Applied Thermodynamics for Engineers

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Lecture - 13
Stirling and Ericsson Cycles

Hello friends. Welcome for the third time in this week where we are talking about the air standard cycles or the so-called gas power cycles, which as per our definition utilizes a gas as a working medium and particular gaseous phase is maintained throughout because generally the working temperature levels or working temperature limits are well above the critical temperature of the corresponding substance.

Whereas practical internal combustion engines or automobile engines uses varieties of fuels, different kinds of hydrocarbons, blends of hydrocarbons, mixture of hydrocarbons and air as a working medium and also during the same cycles, you can have different kinds of working medium like when you are supplying the fuel we can have fresh mixture of fuel and air as the working medium.

Whereas after combustion, it is just burned gases, the combustion products. That makes it nearly impossible to have a simple thermodynamic analysis of those kind of systems. And therefore, we have assumed quite a few things, the most important assumption being the working medium is air, which is treated as an ideal gas. And we are also treating the cycle as a closed system. To facilitate that we have eliminated the inlet and exhaust processes and replaced that by a constant volume heat rejection process. Under this air standard cycles, we have discussed about three cycles so far, these are the three.

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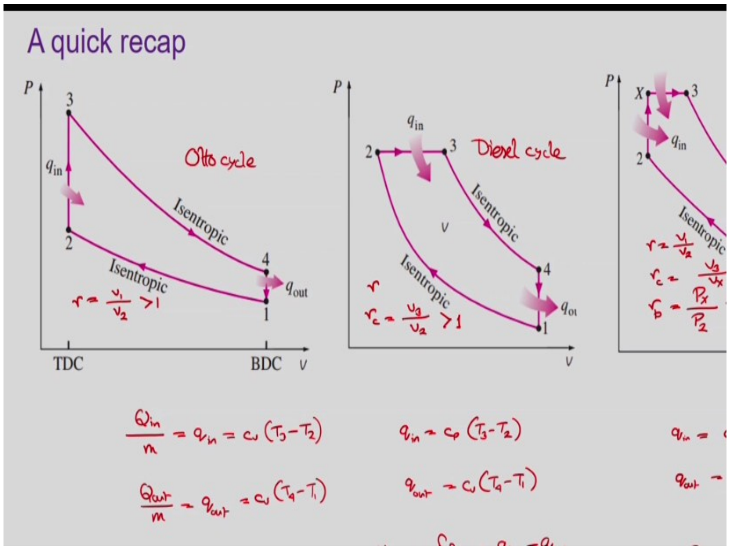

A quick recap

$\frac{Q_{in}}{m} = q_{in} = c_v(T_3 - T_2)$ $\quad q_{in} = c_p(T_3 - T_2)$ $\quad q_{in} =$

$\frac{Q_{out}}{m} = q_{out} = c_v(T_4 - T_1)$ $\quad q_{out} = c_v(T_4 - T_1)$ $\quad q_{out} =$

I am sure by now you are able to recognize them just from their *Pv* diagram or if I show the *Ts* diagram. Like the first one is having two isentropic processes and two constant volume processes. The heat addition being done during process 2-3 and heat rejection during process 4-1, both of these two are isochoric in nature that is volume remains constant, so this is the Otto cycle.

Whereas during the middle one, you have heat addition during a constant pressure process during 2-3 whereas heat rejection is again constant volume, so that is the Diesel cycle. So, if we just write the name, the first one is the Otto cycle, whereas the second one is the Diesel cycle. And the third one is some kind of combination of the two where the heat addition is partly done in constant volume and partly done in constant pressure.

This is while the Otto and Diesel cycles are very much idealization, Otto cycle is the idealization for SI engines, whereas Diesel cycle is the idealization of CI engines. The operation of this third one is much closer to the actual operation where the real combustion is done partly at constant volume and partly at constant pressure. This is called the dual cycle or dual combustion cycle or sometimes also called the mixed pressure cycle or mixed combustion cycles, or there is another name limited pressure cycle.

Like if we are doing it only, the heat addition only at constant volume then the pressure would have been much higher. As you are limiting the pressure to certain range, so sometimes it is also called limited pressure cycle. So, there are several names but we shall be sticking to this dual combustion cycle.

Now, in Otto cycle, the amount of heat addition say, if Q_H is a total amount of heat or $Q_{H\,dot}$ is a rate of heat addition to the system. If we divide that by mass that is sometimes represent by q_{in} that is called we generally remove this dot. So, that is in constant volume, so that is given by:

$$\frac{Q_{in}}{m} = q_{in} = c_v(T_3 - T_2)$$

like we have done. Similarly, the amount of heat rejection:

$$\frac{Q_{out}}{m} = q_{out} = c_v(T_4 - T_1)$$

Whereas during the Diesel cycle heat addition is at constant pressure, so here your q_{in} is:

$$q_{in} = c_p(T_3 - T_2)$$

whereas heat rejection, it remains at constant volume so it is:

$$q_{out} = c_v(T_4 - T_1)$$

And in a dual cycle which is an amalgamation. So, here the heat addition partly at constant volume and partly at constant pressure:

$$q_{in} = c_v(T_x - T_2) + c_p(T_3 - T_x)$$

But heat rejection here again at constant volume, so that remains the same:

$$q_{out} = c_v(T_4 - T_1)$$

Generally, whenever you are analyzing such a cycle, our objective is primarily to identify all these state points, may not be necessary always. But if we have information about all the state points particularly the temperatures and also the properties of the substance, properties of air that is C_p and C_v, then we can calculate everything.

Just like here we have written the expressions for all the heat interactions and once you know the interactions, you can calculate everything. Like if we want to know the net work transfer, net work transfer during the cycle or net work output will be:

$$W_{net} = \oint \delta q = q_{in} - q_{out}$$

Similarly, if you want to calculate efficiency, thermal efficiency that is:

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

$$= 1 - \frac{q_{out}}{q_{in}}$$

This way we can calculate all the parameters. Another parameter sometimes we need to consider, that is called the mean effective pressure. For mean effective pressure $P_{mep,}$ mep is subscript generally used that is called mean effective pressure.

Now the idea of mean effective pressure is whatever net work output your cycle is producing, as you know that during the expansion process both pressure and volume are varying. Now, if we can assume some intermediate pressure or some average hypothetical pressure value such that if we multiply that pressure with the stroke volume that is V_s then we are going to get the net work output. Here, we are writing everything in capital sense.

$$P_{mep} V_s = W_{net}$$

so if we divide both side by mass, then:

$$w_{net} = P_{mep} v_s$$

and referring to any of the diagram this is:

$$w_{net} = P_{mep} (v_2 - v_1)$$

So, the mean effective pressure is represented as:

$$P_{mep} = \frac{W_{net}}{V_s}$$

or in our notation:

$$= \frac{W_{net}}{v_2 - v_1}$$

And we also know that v_2 and v_1 are related by the compression ratio, so you can also characterize or convert this expression in terms of compression ratio. This way, we can calculate all the characterizing performance parameters required. To characterize the cycle operation, we have defined three different ratios like for the Otto cycle we are concerned about only one ratio which we are calling the compression ratio which is defined as just a recap.

$$Compression\,ratio = \frac{Volume\ before\ compression}{Volume\ after\ compression} = \frac{v_1}{v_2}$$

where

v_1 is specific volume before compression

v_2 is specific volume after compression

So, this is always greater than 1. For Diesel cycle, we need the compression ratio along with that we define a new parameter cutoff ratio (r_c) which is defined as the ratio of volume after heat addition to volume before heat addition.

$$r_c = \frac{v_3}{v_2}$$

So, volume after heat addition or specific volume after heat addition is v_3, volume before heat addition is v_2. So, because of heat addition, volume increases so this r_c is also greater than 1. And for the dual cycle, we need r, r_c and we also defined another parameter called r_p i.e., pressure ratio which corresponds to the constant volume heat addition part. It refers to:

$$r_p = \frac{P_x}{P_2}$$

where

P_x is the pressure after constant volume heat addition

P_2 is the pressure before constant volume heat addition

So, during constant volume heat addition process, pressure increases with temperature, so this is also greater than 1. So, all these three ratios are greater than 1. If we rewrite all of them for this dual combustion cycle, then r will be:

$$r = \frac{v_1}{v_2}$$

and what will be your r_c? similar to the Diesel cycle, here the r_c will correspond to the constant pressure heat addition part. So, it is:

$$r_c = \frac{v_3}{v_x}$$

v_3 is the volume after constant pressure heat addition

v_x is the volume before constant pressure heat addition

So, this way we can calculate all the three ratios and then we can use that to identify different state points. Now, we shall be doing three numerical examples each for one of the cycles. Like the first one for Otto cycle, second one corresponds to Diesel cycle and third one for the dual combustion cycle. These are example problems which will show you how to deal with them or how to tackle such kind of numerical situations.

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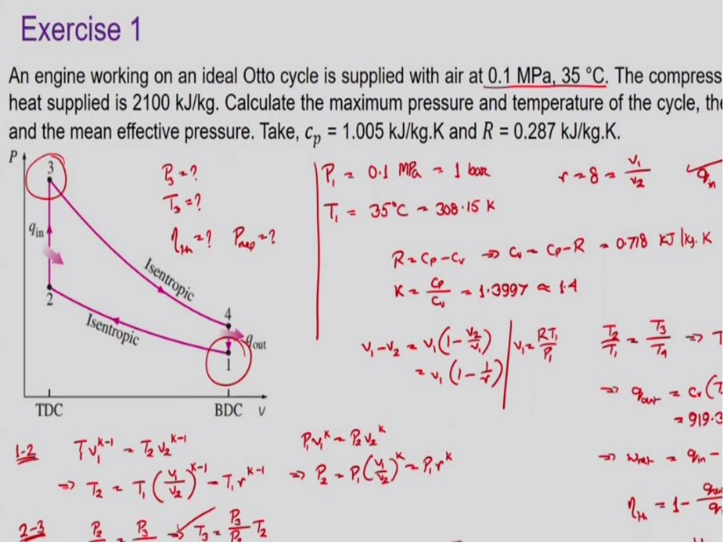

Exercise 1

An engine working on an ideal Otto cycle is supplied with air at 0.1 MPa, 35 °C. The compress
heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, th
and the mean effective pressure. Take, c_p = 1.005 kJ/kg.K and R = 0.287 kJ/kg.K.

$P_3 = ?$
$T_3 = ?$
$\eta_{th} = ?$ $P_{mep} = ?$

$P_1 = 0.1$ MPa $= 1$ bar
$T_1 = 35°C = 308.15$ K

$r = 8 = \frac{v_1}{v_2}$

$R = C_p - C_v \Rightarrow C_v = C_p - R = 0.718$ kJ/kg.K
$K = \frac{C_p}{C_v} = 1.3997 \approx 1.4$

$v_1 - v_2 = v_1\left(1 - \frac{v_2}{v_1}\right) = v_1\left(1 - \frac{1}{r}\right)$ | $v_1 = \frac{RT_1}{P_1}$

$\frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow T$

$\Rightarrow q_{out} = C_v(T$
$= 919.3$

$\Rightarrow w_{net} = q_{in} -$

$\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$

1-2 $T_1 v_1^{K-1} = T_2 v_2^{K-1}$
$\Rightarrow T_2 = T_1\left(\frac{v_1}{v_2}\right)^{K-1} = T_1 r^{K-1}$

$P_1 v_1^K = P_2 v_2^K$
$\Rightarrow P_2 = P_1\left(\frac{v_1}{v_2}\right)^K = P_1 r^K$

2-3 $\frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow T_3 = \frac{P_3}{P_2} T_2$

This is the first one; read the problem statement very carefully. Here, we are talking about an ideal Otto cycle. It is supplied with air at point 1 MPa and 35 ^{0}C, compression ratio is 8 and heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency and the mean effective pressure C_p and r values are given to us.

It is always better to have a cycle diagram whenever you are trying to solve a problem. So, I am showing the *Pv* diagram. I generally emphasize to draw both *Pv* and *Ts* diagram but it is up to you but it is always preferable to have at least one diagram, otherwise it becomes difficult to comprehend which point you are referring to during a particular representation. So, this is a standard cycle for an ideal Otto cycle with standard schematic representation on *Pv* plane.

So, let us now try to relate the given information with the state points. So, the engine is supplied with air at point 1 MPa and 35 ^{0}C. Then, this one corresponds to which state point, the supply point which is your point number 1. So, in this case, your

P_1 = 0.1 MPa = 1 bar

T_1 = 35 ^{0}C.

Now, it is always better to convert temperatures to Kelvin. Because we are going to use ideal gas equation of state which uses temperature in absolute sense. So, we are converting this to 308.15 Kelvin, its SI unit. So, this is for state point 1. What other informations are given? Compression ratio is given to be 8, so

$$r = 8 = \frac{v_1}{v_2}$$

q_{in} = 2100 kJ/ kg

So, heat addition is 2100 kJ/ kg and as the heat is being added at constant volume process. So, this one will be:

$$q_{in} = c_v(T_3 - T_2)$$

So c_v is something that we c_v need to know but here c_p and R are given, not c_v. So, what are the relations? We know

$$R = c_p - c_v$$

So

$$c_v = c_p - R = 0.718\ kJ/kgK$$

Another information that we need to know as we are going to do the isentropic processes that is *K,* which is the ratio of specific heat that is C_p/C_v. So, if you put the numbers *K* is going to come as 1.3997 almost 1.4.

So, we have this information given to us and now we have to identify the maximum pressure and temperature of the cycle and the efficiency of the cycle and also the mean effective pressure. Now, which point corresponds to the maximum pressure and temperature, this is point number 3, so basically you have to calculate P_3 and T_3. So, our objective is to identify

P_3 = ?

T_3 = ?

We have to calculate the thermal efficiency of this cycle.

And you also have to calculate the mean effective pressure of the cycle. Let us try to identify all the state points of the cycle. First, we take process 1-2, for process 1-2, it is being an isentropic process. We know that:

$$T_1 v_1^{k-1} = T_2 v_2^{k-1}$$

From there:

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1} = T_1 r^{k-1}$$

So, you can put the expression for T_1 and r here and do the calculation but probably that is not required. Like if we go to process 2-3, 2-3 is a constant volume process, so for this constant volume process, we can write using the ideal gas equation of state, we have:

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

that is:

$$T_3 = \frac{P_3}{P_2} T_2$$

Now, how to get the value of P2, that we also should get during process 1 to 2. During 1 to 2, we know that:

$$Pv^k = constant$$

So

$$P_1 v_1^k = P_2 v_2^k$$

that means,

$$P_2 = P_1 \left(\frac{v_1}{v_2}\right)^k = P_1 r^k$$

So, from there we can get the values of P_2 and T_2 though we are not directly calculating. Now, we are going to make use of this information. It is given that

$$c_v(T_3 - T_2) = 2100\ kJ/kg$$

So, if we separate them out, then

$$T_3 = T_2 + \frac{q_{in}}{c_v}$$

$$= T_1 r^{k-1} + \frac{q_{in}}{c_v}$$

So, this is going to give you the expression for T_3. So, if you put this, then you are going to get T_3 to be equal to I have pre-calculated the number for you, your T_3 = 3632.29 K, an extremely high temperature that we are talking about that is your first answer the maximum temperature of the cycle.

And how to get the maximum pressure? That relation is here from where we can write:

$$P_3 = \frac{T_3}{T_2} P_2$$

that is

$$= \frac{T_3}{T_2} P_1 r^k$$

and T_2 can be replaced with the help of T_1. So, if you put it there, then you are going to get this:

$$= P_1\left(\frac{T_3}{T_1}\right)r = 9.43\ MPa$$

so that is the second answer that we are looking for.

So, I have got the maximum pressure and temperature of this particular cycle. Now, we have to get the efficiency. To get the efficiency, we need to know the amount of heat rejection. And to know the amount of heat rejection, we need to get the temperature at point number 4. Now for an Otto cycle, we know that:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

So, if you check this,

$$T_4 = T_3\frac{T_1}{T_2} = 1588.63\ K$$

So, that gives you:

$$q_{out} = c_v(T_4 - T_1)$$

so, the amount of heat rejection, if you calculate this way that is coming as:

= 919.382 kJ/kg

Please calculate all these numbers because that will be a good exercise for you. Then,

$$w_{net} = q_{in} - q_{out}$$

$$= 1180.618\ kJ/kg$$

If we want to calculate efficiency, actually this W_{net} was not frequent for efficiency calculation. Because efficiency you can directly get as:

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = \frac{w_{net}}{q_{in}} = 56.22\ \%$$

So, 0.5622 or 56.22% but we calculated work output for two reasons; one is it may be desirable to calculate the work output from the cycle, per unit mass of air involved or the other thing which is exclusively mentioned here that is we have to calculate the mean effective pressure.

Now, mean effective pressure is:

$$P_{mep} = \frac{W_{net}}{V_s}$$

$$= \frac{W_{net}}{v_1 - v_2}$$

Now how to get $v_1 - v_2$? $v_1 - v_2$ can be calculated as now:

$$v_1 - v_2 = v_1\left(1 - \frac{v_2}{v_1}\right) = v_1\left(1 - \frac{1}{r}\right)$$

but still we do not know v_1. How to get v_1? Remember, it is an ideal gas that you are dealing with. Then, using ideal gas equation of state, we can always write:

$$v_1 = \frac{RT_1}{P_1}$$

So if you put everything here, then it is coming as:

$$P_{mep} = \frac{W_{net}}{v_1 - v_2} = \frac{W_{net}}{\frac{RT_1}{P_1}\left(1 - \frac{1}{r}\right)} = 1.525\ MPa$$

That is instead of having such amount of pressure variation during the expansion process, if we can deal with a process during which pressure remains constant at this particular value and you have the same amount of expansion i.e., from the top dead center to the bottom dead centre we are going to get the same work output.

So, this way we can analyze an Otto cycle. You have seen that here something was not required but still we have evaluated all the points, pressure at point 4, we have not calculated but again 3-4 is an isentropic process during which Pv^k remains constant. So, using that idea you can also calculate P_4. So, we have evaluated all the state points.

We have calculated the heat released by the cycle, we have calculated the work output, net work output that is and also we have calculated two performance related parameters, the thermal efficiency and mean effective pressure.

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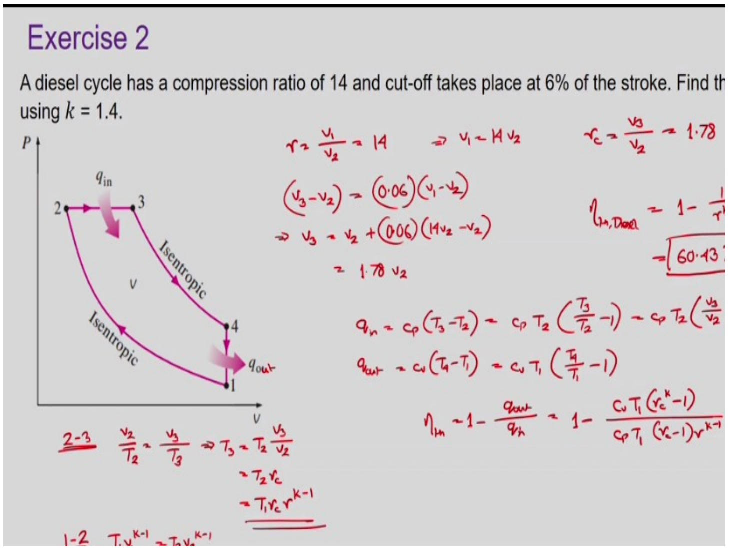

Exercise 2

A diesel cycle has a compression ratio of 14 and cut-off takes place at 6% of the stroke. Find th

using k = 1.4.

$r = \frac{v_1}{v_2} = 14 \Rightarrow v_1 = 14 v_2$

$r_c = \frac{v_3}{v_2} = 1.78$

$(v_3 - v_2) = (0.06)(v_1 - v_2)$

$\Rightarrow v_3 = v_2 + (0.06)(14 v_2 - v_2)$

$= 1.78 v_2$

$\eta_{th,Diesel} = 1 - \frac{1}{r}$

$= \boxed{60.43}$

$q_{in} = c_p(T_3 - T_2) = c_p T_2 \left(\frac{T_3}{T_2} - 1\right) = c_p T_2 \left(\frac{v_3}{v_2}\right.$

$q_{out} = c_v(T_4 - T_1) = c_v T_1 \left(\frac{T_4}{T_1} - 1\right)$

$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_v T_1 (r_c^k - 1)}{c_p T_1 (r_c - 1) r^{k-1}}$

2-3 $\frac{v_2}{T_2} = \frac{v_3}{T_3} \Rightarrow T_3 = T_2 \frac{v_3}{v_2}$

$= T_2 r_c$

$= T_1 r_c r^{k-1}$

1-2 $T_1 v_1^{k-1} = T_2 v_2^{k-1}$

Now, we are going to solve another problem involving the Diesel cycle. The statement is quite small, very small. It has a compression ratio of 14 and cut-off takes place at 6 % of the stroke. Find the cycle efficiency using k=1.4. No information is given about the initial states and others. Only thing is the compression ratio and peculiar information cut-off takes place at 6 % of the stroke.

What does that mean? Now, this is a cycle that we have. So, compression ratio is given:

$$r = \frac{v_1}{v_2} = 14$$

I would like to compare with the previous cycle. Here, you have seen that the compression ratio value was 8, which is quite logical for practical Otto cycles. But in this particular case, the compression ratio value is 14 which can even be lower for typical CI engines, they can go to much higher compression ratio level.

Now, r is given as 14 and it is given that cut-off takes place at 6% of the stroke. Now what does that mean? That means that the change in the volume of the system during the constant pressure heating process or heat addition process is 6 % of the total stroke volume or I mean the change in the volume of the system during constant pressure heat addition. How much is the change in the volume?

$$v_3 - v_2$$

This is the change in the volume during constant pressure heat addition process. This is:

$$(v_3 - v_2) = (0.06)(v_1 - v_2)$$

So, this is the information that is given to us. If we use the compression ratio, from there we can say that

$$v_1 = 14\, v_2$$

So, if we put it there, then:

$$v_3 = v_2 + (0.06)\,(14v_2 - v_2)$$

So, if you combine this, we are going to get as

$$= 1.78\, v_2$$

So what is your cut-off r_c? r_c is defined as the ratio of volume after heat addition to volume before heat addition. So, we are getting the cut-off ratio also using this information. Cut-off takes place at 6 % of the stroke. Be very clear what you are meaning here, the change in the volume of the system during the constant pressure heat addition process occupies 6 % of the total stroke volume.

That is what we are using. Now, we have to identify the cycle efficiency. There are two ways of approaching this. One approach is if you remember the mathematical relation, then you have *r,* you have r_c, you have *K*, you can directly put it. Because the thermal efficiency for an ideal Diesel cycle is given as:

$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c{}^{k-1}}{k(r_c - 1)}\right]$$

This is the expression *r,* r_c*, K* all are available.

You can directly put here, you can get it. And the values going to be coming as

$$= 60.43\ \%.$$

This is the desired answer but as I have suggested earlier, it is always risky to try to remember the formula and calculate the answers from there. If you can remember that is well and good. That will save some time also in certain situations like here but there is every chance that you can make a mistake in any of the terms that you are writing there.

In that case, there is an alternate way of finding this. Now, what is alternate way? We have to go through the definition of efficiency and as per the definition, it is:

$$\eta = 1 - \frac{q_{out}}{q_{in}}$$

So, we have to get q_{out} and q_{in}. Here, this has to be q_{out}. So, q_{in} is it is at constant pressure, so

$$q_{in} = c_p(T_3 - T_2) = c_p T_2 \left(\frac{T_3}{T_2} - 1\right)$$

and there is a constant pressure process.

So, for a constant pressure process, we know that during the process to 2-3 which is a constant pressure process. So,

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

that is what we are going to use here. So,

$$c_p T_2 \left(\frac{v_3}{v_2} - 1\right) = c_p T_2 (r_c - 1)$$

T_2 is not known, C_p is also not known, only information given is K.

Of course, using $r = 0.287$ kJ/kgK, you can calculate C_p by combining with k but that is not required here. Let us see what we are doing now. For

$$q_{out} = c_v(T_4 - T_1)$$

and nothing given about this, so I have to get something about T_4 and T_1. If we take say T_1 common from there okay we shall be doing that shortly. Like if we take:

$$= c_v T_1 \left(\frac{T_4}{T_1} - 1\right)$$

So, something we have to do for this. For that, we have to analyze the process 2-3 again. For process 2-3, we have:

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

which gives this as:

$$T_3 = T_2 \frac{v_3}{v_2}$$

that is

$$T_3 = T_2 r_c$$

and if we go one step back that is process 1- 2, for process 1- 2 we know that:

$$T_1 v_1^{k-1} = T_2 v_2^{k-1}$$

from where we can write:

$$T_2 = T_1 r^{k-1}$$

So, if you put it there, here it becomes:

$$T_3 = T_1 r_c \ r^{k-1}$$

which we shall be using shortly.

And now come to the last process or the third process rather 3-4, 3-4 is again another isentropic process. So, for isentropic process between point 3 and 4, we have:

$$T_3 v_3^{k-1} = T_4 v_4^{k-1}$$

combining this, we can write

$$\left(\frac{v_3}{v_4}\right)^{k-1} can\ be\ written\ as\ \left(\frac{v_3}{v_2}\frac{v_2}{v_1}\right)^{k-1}$$

and 4 to 1 is a constant volume process. So, in that case this v_4 can be substituted by v_1 .so it becomes:

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{v_3}{v_2}\frac{v_2}{v_1}\right)^{k-1} = T_3\left(\frac{r_c}{r}\right)^{k-1}$$

So, if we do that replacement it becomes:

$$T_4 = T_1 r_c^k$$

So, these things we are going to take it back now to get the expression for efficiency. So,

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} == 1 - \frac{c_v T_1 ({r_c}^k - 1)}{c_p T_1 (r_c - 1) r^{k-1}} = 1 - \frac{1}{r^{k-1}}\left[\frac{({r_c}^k - 1)}{K(r_c - 1)}\right] = 60.43\ \%$$

So, see we have developed the expression for Diesel cycle efficiency once more which we actually have done in the previous lecture itself.

Here, for solving this problem again we have developed the same expression to show that with such minimum information available how we can calculate the efficiency when we do not remember the mathematical relation.

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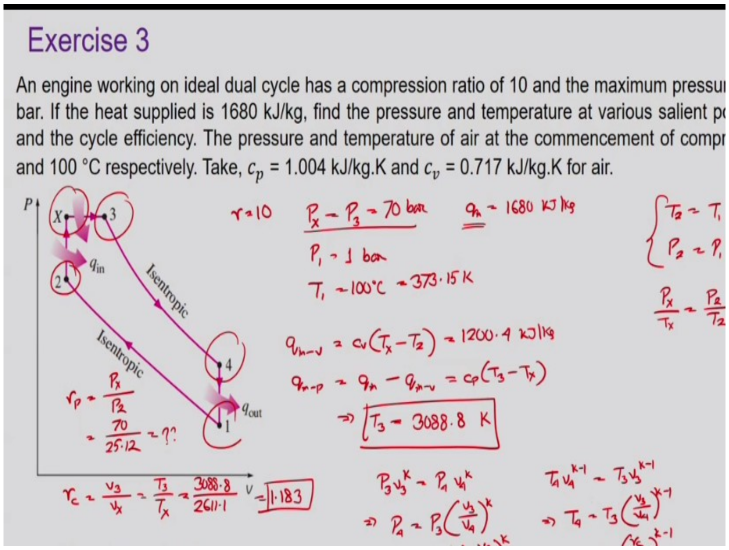

Exercise 3

An engine working on ideal dual cycle has a compression ratio of 10 and the maximum pressu
bar. If the heat supplied is 1680 kJ/kg, find the pressure and temperature at various salient p
and the cycle efficiency. The pressure and temperature of air at the commencement of compr
and 100 °C respectively. Take, c_p = 1.004 kJ/kg.K and c_v = 0.717 kJ/kg.K for air.

$r = 10$ $\underline{P_x = P_3 = 70 \text{ bar}}$ $q_{in} = 1680$ kJ/kg

$P_1 = 1$ bar

$T_1 = 100°C = 373.15$ K

$T_2 = T_1$

$P_2 = P_1$

$\frac{P_x}{T_x} = \frac{P_2}{T_2}$

$q_{in-v} = c_v(T_x - T_2) = 1200.4$ kJ/kg

$q_{in-p} = q_{in} - q_{in-v} = c_p(T_3 - T_x)$

$\Rightarrow \boxed{T_3 = 3088.8 \text{ K}}$

$r_p = \frac{P_x}{P_2} = \frac{70}{25.12} = ??$

$r_c = \frac{V_3}{V_x} = \frac{T_3}{T_x} = \frac{3088.8}{2611.1} = \boxed{1.183}$

$P_3 V_3^k = P_4 V_4^k$

$\Rightarrow P_4 = P_3\left(\frac{V_3}{V_4}\right)^k$

$T_4 V_4^{k-1} = T_3 V_3^{k-1}$

$\Rightarrow T_4 = T_3\left(\frac{V_3}{V_4}\right)^{k-1}$

Third problem now involving the ideal dual cycle. It is a quite lengthy expression. Please read problem statement carefully. There is a diagram here; I would also like to do the solution quite quickly. So, here the information is given are:

compression ratio $r = 10$

maximum pressure is limited to 70 bar. What is your maximum pressure? That is

$P_x = P_3 = 70$ bar.

$q_{in} = 1680$ kJ/kg

But we have to remember this heat has been supplied in two parts; one during 2-x which is constant volume, other during x-3 at constant pressure. C_p and C_v are given and also okay the pressure and temperature of air at the commencement of compression are 1 bar and 100 degree Celsius respectively. So, you have

$P_1 = 1$ bar

$T_1 = 100\ ^0\text{C} = 373.15$ K

So, we have to identify all the salient points of the cycle and the efficiency of the cycle. Of course, if we can calculate all the points, all the state points we can identify, then we can easily identify or calculate terms like work output or mean effective pressure or efficiency and also all the heat interactions. We have to so far so we have to identify all the salient points. One is the point which is completely given. Partial information is given about x and 3, let us try to get 2 first.

Compression ratio is given, so straightforward your T2 will be:

$$T_2 = T_1 r^{k-1}$$

$$P_2 = P_1 r$$

So again, I have pre-calculated the number, your

T_2 = 936.9 Kelvin

P_2 = 25.12 bar.

This is point number 2 so this is now available. For point number 3, the pressure is given, that information we can make use of because 2-3 is a constant volume process.

So, during that particular process, you have:

$$\frac{P_x}{T_x} = \frac{P_2}{T_2}$$

Therefore,

$$T_x = T_2 \frac{P_x}{P_2}$$

$$= 2611.1\ K$$

T_2 is your temperature and pressure for point number 2, you have got temperature at point number *x,* pressure already given here, so this is also known. P_3 is pressure at 70 bar. Now, we have to identify point number 3 for which pressure is given but temperature is not known and for that we shall be making use of this information. Now, as we know point *x* and 2 are rather the temperatures at *x* and 2, we can calculate how much it has been added during this constant volume part.

$$q_{in-v} = c_v(T_x - T_2) = 1200.4\ kJ/kg$$

you can calculate this C_v is given T_x and T_2 you have calculated. That means the amount of heat added during the constant pressure part is total amount of heat added-heat added during the constant volume part and this one should be equal to what?

This is the heat addition at constant pressure, so this should be equal to:

$$q_{in-p} = c_p(T_3 - T_x)$$

So, if you put the numbers, you are going to get:

$$T_3 = 3088.8\ \mathrm{K}$$

So, you now know this particular point, this is the point which corresponds to largest temperature of the cycle but this is not the only point corresponding to the largest pressure because *x* to 3 this entire part corresponds to the highest pressure.

But 3 is the point corresponding to the largest pressure, it also marks the end of the heat addition process. So, 4 is the only one that is left out. So, we have to calculate 4. Before that we can calculate the ratios if required. Like if we want to calculate say the pressure ratio rp, what will be your r_p?

$$r_p = \frac{P_x}{P_2}$$

so P_x is given to be 70 bar and P_2 is 25.12 bar. So, you can put 70/25.12, you can get the number, I do not have the number with me. Similarly, if we have r_c if you want to calculate, r_c is the volume ratio which is:

$$r_c = \frac{v_3}{v_x}$$

but we do not have any information about the volume, then how to get it? You have to remember that x to 3 is a constant pressure process, so for a constant pressure process using the ideal gas equation of state we can write:

$$\frac{v_3}{T_3} = \frac{v_x}{T_x}$$

that is

$$\frac{v_3}{v_x} = \frac{T_3}{T_x}$$

So, rc that is the cut-off ratio is:

$$\frac{v_3}{v_x} = \frac{T_3}{T_x} = r_c$$

and how much we have got for T_3, 3088.8 K and T_x was 2611.1 K. So, whatever you are getting, that number is going to be your *rc* which here 1.183. Somehow, I forget to note the expression for the r_p or rather the value for r_p this one you can calculate. So, once you know r_c then for process 3-4, you can calculate using the isentropic equations.

Because for an isentropic process, we know that:

$$P_3 v_3^k = P_4 v_4^k$$

i.e.,

$$P_4 = P_3 \left(\frac{v_3}{v_4}\right)^k$$

If you put $v_4 = v_1$, then that becomes

$$\left(\frac{v_3}{v_2}\times\frac{v_2}{v_1}\right)^k$$

or whatever we can calculate is now what is your v_3/v_2, v_2 and v_x are equal so in that case we can write this as vx. So, this becomes v_3/v_x is your r_c, v_2/v_1 is $1/r^k$ that is going to be giving your P_4.

So, have I got the number for your P_4, probably not, something again I have missed? Similarly, you calculate the temperature at this point, your:

$$T_4 v_4^{k-1} = T_3 v_3^{k-1}$$

Similarly, this going to have T_4 to be

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{r_c}{r}\right)^{k-1} = 1314.4\ K$$

So, this is again another information I have missed.

So, once you have this, you know point number 4 as well. Then, your total amount of heat rejection:

$$q_{out} = c_v(T_4 - T_1)$$

So, from there you will be getting your q_{out} and efficiency will be:

$$\eta = 1 - \frac{q_{out}}{q_{in}}$$

In this particular problem, it is coming to be 59.82% is the cycle efficiency and you can calculate any other number this way. So, these problems are a bit lengthy but not very difficult once you follow the first principles, you can easily follow the flow of the process and identify all the possible state points.

So, now we have discussed about the three gas power cycles, the most common gas power cycles, Otto cycle, Diesel cycle and the dual combustion cycle. Now, one issue is all three of them are their ideal cycles but not similar to the Carnot cycle, because as we have discussed in our first lecture, all these three cycles are internally reversible but not externally reversible.

Because they can allow heat transfer with finite temperature difference between system and surrounding and therefore allowing the process of external irreversibility, which actually reduces their performance or makes their performance inferior to the Carnot cycle. And that is why the concept of two more cycles comes into picture which are known as the Stirling and Ericsson cycles.

Again, these two are two ideal cycles, less used compared to the ones that we have talked about. But now it is particularly the concept of Stirling cycle has started to gain more importance and that is why I thought about just touching them briefly over next ten minutes.

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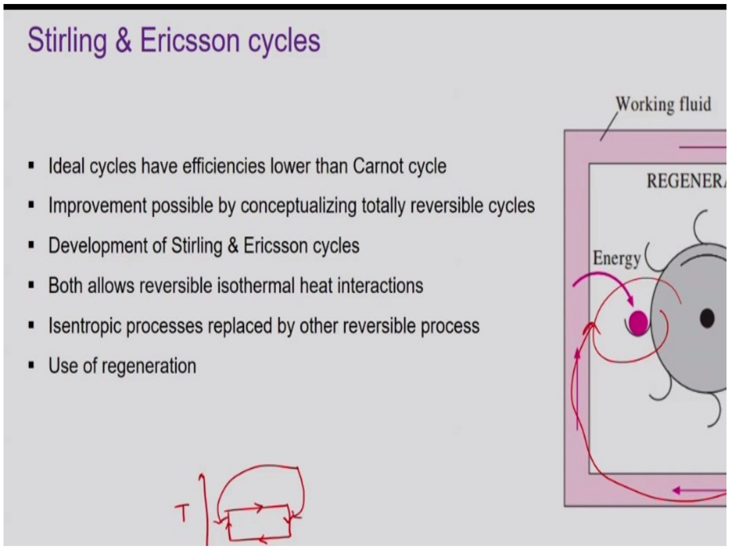

Stirling & Ericsson cycles

- Ideal cycles have efficiencies lower than Carnot cycle
- Improvement possible by conceptualizing totally reversible cycles
- Development of Stirling & Ericsson cycles
- Both allows reversible isothermal heat interactions
- Isentropic processes replaced by other reversible process
- Use of regeneration

So, as you have mentioned the ideal cycles have efficiencies lower than the Carnot cycle because they are internally reversible but externally irreversible. And if you want to get efficiency value similar to the Carnot cycle efficiency or similar to efficiency of a completely reversible cycle, then you have to make the cycle completely reversible.

And that is possible only when you can eliminate any kind of heat transfer with finite temperature difference rather any kind of heat transfer process has to be performed with infinitesimally small temperature difference between system and surrounding and that is why the concept of these two cycles come in, the Stirling cycle and Ericsson cycle. These are two cycles which are totally reversible in nature, though they are not similar to Carnot cycle, they have some differences with the Carnot cycle.

But both of them are totally reversible in nature because they do not allow any heat transfer with finite temperature difference rather whatever heat transfer takes place here are because of infinitesimally small temperature difference between system and surrounding. Both of them allow isothermal heat interactions during which the temperature difference is extremely small and isentropic processes in a Carnot cycle are replaced by other reversible process.

If we draw the Carnot cycle, the *Ts* diagram of the Carnot cycle if I want to draw say the *Ts* diagram looks like rectangle. This is the heat addition process, isothermal heat addition, this is the expansion part, isentropic expansion, this is isothermal heat rejection and this is the compression part, isentropic compression. Now, here two of the processes are isothermal and two of the processes are isentropic.

So, the expansion and compression processes, if I mark these two, this expansion part and the compression part both of these processes are isentropic. In these two ideal cycles that is Stirling and Ericsson cycle, these isentropic processes are replaced with some other process. Actually, they are replaced by constant volume process in case of Stirling cycle and constant pressure process in case of the Ericsson cycle, but the heat addition and heat rejection remain at isothermal. And for that we use a concept called regeneration. Regeneration is a very useful concept where the heat rejected by the system during one part of the cycle is supplied to some kind of energy storage device and during another process part of the cycle, energy is recovered from the storage thing. Like if the system wants to reject heat at this part, it will reject to your storage.

And then when it wants to gain the energy here, then the same storage has moved here to get supply the energy back to this. This concept of regeneration is extremely important; it will again come back when you talk about the gas turbines or when you talk about the steam cycles or Rankine cycle. Here, in conjunction with Stirling cycle we are briefly touching upon this regeneration part.

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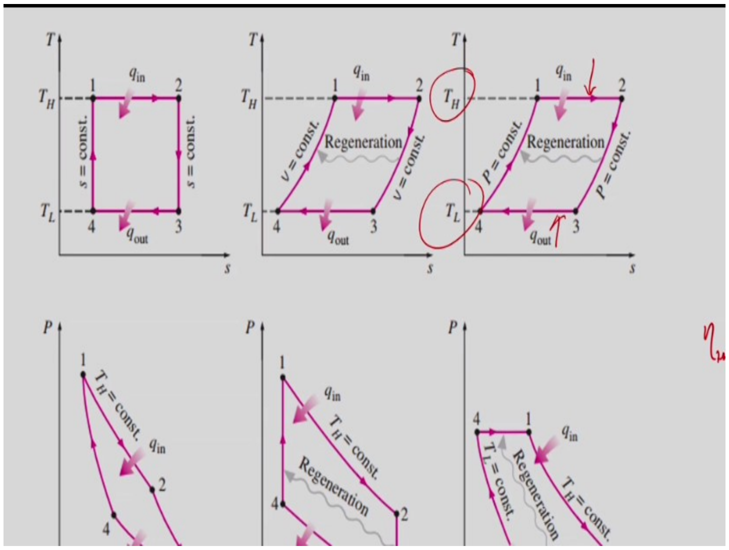

So, these are the conceptual *Pv* and *Ts* diagrams. The first one is the Carnot cycle which we know this the *Ts* diagram, you have a isothermal heat addition and isothermal heat rejection. Then, we have an isentropic expansion and an isentropic compression process. During the Stirling cycle, the heat addition and heat rejection remains at constant temperature but the expansion and compressions are not isentropic during which heat transfer is possible.

Both of them are at constant volume, this is constant volume expansion, this is constant volume compression part. Or if we see on the *Pv* diagram, you can see this is the heat addition part, this is the heat rejection part and then there is a constant volume expansion process, pressure drops, volume remains constant and the constant volume compression process during which the pressure increases but volume remains constant.

Similarly, in case of Ericsson cycle, we again have heat addition and heat rejection constant temperature but other processes are constant pressure like on *Pv* diagram, this is the heat addition, this is the heat rejection but this 4-1 and 2-3 are at constant pressure. So, the difference with the Carnot cycle is that in case of Stirling cycle, instead of isentropic processes, we have constant volume processes, so, it comprises up to isothermal and to isochoric processes. Whereas in case of Ericsson cycle, we have two isothermal and two isobaric processes. If we can design the cycles properly and they are working between the same temperature limits. Like you can see here, all the three cycles are working between two temperature limits. One is the maximum temperature T_H, other is the minimum temperature T_L.

If this T_H and T_L are same for all these three cycles, then it can be shown that the thermal efficiency for all of them is equal to the Carnot cycle efficiency that is:

$$\eta_{th} = 1 - \frac{T_L}{T_H}$$

So, these two cycles are totally reversible because what I have briefly mentioned in the previous slide, this heat addition and heat rejection at constant temperature are done such that the temperature difference between system and surrounding remains infinitesimally smaller during them. Thereby, attaining nearly the total reversibility condition and so we have efficiency to be equal to the Carnot cycle efficiency.

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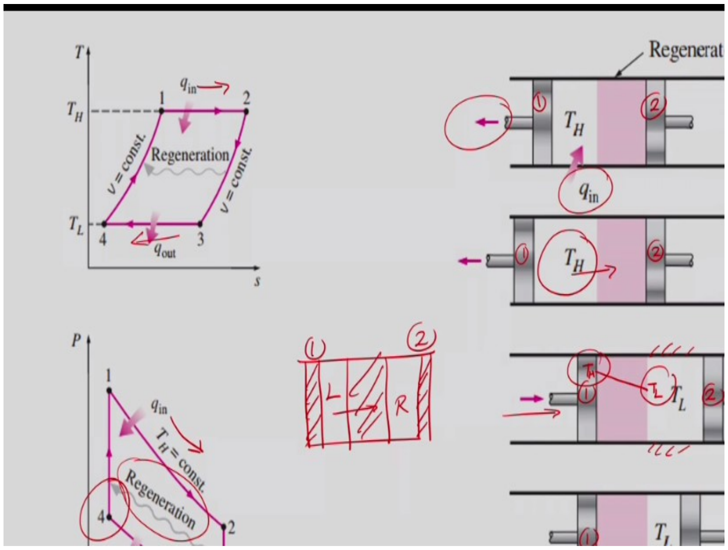

This is how we do Stirling cycle, somewhat practically we can achieve. Truly speaking, exactly this thing is not possible to be done in practice, because isothermal heat addition like in case of Carnot cycle has its own problem. You need either infinite surface area or you need infinite time. But some innovative ways are possible which can give you this near isothermal heat addition without having significant temperature difference between system and surrounding like shown here.

Here, it is a piston cylinder arrangement where there are two pistons. This is piston number 1 and this is a piston number 2. Let me mark this 1 and 2 in all the 4 figures and the one shown in pink is some kind of regenerative material. Regenerative material can be some gel kind of substance, sometimes it is just wire meshes having a porous structure through which the gases can pass through.

So, to start with at the state point 1 that is at this particular point, the situation is shown here, where the porous structure is having the entire mass of its gas trapped on the left of that, left means our left or my left that I am talking about. This is the portion where the entire mass of gas is trapped. As heat is added to the system, as we start to add heat by some external source which is maintained at a temperature very close to the system temperature.

Then, as heat goes in, the volume of the gas, temperature of the gas increases, accordingly its volume also should increase and so it starts to move in this direction. So, this is what is happening, you can see here. The temperature remains constant but because of heat addition is entropy increases like here because of this, its pressure is decreasing but the volume is increasing during this heat addition process.

So, this continues till we are adding this heat and the system finally reaches the state point 2. This is the situation of state point 2 when the system has reached its final position or this piston rather piston number 1 has reached its final position. Now, after this state point 2, now we have to achieve this constant volume pressure reaction thing, i.e., we have to achieve this particular part.

To do this, we have to do the regeneration and how we do this regeneration? During this particular part, both the pistons, 1 and 2 are moved in this direction. Both pistons 1 and 2 are moving in this particular direction at the same rate so that the total volume of the gas that remains constant. Only thing that is happening is that the gas which is now in this particular part of this wire mesh or your regenerating material that is now passing through this porous structure and moving to the other side.

That is, if I draw an intermediate portion say this is your regenerative material, at a particular instant of time between this process 2 to 3, this is piston 1, this is piston 2, so the gas is moving from the chamber on the left to the chamber on the right, I am talking about my left, it is moving from my left towards my right through this. And as it is moving through the regenerative structure, it is transferring heat to the regenerative structure thereby heating it.

And the heating will be done sequentially as it is moving through this, the gas temperature is dropping, accordingly there will be a temperature gradient in this regenerative structure also. So that there is a steep temperature gradient, temperature at this particular end is T_H and this end the temperature will be T_L inside this regenerative structure. At this point, we have reached state number 3.

So, once we have reached state number 3, we have to achieve this constant temperature heat rejection. So this part of the surface of this second chamber that is taken to the contact of another substance whose temperature is again very close to T_L, maybe slightly less than T_L so that the system rejects this Q_{out} amount of heat and as it is rejecting heat, its volume reduces, pressure increases.

So, we have this process 3 to 4, during this process 3 to 4, heat is being rejected at constant temperature, its volume reduces, pressure is increasing slightly. So, as the state point 4 finishes, the entire mass of gas at temperature T_L is trapped within this smaller portion of this tank which is your state point 4. And next as we have to move from 4 to 1, now again both piston number 2 and piston number 1 starts to move in this direction at the same rate so, that the gas is forced to move from the chamber at the right through the regenerative material to the chamber at the left. Now, here the temperature is T_L, here the temperature is T_H as the gas is moving through the regenerative structure it is finding an environment which is slightly hotter than this thereby continuously gaining heat and when it reaches the second chamber, it reaches the state 1, its temperature has increased back to T_H and this is the regeneration.

During process 2 to 3, energy has been transferred by the gas to the regenerative material during process 4 to 1, the system or the gas regains that energy back from the regenerative material. This is a concept of regeneration that is we are storing the energy intermittently to some energy storage option and then when required we are getting it back. Thereby, trying to achieve the near isothermal heat addition process and also a totally reversible process.

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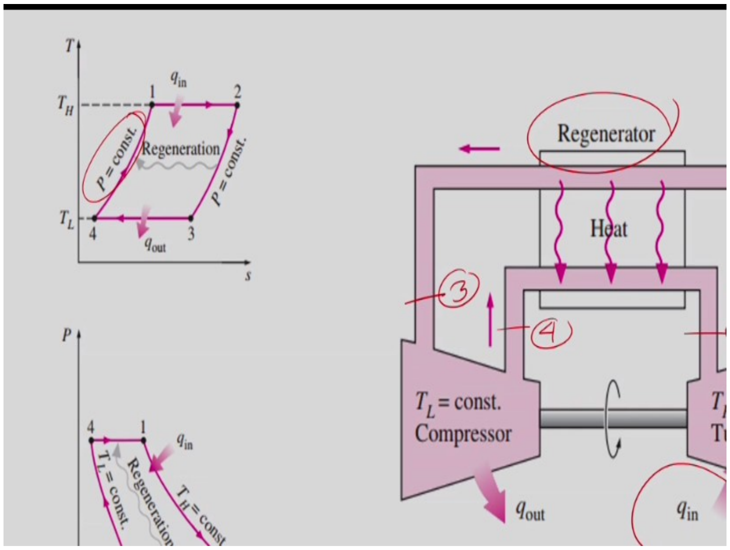

Similarly, this is the Ericsson cycle which is shown here in conjunction with an open system. So, here you have this constant temperature compressor from where heat is being rejected, heat is being added the turbine. As the high-pressure gas is moving from the compressor to this, it actually receives heat from the regenerative material. So, it receives heat from the regenerative material.

So, this can be taken as your point 1, this is your state point 2, then it enters the turbine. In the turbine, constant temperature is maintained sorry actually this is your state point 4, this is your state point 1 during which the pressure remains constant but temperature increases because of this regenerative heating this particular thing. Then, in the turbine, temperature remains constant, heat is being added, we get work output, so this is your 1-2.

Then, as it is going through, it rejects heat to the regenerator thereby coming back to state number 3 and that heat rejected to the regenerator is supplied back to the system during process 4-1 and then 3-4 is constant temperature heat rejection. So, this concept of Stirling cycle and Ericsson cycle have gain importance in recent times. Actually, their concepts are quite old.

But because of the problem associated with this isothermal heat addition and heat rejection, they are virtually discarded. But nowadays scientists are showing renewed interest, very prominent companies have started to develop Stirling engines. Just to name a few something like General Motors, Philips and Philips can be one, Ford Motors can be one. They have started to develop Stirling engines for trucks, buses and even for smaller automobiles as well.

Of course, still lots of design modifications are required but one important advantage for Stirling and Ericsson cycles are they are not internal combustion engines; they are rather external combustion engines. Here, this heat addition is from some external source. So, you can burn the fuel outside, you have more time to burn the fuel thereby you can design your burner whatever the way you want and you can achieve complete combustion.

The problem that we have in case of Diesel cycle about incomplete combustion or in case of SI engines, we have to achieve this combustion process extremely quickly that burying is not there, we can take some time to have to get the combustion process done, and also we can design the burners properly. And the working medium that is gas that truly remains a closed system, nowhere that is coming out of your system.

So, it is a proper closed system not like the internal combustion engines where you actually assuming them to be closed system, but truly they are not closed systems. We need some gases which are chemically stable and have very high thermal conductivity to be the working medium. Hydrogen or helium generally has been found to be very important working medium. But the most important contribution of the Stirling and Ericsson cycle is the concept of regeneration.

Regeneration shows us that how we can increase the efficiency to form the ideal cycles towards the Carnot cycle efficiency. The same thing is possible to be done in case of steam power cycles as well as we shall be seeing later on.

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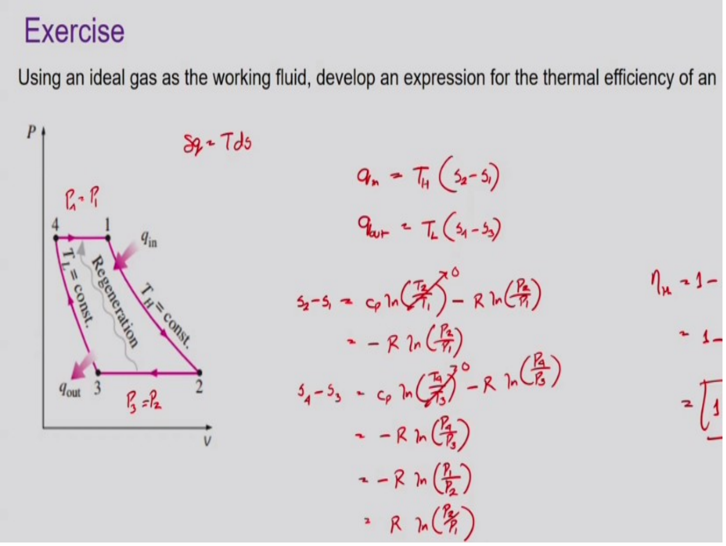

Exercise

Using an ideal gas as the working fluid, develop an expression for the thermal efficiency of an

$$\delta q = T ds$$

$$q_{in} = T_H (s_2 - s_1)$$

$$q_{out} = T_L (s_4 - s_3)$$

$$s_2 - s_1 = c_p \ln \cancel{\left(\frac{T_2}{T_1}\right)}^{0} - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= -R \ln\left(\frac{P_2}{P_1}\right)$$

$$s_4 - s_3 = c_p \ln \cancel{\left(\frac{T_4}{T_3}\right)}^{0} - R \ln\left(\frac{P_4}{P_3}\right)$$

$$= -R \ln\left(\frac{P_4}{P_3}\right)$$

$$= -R \ln\left(\frac{P_1}{P_2}\right)$$

$$= R \ln\left(\frac{P_2}{P_1}\right)$$

$$\eta_H = 1 -$$

$$= 1 -$$

$$= [1$$

To finish the chapter today, we have to show that the expressions of thermal efficiency for an Ericsson cycle is similar to that for a Carnot cycle. So we are using an ideal gas as the working medium and we have to check the efficiency. So, here we have heat addition and heat rejection at constant temperature. So, let us say q_{in}, it is the amount of heat added and heat is being added at constant temperature and is totally reversible heat addition, is not it?

Then, we can write using the *Tds* relation, this can be written as:

$$q_{in} = T_H (s_2 - s_1)$$

because as per the second law of thermodynamics there we developed one relation called *Tds* for totally reversible process that is what we are using is as temperature is constant. So, we are having $T_H (s_2 - s_1)$. Similarly, q_{out}, it is again a totally reversible process and temperature is constant so it can be written as:

$$q_{out} = T_L (s_4 - s_1)$$

Now, here we are treating the working medium as an ideal gas and for an ideal gas we know that we have developed this relation again in our module number 2. That is

$$s_2 - s_1 = c_p \, ln\left(\frac{T_2}{T_1}\right) - R \, ln\left(\frac{P_2}{P_1}\right)$$

Is not it? This is the relation that we know. Now, during process 1-2, the temperature remains constant. So, first term is zero, so this is just

$$= -R \, ln\left(\frac{P_2}{P_1}\right)$$

Similarly, during process 3-4, actually I have written something wrong.

This is between process 3-4 is not it? Q out is happening from 3-4, so it is,

$$s_4 - s_3 = c_p \, ln\left(\frac{T_4}{T_3}\right) - R \, ln\left(\frac{P_4}{P_3}\right)$$

T_4 and T_3 are equal because the temperature is equal to T_L. So, this is because

$$= -R \, ln\left(\frac{P_4}{P_3}\right)$$

Now, look at this *Pv* diagram. P_3 and P_2 are equal to each other. Similarly, P_4 is equal to P_1, so if you put it then it is:

$$= -R \, ln\left(\frac{P_1}{P_2}\right)$$

Now how much is thermal efficiency?

$$\eta = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{R \, T_L ln\left(\frac{P_1}{P_2}\right)}{R \, T_H ln\left(\frac{P_4}{P_3}\right)}$$

So, if this cancels out,

$$= 1 - \frac{T_L}{T_H}$$

which is the same efficiency as that of a Carnot cycle. So, it can be shown that similarly we can prove it for Stirling cycle. I am leaving that to you for an exercise. Try to prove the same thing for Stirling cycle, follow the same procedure just relate the heat addition and heat rejection to the corresponding change in entropy and if we assume that to an ideal gas, follow the same procedure to do it.

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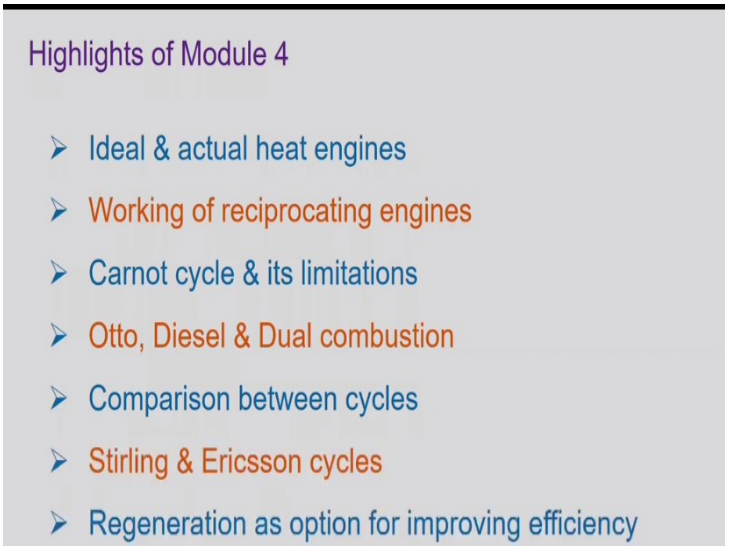

Highlights of Module 4

- Ideal & actual heat engines
- Working of reciprocating engines
- Carnot cycle & its limitations
- Otto, Diesel & Dual combustion
- Comparison between cycles
- Stirling & Ericsson cycles
- Regeneration as option for improving efficiency

So, that takes us to end of module number 4 where we have discussed about the heat engines, the ideal heat engines and their difference with the ideal heat engine cycles and the difference with the Carnot cycle. We discussed about the working of reciprocating engines, the Carnot cycle and its limitation in conjunction with reciprocating engines, then Otto, Diesel and Dual combustion cycles.

Then, comparison between the cycles in terms of different parameters we have talked about. Today, we talked about the Stirling and Ericsson cycles and regeneration as an option of improving efficiency has briefly been touched upon, this will come back more in later modules. The actual heat engine which I have mentioned here, I shall be talking more on actual heat engines next week. We already know that the efficiency of ideal heat engines is lower than Carnot engine.

And in actual heat engines, there are several irreversibilities and complexities come in which reduces their efficiency when further, so that is what we shall be seeing in the next module, as usual we talking about the actual heat engines. Till then, revise this module lectures, do the exercise. There are several interesting problems you can find in your text books also, you can try to solve this and if you have any query, please write back to me. Thank you.