Two-Phase Flow with phase change in conventional and miniature channels

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Lecture - 07
Estimation of pressure drop in two phase flow

We meet once again for Two Phase Flow with Phase Change in Conventional and Miniature Channels. We have discussed two phase flow models and how to calculate pressure gradients for two phase flow using different models. Now, today we will discuss how to find the pressure drop in two phase flow, pressure drop in two phase flow. So, first let us consider the homogeneous model and find the pressure drop using the homogeneous model.

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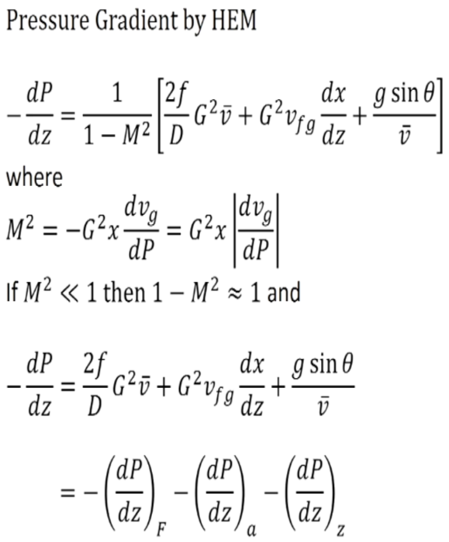

Pressure Gradient by HEM

$$-\frac{dP}{dz} = \frac{1}{1-M^2}\left[\frac{2f}{D}G^2\bar{v} + G^2 v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}\right]$$

where

$$M^2 = -G^2 x\frac{dv_g}{dP} = G^2 x\left|\frac{dv_g}{dP}\right|$$

If $M^2 \ll 1$ then $1 - M^2 \approx 1$ and

$$-\frac{dP}{dz} = \frac{2f}{D}G^2\bar{v} + G^2 v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}$$

$$= -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

The pressure gradient by homogeneous model is given by this expression 1 upon 1 minus M square 2 f upon D G square v bar plus G square v f g d x by d z plus g sin theta upon v bar. And here M square represents the compressibility of the vapor phase and it is equal to G square x absolute value of d v g by d P. And if M square is very small then we can neglect it and then in that case the denominator becomes equal to 1.

And then the pressure gradient becomes simply the quantity the expression in the bracket. And the first term is identified as the pressure gradient due to friction, the second term is the pressure gradient due to acceleration and the third term is pressure gradient due to gravity this we have done before. Now, from pressure gradient how do we find the pressure drop?.

The pressure drop is the decrease in pressure from one point to another point, usually in a pipe we calculate the pressure drop in the pipe that is from the inlet to the outlet of the pipe. Pressure gradient is calculated at a point, in a pipe pressure gradient we can calculate at the inlet or at the outlet or at any point in between. In the numerical examples we have calculated pressure gradient at the midpoint of the pipe.

But, we could have calculated at any other point and we in general we would have got different values of pressure gradient at different points. So, to calculate the pressure drop we will have to integrate the pressure gradient from the inlet to the outlet of the pipe. And since in general it varies from point to point the integral will not be simply the pressure gradient multiplied by the length.

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Pressure drop in an evaporator tube with saturated liquid at inlet

$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$

where

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \int_0^L \frac{2f_{TP}}{D} G^2 \bar{v}\, dz = \int_0^L \frac{2f_{TP}}{D} G^2 \left(v_f + x\, v_{fg}\right) dz$$

$$\Delta P_F = \int_0^L \frac{2f_{TP}}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) dz = \int_0^{x_o} \frac{2f_{TP}}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) \frac{dz}{dx} dx$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2 v_{fg} \frac{dx}{dz} dz = \int_0^{x_o} G^2 v_f \frac{v_{fg}}{v_f} dx$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \int_0^L \frac{g \sin\theta}{\bar{v}} dz = \int_0^L \frac{g \sin\theta}{\left(v_f + x\, v_{fg}\right)} dz$$

$$\Delta P_z = \int_0^L \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} dz = \int_0^{x_o} \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} \frac{dz}{dx} dx$$

Pressure drop in an evaporator tube with saturated liquid at inlet

$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$

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$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \int_0^L \frac{2f_{TP}}{D} G^2 \bar{v}\, dz = \int_0^L \frac{2f_{TP}}{D} G^2 \left(v_f + x\, v_{fg}\right) dz$$

$$\Delta P_F = \int_0^L \frac{2f_{TP}}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) dz = \int_0^{x_o} \frac{2f_{TP}}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) \frac{dz}{dx} dx$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2 v_{fg} \frac{dx}{dz} dz = \int_0^{x_o} G^2 v_f \frac{v_{fg}}{v_f} dx$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \int_0^L \frac{g \sin\theta}{\bar{v}} dz = \int_0^L \frac{g \sin\theta}{\left(v_f + x\, v_{fg}\right)} dz$$

$$\Delta P_z = \int_0^L \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} dz = \int_0^{x_o} \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} \frac{dz}{dx} dx$$

So, let us see how to do this integral. Consider pressure drop in an evaporator tube and assume that at the inlet it is saturated liquid, at the outlet in general it will be a two phase mixture. So, the pressure drop will be equal to the pressure drop due to friction, plus pressure drop due to acceleration, plus pressure drop due to gravity. And here the pressure drop due to friction delta P F is the integral of the pressure gradient due to friction with d P by d z we put a minus sign.

And then we have we integrate and what we get is the decrease in the pressure we; usually calculate the decrease in pressure rather than increase in pressure. Because, usually there is decrease in pressure due to friction and due to gravity also and many times due to acceleration also there is decrease in pressure. So, delta P F is integral 0 to L minus d P by d z F d z and then we substitute the expression for the pressure gradient.

And then v bar is equal to v f plus x v z x v f g v f and x v f g we will be assuming to be constant, but x is not constant in an evaporator tube it changes from point to point at the inlet x is equal to 0. At the outlet we have the maximum value and in between there are intermediate values, so we will have to do an integral. So, delta P F is equal to integral 0 to L 2 f T P upon D G square v f 1 plus x v f g upon v f d z, here we have taken v f common from the quantity in the brackets, so that within brackets the quantities are non dimensional.

Then we can convert from d z to d x in the integral and therefore, instead of d z we write d z by d x into d x. And we will see shortly why we have done this because it will be easier to do the integral with respect to x. Now, d P by d z a the pressure gradient due to acceleration is substituted in the expression for delta P a and we get integral 0 to L G square v f g d x by d z d z.

Here also we want to convert from the integral with respect to x, so we cancel d z d z and then we get G square v f v f g by v f d x integral 0 to x naught. We will see later why we have written instead of v f g we have written v f into v f g by v f. Then the pressure drop due to gravity is integral of the pressure gradient due to gravity and it is equal to integral 0 to L G sin theta upon v bar dz.

And then we substitute for v bar v f plus x v f g and we take v f common, so we get this integral and then we convert from d z to d x, So, finally, we get integral 0 to x naught g sin theta upon v f, in the bracket 1 plus x v f g by v f d z by d x d x. Here x naught is the quality at the outlet at z equal to L x is equal to x naught.

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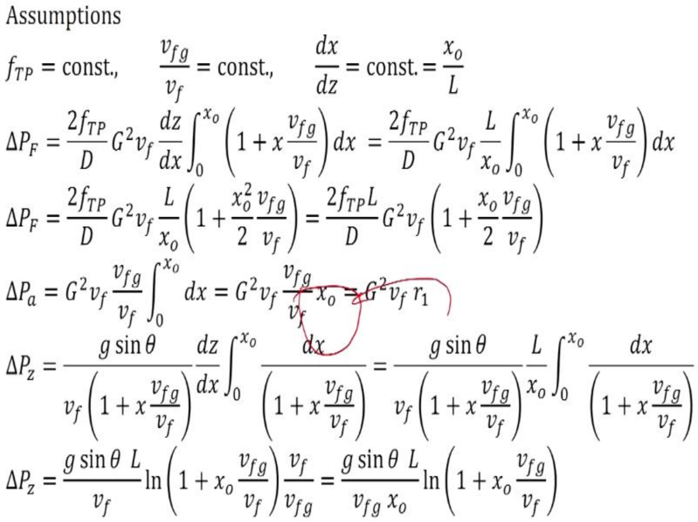

Assumptions

$$f_{TP} = \text{const.},\quad \frac{v_{fg}}{v_f} = \text{const.},\quad \frac{dx}{dz} = \text{const.} = \frac{x_o}{L}$$

$$\Delta P_F = \frac{2f_{TP}}{D}G^2 v_f \frac{dz}{dx}\int_0^{x_o}\left(1+x\frac{v_{fg}}{v_f}\right)dx = \frac{2f_{TP}}{D}G^2 v_f \frac{L}{x_o}\int_0^{x_o}\left(1+x\frac{v_{fg}}{v_f}\right)dx$$

$$\Delta P_F = \frac{2f_{TP}}{D}G^2 v_f \frac{L}{x_o}\left(1+\frac{x_o^2}{2}\frac{v_{fg}}{v_f}\right) = \frac{2f_{TP}L}{D}G^2 v_f\left(1+\frac{x_o}{2}\frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f}\int_0^{x_o} dx = G^2 v_f \frac{v_{fg}}{v_f}x_o = G^2 v_f\, r_1$$

$$\Delta P_z = \frac{g\sin\theta}{v_f\left(1+x\frac{v_{fg}}{v_f}\right)}\frac{dz}{dx}\int_0^{x_o}\frac{dx}{\left(1+x\frac{v_{fg}}{v_f}\right)} = \frac{g\sin\theta}{v_f\left(1+x\frac{v_{fg}}{v_f}\right)}\frac{L}{x_o}\int_0^{x_o}\frac{dx}{\left(1+x\frac{v_{fg}}{v_f}\right)}$$

$$\Delta P_z = \frac{g\sin\theta\ L}{v_f}\ln\left(1+x_o\frac{v_{fg}}{v_f}\right)\frac{v_f}{v_{fg}} = \frac{g\sin\theta\ L}{v_{fg}\,x_o}\ln\left(1+x_o\frac{v_{fg}}{v_f}\right)$$

Now, in order to do this integral we will have to make some assumptions, if we do not make any assumptions then we will have to do numerical integration. Also we have assumed that m square is much less than 1. If m square is not negligible then we will have to keep m square and then all these pressure gradients will have to be multiplied by 1 upon 1 minus m square and then we will have to do numerical integration.

Because m square contains x and then we cannot take it out of the integral sign and therefore, we will have to do numerical integration. Here we want to solve it analytically and get a closed from expression, so therefore, we are considering only those cases where m square is much less than 1 and further we are making some assumptions.

So, we assume that f T P is constant over the length and also v f g by v f is constant over the length. So, unless the pressure is low the change in pressure or the pressure drop will not affect the properties very much and therefore, v f g by v f will be nearly constant. And we also assume that d x by d z is equal to constant and it is equal to x naught upon L this comes from energy balance by assuming that the heat flux is uniform. And then by doing a simple energy balance we get d x by d z equal to constant, so therefore, the quality varies linearly over length and we get d x by d z is equal to x o upon L.

So, we substitute in the expressions d z by d x will be equal to the reciprocal of d z d x by d z and it will be L upon x o. So, after substituting we get delta PF is equal to d z by d x will come out of the integral sign because it is constant and then we have integral of 0 to x naught 1 plus x v f g by v f d x. And after integration we get 2 f T P L upon D G square v f bracket 1 plus x o by 2 v f g by v f, the pressure drop due to acceleration is G square v f v f g by v f 0 to x o d x integral.

So, the integral will be just x o and we get this expression G square v f v f g by v f x o and v f g by v f x o is denoted as r 1 ok. So, we can write delta P a as G square v f r 1, later we will see that when we use the separated flow model we get another expression which is called r 2 and therefore, here this v f g by v f x o is called r 1. The pressure drop due to gravity involves the integral of 0 to x o d x upon 1 plus x v f g by v f and when we integrate we get a logarithmic expression ok.

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$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2} \frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} x_o$$

$$\Delta P_z = \frac{g \sin\theta \; L}{v_{fg} \, x_o} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right)$$

$$\Delta P = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2} \frac{v_{fg}}{v_f}\right) + G^2 v_f \frac{v_{fg}}{v_f} x_o + \frac{g \sin\theta \; L}{v_{fg} \, x_o} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right)$$

Assumptions

$$f_{TP} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad \frac{dx}{dz} = \text{const.} = \frac{x_o}{L}$$

$$\Delta P_F = \frac{2 f_{TP}}{D} G^2 v_f \frac{dz}{dx} \int_0^{x_o} \left(1 + x \frac{v_{fg}}{v_f}\right) dx \; = \frac{2 f_{TP}}{D} G^2 v_f \frac{L}{x_o} \int_0^{x_o} \left(1 + x \frac{v_{fg}}{v_f}\right) dx$$

$$\Delta P_F = \frac{2 f_{TP}}{D} G^2 v_f \frac{L}{x_o} \left(1 + \frac{x_o^2}{2} \frac{v_{fg}}{v_f}\right) = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2} \frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} \int_0^{x_o} dx = G^2 v_f \frac{v_{fg}}{v_f} x_o = G^2 v_f \, r_1$$

$$\Delta P_z = \frac{g \sin\theta}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)} \frac{dz}{dx} \int_0^{x_o} \frac{dx}{\left(1 + x \frac{v_{fg}}{v_f}\right)} = \frac{g \sin\theta}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)} \frac{L}{x_o} \int_0^{x_o} \frac{dx}{\left(1 + x \frac{v_{fg}}{v_f}\right)}$$

$$\Delta P_z = \frac{g \sin\theta \; L}{v_f} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right) \frac{v_f}{v_{fg}} = \frac{g \sin\theta \; L}{v_{fg} \, x_o} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right)$$

So, finally, we get the pressure drop delta P equal to delta P F plus delta P a plus delta P z. And delta P F is equal to 2 f T P L upon D G square v f bracket 1 plus x o by 2 v f g by v f, delta P a is equal to G square v f v f g by v f x o. And delta P z is equal to g sin theta

L upon v f g x o ln of 1 plus x o v f g by v f and by adding these 3 terms we get the total pressure drop, so this was for an evaporator tube.

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Pressure drop in a condenser tube with saturated liquid at outlet, assuming

$$f_{TP} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad \frac{dx}{dz} = \text{const.} = -\frac{x_i}{L}$$

$$\Delta P_F = \frac{2f_{TP}}{D} G^2 v_f \frac{dz}{dx} \int_{x_i}^{0} \left(1 + x\frac{v_{fg}}{v_f}\right) dx = -\frac{2f_{TP}}{D} G^2 v_f \frac{L}{x_i} \int_{x_i}^{0} \left(1 + x\frac{v_{fg}}{v_f}\right) dx$$

$$\Delta P_F = \frac{2f_{TP}}{D} G^2 v_f \frac{L}{x_i} \left(1 + \frac{x_i^2}{2}\frac{v_{fg}}{v_f}\right) = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + \frac{x_i}{2}\frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} \int_{x_i}^{0} dx = -G^2 v_f \frac{v_{fg}}{v_f} x_i = -G^2 v_f\, r_1$$

$$\Delta P_z = \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} \frac{dz}{dx} \int_{x_i}^{0} \frac{dx}{\left(1 + x\frac{v_{fg}}{v_f}\right)} = -\frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} \frac{L}{x_i} \int_{x_i}^{0} \frac{dx}{\left(1 + x\frac{v_{fg}}{v_f}\right)}$$

$$\Delta P_z = \frac{g \sin\theta\ L}{v_f} \ln\left(1 + x_i\frac{v_{fg}}{v_f}\right) \frac{v_f}{v_{fg}} = \frac{g \sin\theta\ L}{v_{fg}\, x_i} \ln\left(1 + x_o\frac{v_{fg}}{v_f}\right)$$

Pressure drop in a condenser tube with saturated liquid at outlet, assuming

$$f_{TP} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad \frac{dx}{dz} = \text{const.} = -\frac{x_i}{L}$$

$$\Delta P_F = \frac{2f_{TP}}{D} G^2 v_f \frac{dz}{dx} \int_{x_i}^{0} \left(1 + x\frac{v_{fg}}{v_f}\right) dx = -\frac{2f_{TP}}{D} G^2 v_f \frac{L}{x_i} \int_{x_i}^{0} \left(1 + x\frac{v_{fg}}{v_f}\right) dx$$

$$\Delta P_F = \frac{2f_{TP}}{D} G^2 v_f \frac{L}{x_i} \left(1 + \frac{x_i^2}{2}\frac{v_{fg}}{v_f}\right) = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + \frac{x_i}{2}\frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} \int_{x_i}^{0} dx = -G^2 v_f \frac{v_{fg}}{v_f} x_i = -G^2 v_f\, r_1$$

$$\Delta P_z = \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} \frac{dz}{dx} \int_{x_i}^{0} \frac{dx}{\left(1 + x\frac{v_{fg}}{v_f}\right)} = -\frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} \frac{L}{x_i} \int_{x_i}^{0} \frac{dx}{\left(1 + x\frac{v_{fg}}{v_f}\right)}$$

$$\Delta P_z = \frac{g \sin\theta\ L}{v_f} \ln\left(1 + x_i\frac{v_{fg}}{v_f}\right) \frac{v_f}{v_{fg}} = \frac{g \sin\theta\ L}{v_{fg}\, x_i} \ln\left(1 + x_o\frac{v_{fg}}{v_f}\right)$$

Now, later consider a condenser tube, in the condenser tube let us consider saturated liquid at the outlet, in the evaporator we considered saturated liquid at the inlet. Here let us consider that after condensing we get saturated liquid at the outlet and at the inlet it is a two phase mixture with a certain quality let us inlet quality x i. So, again we assume f T P equal to constant and v f g by v f equal to constant, d x by d z will be constant by simple energy balance. And it will be equal to minus x i by L, m square we have already assumed to be negligible.

So, now we can do the integration and in the frictional pressure drop we get the integral of x i to 0, 1 plus x v f g by v f d x. And after integration we get the final expression as 2 f T P L by D G square v f, 1 plus x i by 2 v f g by v f. The pressure gradient due to acceleration is also similar to the case of evaporator except that in this case the quality gradient is negative.

So, therefore, we get a minus sign and we get G square v f v f g by v f x i, the pressure gradient pressure drop due to gravity is also similar to that for the case of evaporator. And we get the integral of x i to 0 d x upon 1 plus x v f g by v f and after integration we get a logarithmic expression.

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$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_i}{2} \frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = -G^2 v_f \frac{v_{fg}}{v_f} x_o$$

$$\Delta P_z = \frac{g \sin\theta \ L}{v_{fg} \, x_i} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right)$$

$$\Delta P = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_i}{2} \frac{v_{fg}}{v_f}\right) - G^2 v_f \frac{v_{fg}}{v_f} x_o + \frac{g \sin\theta \ L}{v_{fg} \, x_i} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right)$$

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_i}{2} \frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = -G^2 v_f \frac{v_{fg}}{v_f} x_o$$

$$\Delta P_z = \frac{g \sin\theta \ L}{v_{fg} \ x_i} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right)$$

$$\Delta P = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_i}{2} \frac{v_{fg}}{v_f}\right) - G^2 v_f \frac{v_{fg}}{v_f} x_o + \frac{g \sin\theta \ L}{v_{fg} \ x_i} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right)$$

So, finally, we get the pressure drop as delta P F plus delta P a plus delta P z, where delta P F is equal to 2 f T P L by D G square v f 1 plus x i by 2 v f g by v f. Delta P a is equal to G square v f v f g by v f x i this should be x I, here also x i and here also x i ok. So, the pressure drop due to acceleration is negative in this case; that means, there is pressure rise due to acceleration ok.

Due to friction there is always pressure drop, because friction always opposes motion relative motion and due to acceleration in this case; in this case it is actually deceleration rather than acceleration. And therefore, there is pressure rise due to that, due to gravity whether there is pressure rise or pressure drop or whether delta P z is 0 it will depend on whether the tube is horizontal or vertical, vertically upward flow, downward flow or inclined upward, or inclined downward ok, it will depend on theta. So, the total pressure drop we can get by adding all these terms this is also x i ok.

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Pressure drop for two-phase flow in an adiabatic pipe, assuming

$f_{TP} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad x = \text{const.}$

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \int_0^L \frac{2 f_{TP}}{D} G^2 v_f \left(1 + x \frac{v_{fg}}{v_f}\right) dz = \frac{2 f_{TP}}{D} G^2 v_f \left(1 + x \frac{v_{fg}}{v_f}\right) \int_0^L dz$$

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + x \frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2 v_{fg} (0) dz = 0$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \int_0^L \frac{g \sin\theta}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)} dz = \frac{g \sin\theta}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)} \int_0^L dz$$

$$\Delta P_z = \frac{g \sin\theta \ L}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)}$$

Pressure drop for two-phase flow in an adiabatic pipe, assuming

$$f_{TP} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad x = \text{const.}$$

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \int_0^L \frac{2f_{TP}}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) dz = \frac{2f_{TP}}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) \int_0^L dz$$

$$\Delta P_F = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2 v_{fg}(0) dz = 0$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \int_0^L \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} dz = \frac{g \sin\theta}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)} \int_0^L dz$$

$$\Delta P_z = \frac{g \sin\theta \;\; L}{v_f \left(1 + x\frac{v_{fg}}{v_f}\right)}$$

So, we have considered pressure drop in a evaporator tube and a condenser tube, now let us consider two phase flow in an adiabatic pipe. In an adiabatic pipe there will be no change in quality because we are neither heating it or cooling it; there can we change in quality due to higher order effects which we are not considering right now. So, here we will assume that the quality is constant and then we also assume f T P equal to constant and v f g by v f equal to constant.

So, the frictional pressure drop is equal to the integral of the pressure frictional pressure gradient. And here we see that everything comes out of the integral sign and we are left with only the integral of 0 to L d z which is nothing but the length of the tube L. So, we get delta P F as 2 f T P L by D G square v f in the bracket 1 plus x v f g by v f.

The acceleration pressure drop will be equal to 0, because the acceleration pressure gradient is 0 there is no acceleration, no deceleration, so delta P a is equal to 0. The gravitational pressure drop is equal to the integral of the gravitational pressure gradient and everything comes out of the integral sign and we have only 0 to L integral d z which is equal to L. So, we get delta P z equal to G sin theta L upon v f 1 plus x v f g by v f.

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$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + x \frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = 0$$

$$\Delta P_z = \frac{g \sin\theta \ L}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)}$$

$$\Delta P = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + x \frac{v_{fg}}{v_f}\right) + \frac{g \sin\theta \ L}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)}$$

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + x \frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = 0$$

$$\Delta P_z = \frac{g \sin\theta \ L}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)}$$

$$\Delta P = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + x \frac{v_{fg}}{v_f}\right) + \frac{g \sin\theta \ L}{v_f \left(1 + x \frac{v_{fg}}{v_f}\right)}$$

So, we have delta P equal to delta P F plus delta P a plus delta P z and delta P F is equal to 2 f T P L by D G square v f 1 plus x v f g by v f delta P a is equal to 0. And delta P z is equal to g sin theta L upon v f 1 plus x v f g by v f and by adding all these 3 we get the total pressure drop.

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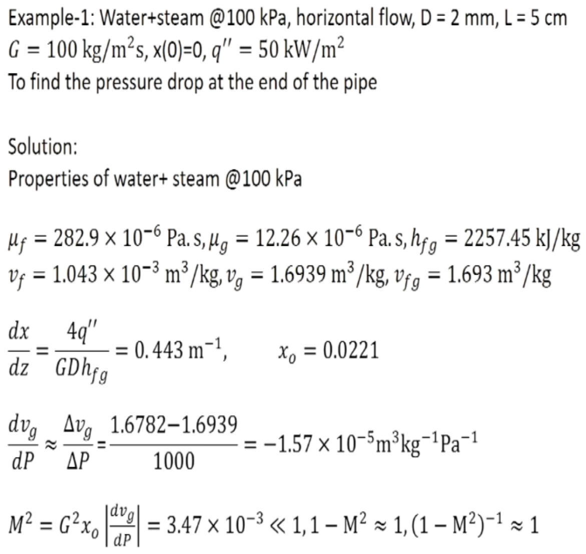

Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 5 cm, $G = 100\ \mathrm{kg/m^2 s}$, x(0)=0, $q'' = 50\ \mathrm{kW/m^2}$. To find the pressure drop at the end of the pipe.

Solution: Properties of water+ steam @100 kPa

$$\mu_f = 282.9 \times 10^{-6}\ \mathrm{Pa.s},\ \mu_g = 12.26 \times 10^{-6}\ \mathrm{Pa.s},\ h_{fg} = 2257.45\ \mathrm{kJ/kg}$$

$$v_f = 1.043 \times 10^{-3}\ \mathrm{m^3/kg},\ v_g = 1.6939\ \mathrm{m^3/kg},\ v_{fg} = 1.693\ \mathrm{m^3/kg}$$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.\ 443\ \mathrm{m^{-1}}, \quad x_o = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782-1.6939}{1000} = -1.57 \times 10^{-5} \mathrm{m^3 kg^{-1} Pa^{-1}}$$

$$M^2 = G^2 x_o \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1-\mathrm{M}^2)^{-1} \approx 1$$

Now, let us consider some numerical examples, so example 1 is the same as the example 1 considered previously for calculating pressure gradients using different models. So, now, we will consider the same example for calculating pressure drop, here we have taken the length of the tube as 5 centimeter earlier it was 10 centimeter.

So, the earlier we were calculating the pressure gradient at z equal to 5 centimeter. Now, we have taken the length as 5 centimeter and we want to calculate the pressure drop from the inlet to the outlet of the pipe the other data is the same ok. So, the properties are same as before and d x by d z is also the same as before the outlet quality is 0.0221 and d v g by d P is the same as before. So, M square is negligible and 1 minus M square is nearly equal to 1, so we can use the expressions which we have discussed which we have derived.

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 1052 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.01521$$

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) = 0.00793\ kPa$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} x_o = 0.374\ kPa$$

$$\Delta P_z = \frac{g \sin\theta\ L}{v_{fg} x_o} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right) = 0\ kPa$$

$$\Delta P = 0.00793 + 0.374 + 0 = 0.382\ \text{kPa}$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 1052 \Rightarrow \text{Laminar flow}$$

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$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} x_o = 0.374\ kPa$$

$$\Delta P_z = \frac{g \sin\theta\ L}{v_{fg} x_o} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right) = 0\ kPa$$

$$\Delta P = 0.00793 + 0.374 + 0 = 0.382\ \text{kPa}$$

Now, the mean viscosity is calculated using McAdams relation and we get R e T P equal to 1052 and it is laminar flow. So, we use the laminar flow friction factor which is 0.01521 and when substitute in the expression and we get delta P F equal to 0.00793 kilo Pascal.

Delta P a, we substitute values in the expression and we get 0.374 kilo Pascal, and delta P z will be 0 because it is a horizontal pipe in this case. So, by adding these pressure drops we get the total pressure drop as 0.382 kilo Pascal.

Now, example 2, you remember that in example 2 M square was not negligible the effect of the compressibility of the vapor phase was not negligible, so therefore, we will not considered that example. If we take that example then we cannot neglect M square and then there will be 1 minus M square in the denominator for every pressure drop term and then we will have to integrate numerically. So, we will have to write a code and then do numerical integration, so it cannot be solved by hand. So, therefore, we will not consider example 2, but we will consider an example 3.

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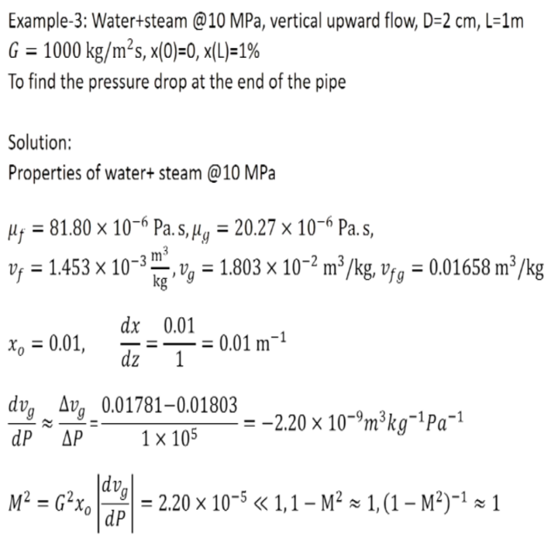

Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=1m, $G = 1000\ \mathrm{kg/m^2s}$, x(0)=0, x(L)=1%. To find the pressure drop at the end of the pipe

Solution: Properties of water+ steam @10 MPa

$$\mu_f = 81.80 \times 10^{-6}\ \mathrm{Pa.s},\ \mu_g = 20.27 \times 10^{-6}\ \mathrm{Pa.s},$$

$$v_f = 1.453 \times 10^{-3}\,\frac{\mathrm{m^3}}{\mathrm{kg}},\ v_g = 1.803 \times 10^{-2}\ \mathrm{m^3/kg},\ v_{fg} = 0.01658\ \mathrm{m^3/kg}$$

$$x_o = 0.01,\ \ \frac{dx}{dz} = \frac{0.01}{1} = 0.01\ \mathrm{m^{-1}}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781-0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x_o \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1 - \mathrm{M}^2)^{-1} \approx 1$$

So, in example 3, all the data is as before except that the length of the pipe here is 1 meter and the outlet quality is 1 percent. The properties are same as before outlet quality x o is equal to 0.01 and d x by d z is equal to 0.01 per meter. So, d v g by d P is calculated and then M square is calculated which is much less than 1. So, therefore, 1 minus M square is approximately equal to 1 and we can use the expressions derived for the pressure drops.

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 79.4 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 2.52 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 3.53 \times 10^{-3}$$

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) = 0.512\ kPa$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} x_o = 0.165\ kPa$$

$$\Delta P_z = \frac{g \sin\theta\ L}{v_{fg} x_o} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right) = 6.39\ kPa$$

$$\Delta P = 0.512 + 0.165 + 6.39 = 7.067\ \text{kPa}$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 79.4 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 2.52 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 3.53 \times 10^{-3}$$

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) = 0.512\ kPa$$

$$\Delta P_a = G^2 v_f \frac{v_{fg}}{v_f} x_o = 0.165\ kPa$$

$$\Delta P_z = \frac{g \sin\theta\ L}{v_{fg} x_o} \ln\left(1 + x_o \frac{v_{fg}}{v_f}\right) = 6.39\ kPa$$

$$\Delta P = 0.512 + 0.165 + 6.39 = 7.067 \text{ kPa}$$

The mean viscosity is calculated and then the Reynolds number for the two phase flow is calculated which is 2.52 into 10 raise to 5. So, therefore, it is turbulent flow and then we use the Blasius relation to calculate the friction factor for turbulent flow in smooth pipe and we get 3.53 into 10 raise to minus 3. And then we substitute the values in the expression for delta P F and we get 0.512 kilo Pascal.

In delta P a also we substitute the values and get 0.165 kilo Pascal and delta P z after substituting the values and calculating we get 6.39 kilo Pascal. Then after adding these pressure drops we get the total pressure drop as 7.067 kilo Pascal ok. Now let us see how to calculate two phase pressure drop using the separated flow model.

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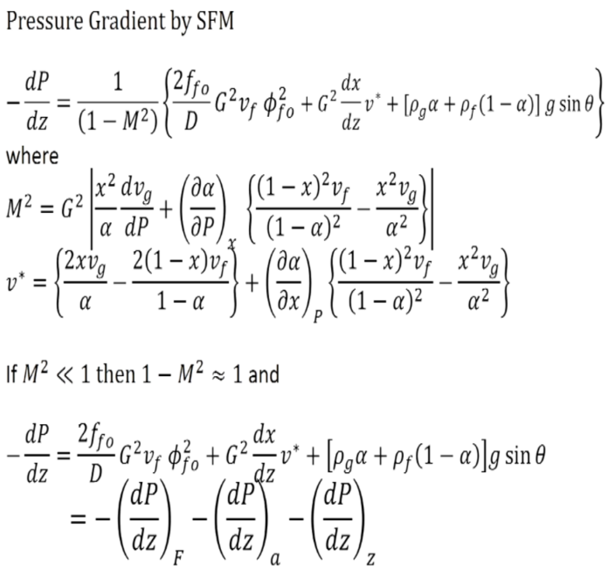

Pressure Gradient by SFM

$$-\frac{dP}{dz} = \frac{1}{(1-M^2)}\left\{\frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 + G^2 \frac{dx}{dz} v^* + [\rho_g \alpha + \rho_f(1-\alpha)]\, g \sin\theta\right\}$$

where

$$M^2 = G^2 \left|\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right|$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

If $M^2 \ll 1$ then $1 - M^2 \approx 1$ and

$$-\frac{dP}{dz} = \frac{2f_{fo}}{D} G^2 v_f \, \phi_{fo}^2 + G^2 \frac{dx}{dz} v^* + \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta$$

$$= -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

So, pressure gradient by separated flow model is given by this expression minus d P by d z is equal to 1 upon 1 minus M square in the bracket 2 f f o by D G square v f phi f o square plus G square d x by d z v star plus rho g alpha plus rho f 1 minus alpha g sin theta. Here, M square is an expression which represents the compressibility of the vapor phase and v star is a short hand for an expression which is given here.

So, if M square is much less than 1 then again we can consider 1 minus M square to be approximately equal to 1. And in case m square is not negligible then we have to retain 1 minus M square in the denominator and in that case we will have to do numerical integration. And we cannot derive any simple expressions, so we will consider only the case where M square is negligible.

Now, by neglecting M square we get minus d P by d z is equal to this expression which was in the bracket. And we identify the first term as the pressure gradient due to friction; the second term is pressure gradient due to acceleration. And the third term is the pressure gradient due to gravity and by integrating these pressure gradients we will get the pressure drops.

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Pressure drop in an evaporator tube with saturated liquid at inlet

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \int_0^L \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 dz = \int_0^{x_o} \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 \frac{dz}{dx} dx$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2 \frac{dx}{dz} v^*\, dz = \int_0^{x_o} G^2\, v^*\, dx$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \int_0^L \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta\, dz$$

$$\Delta P_z = \int_0^{x_o} \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta \frac{dz}{dx} dx$$

Pressure drop in an evaporator tube with saturated liquid at inlet

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \int_0^L \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 dz = \int_0^{x_o} \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 \frac{dz}{dx} dx$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2 \frac{dx}{dz} v^*\, dz = \int_0^{x_o} G^2\, v^*\, dx$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \int_0^L \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta\, dz$$

$$\Delta P_z = \int_0^{x_o} \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta \frac{dz}{dx} dx$$

So, now consider pressure drop in an evaporator tube with saturated liquid at the inlet. The total pressure drop will be equal to delta P F plus delta P a plus delta P z and delta P F is the integral of the frictional pressure gradient and we substitute the expression and convert from d z to d x. So, we get integral 0 to x o 2 f f o by D G square v f phi f o square d z by d x d x, delta P a is the integral of d P by d z a and we substitute the expression, so, we get 0 to x o integral of g square v star d x.

Then delta P z is the integral of d P by d z and after substituting the expression and converting to d x we get 0 to x o rho g alpha plus rho f 1 minus alpha g sin theta d z by d x d x. Now, we will make some assumptions, so that we can do the integration analytically.

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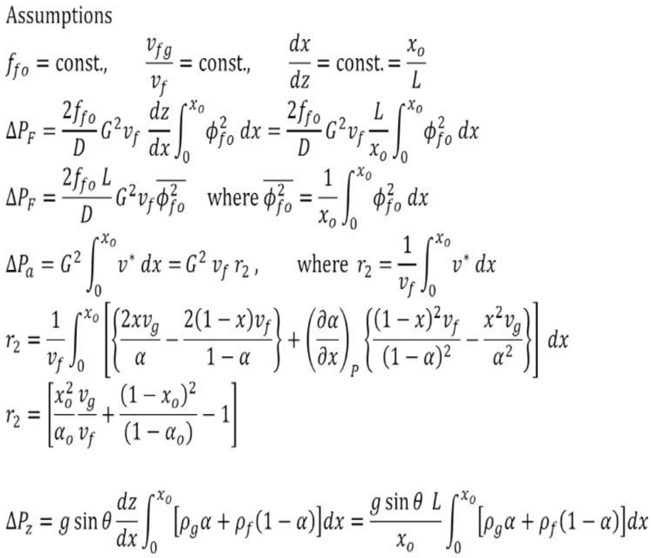

Assumptions

$$f_{fo} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad \frac{dx}{dz} = \text{const.} = \frac{x_o}{L}$$

$$\Delta P_F = \frac{2f_{fo}}{D} G^2 v_f \frac{dz}{dx} \int_0^{x_o} \phi_{fo}^2 \, dx = \frac{2f_{fo}}{D} G^2 v_f \frac{L}{x_o} \int_0^{x_o} \phi_{fo}^2 \, dx$$

$$\Delta P_F = \frac{2f_{fo} L}{D} G^2 v_f \overline{\phi_{fo}^2} \quad \text{where } \overline{\phi_{fo}^2} = \frac{1}{x_o} \int_0^{x_o} \phi_{fo}^2 \, dx$$

$$\Delta P_a = G^2 \int_0^{x_o} v^* \, dx = G^2 v_f r_2, \quad \text{where } r_2 = \frac{1}{v_f} \int_0^{x_o} v^* \, dx$$

$$r_2 = \frac{1}{v_f} \int_0^{x_o} \left[\left\{ \frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha} \right\} + \left(\frac{\partial \alpha}{\partial x} \right)_P \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right] dx$$

$$r_2 = \left[\frac{x_o^2}{\alpha_o} \frac{v_g}{v_f} + \frac{(1-x_o)^2}{(1-\alpha_o)} - 1 \right]$$

$$\Delta P_z = g \sin\theta \frac{dz}{dx} \int_0^{x_o} \left[\rho_g \alpha + \rho_f (1-\alpha) \right] dx = \frac{g \sin\theta \ L}{x_o} \int_0^{x_o} \left[\rho_g \alpha + \rho_f (1-\alpha) \right] dx$$

So, we will assume f f o equal to constant over the length and v f g by v f is equal to constant and d x by d z is equal to constant which is equal to x o upon L. So, most of the quantities will come out of the integration and we are left with the integral of 0 to x o phi f o square d x. And d z by d x is equal to L upon x o and then we define phi f o square bar that is the average phi f o square as 1 upon x o 0 to x o phi f o square d x.

So, then we get delta P f is equal to 2 f f o L by D G square v f phi f o square bar. Delta P a is equal to g square integral 0 to x o v star d x and the integral of v star d x upon v f is called r 2, so we get delta P a is equal to G square v f r 2. The expression for r 2 after integrating v star we get this, x square upon alpha o v g by v f plus 1 minus x o whole square upon 1 minus alpha o minus 1. You can do this integration as c check whether you get the same expression. Now, delta P z here we get g sin theta L upon x o integral 0 to x o rho g alpha plus rho f 1 minus alpha d x.

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$$\Delta P_z = \frac{g\sin\theta\ L}{x_o}\int_0^{x_o}\bar{\rho}dx = g\sin\theta\ L\,\bar{\bar{\rho}}, \text{where } \bar{\bar{\rho}} = \frac{1}{x_o}\int_0^{x_o}[\rho_g\alpha + \rho_f(1-\alpha)]dx$$

$$\bar{\bar{\rho}} = \rho_f - \frac{(\rho_f-\rho_g)}{x_o}\int_0^{x_o}\alpha\,dx = \rho_f - (\rho_f-\rho_g)\,\bar{\alpha}, \text{where } \bar{\alpha} = \frac{1}{x_o}\int_0^{x_o}\alpha\,dx$$

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2f_{fo}\,L}{D}G^2 v_f\,\overline{\phi_{fo}^2}$$

$$\Delta P_a = G^2\,v_f\,r_2$$

$$\Delta P_z = g\sin\theta\ L\,\bar{\bar{\rho}}$$

$$\Delta P = \frac{2f_{fo}\,L}{D}G^2 v_f\,\overline{\phi_{fo}^2} + G^2\,v_f\,r_2 + g\sin\theta\ L\,\bar{\bar{\rho}}$$

$$\Delta P_z = \frac{g\sin\theta\ L}{x_o}\int_0^{x_o}\bar{\rho}dx = g\sin\theta\ L\,\bar{\bar{\rho}}, \text{ where } \bar{\bar{\rho}} = \frac{1}{x_o}\int_0^{x_o}[\rho_g\alpha + \rho_f(1-\alpha)]dx$$

$$\bar{\bar{\rho}} = \rho_f - \frac{(\rho_f-\rho_g)}{x_o}\int_0^{x_o}\alpha\,dx = \rho_f - (\rho_f-\rho_g)\,\bar{\alpha}, \text{ where } \bar{\alpha} = \frac{1}{x_o}\int_0^{x_o}\alpha\,dx$$

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{fo}\, L}{D} G^2 v_f\, \overline{\phi_{fo}^2}$$

$$\Delta P_a = G^2\, v_f\, r_2$$

$$\Delta P_z = g \sin\theta\;\, L\, \bar{\bar{\rho}}$$

$$\Delta P = \frac{2 f_{fo}\, L}{D} G^2 v_f\, \overline{\phi_{fo}^2} + G^2\, v_f\, r_2 + g \sin\theta\;\, L\, \bar{\bar{\rho}}$$

So, this expression within the integral is actually an average density the weighted average density and we call at rho bar. This expression we are calling rho bar and then the integral of rho bar over 0 to x o we are calling it as rho double bar. So, rho double bar is equal to rho f minus rho f minus rho g upon x o integral of 0 to x o alpha d x. And then we call the average void fraction as alpha bar equal to 1 upon x o 0 to x o alpha d x. So, rho double bar is equal to rho f minus in the bracket rho f minus rho g into alpha bar, so if we can calculate this alpha bar then we can calculate delta P z.

So, finally, we have delta P is equal to delta P F plus delta P a plus delta P z where delta P F is equal to 2 f f o L by D G square v f phi f o square bar. Delta P a is equal to G square v f r 2 and delta P z is equal to g sin theta L rho double bar and then by adding these 3 pressure drops we get the total pressure drop delta P. Now, if we can find phi f o square bar r 2 and rho double bar then we should be able to find the pressure drop. So, with separated flow model we will use the Martinelli and Nelson correlation and then evaluate these pressure drops.

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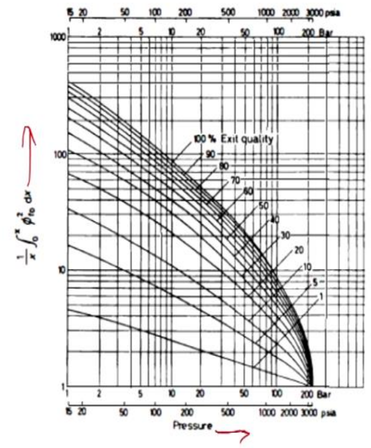

We have seen some graph given by Martinelli and Nelson they had given some more graphs they had plotted this phi f o square bar. So, here we have pressure on the x axis, on the y axis we have phi f o square bar and for knowing the pressure and the exit quality we can find phi f o square bar from this figure. The scales are logarithmic both scales are logarithmic it is a log log plot ok. So, we can find phi f o square bar from this graph and then substitute and get the frictional pressure drop delta P F.

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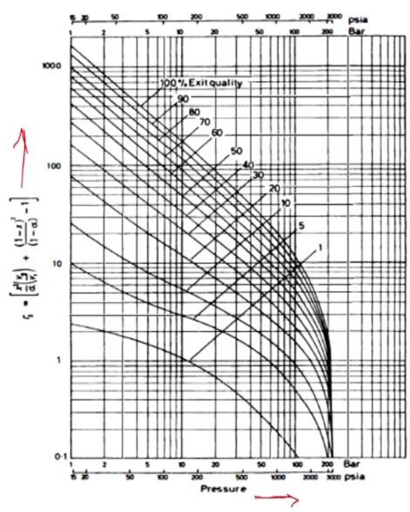

For delta P a we need r 2 Martinelli and Nelson had plated r 2 also, here pressure is on the horizontal axis and r 2 is on the vertical axis and for different pressures. And different exit qualities we can find r 2 and here also the scales are logarithmic it is a log log plot. Using this we can find r 2 and substitute and get the acceleration pressure drop delta P a.

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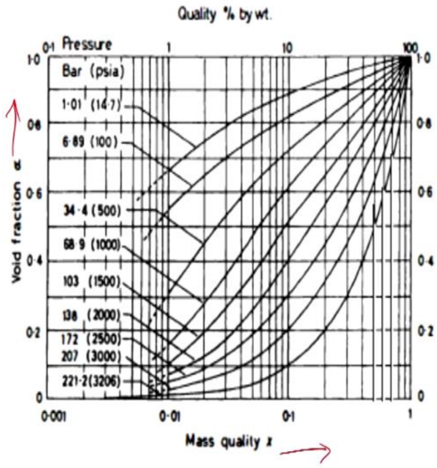

Now, to calculate delta P z we need rho double bar and then to calculate rho double bar we need alpha bar the average void fraction over the length, but there is no graph which directly gives alpha bar. So, we have to find alpha at different points at different locations on the pipe from the inlet to the outlet and then integrate from inlet to the outlet from x equal to 0 to x equal to x o we have to integrate.

So, as a simple approximation we can take the average quality which is the quality at the midpoint and then find the void fraction for that quality. And then assume that alpha bar is approximately equal to that quality that void fraction. Here the scale one scale is logarithmic this the horizontal scale is logarithmic and the vertical scale for alpha is linear.

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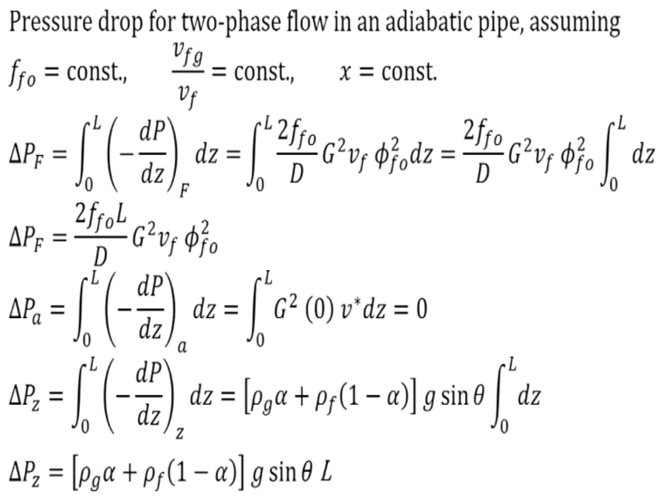

Pressure drop for two-phase flow in an adiabatic pipe, assuming

$$f_{fo} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad x = \text{const.}$$

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \int_0^L \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 dz = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 \int_0^L dz$$

$$\Delta P_F = \frac{2f_{fo}L}{D} G^2 v_f\, \phi_{fo}^2$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2\ (0)\ v^* dz = 0$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta \int_0^L dz$$

$$\Delta P_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta\ L$$

Now, let us consider pressure drop for two phase flow in an adiabatic pipe. So, we assume f f o equal to constant, v f g by v f equal to constant and since it is an adiabatic pipe and we are not considering higher order effects therefore, x will also be equal to constant. So, everything comes out of the integral sign in the expression for delta P F and we are left with only integral 0 to L d z and this is nothing but L.

So, therefore, we get delta P F equal to 2 f f o L by D G square v f phi f o square and phi f o square we can get from the graph given by Martinelli and Nelson. Delta P a because that d P by d z a is 0, so delta P a will also be equal to 0. Delta P z here also everything comes out of the integral and we have only 0 to L integral d z which is equal to L therefore, delta P z is equal to rho g alpha plus rho f 1 minus alpha g sin theta L. So, by finding alpha we can find delta P z and alpha we can find from Martinelli and Nelson graphs.

(Refer Slide Time: 36:37)

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{fo} L}{D} G^2 v_f \, \phi_{fo}^2$$

$$\Delta P_a = 0$$

$$\Delta P_z = \left[\rho_g \alpha + \rho_f (1 - \alpha)\right] g \sin\theta \; L$$

$$\Delta P = \frac{2 f_{fo} L}{D} G^2 v_f \, \phi_{fo}^2 + \left[\rho_g \alpha + \rho_f (1 - \alpha)\right] g \sin\theta \; L$$

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \frac{2 f_{fo} L}{D} G^2 v_f \, \phi_{fo}^2$$

$$\Delta P_a = 0$$

$$\Delta P_z = \left[\rho_g \alpha + \rho_f (1 - \alpha)\right] g \sin\theta \; L$$

$$\Delta P = \frac{2 f_{fo} L}{D} G^2 v_f \, \phi_{fo}^2 + \left[\rho_g \alpha + \rho_f (1 - \alpha)\right] g \sin\theta \; L$$

So, delta P is equal to delta P F plus delta P a plus delta P z, where delta P F is equal to 2 f f o L by D G square v f phi f o square. Delta P a is equal to 0 and delta P z is equal to rho g alpha plus rho f 1 minus alpha g sin theta L.

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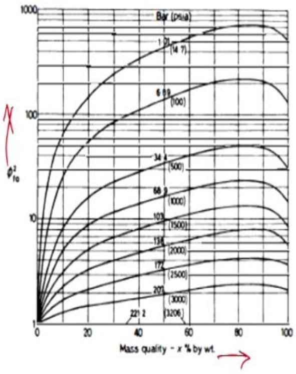

	Pressure, bar (psia)								
Steam quality % by wt.	1.01 (14.7)	6.89 (100)	34.4 (500)	68.9 (1000)	103 (1500)	138 (2000)	172 (2500)	207 (3000)	221.2 (3206)
1	5.6	3.5	1.8	1.6	1.35	1.2	1.1	1.05	1.00
5	30	15	5.3	3.6	2.4	1.75	1.43	1.17	1.00
10	69	28	8.9	5.4	3.4	2.45	1.75	1.30	1.00
20	150	56	16.2	8.6	5.1	3.25	2.19	1.51	1.00
30	245	83	23.0	11.6	6.8	4.04	2.62	1.68	1.00
40	350	115	29.2	14.4	8.4	4.82	3.02	1.83	1.00
50	450	145	34.9	17.0	9.9	5.59	3.38	1.97	1.00
60	545	174	40.0	19.4	11.1	6.34	3.70	2.10	1.00
70	625	199	44.6	21.4	12.1	7.05	3.96	2.23	1.00
80	685	216	48.6	22.9	12.8	7.70	4.15	2.35	1.00
90	720	210	48.0	22.3	13.0	7.95	4.20	2.38	1.00
100	525	130	30.0	15.0	8.6	5.90	3.70	2.15	1.00

Phi f o square we can find from this Martinelli-Nelson graph here we have quality and phi f o square for different pressures and there is a table also which gives the two phase multiplier phi f o square for different pressures and different qualities, so using this we can find delta P F. Now, for delta P z we need alpha the void fraction and that we can find from this graph for any given pressure and given quality we can find alpha from this graph and then we can find delta P z.

(Refer Slide Time: 37:55)

Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 5 cm

$G = 100\ \text{kg/m}^2\text{s}$, x(0)=0, $q'' = 50\ \text{kW/m}^2$

To find the pressure drop at the end of the pipe

Solution:

Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}\ \text{Pa.s}, \mu_g = 12.26 \times 10^{-6}\ \text{Pa.s}, h_{fg} = 2257.45\ \text{kJ/kg}$

$v_f = 1.043 \times 10^{-3}\ \text{m}^3/\text{kg}, v_g = 1.6939\ \text{m}^3/\text{kg}, v_{fg} = 1.693\ \text{m}^3/\text{kg}$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.443\ \text{m}^{-1}, \qquad x_o = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782 - 1.6939}{1000} = -1.57 \times 10^{-5}\text{m}^3\text{kg}^{-1}\text{Pa}^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1, 1 - \text{M}^2 \approx 1, (1 - \text{M}^2)^{-1} \approx 1$$

Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 5 cm, $G = 100\ \mathrm{kg/m^2 s}$, x(0)=0, $q'' = 50\ \mathrm{kW/m^2}$. To find the pressure drop at the end of the pipe

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Now, let us consider examples, in example 1 we have the same data as before except that the length is 5 centimeter here. And the property is at the same as before we get d x by d z and x o and then d v g by d P and then we find m square which is negligible.

(Refer Slide Time: 38:33)

$$Re_{f,o} = \frac{GD}{\mu_f} = 707 \Rightarrow \text{Laminar flow}$$

$$f_{f,o} = 16/Re = 0.0226$$

$$\overline{\phi_{fo}^2} = 5.2 \quad r_2 = 2$$

$$\Delta P_F = \frac{2 f_{fo}\, L}{D} G^2 v_f\, \overline{\phi_{fo}^2} = 0.0613\ kPa$$

$$\Delta P_a = G^2\, v_f\, r_2 = 0.0209\ kPa$$

$$\Delta P_z = 0$$

$$\Delta P = 0.0613 + 0.0209 + 0 = 0.0822\ \mathrm{kPa}$$

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R e f o is equal to 707, so it is laminar flow and f f o is equal to 0.0226. So, phi f o square bar we get from the Martinelli and Nelson graph for the given pressure and outlet quality we know the pressure and outlet quality. And then we get phi f o square approximately 5.2 from the graph and for the same given pressure and given outlet quality we get r 2 is approximately equal to 2.

So, we substitute these values and get delta P f equal to 0.0613 kilo Pascal, and delta P a is equal to 0.0209 kilo Pascal. Delta P z is equal to 0 because it is a horizontal pipe and the total pressure drop delta P is calculated as 0.0822 kilo Pascal.

(Refer Slide Time: 39:49)

Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=1 m

$G = 1000\ \text{kg/m}^2\text{s}$, x(0)=0, x(L)=1%

To find the pressure drop at the end of the pipe

Solution:

Properties of water+ steam @10 MPa

$\mu_f = 81.80 \times 10^{-6}$ Pa. s, $\mu_g = 20.27 \times 10^{-6}$ Pa. s,

$v_f = 1.453 \times 10^{-3}\text{m}^3/\text{kg}$, $v_g = 1.803 \times 10^{-2}\ \text{m}^3/\text{kg}$, $v_{fg} = 0.01658\ \text{m}^3/\text{kg}$

$$x_o = 0.01, \qquad \frac{dx}{dz} = \frac{0.01}{1} = 0.01\ \text{m}^{-1}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781 - 0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1, 1 - \text{M}^2 \approx 1, (1 - \text{M}^2)^{-1} \approx 1$$

$$Re_{fo} = \frac{GD}{\mu_f} = 2.44 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{fo} = 0.079\,Re_{fo}^{-0.25} = 3.55 \times 10^{-3}$$

Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=1 m, $G = 1000\ \mathrm{kg/m^2 s}$, x(0)=0, x(L)=1%. To find the pressure drop at the end of the pipe

Solution: Properties of water+ steam @10 MPa

$$\mu_f = 81.80 \times 10^{-6}\ \mathrm{Pa.s},\ \mu_g = 20.27 \times 10^{-6}\ \mathrm{Pa.s},$$

$$v_f = 1.453 \times 10^{-3}\mathrm{m^3/kg},\ v_g = 1.803 \times 10^{-2}\ \mathrm{m^3/kg},\ v_{fg} = 0.01658\ \mathrm{m^3/kg}$$

$$x_o = 0.01,\ \ \frac{dx}{dz} = \frac{0.01}{1} = 0.01\ \mathrm{m^{-1}}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781 - 0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1 - \mathrm{M}^2)^{-1} \approx 1$$

$$Re_{fo} = \frac{GD}{\mu_f} = 2.44 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{fo} = 0.079\ Re_{fo}^{-0.25} = 3.55 \times 10^{-3}$$

Now, we will not consider example 2 because there M square is not negligible and we will have to do numerical integration by writing a code. So, we will consider example 3, here the data is same as before except that the length is 1 meter and the outlet quality is 1 percent. So, x o is equal to 0.01 and d x by d z is equal to 0.01 per meter, then d v g by d P is calculated M square is calculated and it is very small. So, therefore, 1 minus M square can be taken has 1 R e f o is of the order of 10 raise to 5, so it is turbulent flow, and then we use the Blasius correlation to calculate the friction factor f f o.

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$$\overline{\phi_{fo}^2} = 1.3 \quad r_2 = 0.1 \qquad \bar{\alpha} \approx 0.05$$

$$\bar{\bar{\rho}} = \rho_f - (\rho_f - \rho_g)\bar{\alpha} = 656\ kg/m^3$$

$$\Delta P_F = \frac{2 f_{fo}\, L}{D} G^2 v_f\, \overline{\phi_{fo}^2} = 0.671\ kPa$$

$$\Delta P_a = G^2\, v_f\, r_2 = 0.145\ kPa$$

$$\Delta P_z = g \sin\theta\ \ L\, \bar{\bar{\rho}} = 6.43\ kPa$$

$$\Delta P = 0.0613 + 0.0209 + 6.43 = 6.51\ \text{kPa}$$

$$\overline{\phi_{fo}^2} = 1.3 \quad r_2 = 0.1 \qquad \int_0^{x_o} \alpha\, dx = 0$$

$$\bar{\bar{\rho}} = \rho_f - \frac{(\rho_f - \rho_g)}{x_o} \int_0^{x_o} \alpha\, dx = 688\ kg/m^3$$

$$\Delta P_F = \frac{2 f_{fo}\, L}{D} G^2 v_f\, \overline{\phi_{fo}^2} = 0.671\ kPa$$

$$\Delta P_a = G^2\, v_f\, r_2 = 0.145\ kPa$$

$$\Delta P_z = g \sin\theta\ \ L\, \bar{\bar{\rho}} = 6.74\ kPa$$

$$\Delta P = 0.0613 + 0.0209 + 6.74 = 6.82\ \text{kPa}$$

And then we use the Martinelli and Nelson graphs and for the given pressure and outlet quality we get phi f o square bar equal to 1.3 and r 2 equal to 0.1 and alpha bar is roughly estimated as 0.05. So, this has been done by assuming that alpha bar is approximately equal to the alpha corresponding to the quality at the midpoint of the pipe.

Rho double bar using this alpha bar it is calculated as 656 kg per meter cube which is only slightly less than the density of the liquid rho f. So, after substituting these values we get delta P F equal to 0.67 1 kilo Pascal delta P a is equal to 0.145 kilo Pascal and delta P z is equal to 6.43 kilo Pascal. And then by adding these pressure drops we get the total pressure drop delta P equal to 6.51 kilo Pascal. Now, consider the drift flux model and let us see how two phase pressure drop can be calculated using the drift flux model.

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Pressure Gradient by DFM

$$-\frac{dP}{dz} = \frac{1}{1-M^2}\left[\frac{2f_{TP}}{D}G^2\bar{v} + G^2\frac{dx}{dz}v^* + \left\{\rho_g\alpha + \rho_f(1-\alpha)\right\}g\sin\theta\right]$$

where

$$\alpha = \frac{j_g}{C_0\, j + V_{gj}} = \frac{Gxv_g}{C_0\left\{Gxv_g + G(1-x)v_f\right\} + V_{gj}}, \left(\frac{\partial\alpha}{\partial x}\right)_P = \frac{\alpha}{x} - \frac{\alpha^2 C_0 v_{fg}}{xv_g}$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$$M^2 = G^2\left|\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right|$$

If $M^2 \ll 1$ then $1 - M^2 \approx 1$ and

$$-\frac{dP}{dz} = \frac{2f_{TP}}{D}G^2\bar{v} + G^2\frac{dx}{dz}v^* + \left\{\rho_g\alpha + \rho_f(1-\alpha)\right\}g\sin\theta$$

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$$-\frac{dP}{dz} = \frac{1}{1-M^2}\left[\frac{2f_{TP}}{D}G^2\bar{v} + G^2\frac{dx}{dz}v^* + \left\{\rho_g\alpha + \rho_f(1-\alpha)\right\}g\,sin\,\theta\right]$$

where

$$\alpha = \frac{j_g}{C_0\, j + V_{gj}} = \frac{Gxv_g}{C_0\left\{Gxv_g + G(1-x)v_f\right\} + V_{gj}}, \left(\frac{\partial\alpha}{\partial x}\right)_P = \frac{\alpha}{x} - \frac{\alpha^2 C_0 v_{fg}}{xv_g}$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$$M^2 = G^2\left|\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right|$$

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If using drift flux model we calculate the frictional pressure gradient using the homogeneous model and the expression for the acceleration pressure gradient. And gravitational pressure gradient at the same as those for the separate at flow model, but alpha here will be calculated by using the drift flux model.

So, alpha is equal to j g upon C 0 j plus V g j and then if we substitute for j and j g and then we get this expression G x v g upon C 0 G x v g plus G 1 minus x v f plus V g j. And then we differentiate this with respect to x keeping pressure constant; that means, v g and v f are constant. So, we get deba alpha by deba x at constant pressure equal to alpha by x minus alpha square C 0 v g v f g upon x v g.

V star expression is given here and the expression of M square is also given here these are the same as goes for the separated flow model. And in case M square is much less than 1, we can neglect it and we can take 1 minus M square is approximately equal to 1. So, here we will assume that M square is negligible, so we get the total pressure gradient as 2 f T P by D G square v bar plus G square d x by d z v star plus rho g alpha plus rho f 1 minus alpha g sin theta ok.

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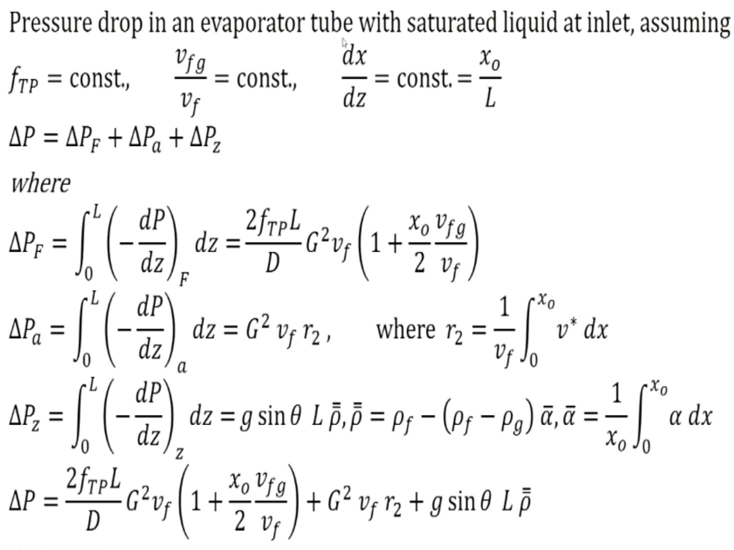


Pressure drop in an evaporator tube with saturated liquid at inlet, assuming

$$f_{TP} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad \frac{dx}{dz} = \text{const.} = \frac{x_o}{L}$$

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

where

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = G^2\, v_f\, r_2\,, \text{ where } r_2 = \frac{1}{v_f}\int_0^{x_o} v^*\, dx$$

$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = g \sin\theta\ L\,\bar{\bar{\rho}},\ \bar{\bar{\rho}} = \rho_f - \left(\rho_f - \rho_g\right)\bar{\alpha},\ \bar{\alpha} = \frac{1}{x_o}\int_0^{x_o} \alpha\, dx$$

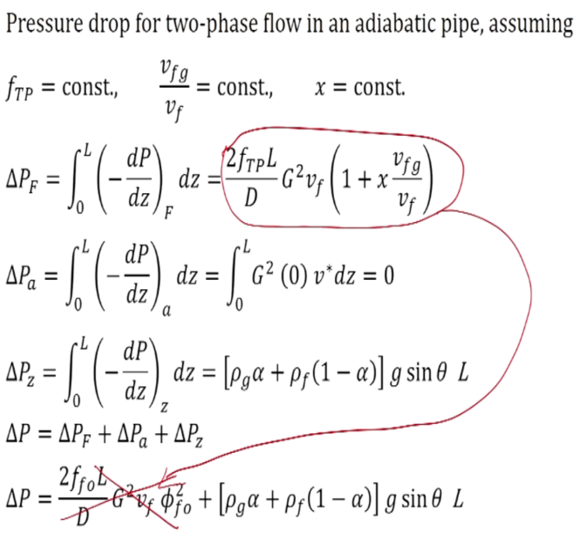
$$\Delta P = \frac{2f_{TP}L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) + G^2\, v_f\, r_2 + g \sin\theta\ L\,\bar{\bar{\rho}}$$

So, now let us consider an evaporator tube in which the inlet is saturated liquid and we will assume f T P equal to constant over the length of the pipe, v f g by v f is equal to constant and d x by d x equal to constant equal to x o upon L. So, the total pressure gradient delta P is equal to delta P F, plus delta P a, plus delta P z and here delta P F is the integral of the pressure gradient due to friction.

And we get this expression which is the same as that for the homogeneous model 2 f T P L by D G square v f 1 plus x o by 2 v f g by v f. And for the acceleration pressure drop we integrate the acceleration pressure gradient and get G square v f r 2 and where r 2 is equal to 1 by v f 0 to x o v star d x integral. Delta P z is the integral of d P by d z and this is equal to g sin theta L rho double bar and this is the same as what we got for the separated flow model.

So, the total pressure drop delta P is equal to 2 f T P L by D G square v f 1 plus x o by 2 v f g by v f plus G square v f r 2 plus g sin theta rho double bar. So, now to calculate this pressure drop we have to find r 2 for that we will have to find alpha and we have to find rho double bar for that also we have to find alpha.

(Refer Slide Time: 46:41)



Pressure drop for two-phase flow in an adiabatic pipe, assuming

$$f_{TP} = \text{const.}, \quad \frac{v_{fg}}{v_f} = \text{const.}, \quad x = \text{const.}$$

$$\Delta P_F = \int_0^L \left(-\frac{dP}{dz}\right)_F dz = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$\Delta P_a = \int_0^L \left(-\frac{dP}{dz}\right)_a dz = \int_0^L G^2\,(0)\, v^* dz = 0$$

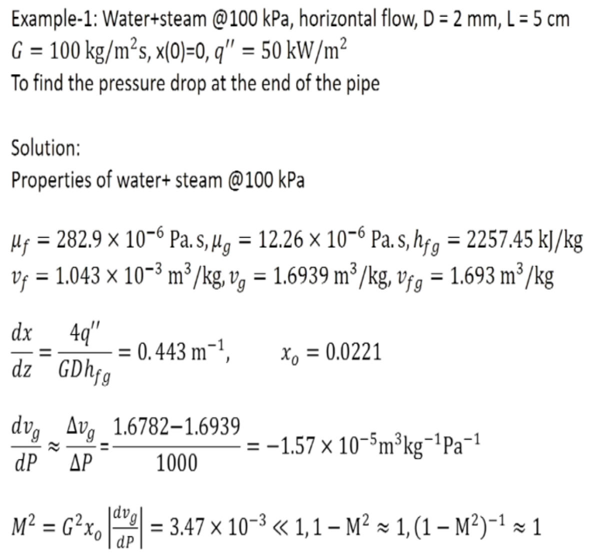
$$\Delta P_z = \int_0^L \left(-\frac{dP}{dz}\right)_z dz = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta\ L$$

$$\Delta P = \Delta P_F + \Delta P_a + \Delta P_z$$

$$\Delta P = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) + \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta\ L$$

So, now consider an adiabatic pipe and assume f T P equal to constant, v f g by v f equal to constant and x will be constant in this case. So, delta P F will be equal to 2 f T P L by D G square v f 1 plus x v f g by v f, delta P a will be 0 in this case and delta P z will be equal to rho g alpha plus rho f 1 minus alpha g sin theta L. And by adding these 3 we get delta P equal to 2 f f o L by D G square v f phi f o square, actually here we do not have to use phi f o square. So, instead of this expression we will use this expression ok and plus rho g alpha plus rho f 1 minus alpha g sin theta L.

(Refer Slide Time: 47:57)



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Solution: Properties of water+ steam @100 kPa

$$\mu_f = 282.9 \times 10^{-6}\ \mathrm{Pa.s},\ \mu_g = 12.26 \times 10^{-6}\ \mathrm{Pa.s},\ h_{fg} = 2257.45\ \mathrm{kJ/kg}$$

$$v_f = 1.043 \times 10^{-3}\ \mathrm{m^3/kg},\ v_g = 1.6939\ \mathrm{m^3/kg},\ v_{fg} = 1.693\ \mathrm{m^3/kg}$$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.\ 443\ \mathrm{m^{-1}}, \quad x_o = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782-1.6939}{1000} = -1.57 \times 10^{-5}\mathrm{m^3kg^{-1}Pa^{-1}}$$

$$M^2 = G^2 x_o \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1 - \mathrm{M}^2)^{-1} \approx 1$$

Now, let us consider some numerical examples, example 1 is the same as before except that the length of the pipe is 5 centimeter. And the property data is the same as before d x by d z is equal to 0.443 per meter, x o is equal to 0.0221 and d v g by d P is of the order of 10 raise to minus 5 s i units and M square is of the order of 10 raise to minus 3, so we can neglect it.

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 1052 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.01521$$

$$\alpha_o = 0.82$$

$$r_2 = \left[\frac{x_o^2}{\alpha_o}\frac{v_g}{v_f} + \frac{(1-x_o)^2}{(1-\alpha_o)} - 1\right] = 5.27$$

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) = 0.00793\ kPa$$

$$\Delta P_a = G^2\ v_f\ r_2 = 0.055\ kPa$$

$$\Delta P_z = 0$$

$$\Delta P = 0.00793 + 0.055 + 0 = 0.0629\ \text{kPa}$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s$$

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$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) = 0.00793\ kPa$$

$$\Delta P_a = G^2\ v_f\ r_2 = 0.055\ kPa$$

$$\Delta P_z = 0$$

$$\Delta P = 0.00793 + 0.055 + 0 = 0.0629\ \text{kPa}$$

The average viscosity is calculated using McAdams relation and R e T P is 1052, so it is laminar flow and f T P is equal to 0.01521. And alpha o is calculated using the outlet quality and using the outlet quality we calculate C 0 and V g j and then after substituting these values of C 0 and V g j we get alpha.

So, we get the value of alpha o as 0.82 and then we calculate r 2 by using this value of alpha o and it turns out to be equal to 5.27. And then we substitute the values and get delta P F equal to 0.00793 kilo Pascal delta, P a is equal to 0.055 kilo Pascal, delta P z is equal to 0. So, adding these pressure drops we get the total pressure drop as 0.0629 kilo Pascal.

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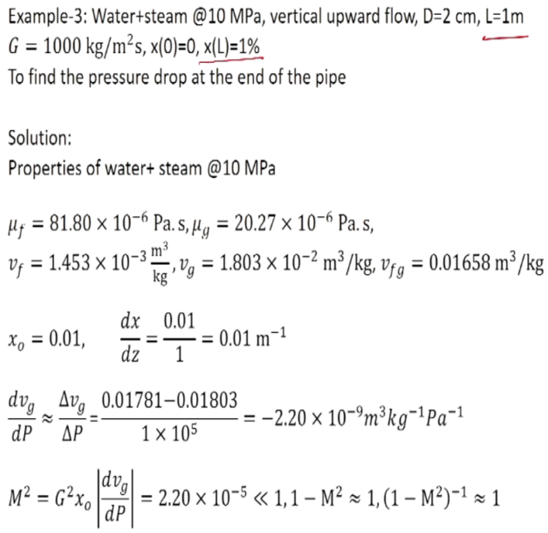

Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=1m, $G = 1000\ \mathrm{kg/m^2s}$, x(0)=0, x(L)=1%. To find the pressure drop at the end of the pipe

Solution: Properties of water+ steam @10 MPa

$$\mu_f = 81.80 \times 10^{-6}\ \mathrm{Pa.s},\ \mu_g = 20.27 \times 10^{-6}\ \mathrm{Pa.s},$$

$$v_f = 1.453 \times 10^{-3}\,\frac{\mathrm{m^3}}{\mathrm{kg}},\ v_g = 1.803 \times 10^{-2}\ \mathrm{m^3/kg},\ v_{fg} = 0.01658\ \mathrm{m^3/kg}$$

$$x_o = 0.01,\ \ \frac{dx}{dz} = \frac{0.01}{1} = 0.01\ \mathrm{m^{-1}}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781 - 0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x_o \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1 - \mathrm{M}^2)^{-1} \approx 1$$

Now, as before we will keep example 2, because M square is not negligible and then we have to do numerical integration by writing a code. So, we will consider example 3, in this the length is 1 meter and the outlet quality is 1 percent, the other data at the same as an example 3 we have considered before. The property data as the same outlet quality is 1 percent and d x by d z is 0.01 per meter d v g by d P is calculated and M square is calculated which is of the order of 10 raise to minus 5, so it is negligible, so 1 minus M square can be taken as 1.

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 79.4 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 2.52 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 3.53 \times 10^{-3}$$

$$\alpha_o = 0.0908, \qquad r_2 = \left[\frac{x_o^2}{\alpha_o}\frac{v_g}{v_f} + \frac{(1-x_o)^2}{(1-\alpha_o)} - 1\right] = 0.0916$$

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) = 0.512\ kPa$$

$$\Delta P_a = G^2\ v_f\ r_2 = 0.133\ kPa$$

$$\bar{\alpha} = \frac{1}{x_o}\int_0^{x_o} \alpha\ dx = 0.0476$$

$$\bar{\bar{\rho}} = \rho_f - (\rho_f - \rho_g)\ \bar{\alpha} = 658\ kg/m^3$$

$$\Delta P_z = g \sin\theta\ \ L\,\bar{\bar{\rho}} = 6.49\ kPa$$

$$\Delta P = 0.512 + 0.133 + 6.49 = 7.135\ \text{kPa}$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 79.4 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 2.52 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 3.53 \times 10^{-3}$$

$$\alpha_o = 0.0908, \qquad r_2 = \left[\frac{x_o^2}{\alpha_o}\frac{v_g}{v_f} + \frac{(1-x_o)^2}{(1-\alpha_o)} - 1\right] = 0.0916$$

$$\Delta P_F = \frac{2 f_{TP} L}{D} G^2 v_f \left(1 + \frac{x_o}{2}\frac{v_{fg}}{v_f}\right) = 0.512\ kPa$$

$$\Delta P_a = G^2\ v_f\ r_2 = 0.133\ kPa$$

$$\bar{\alpha} = \frac{1}{x_o}\int_0^{x_o} \alpha\ dx = 0.0476$$

$$\bar{\bar{\rho}} = \rho_f - (\rho_f - \rho_g)\ \bar{\alpha} = 658\ kg/m^3$$

$$\Delta P_z = g \sin\theta\ \ L\,\bar{\bar{\rho}} = 6.49\ kPa$$

$\Delta P = 0.512 + 0.133 + 6.49 = 7.135$ kPa

Then mu bar is calculated and R e T P is calculated and it is turbulent flow, so using Blasius correlation we get the friction factor and alpha o we get from the expression for the drift flux model. We get the C 0 and V g j and substitute and then get alpha o corresponding to x o which is 0.0908. And then we substitute it in order to and we get r 2 equal to 0.0916.

Then after substituting the values we get delta P F equal to 0.512 kilo Pascal, delta P a is equal to 0.133 kilo Pascal, and alpha bar is the average alpha and that is equal to 0.0476. And this we have taken as the void fraction corresponding to the average quality, average quality is taken as the quality at the midpoint which is half of the outlet quality. And corresponding to that the void fraction alpha has been calculated using the drift flux model and we get alpha bar equal to 0.0476.

Then we substitute this alpha bar and get rho double bar equal to 658 kg per meter cube and then by substituting it we get delta P z equal to 6.49 kilo Pascal, so the total pressure drop by adding these pressure drops we get 7.135 kilo Pascal. So, as I have mentioned before in the earlier numerical examples that these examples are for the purpose of illustration only, please check the calculations there maybe errors in the calculations, so please do your own calculations and check.

But, the methodology I hope is clear and using this methodology you should be able to calculate pressure drop with homogeneous model, or separated flow model or drift flux model whichever model you choose. And for a evaporator tube, or condenser tube or adiabatic two phase flow are all these 3 types of cases you should be able to calculate pressure drop.

If M square is negligible and if you make simplifying assumptions, then the expressions which we have derived by analytical integration can be used and then you can calculate pressure drops by hand. Otherwise if the simplifying assumptions cannot be made then you will have to do numerical integration by writing a code.

Thank you.