

Two-Phase Flow with phase change in conventional and miniature channels
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Lecture – 08
Two phase flow and pressure drop in miniature channels

Today we are meeting for the last time for the course Two-Phase Flow with Phase Change in Conventional and Miniature Channels. We have discussed two phase flow regimes, in conventional channels. And we have discussed modelling of two phase flow using different models homogeneous model, separated flow model and drift flux model. We have discussed how to evaluate pressure gradients; and then how to integrate the pressure gradient to get the pressure drop.

Mostly in conventional channels, in the numerical example we also considered a channel of 2 millimetre diameter, which can be considered as a miniature channel. But, there is some difference between the flow regimes. And conventional channels and miniature channels and the methods to evaluate the pressure gradients and pressure drop. So, therefore, it merits a separate discussion. So, today we will discuss Two phase flow and pressure drop in miniature channels. We will further classify miniature channels into two categories minichannels and microchannels.

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Two-Phase Flow Regimes in Miniature Channels

- Size range considered $10 \mu m \leq D_H \leq 1 mm$:
 - Microchannels ($10 \mu m \leq D_H \leq 100 \mu m$)
 - Minichannels ($100 \mu m \leq D_H \leq 1 mm$)

We will consider the size range of 10 micron to 1 millimeter hydraulic diameter. And we will divide it into two categories; microchannels when we say microchannels we will mean approximately 10 micron to 100 micron hydraulic diameter. And when we say mini channel, we will mean the hydraulic diameter ranging from 100 micron to 1 millimeter. There is a difference in the flow regimes observed and the hydrodynamic behaviour in these two categories minichannels and microchannels.

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Two-Phase Flow Regimes in Minichannels ($100 \mu m \leq D_H \leq 1 mm$):

- Except stratified flow, all major flow regimes (bubbly, slug, churn, annular etc.) occur in minichannels.
- Bubbly flow: distinct and distorted (nonspherical) bubbles, considerably smaller than the channel.
- With $j_G \uparrow$ and $j_L = \text{const.}$ ($\Rightarrow \alpha \uparrow$) \rightarrow crowding of bubbles \rightarrow plug/slug flow (difference in nomenclature).
- Churn flow:
 - Pseudo-slug / churn / frothy-slug
 - Churn / frothy slug-annular
- Slug-annular flow
- Annular flow

So, let us consider two phase flow regimes in minichannels, that is hydraulic diameter ranging from 100 micron to 1 millimeter. The researchers have done experiments and they have observed all major flow regimes except stratified flow. You remember the classification of miniature channels in that, confinement number is used or the bond number base criterion is used.

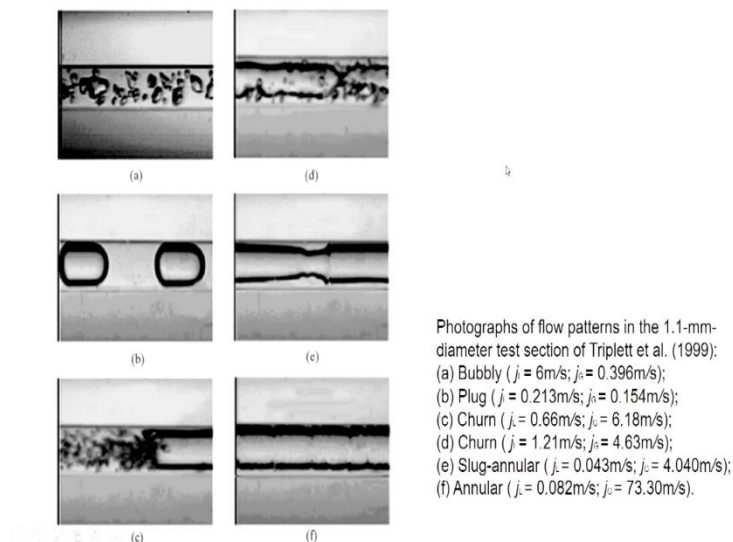
And the criterion is based on the phenomenon that below a certain size stratified flow becomes impossible. The surface tension become so dominant over buoyancy that stratified flow becomes impossible. In mini channels people have not observed stratified flow, they have observed all other major flow regimes; bubbly flow, slug flow, churn flow, annular flow etcetera.

So, at low gas velocities or gas volumetric fluxes, we observe bubbly flow which contains distinct and distorted bubbles which are non-spherical; and they are considerably smaller than the channel. When j_G increases, keeping j_L constant which means that the void

fraction increases, then the bubbles get crowded and because of the crowding it results in plug flow or slug flow. Some researchers have called it plug flow while, some others have called it slug flow and there is a difference in nomenclature.

If the j_G increases further keeping j_L constant then, churn flow is observed. And some researchers have called it pseudo slug flow, some have called it churn flow some have called it frothy slug flow. And there is another kind of churn flow, which some researchers have called churn flow and some have called frothy slug annular flow. Then if j_G increases further, then slug annular flow is observed; and further increase of j_G results an annular flow.

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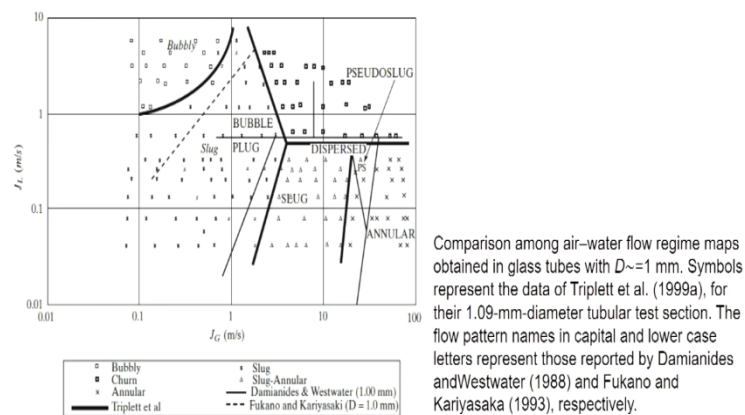
Here flow regimes observed by triplet et al are shown in the first figure a; we see bubbly flow. Here j_L is equal to 6 meter per second; and j_G is equal to 0.396 meter per second. And in figure b; plug flow is shown, here j_L is less but j_G is also less. And then in figure c; churn flow is shown. And figure d; is shows a different kind of churn flow, figure e; shows slug annular flow.

Here you see it is essentially annular flow but, there is a slight resemblance with slug flow. Because, there is a thinning of the vapour bubble; and if it added thinned further, then it it could have resulted in splitting of the bubble and there would have been a liquid slug between the bubbles. But, here it does not happen and it remains like annular flow. So, this is something between slug flow and annular flow and it is named as slug annular flow.

Then, the last figure f shows annular flow, where there is liquid annulus near the walls at the centre of the channel, there is vapour with of course, entrained droplets. It has also been observed that flow regimes in minichannels and also microchannels are independent of the orientation whether it is vertical flow or horizontal flow, whether it is inclined or not inclined or whatever is the angle of inclination.

Depending on the mass fluxes and volumetric fluxes of the liquid and vapour phase is the there are different flow regimes observed but, for the same conditions if the orientation is changed; then the flow regime as not change. And this is understandable because, in such small channels the effect of gravity is not dominant. And therefore, whether it is vertical or horizontal or inclined should not change the flow regime.

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So, there is a flow regime map and actually different researchers have given different flow regime maps. And here a comparison of these flow regime maps is shown; and experimental data of different researchers are also shown. So, there is bubbly flow, plug flow and there is pseudo slug flow and dispersed flow, slug flow and annular flow different kinds of flow regimes are there. And there are transitions, transition lines between the flow regimes. There is a difference between flow regime maps obtained by different researchers, but there is similarity also.

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Void fraction in minichannels:

Mishima and Hibiki (1996) correlation (for drift flux model)

$$C_0 = 1.2 + 0.510 e^{-0.692 D_H}, \quad V_{gj} = 0$$

where D_H is in mm.

Other correlations

$$\alpha = 0.8 \beta$$
$$\alpha = \frac{C_1 \beta^{0.5}}{1 - C_2 \beta^{0.5}}$$

(The constants are sensitive to channel size.)

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(The constants are sensitive to channel size.)

Now, how to calculate pressure gradient and pressure drop in minichannels? So, for that there are correlations for void fraction; and Mishima and Hibiki gave a correlation in 1996 for drift flux model which we have mentioned before. And it gives C_0 equal to 1.2 plus 0.510 e raise to minus 0.692 D_H and V_{gj} is equal to 0.

And here the hydraulic diameter is in millimeter V_{gj} has been taken as 0; because, in minichannels it is observed that there is very small slip between the phases. So, the effect of local slip on the void fraction is negligible; and we know that v_{gj} is the gas drift

velocity. And, it represents the local slip between the phases and therefore, v_{gj} is taken as 0.

And C_{naught} is evaluated from this expression. Then there are other correlations given by different researchers, when give α is equal to 0.8 β linear relation between α and β . And other gave α equal to $C_1 \beta$ raise to 0.5 upon $1 - C_2 \beta$ raise to 0.5 so, this is a non-linear relation. And the constants C_1 and C_2 are also sensitive to channel size. And the same researcher has done experiments on different channel sizes and has given different values of C_1 and C_2 for different channel sizes.

So, in minichannels we see that different researchers have given different flow regime maps and different correlations for the void fraction. And, even the same researcher has given different correlations for different channel sizes.

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Two-Phase Flow Regimes in Microchannels ($10 \mu m \leq D_H \leq 100 \mu m$):

Experiments on 100 μm dia tube:

- Bubbly flow
- Slug flow
- Liquid ring flow
- Liquid lump flow
- Droplet flow

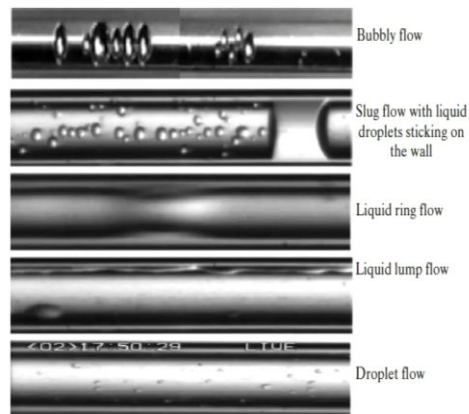
Experiments on ~~100~~²⁵ μm dia tube:

- Bubbly flow
- Slug flow
- Liquid ring flow
- Liquid lump flow

So, now, consider flow régimes, in microchannels that is the hydraulic diameter ranging from 10 micron to 100 micron. Experiments on 100 micron diameter tube, in these experiments the following flow regimes were observed. Bubbly flow, slug flow and there new flow regimes which are not observed in conventional channels; and even in minichannels these are liquid ring flow and liquid lump flow.

And then droplet flow regime was also observed. Then experiments were done on 25 micron, this is 25 micron 25 micron diameter tube. And in this the following flow regimes were observed bubbly flow, slug flow, liquid ring flow and liquid lump flow.

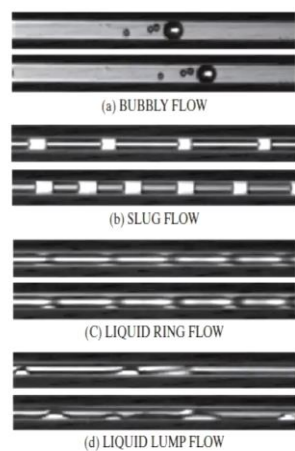
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Air-water two-phase flow regimes in a 100- μm inner diameter quartz tube. (From Serizawa et al., 2002.)

Here the flow regimes are shown. So, there is bubbly flow in which there are small bubbles. And then there is slug flow and it contains liquid droplets also. Then there is liquid ring flow, in which there is a ring of liquid then there is a liquid lump flow and droplet flow.

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Air-water two-phase flow patterns in a 25- μm inner diameter silica tube. (From Serizawa et al., 2002.)

The same researcher did experiments on 25 micron diameter tube and observed these flow patterns; bubbly flow, slug flow, liquid ring flow and liquid lump flow. Other researchers have also done experiments and they have observed different flow regimes. There is no common agreement between different researchers, in case of microchannels; and different researchers have observed different flow regimes and they have given different correlations.

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Void fraction in microchannels:

Serizawa et al. correlation for bubbly and slug flow regimes

$$\alpha = 0.833 \beta$$

$$V_{gj} = 0$$

Void fraction in microchannels:

Serizawa et al. correlation for bubbly and slug flow regimes

$$\alpha = 0.833 \beta \quad V_{gj} = 0$$

One correlations for void fraction given by Serizawa et al is for bubbly flow and slug flow regimes; and it is a linear relation between alpha and beta. Alpha is equal to 0.833, beta and of course V_{gj} is equal to 0 ok. So, far while evaluating pressure drop, we have assume the properties to be constant so, that we can integrate the expressions easily. We integrated the expressions for the pressure gradients to obtain pressure drops due to friction acceleration and gravity.

And there we assume the property is like v g v f g h i f etcetera all these properties we assumed to be constant. And also when we did the energy balance, we assumed that the

vapour generation is only due to heating. So, we did a simple energy balance and then if there is a uniform heat flux, then we get a linear quality profile $\frac{dx}{dz}$ is equal to constant. And if the heat flux is 0; that is adiabatic channel then we get a constant quality x equal to constant.

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- If the pressure drop is small compared to the system pressure then properties at the inlet and outlet pressures are not quite different.
- If so, properties can be evaluated at the system pressure and can be treated as constant.
- Vapour generation is due to heating only, and can be evaluated by simple energy balance.
- A uniform heat flux implies a linear quality profile and no heat flux implies a constant quality.
- The above approximations make it easy to integrate pressure gradient to evaluate pressure drop.

If the pressure drop is small compared to the system pressure, then properties at the inlet and outlet pressures are not quite different. Suppose consider the example 3; in which the pressure is 10 mega Pascal. We have taken all properties at 10 mega Pascal, suppose the pressure drop is 50 kilo Pascal.

So, 10 mega Pascal is 10,000 kilo Pascal suppose that is the outlet pressure. Then the inlet pressure will be 10,050 kilo Pascal. And if we evaluate properties at 10,000 kilo Pascal and 10050 kilo Pascal, we will get almost the same values. You can check from steam tables are from the program steam tuff dot e x c. So, it does not matter at which pressure we evaluate the properties, it can be anything between 10 mega Pascal and 10,000 kilo Pascal and 10,050 kilo Pascal.

So, we can evaluate the property at an average pressure called system pressure. And then we can use those properties and treat them as constant, that will not result in much error. And another thing is that the vapour generation is only due to heating; and we can be evaluated by simple energy balance as we have done in the numerical examples. So, if the

heat flux is uniform then dx by dz is equal to constant and if heat flux is 0; then x is equal to constant. And we have used it in our derivations and numerical examples.

But, if there is a significant pressure drop then what happens? In case of miniature channels, the since the hydraulic diameter is small the pressure gradients are large. Specially, the frictional pressure gradient is large because the hydraulic diameter is small also the because of change in quality significant quality there is a significant pressure drop due to acceleration also. And another thing is that in miniature channels, we usually it is not possible to use very high pressures. So, purpose for common applications the system pressure is not very high. So, suppose the pressure at the outlet is 100 kilo Pascal, nearly atmospheric pressure and the pressure drop is 50 kilo Pascal, then the inlet pressure will be 150 kilo Pascal.

Now, if we evaluate properties at 100 kilo Pascal and 150 kilo Pascal, we will see that there is significant difference between the properties. And in the pressure gradient we have these properties v f v g v f g etcetera; and these properties when we evaluate along the length for different values of z ; we will get different values. And therefore, pressure gradient at every z has to be evaluated using the local properties, that is the properties at the local pressure. And then only can we hope to get an accurate value.

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- If the pressure drop is not small compared to the system pressure then properties need to be evaluated at the local pressure for every axial location.
- Properties vary with z and cannot be treated as constant while integrating the pressure gradient.
- Vapour generation is due to heating as well as flashing.

$$\frac{dx}{dz} = \frac{1}{h_{fg}} \left(\frac{q'' P_H}{G A} - \frac{dh_f}{dP} \frac{dP}{dz} \right) \checkmark$$

- A uniform heat flux does not necessarily imply a linear quality profile and no heat flux does not imply a constant quality.

$$\frac{dx}{dz} = \frac{1}{h_{fg}} \left(\frac{q'' P_H}{G A} - \frac{dh_f}{dP} \frac{dP}{dz} \right)$$

So, if we treat properties as constant so, if the pressure drop is not small compared to the system pressure then properties need to be evaluated at the local pressure for every axial location. So, properties vary with z ; and we cannot treat them as constant while integrating the pressure gradient. And therefore, we cannot integrate the pressure gradient by hand and then it becomes necessary to do numerical computation while writing a computer code. Another complication is that vapour generation is not only due to heating but also due to flashing.

You know from basic thermodynamics that when the pressure of a liquid vapour mixture is decreased, then the saturation temperature also decreases. And because of that some liquid gets evaporated and there is vapour generation due to that; even if there is no heating the quality will increase. So, the vapour generation is due to heating as well as due to flashing. And this should be taken into account in the energy balance.

So, if we consider the vapour generation due to flashing in the energy balance, then we get this equation $\frac{dx}{dz}$ is equal to $\frac{1}{h_{fg}} \frac{dP}{dz} + \frac{P_H}{G A}$. Here P_H is the heated perimeter and minus $\frac{dh_f}{dP} \frac{dP}{dz}$. So, this first term is the term which we have considered before, this is vapour generation due to heating. And the second term is the vapour generation due to flashing. If the pressure gradient is large from here we can see that $\frac{dP}{dz}$ is large, then the second term will be significant and cannot be ignored.

Another thing to be noted is that $\frac{dP}{dz}$ is unknown, we have usually trying to evaluate the pressure gradient and then we have to integrated to evaluate the pressure drop. So, this is an unknown term and it has to be taken to the left hand side and combined with the other terms containing the pressure gradient $\frac{dP}{dz}$. And then finally, we have to express $\frac{dP}{dz}$, in terms of other quantities and then we have to integrate and find the pressure drop ΔP . And as mentioned before we will have to write a computer code and do numerical computation to find the pressure drop.

So, because there is vapour generation due to heating as well as due to flashing uniform heat flux does not necessarily imply a linear quality profile. Previously, we have assumed a linear quality profile. But if this effect a significant then we cannot assume it. And even if there is no heat flux even if the channel is adiabatic, there will be change in quality there will be increase in quality due to flashing ok.

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Example-1: Water+steam @100 kPa, horizontal flow, D = 0.5 mm, L = 5 cm

$$G = 100 \text{ kg/m}^2\text{s}, x(0)=0, q'' = 50 \text{ kW/m}^2$$

To find the pressure drop at the end of the pipe using

- Homogeneous flow model
- Separated flow model
- Drift flux flow model

Solution:

Properties of water+ steam @100 kPa

$$\mu_f = 282.9 \times 10^{-6} \text{ Pa.s}, \mu_g = 12.26 \times 10^{-6} \text{ Pa.s}, h_{fg} = 2257.45 \text{ kJ/kg}$$

$$v_f = 1.043 \times 10^{-3} \text{ m}^3/\text{kg}, v_g = 1.6939 \text{ m}^3/\text{kg}, v_{fg} = 1.693 \text{ m}^3/\text{kg}$$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 1.772 \text{ m}^{-1}, \quad x_o = 0.0886$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782 - 1.6939}{1000} = -1.57 \times 10^{-5} \text{ m}^3 \text{ kg}^{-1} \text{ Pa}^{-1}$$

$$M^2 = G^2 x_o \left| \frac{dv_g}{dP} \right| = 3.47 \times 10^{-3} \ll 1, 1 - M^2 \approx 1, (1 - M^2)^{-1} \approx 1$$

Example-1: Water+steam @100 kPa, horizontal flow, D = 0.5 mm, L = 5 cm, G = 100 kg/m²s, x(0)=0, q'' = 50 kW/m². To find the pressure drop at the end of the pipe using

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$$M^2 = G^2 x_o \left| \frac{dv_g}{dP} \right| = 3.47 \times 10^{-3} \ll 1, 1 - M^2 \approx 1, (1 - M^2)^{-1} \approx 1$$

So, now let us consider some numerical examples; in the example 1, we have the same data as before except that the length here is 5 centimeter. And you have to find the pressure drop in the pipe using homogeneous model separated flow model and drift flux model.

The properties as same as before d x by d z, here we will calculate from the simple energy balance as we have done before otherwise it cannot be solved by hand. So, this is the same calculation as before 1.772 per meter; and the outlet quality 0.0886. And d v g by d P is of the order of 10 raise to minus 5. So, M square is of the order of 10 raise to minus 3 which is much less than 1 so, it can be neglected.

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 95.7 \times 10^{-6} \text{ Pa.s}$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 522 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.0306$$

$$\Delta P_F = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + \frac{x_o v_{fg}}{2 v_f} \right) = 0.064 \text{ kPa}$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} x_o = 1.5 \text{ kPa}$$

$$\Delta P_z = \frac{g \sin \theta L}{v_{fg} x_o} \ln \left(1 + x_o \frac{v_{fg}}{v_f} \right) = 0$$

$$\Delta P = 0.064 + 1.5 + 0 = 1.564 \text{ kPa}$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 95.7 \times 10^{-6} \text{ Pa.s}$$

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$$\Delta P_z = \frac{g \sin \theta L}{v_{fg} x_o} \ln \left(1 + x_o \frac{v_{fg}}{v_f} \right) = 0$$

$$\Delta P = 0.064 + 1.5 + 0 = 1.564 \text{ kPa}$$

So, now first we use the homogeneous model μ bar is calculated as 95.7 into 10 raise to minus 6 Pascal point second. And the two phase Reynolds number is calculated as 522 so, it is laminar flow. And the friction factor is calculated from the laminar flow relation and it is 0.0306.

So, the frictional pressure drop is calculated from this expression; and this is the same expression as we have used before assuming constant properties and linear quality profile. So, ΔP_F is equal to 0.064 kilo Pascal; and pressure drop due to acceleration is $G^2 v_f v_f g$ by $v_f x o$ and it is equal to 1.5 kilo Pascal. ΔP_z is 0; because the channel is horizontal and then by adding these pressure drops we get the total pressure drop as 1.564 kilo Pascal.

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$$Re_{f,o} = \frac{GD}{\mu_f} = 176 \Rightarrow \text{Laminar flow}$$

$$f_{f,o} = 16/Re = 0.0905$$

$$\overline{\phi_{f,o}^2} = 30 \quad r_2 = 25$$

$$\Delta P_F = \frac{2f_{f,o} L}{D} G^2 v_f \overline{\phi_{f,o}^2} = 5.66 \text{ kPa}$$

$$\Delta P_a = G^2 v_f r_2 = 0.26 \text{ kPa}$$

$$\Delta P_z = 0$$

$$\Delta P = 5.66 + 0.26 + 0 = 5.92 \text{ kPa}$$

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$$Re_{f,o} = \frac{GD}{\mu_f} = 176 \Rightarrow \text{Laminar flow}$$

$$f_{f,o} = 16/Re = 0.0905$$

$$\overline{\phi_{f,o}^2} = 30 \quad r_2 = 25$$

$$\Delta P_F = \frac{2f_{f,o} L}{D} G^2 v_f \overline{\phi_{f,o}^2} = 5.66 \text{ kPa}$$

$$\Delta P_a = G^2 v_f r_2 = 0.26 \text{ kPa}$$

$$\Delta P_z = 0$$

$$\Delta P = 5.66 + 0.26 + 0 = 5.92 \text{ kPa}$$

Now, we use the separated flow model for the same problem. Re_{fo} is calculated as 176 so, it is laminar flow. And, f_{fo} is 16 by Re equal to 0.0905; and the average two phase multiplier ϕ_{fo}^2 is found from the Martinelli Nelson graphs approximately equal to 30 and r_2 is also found from the Martinelli Nelson graph and it is approximately 25.

ΔP_F is equal to $2 f_{fo} L$ by $D G^2 v_f \phi_{fo}^2$ and it is 5.66 kilo Pascal. ΔP_a is equal to $G^2 v_f r_2$ and it is 0.26 kilo Pascal. ΔP_z is equal to 0, because the channel is horizontal and then we add these pressure drops and get the total pressure drop as 5.92 kilo Pascal.

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 95.7 \times 10^{-6} \text{ Pa.s}$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 522 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.0306 \quad \alpha_o = 0.637$$

$$r_2 = \left[\frac{x_o^2 v_g}{\alpha_o v_f} + \frac{(1-x_o)^2}{(1-\alpha_o)} - 1 \right] = 21.3$$

$$\Delta P_F = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + \frac{x_o v_{fg}}{2 v_f} \right) = 0.064 \text{ kPa}$$

$$\Delta P_a = G^2 v_f r_2 = 0.222 \text{ kPa}$$

$$\Delta P_z = 0 \text{ kPa}$$

$$\Delta P = 0.064 + 0.222 + 0 = 0.286 \text{ kPa}$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 95.7 \times 10^{-6} \text{ Pa.s}$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 522 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.0306 \quad \alpha_o = 0.637$$

$$r_2 = \left[\frac{x_o^2 v_g}{\alpha_o v_f} + \frac{(1-x_o)^2}{(1-\alpha_o)} - 1 \right] = 21.3$$

$$\Delta P_F = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + \frac{x_o v_{fg}}{2 v_f} \right) = 0.064 \text{ kPa}$$

$$\Delta P_a = G^2 v_f r_2 = 0.222 \text{ kPa}$$

$$\Delta P_z = 0 \text{ kPa}$$

$$\Delta P = 0.064 + 0.222 + 0 = 0.286 \text{ kPa}$$

Finally we use the drift flux model for the same pressure drop. So, μ bar is the same then $R e T P$ is 522. So, we get f_{TP} equal to 0.0306; and α_o is calculated using the drift flux model. And with drift flux model, we have use the Mishima and Hibiki correlation. This correlation has been used C naught has been obtained from this correlation and v_{gj} is taken as 0. And then using this C naught and v_{gj} , we have evaluated α_o corresponding to the outlet quality.

So, it is α_o ; the α_o is equal to 0.637. And using this value of α_o ; we calculate r_2 from this expression and we get r_2 equal to 21.3. The frictional pressure drop ΔP_F is calculated from the homogeneous model and it is 0.064 kilo Pascal. The acceleration pressure drop is equal to the $G^2 v_f r_2$ and r_2 we have calculated. So, we get ΔP_a equal to 0.222 kilo Pascal; and ΔP_z is equal to 0 because the channel is horizontal.

And then we add these three pressure drops and get the total pressure drop is 0.286 kilo Pascal. As I have mentioned for previous examples here also these examples are given only for illustration to illustrate the methods and application of the methods. And because the calculations a lot of calculations are involved, there can be errors in calculations. So, please do your own calculations and check. So, with this the course comes to an end. And we are going to have a live session and then that we can discuss doubts. And hopefully we will meet again in other course so.

Thank you very much and good bye.