Applied Thermodynamics for Engineers
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Lecture – 17
Ideal Brayton Cycle

Good morning friends welcome to week number 6 where we are going to talk about the gas turbine cycles. In the first three weeks of this course we have talked about some basic concepts and also we have prepared the backdrop for going to the details of applied thermodynamics or application of thermodynamics to practical systems by discussing topics like thermodynamic property relations or the ways of using thermodynamic tables for identifying the properties of pure substances.

In week number 4 we have discussed about the first applied concept of ours which is the gas power cycles or air standard cycles which are the cycles associated with reciprocating internal combustion engines. And in week number 5 that is in the previous week we have talked about some more realistic aspects that we should consider along with the air standard cycles. Now before we start the discussion on gas turbine cycles, I would like to take you back to the concept of heat engines.

Now we know that whenever you talk about engines, we are generally talking about an energy conversion device.

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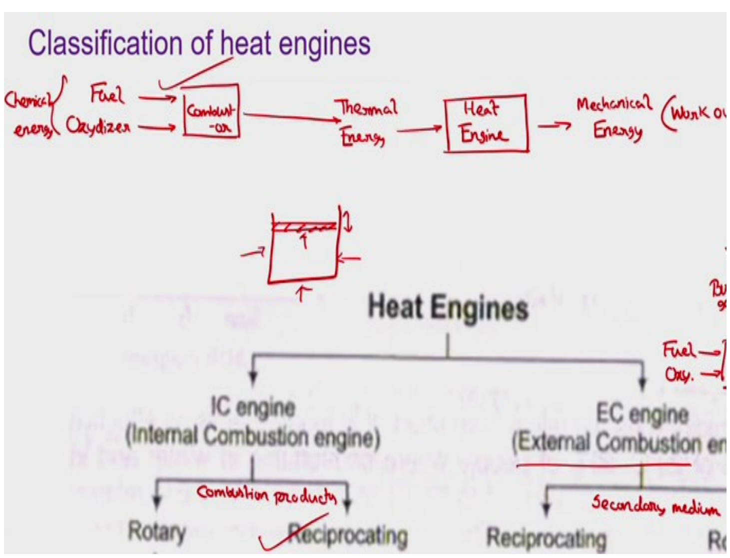

That is, if we just represent in a block sense, an engine always corresponds to the receiving energy of a certain form from some source and converting that to another form of energy. And generally, the term engine is primarily associated when the output side energy is in mechanical form, which can be a change in kinetic energy or potential energy or maybe in the form of some thrust or shaft movement, thereby giving you work output. So, for an Indian commonly our objective is to convert some form of energy to get some useful work output. And depending upon the nature of this input energy we generally can have different kinds of engines. Like when your input is an electrical energy, then we can easily call this to be an electrical engine; common example can be a motor. A motor can easily be considered to be an electrical engine where we are giving some electrical energy as an input and we are getting mechanical work as the output. And they are similarly instead of having electrical or some other form of energy whenever we are having thermal energy as the input, then we call them thermal engines or more commonly more popularly they are called heat engines.

So, heat engines are devices which takes thermal energy or heat energy as the input and converts this to work. And as from the second law of thermodynamics, we know that complete conversion of thermal energy to work output is not possible because heat is a lower grade concept type of energy compared to work. And therefore, only a portion or only a fraction of this input heat can be converted to work.

And we have already discussed about the concept of Carnot cycles etc, which are ideal heat engines providing the upper limit of this conversion of thermal energy to work. But the issue is that thermal energy apart from some certain sources, thermal energy is not freely available in nature. Like solar energy can be one source of thermal energy which is quite freely available but again corresponding temperature is quite low and we know that for Carnot cycle kind of heat engines or at least from the concept of Carnot cycle we can say that higher the temperature corresponding to the heat addition higher will be the efficiency of the heat engine. So, you always want to add energy to the heat engine from a higher temperature source. And from that point of view thermally solar energy rather is not the most ideal source because the effective temperature of the solar radiations can be quite low.

Therefore, we have to produce thermal energy from some other source and most common source for that is burning some kind of fuel. Commonly it can be hydrocarbon fuel, but it can be some other kind of fuel also. Or to be more general we can say that to have some kind of oxidation reaction or some kind of exothermic oxidation reaction has to take place. So, what we do? We have some kind of device where we supply a fuel and we supply some kind of oxidizer.

There goes the combustion reaction which we can call a combustor, and because of this combustion reaction or because of this exothermic chemical reaction, the chemical energy stored inside the fuel and oxidiser gets converted to thermal energy which we can subsequently use in the heat engine. So, most of the practical heating is actually are associated with two different steps of energy conversion.

They always take some fuel or chemical energy as the input. So, this fuel and oxidizer together this thing we can classify as or we can combine them to represent as a form of chemical energy. So, any heat engine commonly starts with a form of chemical energy through the combustion reaction it gets converted to thermal energy, and in the second step this thermal energy is converted to mechanical energy or work output.

So, it is not one that two steps of energy conversion involved in any heat engine. And exactly where this combustion reaction is going on depending upon that we can commonly classify heat engines into two categories as shown here. Heat engines can commonly be classified into an internal combustion engine and external combustion engine. Internal combustion engine refers where this combustion reaction is happening inside the engine itself.

Whereas external combustion engine refers to where the combustion is not taking place inside the engine rather combustion is taking place somewhere else, and by some means the thermal energy generated because the combustion is taken to your heat engine. Examples of internal combustion engines are automobile engines where you have one cylinder and you are directly supplying your fuel and oxidizer to the cylinder, combustion reactions happening inside the cylinder and also the cylinder is producing the work output. Whereas the external combustion engine examples can be steam engines or steam turbines where you have a boiler. In the boiler we burn the fuel and the thermal energy released because of this combustion reaction is first supplied to a steam or first applied to produce steam from water. Subsequently we take this steam to an engine or turbine where we get the work output.

So external combustion engines always have a separate combustor which is not required for internal combustion engines. Therefore, the internal combustion engines, for a given power output they are of much smaller size. But at the same time external combustion engines are more suitable for large-scale power generation. Now both internal combustion engines and external combustion engines can be classified into rotary and reciprocating.

Reciprocating means where we have just one device or like a cylinder in a in an IC engine and inside the cylinder, we supply the fuel and oxidizer, we have the combustion, we have the expansions etc, everything happening inside this because of the reciprocating motion of some piston. Because or we can say in more general way because of the reciprocating motion of the system boundary. Because the piston always forms a part of the system boundary.

Like if we think of an SI engine where you have the cylinder and now this is the piston. Now while the three walls of the cylinder like this one this, one this, one they are all fixed, this particular piston is not fixed that keeps on moving up and down and that from the fourth boundary of the system. So, the boundary of the system is flexible it is movable and as that keeps on moving following a reciprocating pattern, we have different kinds of operations done for a cycle.

Whereas in case of a rotary engine it is not one, rather it is a combination of multiple devices. So, reciprocating engines primarily corresponds to some kind of closed systems which of course allows the supply of fuel and air and also the exhaust of burned gases during some part of your operation, but during the other parts it acts like a closed system. Whereas rotary kind of this engines they are combination of multiple systems or multiple components.

Each of the components does one specific work and supplies that, or the output of that is taken to the subsequent one. Like suppose the combustion reaction takes place inside one, so whatever thermal energy is generated the burnt gaseous that is taken to another one where the work output is done. Then that exhaust coming out of that engine or the component producing work output that is taken to another where the exhaust or heat released or heat removal takes place.

This way different operation takes place in different components. If there are suppose four processes involved in a cycle then there will be about four components each of the component performing one of the cycles of the stroke. And they generally have a rotary kind of motion they have fixed boundaries, no moving boundary work involved, commonly they have shaft work which gives the name this rotary kind of thing.

Now the example of reciprocating as we have already discussed the gasoline engines or more commonly the spark ignition engines, we have talked about the Otto cycle which are ideal cycles for this spark ignition engine and diesel engines or more commonly the compression ignition engine. We have talked about the diesel cycle which is ideal cycle for diesel engines. They are examples of reciprocating engines and by now we know their working principle.

A rotary example can be the open cycle gas turbine which is the objective for this particular discussion. Wankel engine can be another example but we are not going to talk about that. If anyone is interested you can search on over net to get more idea about Wankel cycle, whose working principle is somewhat related to gas turbine because that is also a rotary kind of internal combustion engine.

Now all these examples that we have talked about a SI and CI engines or gas turbine or Wankel engines there the combustion takes place inside at least one of the components. Like in case of reciprocating that is there is only one component, in case of the rotary kinds of things there are several components and combustion takes place in one of them which primarily we call the combustor. And the products of combustion are taken to the next component where the word production is done.

So, the combustion gases they actually act as the working medium during the power producing process. So, we can say that for all these internal combustion engines your working medium at the time of power production is the combustion gases or combustion products. However, in case of external combustion engine, you have a separate combustor commonly called a boiler where we burn the fuel, produces a thermal energy and then we use the thermal energy to convert certain liquid to gaseous stage.

Common choice of liquid is water of course but we can use some other kind of liquids also. And subsequently that gaseous phase of that particular working fluid is taken to your work producing component, where we have the work output. Here also we can have both reciprocating and rotary type. Reciprocating examples can be steam engines which is virtually obsolete nowadays where we have the steam production and work output done in the same device.

Because the steam is supplied to a piston cylinder kind of arrangement which has a reciprocating motion. That is why, it is a reciprocating kind of thing. Stirling engine however is a more recent concept which acts on the Stirling cycle which we have discussed earlier. It is another example I think in case of Stirling engine I have mentioned about this external combustion part also.

Whereas the example of rotary kind of external combustion engines most common example is steam turbine. In case a steam turbine you have a boiler. In the boiler we supply fuel, we supply oxidizer and we also supply water. Some liquid that commonly is water. Now because of the combustion reactions whatever thermal energy is released that thermal energy is used to convert this water to steam.

And the products of combustion that also comes out separately. These burnt out gases are subsequently taken to the chimney of your plant from where that goes off the chimney or stack, where this steam is taken to the turbine where we have the work output taken. So, the combustion gaseous are not used as the working media, the combustion gases are going out whereas we are using a second medium to produce the real work output inside the turbine.

Same in case of a steam engines also. Closed cycle gas turbine, the other variation of the gas turbine that also works in the same way where we may not be using steam we may be using some other gas but there is a secondary medium present. So, here the working medium at the time of power production is the secondary media. So, the thermal energy released because of combustion or which is contained in the combustion products or burnt gaseous that is supplied or transferred to a second medium which acts as the working medium for the rest of the cycle.

That is the major difference between external combustion and internal combustion engines. Now we have already discussed about the reciprocating kind of internal combustion engine, we shall be talking about the steam turbines later on, in one of the later modules. Here we, Wankel engines or Stirling engines are not within our scope. Here we shall be discussing about gas turbines both closed cycle and open cycle. Because our cycle consideration is primarily in the form of a closed cycle though most of the gas turbines work in an open cycle form.

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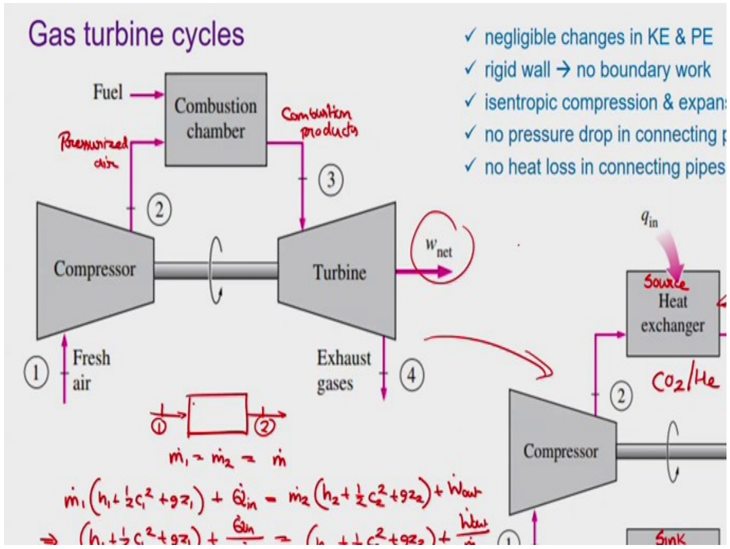

So, this is a typical open cycle kind of gas turbine where we are supplying fresh air in a compressor first. There this air is compressed to a much higher-pressure level and also temperature increases simultaneously. Now this high-pressure air is taken to the combustion chamber where you are also spraying the liquid fuel inside the combustion chamber. When combustion happens thermal energy is released and this high temperature combustion gases are subsequently taken to the turbine.

So, here in compressor your working medium is fresh air. In the combustion chamber you are receiving pressurized air and your fuel and from here we are having the combustion products that is coming out. The combustion products are coming out which are subsequently taken to the turbine and in the turbine, we are getting the real work output. Finally, the exhaust gases are released to, the may be there can be some downstream application or maybe to the surrounding.

This is the open cycle form of gas turbine whereas this is a closed cycle form, where instead of releasing the exhaust gasses to the turbine here we are supplying that to some heat exchanger. In fact, for as we have mentioned close cycle gas turbines work as more like external combustion engines where the there is no combustion chamber rather combustion chamber may be somewhere here where we are supplying the fuel and oxidiser.

The combustion is happening and whatever thermal energy is released that is being supplied to a heat exchanger. And inside the cycle we are having some other kind of working media. Common choice for working medium can be carbon dioxide, can be helium these are quite common choice for closed cycle gas turbine. So, this medium is first passed through a compressor where that gets compressed to higher pressure temperature level then it receives the heat of combustion in the heat exchanger.

Then it is taken to the turbine to keep the work output and finally there is a second heat exchanger a low temperature heat exchanger you can call this to be the sink also. This acts like the sink of your heat engine where the exhaust heat is released to certain cooling medium and then it is taken back to the compressor. So, whatever your working medium, carbon dioxide or helium or something else that is not going out of your system it is a closed system.

And there is no direct combustion so it is an external combustion kind of option. So, your open cycle gas turbine is an internal combustion device, where you have a combustor present and combustion gases are used as a working medium in the turbine. In case of a closed cycle gas turbine we do not have a combustor, rather we are using a second medium which is receiving the combustion heat or heat release during combustion in this heat exchanger.

But what is common then in between? The common is the three parts if we assume like in case of air standard cycles, we have assumed there is no combustion rather we are replacing the combustion process by a heat addition process and also there we have replaced the supply and exhaust process by a heat rejection process. If we put those air standard emissions on the open cycle then that turns to be a closed cycle. Is not it?

In case of an open cycle just think about the air standard assumptions again. We are not considering any combustion rather we are replacing the combustion by a heat addition process. So, the combustion chamber gives to this high temperature heat exchanger which can also be termed as the source. And the inlet and exhaust processes are eliminated and substituted by a low temperature heat exchanger which is allowing the heat to release.

So, that the system can go back to the initial process part that is at the beginning of compression. So, we have to apply the air standard assumptions on the gas turbine cycles then the open cycle can also be viewed like a closed system or closed cycle. And that is the way we are going to analyze the gas turbine cycle though it is very much possible and most of the practical gas turbines work in the open cycle mode. But we shall be considering them in more closed cycle mode and trying to get their performance parameters.

And I also have to add to that the gas turbine cycle that I am going to talk about here is also one kind of air standard cycles. So, the Otto, Diesel and dual combustion cycles that we have discussed is an addition to that. The name of this one is coming in the next slide. But before that just look at the closed cycle now. I am not going to talk any more about the open cycle gas turbine configuration rather I am focusing only on this closed cycle mode.

Here there are four components: compressor, high temperature heat exchanger source, turbine and the sink or low temperature heat exchanger. Now each of them is open systems because all of them are having mass input and output. And therefore, we have to analyze them by considering each of them as open system. And therefore, by solving the mass and energy conservation equation for each of these open systems and subsequently combining them.

For that quite a few considerations have to be taken. Firstly, we are going to neglect any changes in kinetic and potential energies of the working medium. Secondly, we are going to consider a rigid wall which is practically true. Because compressor and turbine both involve shaft work compressor receives shaft work as input and turbine gives work output in the form of shaft work. So, there is no rigid boundary or so rather there is a no moving boundary. All walls are rigid for all the four components and therefore no boundary work involved.

Isentropic compression and expansion are the assumption that we are going to put in. So, to start with, you are assuming that there are no frictional losses anywhere in the system and also the compression and expansion processes i.e., the compressor and turbine in there ideally adiabatic therefore giving them reversible adiabatic or frictionless adiabatic to be isentropic processes, isentropic compression and isentropic expansion.

No pressure-drop in the connecting pipes, the working medium that has to get transferred from one component to another. That is, from the source it has to go to the turbine from turbine it has to go to the sink. So, there will be certain kind of connecting pipe lines and as the fluid passes through the pipe lines because of the presence of friction, there can be some amount of pressure drop. But as we are neglecting any kind of friction and any kind of heat leakage. So, there is no pressure drop in the connecting pipes and no heat loss in the connecting pipes. These are important assumptions that we are adding along with the air standard assumptions. Now before I go to the cycle for a gas flow system gas turbine very briefly let us review how we analyze an open system. If we talk about an open system, like this which is receiving 1 as input state and 2 as output state, then if we understand state consideration if we apply the mass conservation on this then what we are going to get?

From mass conservation,

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

under steady state.

Now we apply the first law of thermodynamics for an open system on this then what we can write so,

$$\dot{m}_1\left(h_1 + \frac{1}{2}c_1^{\ 2} + gz_1\right) + \dot{Q}_{in} = \dot{m}_2\left(h_2 + \frac{1}{2}c_2^{\ 2} + gz_2\right) + \dot{W}_{out}$$

where as per our convention we are considering heat to be supplied to the system as positive and work being done by the system as positive.

Here and the state 1 is the input state, state 2 is the output state and so the corresponding energy of the flowing fluid stream is a combination of enthalpy, kinetic energy and potential energy. Now from mass conservation we know

$$\dot{m}_1 = \dot{m}_2$$

So, we can write as

$$\left(h_1 + \frac{1}{2}c_1^{\ 2} + gz_1\right) + \frac{\dot{Q}_{in}}{\dot{m}} = \left(h_2 + \frac{1}{2}c_2^{\ 2} + gz_2\right) + \frac{\dot{W}_{out}}{\dot{m}}$$

Now look at this assumption, we are neglecting any changes in kinetic and potential energies. The changes in kinetic and potential energies are negligible. So, we can almost say that c_1 and c_2 are nearly equal, z_1 and z_2 are equal and this at least for gas turbine cycle there is not a bad assumption because the magnitude of enthalpy can be 1000 times larger than the magnitude of kinetic and potential energy. Or I should say the changes in enthalpy can be several orders higher than the changes in kinetic and potential energies.

And it is a quite realistic assumption. So

$$(h_1 - h_2) + q_{in} = w_{out}$$

here

$$q_{in} \rightarrow \frac{\dot{Q}_{in}}{\dot{m}}$$

and

$$w_{out} \rightarrow \frac{\dot{W}_{out}}{\dot{m}}$$

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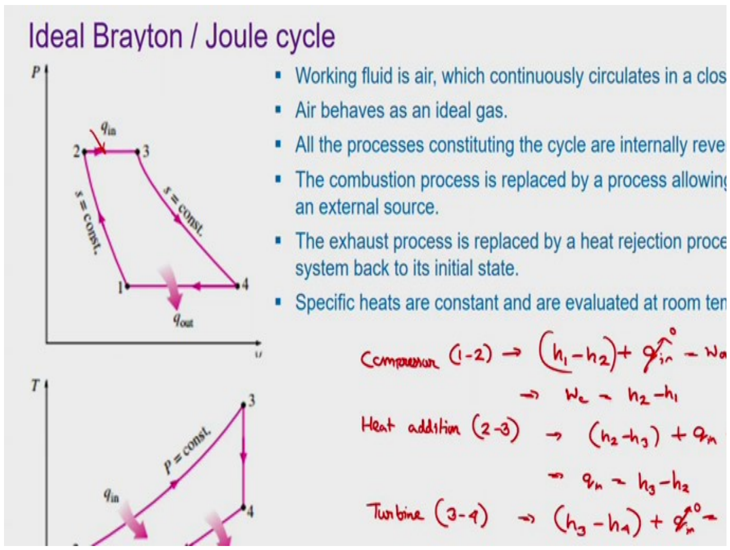

Now this is the ideal cycle that I am talking about which is called the Brayton cycle or Joule cycle. Brayton cycle is the most common name and therefore shall be sticking to that. Look at the processes there plot on *Pv* and *Ts* diagrams are shown. Here 1 to 2 is an isentropic compression process which is being done inside the compressor. Then 2 to 3 is a constant pressure heat addition process. Just going back to the previous one, let us try to get the cycles or nature of the processes from here.

So, 1 to 2 is a compression process and we have as even the compression to be happening at isentropically, so 1 to 2 is an isentropic compression process. 2 to 3 there is a heat exchanger, heat addition going on and there is no change in volume or I should say there is no change in volume of the component ,but as the gas is flowing through a heat exchanger there can be change in the volume on the gas because of changing its temperature. It is not like a piston cylinder kind of arrangement.

So, it is more realistic to assume this heat exchange to be happening at constant pressure. Now turbine, again it is isentropic expansion and finally the heat exchanger low temperature side again we can assume that to be heat released or heat rejection at constant pressure.

Because that is the most realistic way of assuming this, we have neglected any kind of pressure drop in connecting pipes and also inside the components. So, we can safely assume the heat exchange processes to be at constant pressure. And so we are getting this Brayton cycle. 1 to 2 is isentropic compression, 2 to 3 is constant pressure heat addition 3 to 4 isentropic expansion done in the turbine and 4 to 1 is heat rejection. Here $Q_{dot\ in}$ is being added to the system which is missing. Similarly, in *Ts* diagram, 1 to 2 is the isentropic compression process, so during which there is no change in entropy but temperature increases because it is a compression. 2 to 3 is heat addition process during which temperature entropy both increases. Then 3 to 4 is expansion process in the turbine and 4 to 1 the heat rejection.

To analyse this, we are going to just rehearse the air standard assumptions that we have assumed earlier. We are considering the working fluid to be air which contains the circulates in a closed loop which is very much valid for a closed type gas turbine cycle. We are assuming air as an ideal gas and that is the working medium so that is one serious assumption.

As we have already seen in case of fuel air cycle how that can affect the engine performance. So, that same is true for gas turbine also. But still for identifying the ideal performance we shall be sticking with air as the working medium. All the processes constituting the cycles are internally reversible. The combustion process is replaced by a process allowing heat addition from an external source as I have mentioned earlier.

Similarly, there is no exhaust it is being replaced by heat rejection process and we are assuming the specific heats to be constant. So, let us analyze each of the processes separately. So, first one, first component is compressor which is associated with this 1 to 2. So, if we use the earlier equation here we had:

$$(h_1 - h_2) + q_{in} = w_{out}$$

Now look at the compressor, it is isentropic so there is no q_{in} and there is no work output rather is work input. So, this w_{out} can be replaced by $-\ w_c$, where w_c refers to the compressor work and so the compressor work w_c can be written as:

$$w_c = h_2 - h_1$$

Next is the heat addition part which is 2 to 3, so for this part,

$$(h_2 - h_3) + q_{in} = w_{out}$$

during this process is happening at constant pressure there is no work interaction involved. So, w_{out} can be taken to be 0, q_{in} is there and q_{in} is the amount of heat added.

So

$$q_{in} = h_3 - h_2$$

Next is turbine which is facilitating the process 3 to 4. So similarly, for this turbine we can write it to be as we are writing as

$$(h_3 - h_4) + q_{in} = w_{out} = w_t$$

and in this case there is no heat transfer because it is isentropic and w_{out} is the turbine work which is our actual target. So, your w_t is

$$w_t = h_3 - h_4$$

and finally, is the heat rejection which corresponds to process 4 to 1 so

$$(h_4 - h_1) + q_{in} = w_{out}$$

again during the heat rejection process 4 to 1 there is no work transfer involved.

But there is heat rejection that is q instead of q_{in} there is $-q_{out}$. Here we can say

$$q_{in} = q_{out}$$

in this particular case. So, if you put that your

$$q_{out} = h_4 - h_1$$

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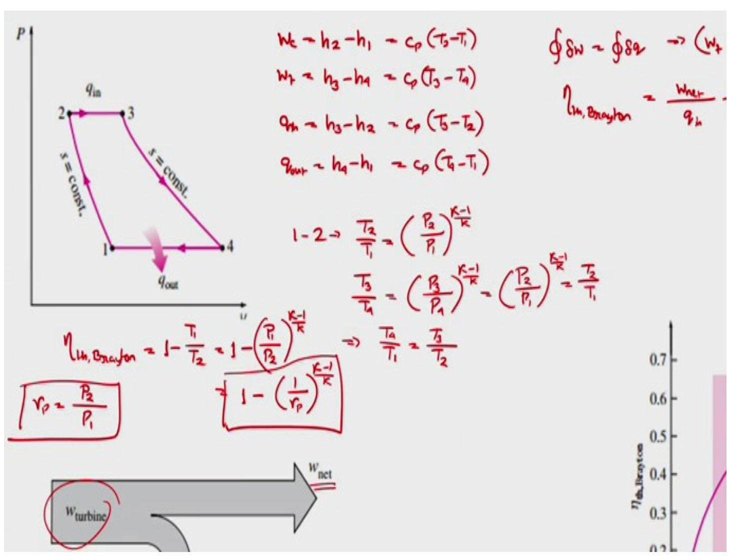

So, we are moving on with this so I am just summarizing whatever we have got. So, we have got the compressor work

$$w_c = h_2 - h_1$$

and the turbine work

$$w_t = h_3 - h_4$$

just check from the previous slide and similarly the heat input and heat output so we have

$$q_{in} = h_3 - h_2$$

$$q_{out} = h_4 - h_1$$

just check from here

$$q_{in} = h_2 - h_1$$

$$q_{out} = h_4 - h_1$$

Now we have assumed the working fluid to be here as air and air is assumed to be an ideal gas with constant specific heats. Then for an ideal gas what is the change in enthalpy? That can be directly equal to the change in temperature as

$$w_c = h_2 - h_1 = c_v(T_2 - T_1)$$

Similarly, this becomes

$$w_t = h_3 - h_4 = c_p(T_3 - T_4)$$

and this becomes

$$q_{in} = h_3 - h_2 = c_p(T_2 - T_3)$$

and this becomes

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

These are the four of interactions heat or work interactions involved in the system. And applying the first law of thermodynamics for a cycle we can write the:

$$\oint \delta w = \oint \delta q$$

that is total work interaction or net work output which is

$$w_t - w_c = q_{in} - q_{out}$$

So, what will be the efficiency for this cycle? The thermal efficiency for this Brayton cycle will be equal to:

$$\eta_{th,Brayton} = \frac{w_{net}}{q_{in}} = \frac{w_t - w_c}{q_{in}}$$

And if we use the first law then you can use that write this has to be equal to

$$= 1 - \frac{q_{out}}{q_{in}}$$

So, if you pull the expressions in terms of c_p then what you have q_{out} and q_{in} expressions you have so you can write this to be:

$$= 1 - \frac{T_1\left({T_4}/{T_1} - 1\right)}{T_2\left({T_3}/{T_2} - 1\right)}$$

Now look at the other two processes process 1 to 2 and 3 to 4 these are isentropic processes. So, for them isentropic processes involving some ideal gas so for them

$$Pv^k = constant$$

is valid. Therefore, for process 1 to 2 we can write that:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

that is true for any isentropic processes. Similarly,

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \frac{T_2}{T_1}$$

which allows us to now write that

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

So if we take it back to the expression for the thermal efficiency we have

$$\eta_{th,Brayton} = 1 - \frac{T_1}{T_2}$$

Now here it is important to know that or important note that, the network output can be significantly different from the work output that you get from the turbine. Because here the working medium is a gas and to compress the gas we generally have to use significant portion of the work produced by the turbine. And therefore, this compressor work can eat up a significant portion of the turbine work thereby giving a much lesser amount of network and this work is often referred to as a back work. So, the ratio of these two of compressor and turbine work is often refer to as the back work ratio.

So, the performance parameters now we are getting that as the thermal efficiency for a Brayton cycle is equal to

$$\eta_{th,Brayton} = 1 - \frac{T_1}{T_2}$$

and from the expressions of that we have got we can write this to be equal to

$$= 1 - \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} = 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

where

r_p is called the pressure ratio which is the pressure after combustion or rather pressure after compression by pressure before compression.

$$r_p = \frac{P_2}{P_1}$$

and this is the expression for your efficiency for a Brayton cycle:

$$= 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

it is quite similar to your compression ratio that we use in SI engines or CI engines. But as here the heat addition processes and heat rejection process at constant pressure. So, instead of working in volume ratio it is much more comfortable to work with the pressure ratio. That is why we are using this pressure ratio term here.

And we are getting

$$= 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

and the term back work is often related to the ratio of the compressor work by the turbine work is called the back work ratio. Quite often this backward ratio can be significant in case of compressors, in case of gas turbines as I mentioned it can even be greater than 50% that is whatever power your turbine is producing your compressor may eat up almost half of that. Therefore for a given power output we need to have a very large sized compressor, so that we can keep this effect of back work quite small.

And this is we are talking in terms of ideal cycle you will be solving some numerical problems later on where you can see the magnitude of this back work ratio. Even for ideal cycles we can have 40 to 50% loss in form of compressor work and even much more in case of a real cycle. So, if we plot the efficiency in terms of the pressure ratio, then this will look like just what we got as a efficiency variation each compression ratio for Otto and Diesel cycle, as pressure ratio increases on the Brayton thermal efficiency of a Brayton cycle, that also keeps on increasing.

And commonly we work in the range of r_p 11 to 16, because higher pressure we go of course we always want to have work at a higher pressure level, but higher pressure that we go there we increased pressure level to deal with which makes the design more complicated. So, to wrap up this particular slide the characteristic parameter that we use in case of a Brayton cycle or gas turbine that is called the r_p the pressure ratio which is pressure after compression by pressure before compression.

So, there is a pressure ratio associated with the compression process and as pressure increases during compression. So, this is also always greater than 1. Then the thermal efficiency for a Brayton cycle that is:

$$= 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

where k is the ratio of specific heats and the back work ratio is given as the ratio of compression work to the turbine work.

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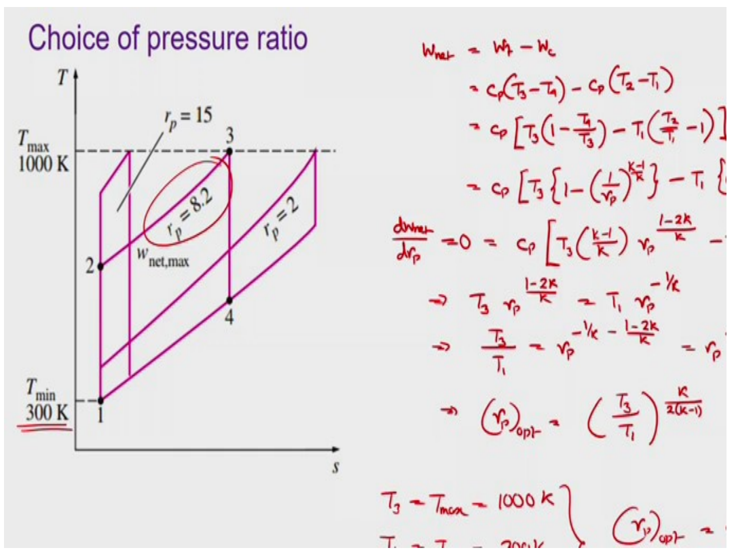

Now as I just mentioned that as the pressure issue increases the efficiency increases but still there are some practical limits of going to higher pressure ratios but there is another thing to be considered. Look at this here okay before going there I am just going back to the previous slide once. Here at which point you expect to have the smallest temperature? Of course, point number 1 there point before state before compression where the temperature is the lowest which is commonly close to the atmospheric temperature.

And which point you should have the highest temperature? Of course at point 3 which is the state at the end of combustion. So, and these two temperatures are quite often limited by the practical considerations because T_1 cannot be lower than atmospheric temperature. So, the lowest we can have is the atmospheric temperature. The T_2 or T_3 rather the maximum temperature the difference of a material consideration and therefore we cannot go beyond certain limits.

Now if we maintain T_{mean} and T_{max} i.e., T_1 and T_3 add some case values and vary our P then it changes like this like when your r_p is very small that is we are having a very small pressure rise then to there will be quite long heat addition process to reach to this highest temperature level and you can see only a small area is enclosed by this curve on a Ts plane so net heat transfer is quite small and therefore net work transfer also will be quite small.

Similarly, if we are talking about very high value of r_p then there is a significant amount of compressor work involved and heat addition is quite small during this process. So, total area enclosed by the curve again is quite small. But for san intermediate value of r_p you are getting much higher amount of work output. Therefore, for a given maximum and minimum temperatures there has to be some kind of optimum level of this pressure ratio.

And let us try to identify that optimum level here. Now we know that the net work output will be equal to the turbine work minus compressor work.

$$w_{net} = w_t - w_c = c_p(T_3 - T_4) - c_v(T_2 - T_1)$$

So, from there,

$$= c_p\left[T_3\left(1 - {T_4}/{T_3}\right) - T_1\left({T_2}/{T_1} - 1\right)\right]$$

Now I would like to take you back to the previous slide again look at this expressions that was developed here. And the expression is as follows:

$$= c_p\left[T_3\left\{1 - \frac{1}{r_p}^{\frac{k-1}{k}}\right\} - T_1\left\{(r_p)^{\frac{k-1}{k}} - 1\right\}\right]$$

So, this is your net work output. In order to maximize this work output for with r_p then what we have to do we have to differentiate this expression with respect to r_p,

$$\frac{dw_{net}}{dr_p} = 0 = c_p\left[T_3\left(\frac{k-1}{k}\right)r_p^{\frac{1-2k}{k}} - T_1\left(\frac{k-1}{k}\right)(r_p)^{\frac{k-1-k}{k}}\right]$$

So, if we take c_p and $k - 1/k$ outside then that goes off so we have:

$$T_3 r_p^{\frac{1-2k}{k}} = T_1 r_p^{-1/k}$$

So if you take all r_p on one side and rest on the other side. So, it becomes:

$$\frac{T_3}{T_1} = r_p^{-1/k - \frac{1-2k}{k}} = r_p^{-\frac{2-2k}{k}} = r_p^{\frac{2(k-1)}{k}}$$

So, the optimum value of r_p then becomes:

$$(r_p)_{opt} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

so that is the optimum value of r_p for a given maximum and minimum temperature. And like the example shown here you can try to find out like say when we have

$$T_3 = T_{max} = 1000\,K$$

$$T_1 = T_{min} = 300\,K$$

$$k = 1.4$$

which is for common value for air then your r_p optimum you can try to find out this it will be coming quite close to this value of 8.2.

So this way we can see that as r_p increases your efficiency increases, but that does not mean that work output is also increased because of work out becomes maximum and only for an optimum value of r_p once you are fixing that two temperatures. So, in general you have to go for some kind of compromise or some kind of trade-off between the efficiency and work output. Of course, for lesser power requirement, once we are having a very high value of r_p, we are getting less net work output from a cycle. And therefore if your plant needs to given power output we need much larger mass flow-rate we are going to large much larger size of compressor and turbine which is also not desirable.

So, primarily we go for maximizing the work output for this. So, what are the applications of gas turbines? There are two typical types of applications. One for electric power generation. Electric power generation again can be done in two ways: one where we are using the gas turbine as standalone systems and other where we are using gas turbine as a part of a cogeneration plant that is we are combining a gas turbine and a steam turbine.

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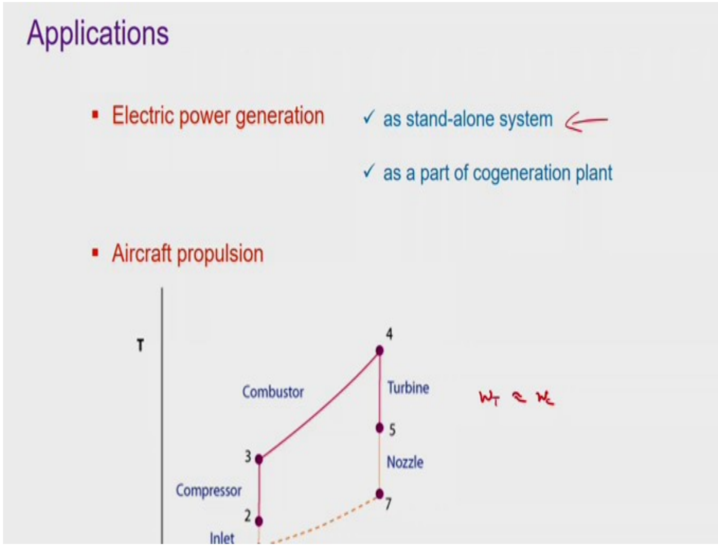

The exhaust from a gas turbine which is generally at a very high temperature that is used to raise steam to be used in the steam turbine. We shall be discussing about cogeneration also in later module. One problem with electrical power generation or using gas turbine for electric commercial power generation is that the fuel can be very costly.

So, it is not that widespread the use of stand-alone systems, quite often it is used in places where commercial fuels or common fossil fuels are not available or they are far away from locations like from Indian perspective I can mention about the places like Andaman and Nicobar islands or some remote places of Sunderbans where gas turbines can be feasible options for power generation because there are no other fuels available.

And second options aircraft propulsion. The aircraft runs on the jet propulsion cycle or aircraft propulsion cycles which are just modifications of the gas turbine cycles, something like this, where you have the compressor you have the combustor you have the turbine. But in the turbine we do not allow the gas to expand to the maximum level possible rather the turbine work is produced such that it is nearly equal to the compressor work.

Both turbine and compressor are always mounted on the same shaft. So, that the work produced by the turbine a part of that can be taken to the compressor can use to run the compressor. In case of aircraft propulsion, we allow the turbine to produce exactly the same amount of work which is able to run the compressor or maybe slightly higher amount of work because we also need to run the air conditioners inside the aircraft the electrical appliances etc.

So, whatever is the power required by the compressor plus some additional power for the electrical accessories that is what the turbine is allowed to produce. Then how the vehicle is running or how your aircraft is running? That is the hot gases that are coming out of the turbine that is allowed to pass through the nozzle, so that the it goes out at very high velocity and the thrust or the reverse thrust that is produces. Just think about Newton's third law of motion using that reverse thrust only the aircraft is able to move in the forward direction. If you are interested you can explore more on this aircraft propulsion cycles.

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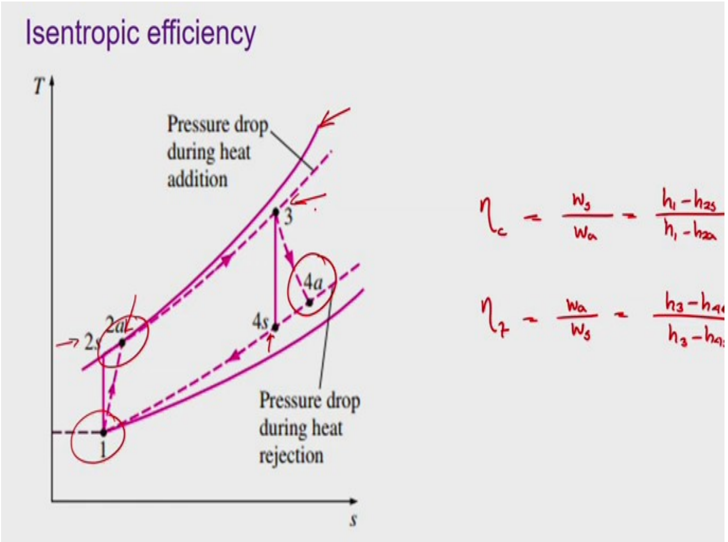

And before we close off, just a glimpse about what we have in practice. Ideal cycles or ideal Brayton cycle we have talked about, but practically the frictional losses can never be neglected and similarly the pressure drop while it flows to the heat exchangers. So, in a practical Brayton cycle our *Ts* diagram may look something like this, where 1 is the starting point at the beginning of compression.

Had it been an ideal cycle you should have got 2*s*, but because of the presence of irreversibilities primarily because of friction, we are reaching $2a$. So, we are actually losing a bit of work there. The effect of the presence of irreversibilities is that whatever work output or rather want input that we need to be given to the compressor we have to give higher work input. And accordingly, we define something known as the isentropic efficiency.

The isentropic efficiency for a compressor is defined as:

$$\eta_c = \frac{w_s}{w_a} = \frac{h_1 - h_{2s}}{h_1 - h_{2a}}$$

where

w_s is the ideal work requirement

w_a is the actual work requirement

Actual work requirement will always be higher because of the presence of irreversibilities so this can be related in terms of enthalpy as shown above. Similarly on the turbine side had it been isentropic expansion we should have reached point 4s but because of the irreversibilities we are not able to reach there we are reaching point 4a.

So, again the net work output from the turbine will be lower. Similarly, we define an isentropic efficiency for the turbine which is given as actual work output that we are getting from the turbine minus the work output that you should have got if it is a reversible expansion. So, it is:

$$\eta_t = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

both cases the definition of short such that the values are less than 100%. So, we have to understand that because of the presence of irreversibilities the work input required for the compressor is more than the ideal situation.

Similarly, because of the presence of irreversibility the work output that you are getting from the turbine is less than the actual scenario thereby we are losing in both account. And there also can be pressure drop present because of the heat addition in the heat exchanger. So, we are not able to follow this line rather while it starts from this particular point it deviates and reach to this particular portion there is a pressure drop between the continuous and these dotted lines the same applies on the other side as well.

So the effect of isentropic efficiency shall be concerned in the next lecture while solving a numerical problem. Let us end up today by solving a numerical problem involving an ideal Brayton cycle.

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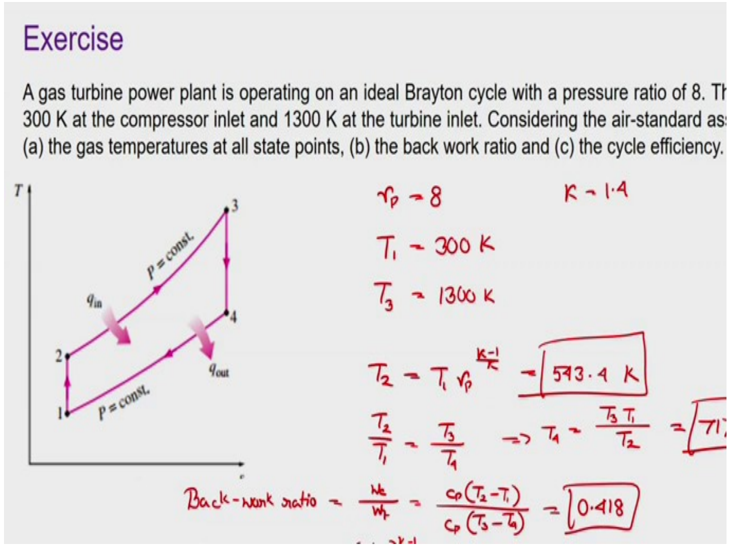

An ideal Brayton cycle is just this that is both compression and expansion processes are isentropic and there is no pressure drop anywhere in the cycle. So, just read the problem here the pressure ratio is 8 so we have

$$r_p = 8$$

$$Compressor\ inlet\ temperature, T_1 = 300\ K$$

$$Turbine\ inlet\ temperature, T_3 = 1300\ K$$

$$k = 1.4$$

So, you have to be considering the air standard assumptions you have to determine the gas temperatures at all state points two of them already given. So basically we have to found in T2 and T4, the back work ratio and the cycle efficiency. So, what will be the magnitudes we know that following the compression process:

$$T_2 = T_1 (r_p)^{\frac{k-1}{k}} = 543.4\ K$$

And as always I have pre calculated the numbers so it will becoming something like 543.4 K. Similarly T_3 and T_4 can be used to do that we can use the ratio or r_p value there as well or there is another way of calculating this because we have used earlier that:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

So, what we can do

$$T_4 = \frac{T_1 T_3}{T_4} = 717.6\ K$$

So if you put the numbers it will be coming as 717.6 K so is other two temperatures. You have to calculate the back work ratio. Back work ratio is equal to the compressor work by turbine work.

$$Back\ work\ ratio = \frac{w_c}{w_t} = \frac{c_p(T_2 - T_1)}{c_p(T_3 - T_4)} = 0.418$$

So, even in this ideal cycle also you can see 42% of work produced by turbine is being eaten by the compressor. And finally the thermal efficiency for this Brayton cycle, there are two ways you can do. One is if you remember the formula

$$\eta_{th,Brayton} = 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

that is one way or other way which I always suggest use

$$\eta_{th,Brayton} = 1 - \frac{q_{out}}{q_{in}}$$

$$\eta_{th,Brayton} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_2 - T_3)}$$

Again, you know all the four temperatures, c_p cancels out and if you put the values your efficiency will becoming something like 44.8 %.

So that is the thermal efficiency you can cross check that one by you calculating following expression as well. So, this is a very simple formula just to use simple numerical example where we are using basic principles to find the efficient and back work ratio for Brayton cycle.

Remember here we are not in a position to calculate the exact work output from this because for that we need more information. We need to know the mass flow rate, we need to know the specific heat also. We are just given with the information of *k* not *Cp*. So, we cannot calculate q_{in}, q_{out}, w_c or w_t but we can get their ratios which exactly what we have done here. So, that takes us to the end of today's lecture.

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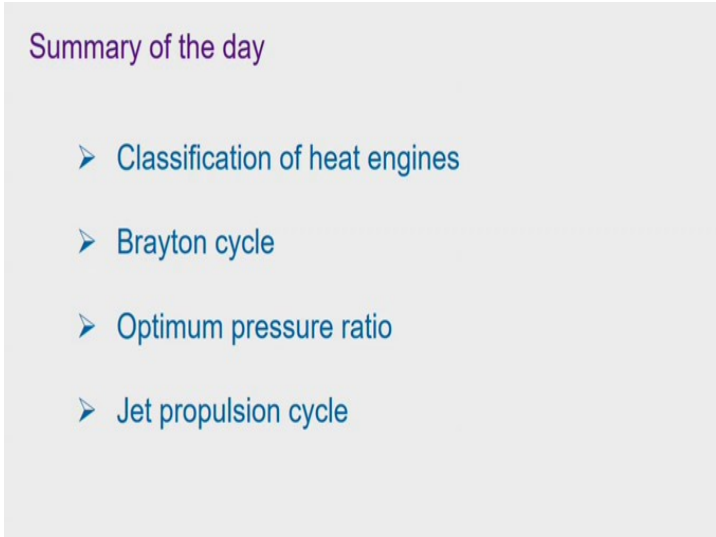

Just to summarize our day, we have discussed about the classification of heat engines to identify the difference between external combustion and internal combustion engines. Then we talked about the Brayton or Joule cycle, the ideal cycles, the concept of optimum pressure ratios for given maximum and minimum temperature for the cycle. Then we talked about the application of Brayton cycle in the form of jet propulsion cycle, where we curtail the turbine or power output just to satisfy the compressor input requirement. And finally we have talked about the isentropic efficiencies and accordingly the modification required in or practical modification that automatically comes into the ideal Brayton cycle. So, in the next lecture we shall be first solving the solving a numerical example using the concept of isentropic efficiency. And then we shall be moving on to identify the ways of improving the performance of the ideal Brayton cycle. Till then revise this lecture and if you have any query please write to me. Thank you.