasing the number of expansion and compression stages, we can have at most a further 8% gain, which may not worth while in most practical scenario considering the complexity requirement to have so many numbers of stages in out of the space requirement for this.

So generally, it is restricted only two or three stages of compression and expansion. Now, we have a more complicated problem, which is the really very large and read the problem statement very, very carefully.

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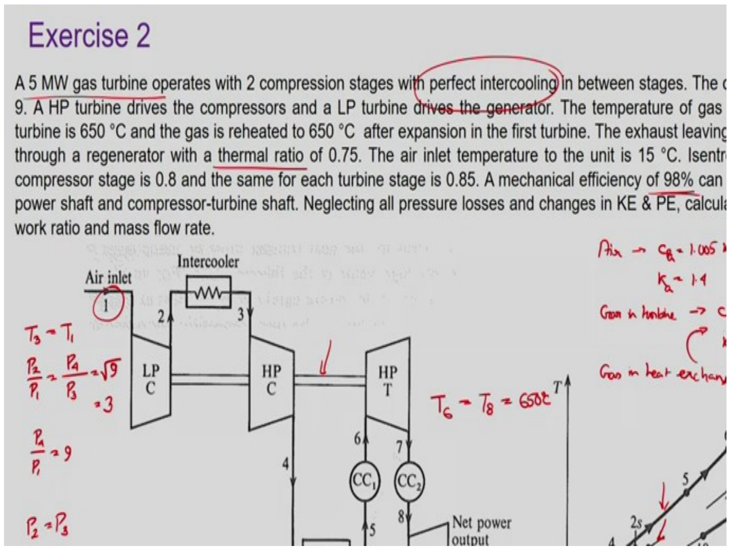

I am giving the diagram first so that you can directly compare the diagram. Here we have 5 MW gas turbine plant. It is operating with two compression stages, so this is the LP compression stage and this is the HP compression stage and perfect intercooling in between the stages. So, you are having perfect intercooling that is once it is perfect intercooling, so we can easily say that:

$$T_3 = T_1$$

and also the pressure ratio is equal that is:

$$\frac{P_2}{P_1} = \frac{P_4}{P_3}$$

the overall pressure ratio is given as 9.

So, here it is given as:

$$\frac{P_4}{P_1} = 8$$

So, pressure ratio for each stage is equal to √9 that is equal to 3. Now look at the turbine side, what other information are given? The temperature of gas at the entry to the HP turbine is

650 ^{0}C and is reheated to 650 ^{0}C after expansion in the first turbine. So, you are having ideal heating as well. So, in this case you have

$$T_6 = T_8 = 650 \quad {}^0C$$

the exhaust leaving the LP turbine passes through the regeneration where the thermal ratio of 0.75 or effectiveness of 0.75.

I intentionally have used the term thermal ratio even though e effectiveness is the common name, but in several cases thermal ratio is the one that we can see. So, the regeneration is done. The air inlet temperature to the unit is 15 ^{0}C, which refers to this state number 1. Isentropic efficiencies for compression stages 0.8 and turbine stage 0.85 and the mechanical efficiency of 98 % which is considered for both power shaft and the compressor turbine shaft.

In this case both the compresses are coupled with HP turbine, so the HP turbine is a driving both the compressors as mentioned in here. The HP turbine is driving the compressor and LP turbine is driving the generator. Now, the mechanical losses need to be considered and we have a mechanical efficiency of 98 % for this particular shaft and also for this particular power shaft. This also we have to consider.

We can neglect all the pressure losses and changes in kinetic and potential energies. The pressure losses are neglected. We can easily say that pressure in the intercooler remains constant, i.e.,

$$P_2 = P_3$$

and also pressure inside the combustion chamber remains constant, i.e., we have:

$$P_4 = P_5 = P_6$$

Similarly, have

$$P_7 = P_8$$

and also, as the *Ts* diagram is shown there.

From the *Ts* diagram we can also say that the pressure of air at the inlet to the LP turbine stage or LP compression stage rather should be equal to the pressure at the end point i.e., we have:

$$P_1 = P_9 = P_{10}$$

So here you are talking about at most four pressure stages. One pressure stage which corresponds to the air inlet pressure and the gas exit pressure passing through the regenerator.

The second pressure stage is the LP compressor to HP compressor stage. The third pressure stage from HP compressor to the HP turbine stage and the fourth pressure stage is from HP turbine to the LP turbine stage. The *Ts* diagram is also shown accordingly. So we have clearly see that this is pressure stage is number 1, which corresponds to the inlet air and also the exhaust gas. Then this is a second pressure stage from LP compressor to HP compressor, the third pressure stage is from HP compressor to the HP turbine and the fourth pressure stage from HP turbine to the LP turbine.

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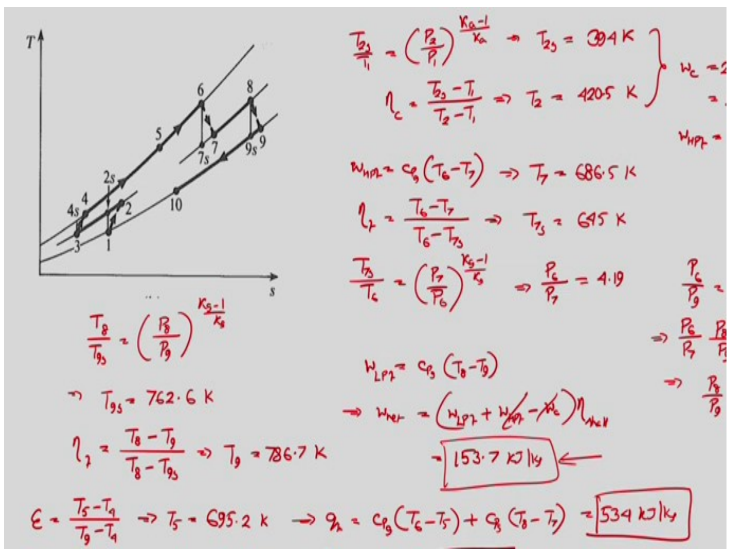

So, this is a diagram now, we have to solve this one carefully. We know the pressure ratio for the compression side, but not given about the turbine side, because it is mentioned about perfect intercooling but for regeneration nothing is given. Of course, temperature is mentioned that is heated at the same temperature, but that does not mean that perfect reheating as no information is given about the pressure ratios. So, now as the intercooling is perfect so we can go step by step. For process 1-2 we can easily write:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

So, some additional information which I have not noted here, that information is given for air we have:

$$c_p = 1.005\ kJ/kgK$$

and

$$k = 1.4$$

for the compression side i.e., for both LP and HP compressor. For gas on the turbine side we have:

$$c_{pg} = 1.15\ kJ/kgK$$

and

$$k_g = 1.333$$

where

c_{pg} is the specific heat for gas

k_g is the k value for gas for this expansion process, in both the stages of turbine is 1.333.

And gas in heat exchanger here the properties are similar to these values.

So, gas is having the specific 1.15 and exponent of 1.333 whereas for the air the c_p is 1.005 and for k equal to 1.4. So, here as you talking about air, so we should write it properly as:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k_a - 1}{k_a}}$$

give you some value of:

$$T_{2s} = 394\ K$$

And now the isentropic efficiency is given for the compressor. For isentropic efficiency what should be the expression for this? It should be the ideal temperature rise, i.e.,

$$\eta_c = \frac{(T_{2s} - T_1)}{(T_2 - T_1)}$$

which gives you T_2 which will be slightly greater than T_{2s}. And in this for particular problem it is:

$$T_2 = 420.5\ K$$

I am writing approximate numbers you please do all these calculations on your own. So, the work input requirement in this first stage of compression:

$$W_c = c_{pa}(T_2 - T_1)$$

and as we are talking about perfect intercooling in between multistage compressor, the total compressor work or should be just twice of this.

$$W_c = 2\ c_{pa}(T_2 - T_1)$$

And this work input requirement for the combustion stage is then coming to be equal to:

$$= 2 \times 133.1\ kJ/kg$$

Therefore, the work that the HP turbine used to produce should be equal to:

$$W_{HP} = \frac{W_c}{\eta_{shaft}}$$

which is I am talking about this compressor turbine shaft which is having and mechanical efficiency of 98%. So, this should be:

$$= \frac{2 \times 133.1}{0.98}$$

$$\approx 272\ kJ/K$$

This is the work output from the HP turbine that you are getting, 2% of that actually you are losing because of the frictional behaviour of the corresponding shaft.

Look at the process 6-7 which is happening in the HP turbine. So, there we can easily write W_{HP} turbine should be equal to:

$$W_{HP} = c_{pg}(T_6 - T_7)$$

from there T_6 is given because it is given to 650 ^{0}C. So, if you put the values then you will be getting:

$$T_7 = 686.5\ K$$

Now isentropic efficiency for the turbine is also given, which is the actual temperature loss, which is:

$$\eta_t = \frac{(T_6 - T_7)}{(T_6 - T_{7s})}$$

from where we can get:

$$T_{7s} = 645\ K$$

Now the actual temperature at the end of HP compression process 686 K but had there been an isentropic compression the temperature would have been 645 K. Now, what is the usefulness of this T_{2s}. T_{2s} is the one which can be utilised to get the pressure at the end of HP expansion because that information is not given to us. For the compression side we know that both stages have equal pressure ratio from here we can calculate the pressure P_2 or P_3 but that information is not given for turbine that we can get from air.

So, we can write:

$$\frac{T_{7s}}{T_6} = \left(\frac{P_7}{P_6}\right)^{\frac{k_g - 1}{k_g}}$$

to get:

$$\frac{P_6}{P_7} = 4.19$$

exact value is not required, not given also. So,

$$\frac{P_6}{P_7} = 4.19$$

and we know that:

$$\frac{P_6}{P_9} = overall\,pressure\,ratio$$

which is:

$$\frac{P_6}{P_9} = \frac{P_4}{P_1} = 9$$

So, we can write this to be equal to:

$$\frac{P_6}{P_9} = \frac{P_7}{P_9} = 9$$

And there is no pressure losses P7 and P8 should be equal so dropping this we have:

$$\frac{P_8}{P_9} = \frac{9}{4.19} = 2.147$$

So, the two stages of turbine are actually not having the same pressure ratio, the pressure ratios are different. In the first stage we are having 4.19 as the pressure ratio and in the second stage it is almost half of the previous level 2.15 only. So, use this pressure ratio now to get the information about the temperature at the end of LP compression stage or LP expansion I should say.

So, we can write:

$$\frac{T_8}{T_{9s}} = \left(\frac{P_8}{P_9}\right)^{\frac{k_g - 1}{k_g}}$$

from there, we are getting:

$$T_{9s} = 762.6\,K$$

and again using the isentropic efficiency for this LP turbine stage to be, we have:

$$\eta_t = \frac{actual\ temperature\ change}{ideal\ temperature\ change} = \frac{(T_8 - T_9)}{(T_8 - T_{9s})}$$

getting:

$$T_9 = 786.7\ K$$

so, we have the temperature. So, the work from the HP turbine. Work from the LP turbine then is coming to be equal to:

$$W_{LPt} = c_{pg}(T_8 - T_9)$$

and that and so:

$$W_{net} = W_{LPt} + W_{HPt}$$

So,

$$W_{net} = (W_{LPt} + W_{HPt} - W_c)\eta_{shaft}$$

these two works are cancelling each other and the network output which is coming to be:

$$W_{net} = (W_{LPt} + \cancel{W_{HPt}} - \cancel{W_c})\eta_{shaft}$$

$$= 153.7 kJ/kg$$

So, we have to calculate the cycle efficiency. We have got the net work output from the cycle for this. So, we have to get efficiency now. To get the efficiency we have to make use of the information about the reheater. So, for the reheater, the effectiveness is defined as:

$$Effectiveness\ (\varepsilon) = \frac{Actual\ heat\ transfer}{Maximum\ possible\ heat\ transfer}$$

$$= \frac{(T_5 - T_4)}{(T_9 - T_4)}$$

from where we can get:

$$T_5 = 695.2\ K$$

so, for q_{in} we are adding heat of gas in the combustion chamber and in the reheater. It can be mathematically expressed as:

$$q_{in} = c_{pg}(T_6 - T_5) + c_{pg}(T_8 - T_7) = 534\ kJ/kg$$

So, the thermal efficiency for the cycle is:

$$\eta_{th} = \frac{W_{net}}{q_{in}} = 28.8\ \%$$

This is the thermal efficiency for this particular cycle.

And we have to calculate the work ratio as well. So, to calculate the work ratio, we have to calculate the net work output by the gross work output. Now, how much is the gross work output? The gross work output is the work produced by HP turbine plus work produced by the LP turbine.

$$r_w = \frac{W_{net}}{W_{LPt} + W_{HPt}} = 0.358$$

We could also calculate the back work ratio from there.

And the final answer that we have to calculate is a mass flow rate. And how to get the mass flow rate? This is the information that we to make use of, 5 MW is the net output that you are getting from a gas turbine plant. And this is the net work output given in kJ/kg, i.e., for 1 kg per second flow rate we are getting 153.7 kilowatt of power output. And total output from this plant is 5 MW. So just use that information, that is going to give you the mass flow rate to be equal to:

$$\dot{m} = \frac{5 \times 10^3}{153.7} = 32.6\ kg/s$$

This is the way we can solve quite complicated problems with gas turbines also. It is a quite practical problem. Of course, one thing we have neglected that is, we have assumed no pressure loss in the component but we have considered intercooling, a perfect intercooling. We have considered an imperfect reheating though that reheated gases return back to the initial temperature and pressure ratios are not equal and we have also considered the regeneration to get the final results for this.

This way please try to solve some more problems from the textbook. Here I cannot remember whether I have mentioned in my previous lectures or not. While for the earlier weeks I have primary followed the books of Cengel and Boles and in certain situations the book of Sonntag. Here I am mostly following the third textbook that we have suggested for this course the book of applied thermodynamics by Eastop and McConkey. You can check the details that is provided in the concerned webpage where the entire physical details of that particular book is available.

You can purchase that book if required at all, may not be required too. If you are following the lecture, I think that should be sufficient and you can also get lots of material on internet for this. Try to solve a few more problems to clarify or to get a hold on the process that you have followed here this. So, that take us to the end of week number 6 where we have focused on the gas turbine cycles.

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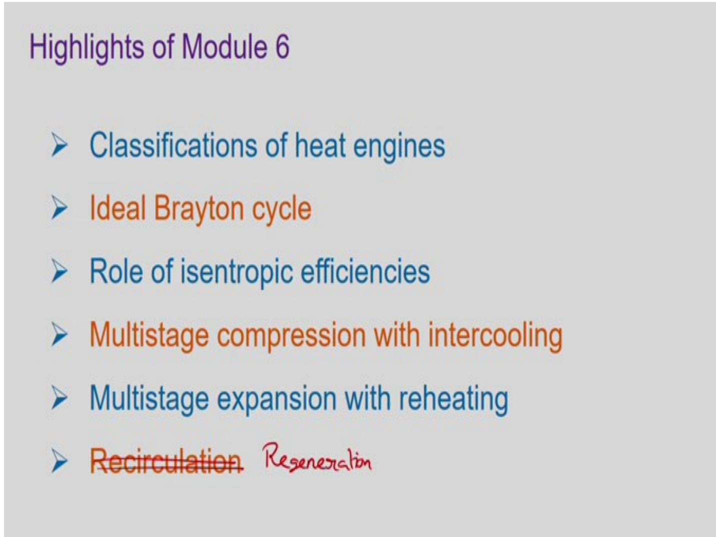

So, we started with the classification of heat engines particularly the reciprocating and rotary type of heat engines, then we introduced the ideal Brayton cycle that is ideal cycle for gas turbines and also, we discussed the role of isentropic efficiency for both the turbine and compressor for the Brayton cycle. Then we discussed three different ways of improving the performance of the ideal Brayton cycle. First is multistage compression with intercooling, then multistage expansion with reheating and third is recirculation or regeneration. Actually, I should not have written recirculation and the better term is to write regeneration.

Do not use the term recirculation that is a typographical error, it is regeneration, it should be regeneration. And finally we talked about the practical gas turbine cycles where all these three kinds of methods of present. We can have intercooling with multiple compression stages 2 or 3 stages of compression, we can have a reheating with 2 or 3 stages of expansion and definitely we can have regeneration. Only we have not solved numerical problems which involves special properties in the components also, but that can also be their which primarily affects the properties of the gas mixture and also the properties air.

You may get problems involving the pressure losses in your textbooks particularly in the book of Eastop and McConkey. So, that is it for week number 6. In the previous three weeks or I should say from week 4, 5 and 6 we have talked about the power cycles involving gas as a working medium. You talked about the air standard cycle for reciprocating engines and here we have talked about the cycle for rotary engine or gas turbine.

The next week onwards, we are going to enter into the steam side and we shall talk about the steam power cycle. So, thank you till then.