Applied Thermodynamics for Engineers
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Lecture – 18
Intercooling and Reheating in Brayton cycle

Good morning everyone, so you are into the second lecture for week number 6 where we are talking about the gas turbine cycles. During yesterday's lecture you were introduced to the concept of a gas turbine both open cycle based and closed cycle based gas turbines. And then I have talked about the ideal cycle for a gas turbine which is a Brayton cycle or Joule cycle. Where we have seen that quite similar to the other air standard cycles that we have talked about like Otto or Diesel cycle here also you have four processes two of them are isentropic process: an isentropic compression and another isentropic expansion process.

However, the heat addition and heat rejection both are taking place at constant pressure. Here actually you can draw an analogy between the Otto cycle, Diesel cycle and this Brayton cycle. In all these three we have four processes: we have an isentropic compression and an isentropic expansion process, their difference lies only in the mode of heat addition and heat rejection.

Like heat addition is at constant volume in case of Otto cycle, is at constant pressure in case of both Diesel cycle and Brayton cycle. Whereas heat rejection is at constant volume in case of both Otto and diesel cycle however it is at constant pressure in case of Brayton cycle. So, this way these three cycles are related and also this is an air standard cycles at least the ideal one that is the way we mathematically analyze this.

However, there is one major difference because the Brayton cycle is associated with gas turbines which operate on rotary machinery not on reciprocating one which are associated with the SI and CI engines i.e., Otto and diesel cycles. So, here we are talking about a cycle or an air standard cycle which is suitable for rotary devices therefore each of the four processes that I just mentioned all of them are performed in four different components.

So, we have the compression process done in a compressor, then the heat addition is done inside a combustor or maybe a heat exchanger in closed cycle. Then we have a turbine to perform the expansion and then to get the system back to the initial part we have another heat exchanger which facilitates the heat rejection to a low temperature sink. So, there are four components. Each of them can be modelled as a separate open system or system having mass in and mass outflow from only a single inflow and single outflow.

And combining the individual analysis for each of these four we can get the total cycle performance, which we have already done yesterday. We have calculated the compressor work, the turbine work, the heat input and heat output from there we have got an expression for the efficiency. And there we have seen that like in case of |Otto cycle the major parameter was the compression ratio which is a volume ratio actually volume before compression and volume after compression.

Here we have a pressure ratio which is a pressure ratio as the name suggests it is pressure after compression to pressure before compression. So, there is a difference with compression ratio, in compression ratio, you have volume before compression divided by volume after compression. Whereas here in pressure ratio, we have pressure after compression by pressure before compression. You can logically think also because they are defined such that both of them are greater than one, so, accordingly you can identify their definitions. And as pressure ratio increases the thermal efficiency also increases. However, the work output may not keep on increasing continuously rather we have seen that once we have fixed up the minimum and maximum cycle temperatures there is an optimum pressure ratio, which we have identified in the last class. And then some practical aspect was incorporated the concept of isentropic efficiencies of the compressor and turbine that we have touched upon.

So, today we shall be starting with a numerical example where we are going to use that concept of the isentropic efficiency.

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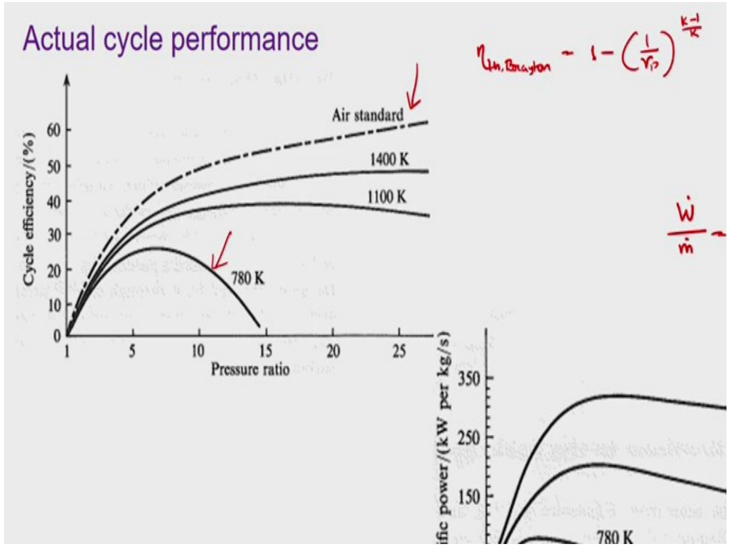

But before that just check out the actual cycle graphs. This is the variation of cycle efficiency with pressure ratio. The dotted line is the air standard one which follows the analysis that we have done yesterday. Because we know that the thermal efficiency for Brayton cycle is:

$$\eta_{th,Brayton} = 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

where

k is the ratio of specific heat

if we take k = 1.4, then we are going to get this particular line. 1.4 is a typical value for air under normal atmospheric condition.

However, as the maximum cycle temperature changes truly speaking, we are not getting such change rather cycle efficiency keeps on increasing till a certain level and that decreases very prominently, you can see from this particular curve. Of course, the pressure ratio corresponding to this optimum cycle efficiency that keeps on increasing with the highest temperature but still there is some kind of optimum.

And the same thing we can see we have already discussed this, this is for the power output. Specific power is nothing but the total power output divided by the mass flow rate which we have denoted by the w. So, or sometimes were unit can be kilo Joule per kg or just the way it is presented here kilo Watt divided by kg per second which is for the mass flow rate. So, specific power is nothing but rate of work output per unit mass flow rate.

So, here also we know that there will be an optimum pressure ratio corresponding to the maximum power output and that is a function of the maximum temperature. But interestingly the location of the optimum pressure ratio for the efficiency and for the power output they are not same, rather they are different and therefore we have to go for some kind of optimization or trade-off.

Because it is not possible to identify pressure ratio which is going to give simultaneously highest efficiency and highest power output. So, we have to choose one of them as per the requirement.

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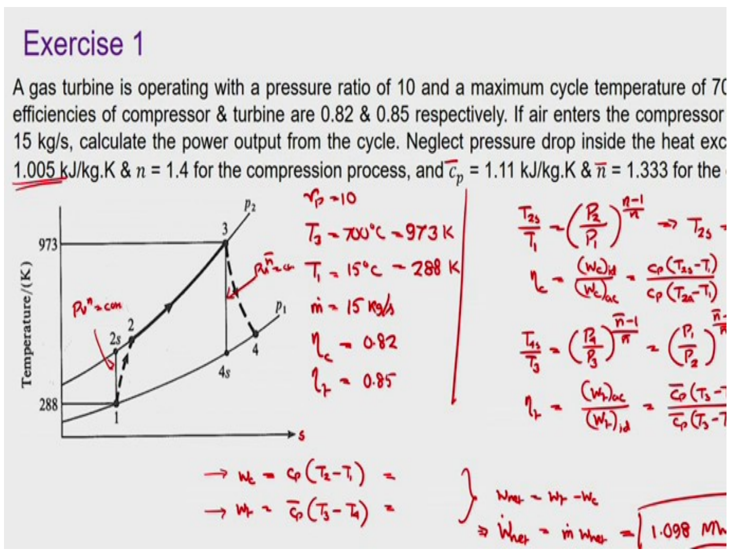

Now let us see what we have as an effect of the isentropic efficiencies. Here we have a gas turbine operating with the pressure ratio of 10 and a maximum temperature of 700 ^{0}C. Isentropic efficiencies for compressor and turbine are given and they are separate in this case. Air enters the compressor at 15 ^{0}C at a rate of 15 kg/s, so you calculate the power output.

So, this is the typical cycle which is a here 1 is the point before compression 2s refers to a point had there been isentropic compression but because of the presence of irreversibilities it shifts towards right to point number 2, here the axes are missing which is temperature the vertical axis it is entropy. Similarly, the turbine also a have a showing and irreversible expansion, had it been an ideal expansion isentropic expansion it would have reached point 4s but because of the presence of irreversibility there is entropy generation during this process so it is going to shift it towards right to point number 4.

Now here of course it is mentioned that we are neglecting any pressure drop in the heat exchangers so 2 to 3 and 4 to 1 they just follow the constant pressure line, they are constant pressure processes. This is some simplification we are doing because we are just looking to check out the effect of the isentropic efficiencies only. And is also given that the c_p and n for the compression process and expansion process they are separate that is also logical because we have to think about that during the compression process it is generally only air which is getting compressed whereas during the expansion process it is the combustion products gets expanded. So, their characteristics their properties can be different. So, we may have different values of the properties. So, here as we have this n mentioned so it is not isentropic process rather we shall be treating this one as a polytropic process.

So, let us try to solve it using the definition of isentropic efficiencies. Let us note down what temperatures or what information are there.

$$r_p = 10$$

$$T_3 = 700^0C = 973\ K$$

$$T_1 = 15^0C = 288\ K$$

$$\dot{m} = 15\frac{kg}{s}$$

$$\eta_c = 0.82$$

$$\eta_t = 0.85$$

So, these are the known information. So, let us look at the process 1 to 2 or first 1 to 2*s*, so 1 to 2*s* represents an isentropic process. So, there we can always write:

$$\frac{T_{2s}}{T_1} = \frac{P_2}{P_1}^{\frac{n-1}{n}}$$

So, here because this is an isentropic process which follows the relation like if we are talking about this particular process it follows the relation:

$$Pv^n = constant$$

same about this one as well, however the n values are different. So, putting this we have

$$T_{2s} = 556\ \text{K}$$

So, had the compression process being isentropic this would have been a temperature at the end of compression. But this is not isentropic, so we have to make use of the definition of this isentropic efficiency.

$$\eta_c = \frac{(w_c)_{id}}{(w_c)_{act}} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)}$$

So, it gives us a relation of the isentropic efficiency solely in terms of temperature here we already know

$$T_1 = 15^0 C$$

T_{2s} we have just calculated. So, from there we can calculate T_{2a} which is coming to be 614.8 K. So, we know the actual temperature, now we have to calculate point number 4. Similarly, we follow the process 3 to 4 is first so we follow that:

$$\frac{T_{4s}}{T_3} = \frac{P_4}{P_3}^{\frac{\bar{n}-1}{\bar{n}}} = \frac{P_1}{P_2}^{\frac{\bar{n}-1}{\bar{n}}}$$

always put SI units or temperature in Kelvin because you are talking about absolute temperatures. So, T_{4s} is coming as 547.4 K. You please do all these calculations on your own to check the correctness of these numbers, these are just approximate numbers. Now we are using the definition of the turbine isentropic efficiency:

$$\eta_t = \frac{(w_t)_{id}}{(w_t)_{act}} = \frac{\bar{c_p}(T_3 - T_4)}{\bar{c_p}(T_3 - T_{4s})}$$

So, putting this we get T_{4a} in the actual temperature at the end of this expansion process is:
T_{4a} = 611.2 Kelvin.
Here you have to keep in mind the here this c_p is also different let us put into c_p *bar* because the c_p *bar* is this one whereas c_p is this particular thing. Similarly,
n bar = 1.333
So for compression or isentropic process, compression and expansion processes the magnitude of specific heat and the polytropic coefficients they are different or polytropic index you have to be careful during calculation.

So now we know the temperature at all the four points and therefore it is much easier to calculate all the corresponding work. So, how much will be your specific compression work requirement so

$$w_c = c_p(T_2 - T_1)$$

$$w_t = \bar{c_p}(T_3 - T_4)$$

So, you can calculate both of them separately. And once you combine them then

$$w_{net} = w_t - w_c$$

and net power output then should be equal to

$$\dot{w_{net}} = \dot{m}w_{net}$$

once you put all the numbers, the net power output will be:

$$\dot{w_{net}} = 1.098\ MW$$

Again remember while calculating this one you have to put c_p equal to 1.005 while calculating this one you have to put c_p *bar* is equal to 1.11. So, this way using isentropic efficiencies we can calculate the work requirement. And definitely there is a change if we solve the same problem, I would request you to solve the same problem by assuming both compression and expansion process to be isentropic that is there is 2*s* and 4*s* at the correct points and see what values you get.

Of course, we are not in a position to calculate the efficiency here because for calculation of efficiency we need to know the total heat input given. Because the thermal efficiency should be equal to the net work output divided by net heat input given.

$$\eta_{th} = \frac{\dot{W_{net}}}{\dot{Q_{in}}} = \frac{\dot{W_{net}}}{\dot{m}c_p(T_3 - T_2)}$$

Now in this case we know *mdot* ,T_2 and T_3. But we do not know the c_p associated with this combustion process. So, if we have some idea about the magnitude of specific heat during the heat addition process then you can calculate the amount of heat added during the heat addition and subsequently you can get the efficiency as well. But in this problem as the expression or magnitude for this one is not given so we are not in a position to get the thermal efficiency.

Now we have seen that the back work ratio can be a significant portion actually in this problem we could have calculated a back work ratio also. The back work ratio is represented by

$$r_{bw} = \frac{w_c}{w_t}$$

and often there is something called work ratio. Work ratio is the:

$$r_w = \frac{w_{net}}{w_t}$$

So, if we put the expression for network it just becomes:

$$= 1 - \frac{w_c}{w_t} = 1 - r_{bw}$$

So, in this case I have not put the numbers here you can get the compressor specific compression work and specific expansion what from there you can easily get the back work ratio or the work ratio.

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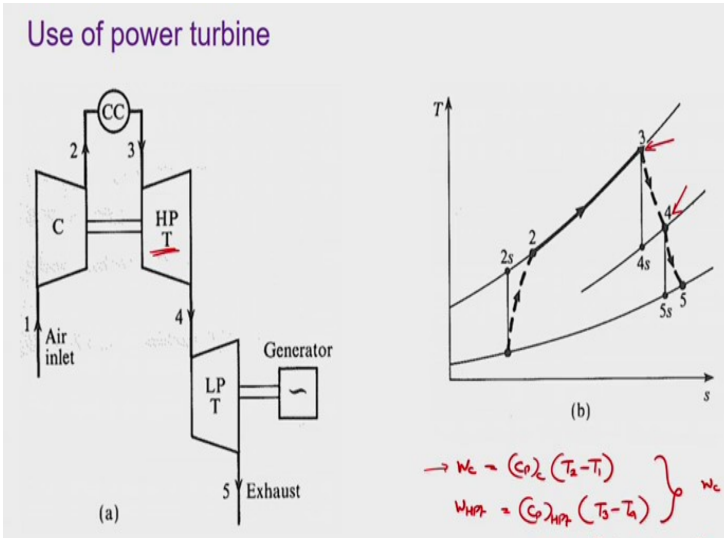

And we know that you can see we have solved one problem yesterday and you can take from here also this back work ratio can be quite significant, it can even be in the range of 40 to 50% and therefore a significant portion of the energy produced by the turbine is getting consumed in the compressor. Therefore, quite often instead of mounting the compressor and turbine on the same shaft we divide the turbine into two components.

The first one which is often referred as HPT or high pressure turbine which directly receives the input from the combustion chamber that is mounted on the same shaft on the compressor. Whereas the exhaust from the high pressure turbine the supplied to the low pressure turbine in LP turbine and that is the one that is mounted on the generator to get the work output for this.

This can be a *Ts* representation, so here this 3-4, this particular process corresponds to the expansion in the high pressure turbine and 4-5 is the expansion in the low pressure turbine. Here the work output in the high pressure turbine is modulated such that it exactly matches the work input required for the compressor. So, the work input required for the compressor should be equal to specific work that is should be:

$$w_c = (c_p)_c (T_2 - T_1)$$

and work produced by the high pressure turbine should be equal to:

$$w_{HPt} = (c_p)_{HPt} (T_3 - T_4)$$

Now these two should be exactly equal to each other there is this:

$$w_c = w_{HPt}$$

Then the network output that we are going to get that is exactly equal to what is being produced by the second turbine the low-pressure turbine which can be calculated as:

$$w_{net} = w_{LPt} = (c_p)_{LPt} (T_4 - T_3)$$

So, this way we can sometimes use a power turbine to have one turbine producing only the compressor work requirement and other turbine is coupled to the generator to give the commercial power output that we are looking for. And this c_p values we have used separate c_p values in the previous problem same applicable here like during the compression process the c_p corresponds to c_p for air.

$$(c_p)_c = (c_p)_{air}$$

Whereas for high pressure turbine and low-pressure turbine is c_p for the gas and their values also can be different because we know specific heat can be a strong function of temperature. High pressure turbine definitely works at a higher pressure and higher temperature level as can be seen from the *Ts* diagram. So, its c_p can be higher compared to the c_p for the gas in the low-pressure turbine.

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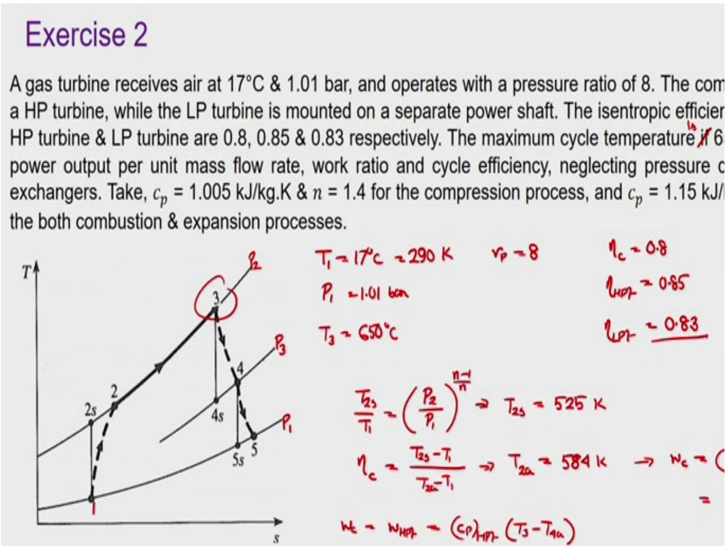

Let us solve another numerical problem to use the same concept of high pressure and low-pressure turbines. So, here we have a gas turbine which is receiving air at 70 ^{0}C and 1.01 bar the diagram is shown. So, point number 1 is missing in the diagram, so this is your point number 1 so we have

$$T_1 = 17^0C = 290\ K$$

$$P_1 = 1.01\ bar$$

$$r_p = 8$$

It is driven by a HP turbine where LP turbine is mounted on a separate power shaft just as we have seen in a previous slide. The isentropic efficiencies of the compressor HP turbine in LP turbine are given so we have:

$$\eta_c = 0.8$$

$$\eta_{HPt} = 0.85$$

$$\eta_{LPt} = 0.83$$

Maximum cycle temperature should be 650 ^{0}C maximum cycle temperature corresponds to which point T_3 clearly seen from the *Ts* diagram. So, your

$$T_3 = 650^0C$$

and now we have to calculate the power output per unit mass flow rate work ratio and the cycle efficiency neglecting the pressure drop inside the heat exchangers c_p again it is given to be 1.005 and n equal to 1.4 for the compression process. And c_p and n in for both the stages of expansion and also the c_p for combustion they are same which is given.

So, you have to calculate first the temperature at all the points. So, we shall be following some process quite similar to the previous exercise, so I shall be do trying to do a bit quickly. So, we first follow process 1 to 2s so:

$$\frac{T_{2s}}{T_1} = \frac{P_2}{P_1}^{\frac{n-1}{n}}$$

here the pressures are not shown let us say this is P_1 this pressure level is P_2 and this pressure level is some intermediate P_3. So, from there we are going to get

$$T_{2s} = 525\ K$$

then we use the definition of compressor efficiency, which like we have seen it is a ratio of ideal work requirement for the compressor to the actual work requirement which will be coming as:

$$\eta_c = \frac{(T_{2s} - T_1)}{(T_{2a} - T_1)}$$

from there we get:

$$T_{2a} = 584\ K$$

So, once we know this then we can say that specific work requirement for the compression process should be:

$$w_c = c_{p_c}(T_2 - T_1) = 295.5\ kJ/kg$$

So, we know that compressor work requirement or specific compression work requirement. Now we have to shift to the turbine side. So again, quite similarly use the concept of isentropic expansion for the HP turbine and the LP turbine. But before that look at whatever the work input that are required for the compressor that we have calculated that has to be supplied by the HP turbine that is

$$w_c = w_{HPt}$$

Because that is the entire idea of having two different stages of expansion and this should be equal to:

$$w_c = w_{HPt} = (c_p)_{HPt}(T_3 - T_{4a})$$

So we know T_3 that is given to be equal to 650 ^{0}C we have to convert that to Kelvin I am not doing here but please convert T_3 to absolute temperature. From there we are going to get:

$$T_{4a} = T_3 - \frac{w_c}{(c_p)_{HPt}} = 666\ K$$

Now we know the isentropic efficiency of the HP turbine using that we can also calculate T_{4s} but that is not required as of now. So, we know the compression work and we have also used the equivalence of the compression work an HP turbine work from there we have got the temperature T_4.

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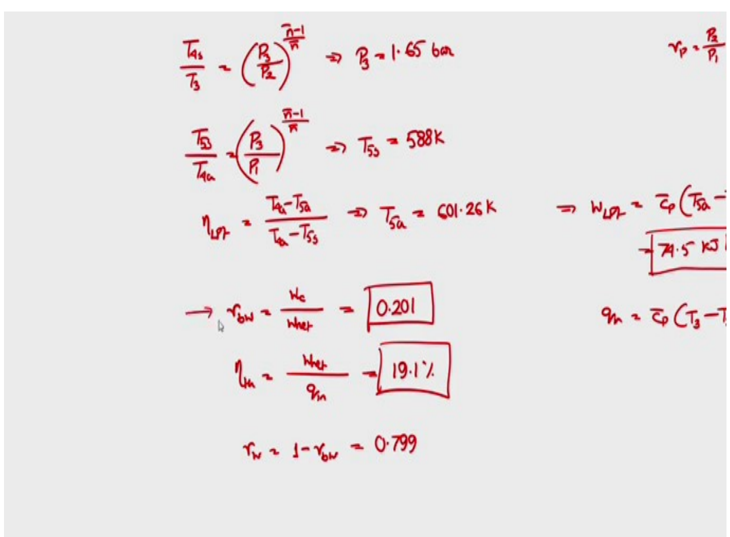

So, now look to the LP turbine, But before we move to the LP turbine look at what we have for the LP stages. We have an isentropic efficiency 0.83 and other values are also given. So, before that actually T_4 is I have mentioned that we do not need to have T_{4s} but there is something that we still need because P_3 the intermediate pressure is not known and that can be obtained only from the value of T_{4s}.

So, we use the definition of the isentropic efficiency for the HP turbine which should be equal to actual work output by the ideal work output that is:

$$\eta_{HPt} = \frac{(T_3 - T_{4a})}{(T_3 - T_{4s})}$$

which gives you

$$T_{4s} = 620.5K$$

So, now we know T_{4s} from there we have to and consider that T_3 or 3 to 4 is an isentropic process then,

$$\frac{T_{4s}}{T_3} = \left(\frac{P_3}{P_2}\right)^{\frac{\bar{n}-1}{\bar{n}}}$$

where *n bar* is the one corresponding to the expansion processes.
From there what we can get how we can get P_2,

$$r_p = \frac{P_2}{P_1}$$

$$r_p = 8$$

$$P_1 = 1.01\, bar$$

So

$$P_2 = 8.08\, bar$$

From there we get

$$P_3 = 1.65\, bar$$

Now we know the pressure at this particular level P_3. So, from there we know for temperature at point 4 we know pressure P_3 and P_1 so from there we get T_{5s} first following the previous procedure.

So,

$$\frac{T_{5s}}{T_{4a}} = \left(\frac{P_3}{P_1}\right)^{\frac{\bar{n}-1}{\bar{n}}}$$

there we are going to get

$$T_{5s} = 588\ K$$

then we have to use the explanation for the efficiency of LP turbine which is the actual work produced by the LP turbine why the ideal work produced by the LP turbine. Look at the diagram actual work should be proportional to $T_4 - T_5$ ideal work should be proportional to $T_4 - T_{5s}$.

So, it is equal to

$$\eta_{LPt} = \frac{(T_{4a} - T_{5a})}{(T_{4a} - T_{5s})}$$

which will be giving you

$$T_{5a} = 601.26\ K$$

So, using this we can now calculate the work produced by the LP turbine to be equal to:

$$w_{LPt} = \bar{c_p}\,(T_{5a} - T_{4a})$$

$$= 74.5\ kJ/kg$$

So, now we have the work required by the compressor specific work. Work produced by the HP turbine which should be equal to the compression work and work produced by the LP turbine. So we can calculate whatever we need like our first object is to get the back work ratio. So, back work ratio will be equal to:

$$r_{bw} = \frac{c}{w_{net}}$$

So, putting this we can get the back work ratio in this particular case is coming to the 0.201. And thermal efficiency should be equal to:

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

Now q_{in} is something that we have not calculated yet. So, how we can get q_{in}, q_{in} corresponds to the heat addition process that is process 2-3 and during this combustion process it is given that c_p is 1.15 and we already know that temperature at point 3 and 2. So, it be equal to:

$$q_{in} = \bar{c_p}(T_3 - T_{2a})$$

so the heat addition you can easily calculate from there the thermal efficiency in this case will becoming something like 19.1%.

So, these are the two results that we were looking for. Just check there what we are looking to calculate. We are looking to identify the power output per unit mass flow rate, work ratio and cycle efficiency. So, power output will be this one power output per unit mass flow rate back work ratio and cycle efficiency we have calculated. And if you are interested to get the work ratio that is nothing but:

$$r_w = 1 - r_{bw} = 0.799$$

Back work ratio or work ratio either of them are used conveniently but generally this one is much more popular. So, this way we can use two different turbines and calculation is quite longish, but you have to keep in mind that always you have to use the isentropic efficiency. And for the isentropic process we have to make use of this kind of relation between temperature and pressure, so that we can identify both actual and ideal performance.

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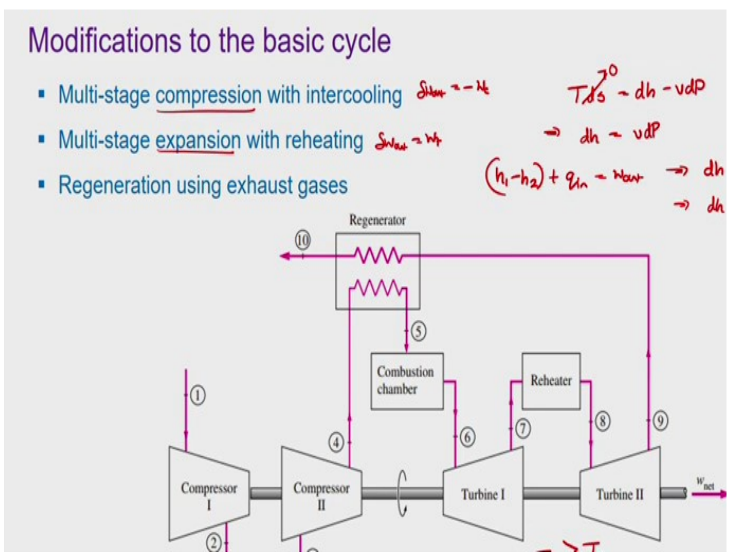

Now let us move to the modifications in the basic cycle. We have seen that back work ratio can be quite significant. Of course, in this case the back work ratio is smaller but in practical cases back work ratio can be in the range of 40-50-60%. And therefore we need to make certain kind of modification in the basic cycle so that we can increase the work ratio or we can reduce the compressor work requirement.

For this several modifications have been done historically on the gas turbine cycles or on the ideal Brayton cycles. And the three most common kind of modifications are multistage compression with intercooling, multistage expansion with reheating and the use of regeneration using the exhaust gaseous. Now during the compression process, we know that the work input requirement for the compressor, like if we just think about the second law of thermodynamics and apply that to an isentropic compressor. Now following second law we know there was one of the *T*d*s* relation giving

$$TdS = dh - vdP$$

Now if we apply this one on an isentropic compression or expansion process then we know that due to isentropic nature ds is equal to 0 that is

$$dh = vdP$$

Now just think about the form of the first law of thermodynamics that we have used yesterday. What we had for a process from 1- 2 we had:

$$(h_1 - h_2) + q_{in} = w_{out}$$

or if we write in differential form

$$dh + \delta q_{in} = \delta w_{out}$$

Now for the compression process we have, because of the isentropic nature of the compression expansion there is this is not present so we have a direct relation between dh and output work. Now for the compression process your output work is equal to $-\ W_c$ because what input is required. Whereas during the expansion process the expansion process we have and work output so this is equal to your W_t.

So, if we compare this then what we have for the compression process

$$\delta w_c = -dh = -vdP$$

$$\delta w_c = -\int vdP$$

Similarly for the turbine part putting

$$\delta w_t = dh = vdP$$

$$\delta w_t = \int vdP$$

So we can see in both this case is the work input required by the compressor or work produced by the turbine they are directly proportional to a specific volume of the working medium. So, what we need for a compressor and what we need for a turbine. For a compressor our objective is to keep the work input requirement to small to the smallest possible. Then the specific volume of the medium should be smaller.

Whereas in case of a turbine we want to have as much work output as we can then the specific volume also has to be larger. Therefore, for the compression stage we want to keep the specific volume smaller and for the expansion stage we want to keep the specific volume larger. Now if we are using a liquid during compression that is probably the best possible solution because specific volume is very small that is what is done in a steam-based cycle which we shall be talking in next week.

However when we are working in a gas turbine where working medium is gas then what is the way we can keep the specific volume of gas to a smaller value only by reducing this temperature. And that is the idea of this multistage compression with intercooling. Here we break the compression into several stages and in between two successive stages we cool down the gas so that is temperature reduces and thereby this volume also reduces. So, that we can reduce the total compression work requirement. Similarly, in the turbine we want the specific volume of the working medium to be larger as large as possible.

And therefore, we also break the expansion stage into several stages of turbine and in between each of the stages we reach it back to higher temperature. So, the specific volume again increases so we can get higher work output from the next stage. This is the idea of having multistage compression and multistage expansion. And regeneration is something that we shall be talking in the next lecture but today we are focusing on compression and expansion.

This is a diagram of our typical gas turbine cycle with all these three modifications. Look at this, here the air enters compressor 1 and comes out as stage 2 then we have an intercooler where heat is being rejected by the air. So, that its temperature reduces and therefore temperature at point 3 is lower than temperature at point 2. So, in the compression stage 2 the work input requirement that there is a reduction because of the lowering in the temperature. So, it comes out at temperature 4, then there is a regeneration I am not going to talk about that now. So, we have combustion chamber after combustion is coming out at state 6 and getting expanded to stage 7 in turbine 1. Then we have a reheater where we are reheating it back to higher temperature. That is temperature at 0.8 is greater than temperature at 7 accordingly the specific volume also will be larger. And therefore, we are getting higher work output from turbine stage 2.

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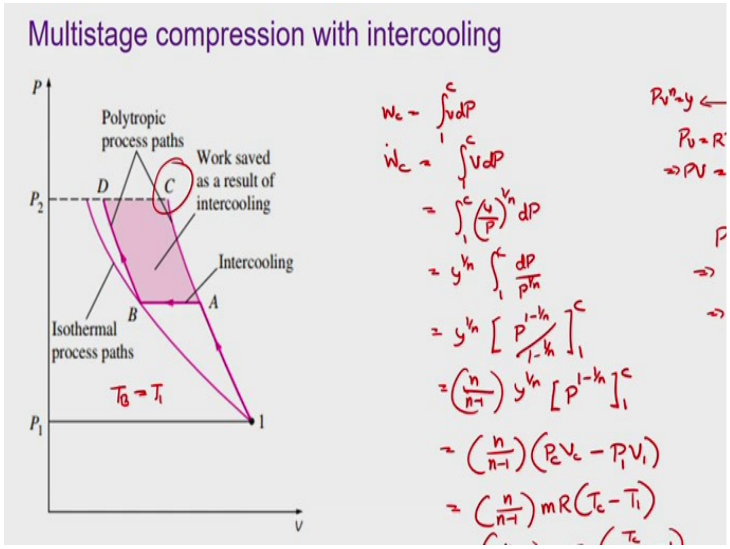

So, let us explore the first idea which is multistage compression with intercooling. Here the diagram is shown for a compressor which is looking to compress the gas from pressure P_1 to pressure P_2. Now if we are following a polytropic process a single stage compression then we are starting from point 1 and we are finishing at point C. However, our idea is to break this compression into multiple stages let us just focus on two stages and in between you are going to cool it. If we are following this single stage compression in isothermal way then that is a situation where we need the least possible work input requirement. It can be proved also from their expressions. So had it been an isothermal process it was started from point 1 it will finish at somewhere here and I am just going back to the previous slide.

Look at this work input requirement is $\int v dP$ then what it represents on diagram? The same Pv diagram we are drawing, but work input requirement is not a projection like this because that is true when we are talking about $\int P dv$. Here we are talking about $\int v dP$ i.e., the projection is this much on the v axis that is going to give you the work input requirement. So, for the polytropic case we are ending up at point C and for isothermal you are ending up here definitely much less amount of work input requirement. So, intercooling what we are doing we are expanding up to stage a then we are doing the intercooling so that is temperature comes back to the initial temperature. Here as per this idea T_B is equal to T_1, when we can have this, this is called a perfect intercooling. There may be certain issues before where the T_B cannot be lowered up to T_1 what we try to go as close as we can.

And then we have the second stage of compression from point B to point D and you can see the shaded area shown in pink which represents the work saved as a result of this intercooling. So, let us try to get a mathematical idea about the work input requirement or work saving and also try to identify exactly at what intermediate pressure we should do this. Because P_1 and P_2 are the two extreme pressure and let us say P_I is this intermediate pressure where we are doing this intercooling.

So, let us see at what choice of P_I gives us the maximum saving in the work input requirement. For that first we have to analyze a single stage compression. And so initially we are focusing on a single stage compression that is you are starting from point 1 and going up to point C. As we have seen in the previous slide the work input requirement for the compressor is equal to $\int v dP$ starting from point 1 to point C in this case.

$$w_c = \int_1^C v dP$$

or

$$\dot{w}_c = \int_1^C V dP$$

Now we have to assume a few things. Firstly we are going to assume isentropic compression and secondly we are going to assume an ideal gas as the working medium. Ideal gas means we can use

$$Pv = RT$$

or

$$PV = mRT$$

Similarly, the for isentropic compression we can make use of the relation

$$Pv^n = constant$$

Actually, instead of isentropic let us do in a polytropic way so we are putting the index n. If it is isentropic then the n would be by replaced by k.

So, to get the total work output

$$\dot{w}_c = \int_1^C v dP$$

Now let us use the ideal gas equation of state so that we can represent this isentropic process as using the relation:

$$Pv^n = constant$$

Let y be a constant

$$PV^n = y$$

or we can write

$$V^n = \frac{y}{P}$$

$$V = \left(\frac{y}{P}\right)^{1/n}$$

Substituting this value of V in the expression for work input requirement for the compressor , we have:

$$\dot{w}_c = \int_1^C V dP$$

$$= \int_1^C \left(\frac{y}{P}\right)^{1/n} dP$$

$$= y^{1/n} \int_1^C \frac{dP}{P^{1/n}}$$

So, if we perform this integration now it is:

$$= y^{1/n} \left[P^{1-1/n} / 1 - 1/n \right]$$

so we have:

$$= \left(\frac{n}{n-1}\right) y^{1/n} \left[P^{1-1/n}\right]_1^C$$

and integration limits are 1 to C

and if you combine this one now with the equation

$$PV^n = y$$

and then this will be coming as

$$= \left(\frac{n}{n-1}\right)(P_c V_c - P_1 V_1)$$

and now using the ideal gas equation of state we have:

$$= \left(\frac{n}{n-1}\right) mR(T_c - T_1)$$

this is coming to be your ideal gas form of the equation.

However, we are not going to use this rather we are going to use in a separate form as follows:

$$= \left(\frac{n}{n-1}\right) mRT_1 \left(\frac{T_c}{T_1} - 1\right)$$

going back to the ideal gas equation of state then:

$$= \left(\frac{n}{n-1}\right) mRT_1 \left(\frac{P_c V_c}{P_1 V_1} - 1\right)$$

So, this is for a single stage compression. We are going to you go for the multi stage compression now we are expanding again from point 1 to A and then from B to D assuming T_B and T_1 to be equal.

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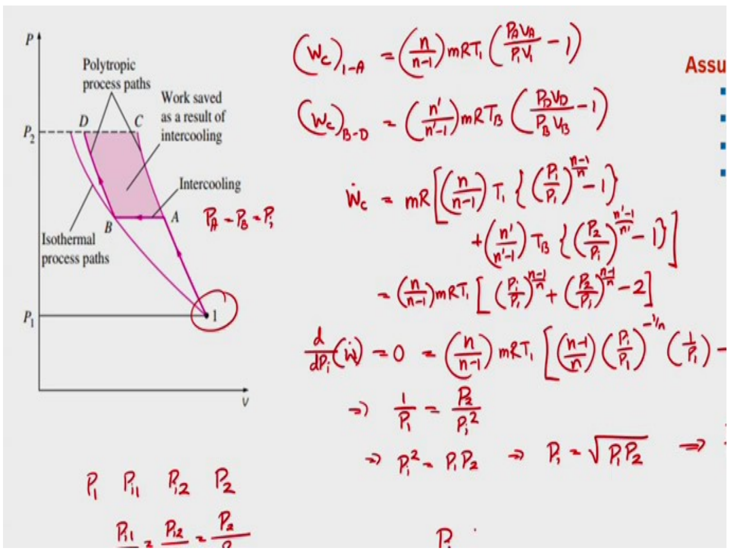

So, work input requirement for stage A, 1-A:

$$(w_c)_{1-A} = \left(\frac{n}{n-1}\right) mRT_1 \left(\frac{P_A V_A}{P_1 V_1} - 1\right)$$

And work input requirement for the second stage B to D should be equal to:

$$(w_c)_{B-D} = \left(\frac{n'}{n'-1}\right) mRT_B \left(\frac{P_D V_D}{P_B V_B} - 1\right)$$

here we are continuing to assume isentropic or polytropic rather instead of isentropic we should call it polytropic compression and also the ideal gas equation of state. Now working with V is quite complicated so instead of working with V_A/V_1 and V_D/V_B we are going to convert everything in terms of pressure. And also, you have to note that:

$$P_A = P_B = P_I$$

and now here we are using the relation for the first stage of compression

$$P_1 v_1^n = P_A v_A^n$$

Similarly for the second stage of compression we have this relation.

$$P_D v_D^{n'} = P_B v_B^{n'}$$

Applying the above relations, the net compressor work requirement now comes out to be:

$$\dot{w}_c = mR\left[\left(\frac{n}{n-1}\right)T_1\left\{\left(\frac{P_I}{P_1}\right)^{\frac{n-1}{n}} - 1\right\} + \left(\frac{n'}{n'-1}\right)T_B\left\{\left(\frac{P_2}{P_I}\right)^{\frac{n'-1}{n'}} - 1\right\}\right]$$

To simplify our analysis, we put another assumption that we assume *n* and *n'* to be equal to each other. And as a final assumption we have assume a perfect intercooling that is T_I and T_B are equal to each other. So, in that case we can write the above expression as:

$$= \left(\frac{n}{n-1}\right)mRT_1\left[\left(\frac{P_I}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_I}\right)^{\frac{n'-1}{n'}} - 2\right]$$

this is the work input required by the compressor after we have put this intercooling.

So, we have to identify for which value of once we have to accept the P_I and P_2 that is the initial pressure level and final pressure level and also initial state point that is both P_I and T_I, then for which P_I we need the minimum possible work input. For that we have to optimize this that is we have to differentiate this:

$$\frac{d}{dP_i}(\dot{w}_i) = 0 = \left(\frac{n}{n-1}\right)mRT_1\left[\left(\frac{n}{n-1}\right)\left(\frac{P_I}{P_1}\right)^{\frac{-1}{n}}\left(\frac{1}{P_1}\right) - \left(\frac{n-1}{n}\right)\left(\frac{P_2}{P_I}\right)^{\frac{-1}{n}}\left(\frac{P_2}{P_I^2}\right)\right]$$

So, removing the constant terms and taking that *(n – 1)/n* outside, so we have a very simple form actually:

$$\frac{1}{P_1} = \frac{P_2}{P_I^2}$$

you can just check out the calculation or

$$P_I^2 = P_1 P_2$$

giving

$$P_I = \sqrt{P_1 P_2}$$

So, probably a better way of writing this one is:

$$\frac{P_I}{P_1} = \frac{P_2}{P_I}$$

that is we can see that the intermediate pressure is just the geometric mean of the two extreme pressure. And in order to have the minimum possible work input requirement the pressure ratio for both the stages has to be equal to each other. If they are equal then correspondingly we can put this into back into the expression for the compressor work to get the values for the corresponding pressure ratio or the minimum possible work output.

If you have any doubt whether this is minima or maxima you can differentiate it once more and check the sign of that second derivative putting:

$$P_I = \sqrt{P_1 P_2}$$

and you can confirm that this represent the minimum possible work input requirement. So, when the compressor is subjected to multi staging with intercooling then we have to put identical pressure ratio for both the stages of compression and that can ensure that we need the minimum possible work input requirement for this.

If we are going for several stages instead of two if we have more than two stages then again the same principle is applicable. Like suppose we have three stages where it starts with the pressure P_1 then we have a intermediate P_{i1} then it goes to in the second stage he goes to intermediate P_{i2} and finally goes to P_2 then again the pressure ratios to be equal for all the stages that is:

$$\frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{iI}} = \frac{P_2}{P_{i2}}$$

So, the pressure ratio has to be equal for all the stages. And without proving I can show that if this is not perfect intercooling but *n* and *n'* are equal in that case it will become equal to:

$$P_i = \sqrt{P_1 P_2}\left(\frac{T_B}{T_1}\right)^{\frac{n}{2n-1}}$$

So, this one I am leaving to you to prove this. Here this T_1 and T_B are equal and we have taken that out or this ratio will also come into picture.

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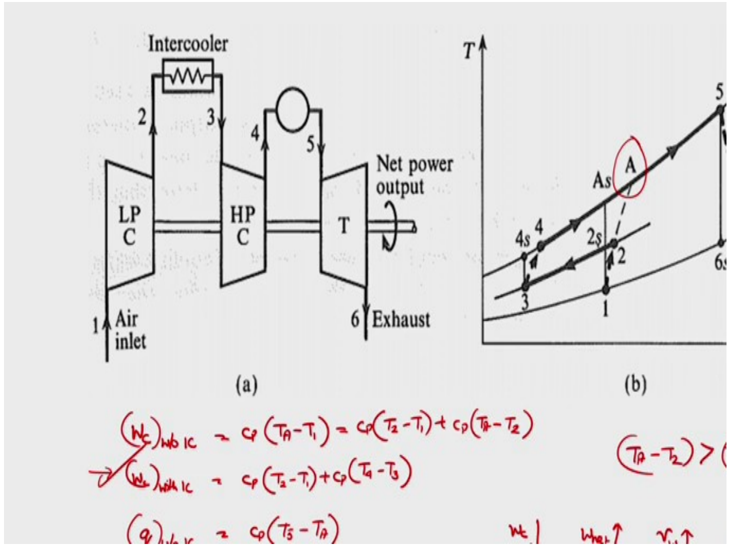

So, this is what we get when the system is having two stages of compression and an intercooler in between. First is an LP compression stage, then its temperature is lowered in the intercooler, then it goes to an HP compression stage and subsequently it goes to the combustion chamber. So, 1- 2 is in the *Ts* plane represents the 2s is the ideal scenario for LP compression practically it goes to point 2’, then it lowers to point 3. T_1 and T_3 has to be equal in their ideal scenario but practically T_3 may not be as low as T_1.

So, from there we have the second stage of compression where it is going from T_3 to T_4 because of the presence of irreversibilities. And then we have the heat addition process from point 4 to 5 and then the expansion in the turbine. Here the point A or A*s* rather represents if we are going for a single stage compression then in ideal scenario we would have would have ended at Point A*s* and in actual case would have ended at Point A.

So, if there is no intercooling, no multistaging then the compression would be from T_1 to T_A. But with multistaging the compression is first from 1 to 2 and then from 3 to 4. Then without intercooling total compression work requirement in reference to this diagram, when there is no intercooling then compression work requirement would be:

$$(w_c)_{without\ IC} = c_p(T_A - T_1)$$

and compression work requirement with intercooling in this case is

$$(w_c)_{with\ IC} = c_p(T_2 - T_1) + c_p(T_4 - T_3)$$

The first one without intercooling the work recommend if we write this way as:

$$(w_c)_{without\ IC} = c_p(T_2 - T_1) + c_p(T_A - T_2)$$

then the work requirement with intercooling can be lower compared to the work requirement without intercooling only when this

$$(T_A - T_2) > (T_4 - T_3)$$

and that is true because on *Ts* diagram these lines the constant pressure lies diverge from each other as we are moving towards right.

As we are moving to higher entropy these lines diverge away from each other and so $(T_A - T_2)$ always has to be greater than $(T_4 - T_3)$. And therefore this one always give you less work input requirement when the intercooling is present. However just check out what is the heat input requirement:

Without intercooling your heat input requirement:

$$(q)_{without\ IC} = c_p(T_5 - T_A)$$

However, when there is intercooling

$$(q)_{with\ IC} = c_p(T_5 - T_4)$$

So, this one definitely is much larger. Total heat input requirement definitely is much larger. And so the effect of intercooling is a reduction in the compression work and turbine part remains same so there is an increase in the net work output, subsequently we have an increase in the work ratio and a reduction in the back work ratio. However, for efficiency we cannot comment while the net work output is increasing heat input requirement is also increasing simultaneously.

In fact, if we can do some calculation can be shown that efficiency actually decreases however the work ratio that increases back work ratio decreases. So, intercooling helps us in increasing the work output but may not help from efficiency point of view.

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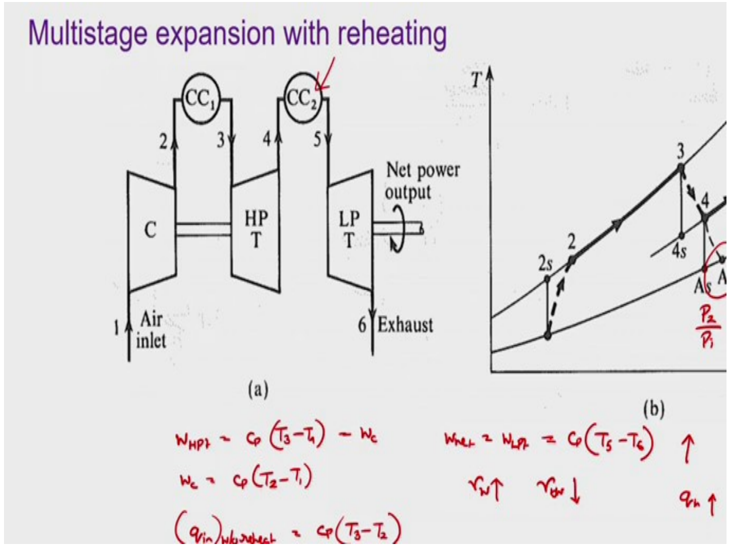

The opposite of intercooling is multi-stage expansion with reheating. Because of the same reason just to increase the specific volume, we can separate the turbines into 2 stages. Look at it is quite similar to the power turbine that you have shown here. The HP stage of expansion is mounted on the same shaft as the compressor but the LP stage is separate which is mounted or on the same shaft as the generator.

The difference with the power turbine is here we have a stage of reheating between the two stages of turbine. Look at the diagram 3 to 4 refers to the HP expansion stage and point 4 instead of continuing to point A we are going to point number 5, for this which is the reheating stage. So, this 4 to 5 is a reheating stage, 5 to 6 is the second stage of expansion and we are definitely getting higher work output from this.

Here again we can do the calculation to identify the optimum value of this intermediate pressure like if this is P_1 and this is P_2 and this is the intermediate pressure P_i. In this case also it can be shown that the maximum work output from the turbine we can get when this pressure ratios are equal that is:

$$\frac{P_2}{P_i} = \frac{P_i}{P_1}$$

Then we are going to get the maximum value of turbine work output. In this case the work produced by the HP turbine will be equal to:

$$w_{HPt} = c_p(T_3 - T_4)$$

and the compression work is equal to:

$$w_c = c_p(T_2 - T_1)$$

Ideally these two should be equal so that they can be mounted on the same shaft. So, your net work output will be equal to:

$$w_{net} = w_{LPt} = c_p(T_5 - T_6)$$

this one definitely gives an increase in the net work input requirement.

Subsequently the work ratio increases the back work ratio that also decreases. But look at total heat input requirement. Without reheat the total heat input requirement would be

$$(q)_{without\ reheat} = c_p(T_3 - T_2)$$

But heat input requirement when reheat is also present in the system

$$(q)_{with\ reheat} = c_p(T_3 - T_2) + c_p(T_5 - T_4)$$

first part of the above expression is still there because of the heat input in the combustion chamber, but there is also the reheating part there is a second combustion chamber where we also had to add the second part i.e.,

$$c_p(T_5 - T_4)$$

So, your q_{in} that is also increasing so again we are not sure about the efficiency. Commonly efficiency again decreases efficiency may increase efficiency may decrease depending upon the amount of reheat that has been provided. Ideally, we want T_3 and T_5 to be equal but practically if I may not reach up to T_3 it may be slightly lower and depending upon the amount of reheat we have provided efficiency may increase may decrease.

Generally, we have again and decreasing the efficiency. So, both intercooling and reheating can cause significant increase in the network output and work ratio, but heat input requirement also increases in both the cases and therefore the efficiency of the gas turbine system may suffer. Another interesting point to note in case of reheating is that if there is no reheating then the temperature at the end of expansion would have been T_A.

However, with reheating the temperature at the end of expansion is T_6. And T_6 definitely significantly greater than this T_A. So, the temperature of the exhaust gas is much larger with the reheating and we have to find a way of utilizing this high temperature. And that is what is done in the regeneration which allows us to use the exhaust heat and thereby reducing the total heat input requirement and cause an increase in the efficiency.

Regeneration we shall be discussing in the next lecture today we are keeping it up to intercooling and reheating. You please study this concept carefully and check also try to develop all these relations that we have done. For compression parts or for intercooling with multistage compression we have derived the optimum pressure ratio. You please try to do that for this multistage expansion with reheating also.

Assuming a perfect reheating that is T_3 equal to T_5 try to prove that the pressure ratio has to be equal in both cases you know to maximize is the turbine work output.

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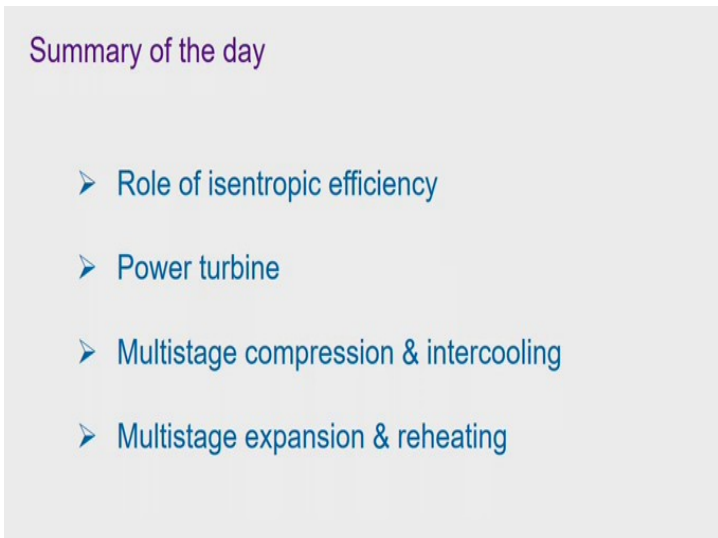

So, now we can wrap up the day, where we have discussed about the role of the isentropic efficiency through a numerical example. Then we talked about the concept of a power turbine that is having two separate turbine want to drive the compressor and other to produce the commercial power output. Then we have discussed about two possible modes of gas turbine, basic gas turbine cycle modification to increase the work output.

One is multistage compression with intercooling and other is multistage expansion with reheating. The third option which may not affect the work ratio that much, but definitely can increase the efficiency that which is regeneration that we shall be discussing in the next lecture, so till then you try to solve the problems that we have done here. And also try to develop the expressions or the mathematical derivations for multistage compression and multistage expansion on your own. Thank you.