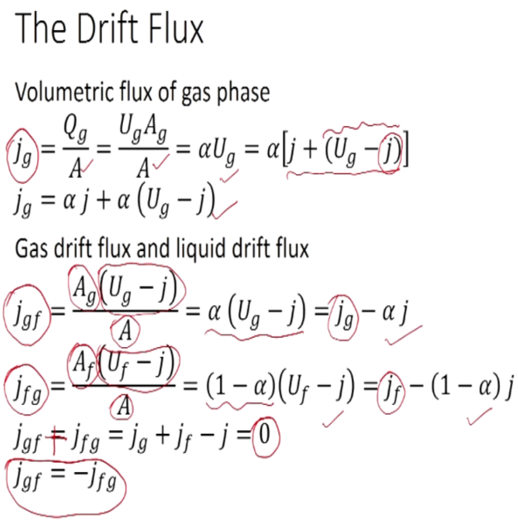
Two-Phase flow with phase change in conventional and miniature channels
Prof. Manmohan Pandey
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 06
The Drift Flux Model

We meet again for the course on Two-phase flow with phase change in conventional and miniature channels. We were discussing the modeling of two phase flow in that we have discuss the homogeneous model, the separated flow model with two correlations Lockhart Martinelli correlation and Martinelli Nelson correlation and now we will discuss another model called the Drift Flux Model.

(Refer Slide Time: 01:03)



Volumetric flux of gas phase

$$j_g = \frac{Q_g}{A} = \frac{U_g A_g}{A} = \alpha U_g = \alpha\left[j + (U_g - j)\right]$$

$$j_g = \alpha\, j + \alpha\,(U_g - j)$$

Gas drift flux and liquid drift flux

$$j_{gf} = \frac{A_g(U_g - j)}{A} = \alpha\,(U_g - j) = j_g - \alpha\, j$$

$$j_{fg} = \frac{A_f(U_f - j)}{A} = (1-\alpha)(U_f - j) = j_f - (1-\alpha)\, j$$

$$j_{gf} = j_{fg} = j_g + j_f - j = 0$$

$$j_{gf} = -j_{fg}$$

The drift flux model combines the simplicity of homogeneous model, but it takes into account the relative motion between the phases like the separated flow model. So, it is simple as well as relatively accurate, it involves the concept of drift flux. So, let us first understand what is drift flux.

The Drift Flux before we define the drift flux let us consider the volumetric flux of the gas phase which is j g it is Q g upon A and Qg is U g A g. So, it is U g A g upon a and A g by a is equal to alpha. So, this is equal to alpha U g and U g we can rewrite as j plus U g minus j. So, it becomes alpha into j plus U g minus j; now, why I have written U g in this form? Because this U g minus j is the difference between the gas velocity and the total volumetric flux, this j is the total volumetric flux and it is also equal to the mean velocity of the two phase mixture. And therefore, U g minus j is the relative velocity of the gas phase with respect to the plane which is moving with the mean velocity j ok.

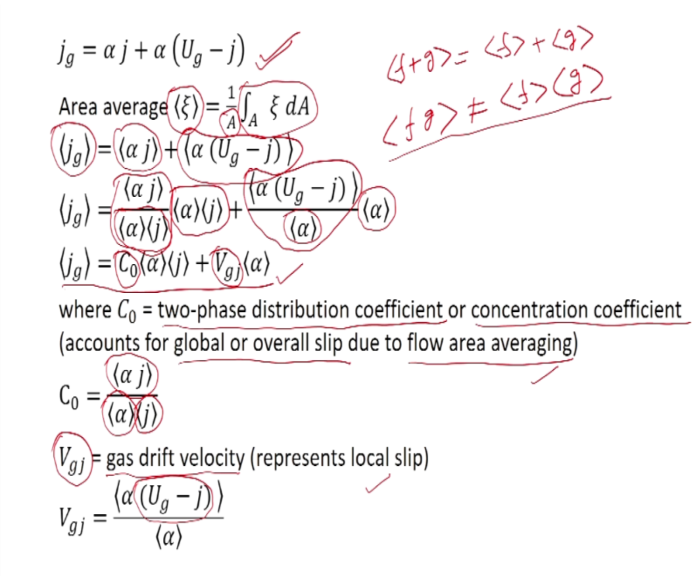
Now j g can be written as this alpha j plus alpha times U g minus j, now consider the gas drift flux. The gas drift flux is denoted by jgf and it is the volumetric flux of the gas phase with respect to the plane moving with the mean velocity j. The volumetric flow rate of the gas phase with respect to this plane which is moving with the mean velocity j a volumetric flow rate will be equal to the area A g occupied by the gas phase multiplied by the relative velocity of the gas phase with respect to the plane moving with mean velocity j.

And this whole thing has to be divided by the total cross sectional area A. But A g by A is nothing, but alpha; so, this is equal to alpha times U g minus j. And we recognized that alpha U g is nothing, but j g; do, get j g minus alpha j. In a similar manner j fg the liquid drift flux is defined as the volumetric flux of the liquid phase with respect to the same plane, which is moving with mean velocity j. So, this will be equal to A f multiplied by the relative velocity U f minus j divided by the total cross sectional area A, but A f by A is nothing, but 1 minus alpha.

So, we get 1 minus alpha times U g minus j and 1 minus alpha U f is nothing, but j f. So, this is equal to j f minus 1 minus alpha j. So, if we add j gf plus j fg then we get 0 ok. So, therefore, j gf is the negative j fg. This means that the gas drift flux and liquid drift flux are equal and opposite that is the volumetric flux of the gas phase with respect to the plane moving with mean velocity j is equal and opposite of the volumetric flux of the liquid phase with respect to the same plane.

Now let us come back to this equation j g is equal to alpha j plus alpha times U g minus j.

(Refer Slide Time: 06:01)



$$j_g = \alpha j + \alpha \left(U_g - j\right)$$

Area average $\langle \xi \rangle = \frac{1}{A}\int_A \ \xi \, dA$

$$\langle j_g \rangle = \langle \alpha j \rangle + \langle \alpha (U_g - j) \rangle$$

$$\langle j_g \rangle = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}\langle \alpha \rangle \langle j \rangle + \frac{\langle \alpha (U_g - j) \rangle}{\langle \alpha \rangle}\langle \alpha \rangle$$

$$\langle j_g \rangle = C_0 \langle \alpha \rangle \langle j \rangle + V_{gj} \langle \alpha \rangle$$

where C_0 = two-phase distribution coefficient or concentration coefficient (accounts for global or overall slip due to flow area averaging)

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}$$

V_{gj} = gas drift velocity (represents local slip)

$$V_{gj} = \frac{\langle \alpha \, (U_g - j) \, \rangle}{\langle \alpha \rangle}$$

Let us define area average and denote it by this angular brackets suppose psi is some variable, then the area average of this variable is equal to the integral of psi over the area A divided by the area A. And from this we can clearly see that the sum of the area averages of two quantities will be equal to the area average of the sum of the two quantities. Suppose f and g are two functions, then area average of f plus g will be equal to area average of f plus area average of g.

But if we multiply two functions then the area average of the product of the two functions will not be equal to the product of the area averages. So, in an equation we can take area averages of each term and the equation will still be valid. So, now, in this equation we take area average of each term. So, we get area average j g or angular bracket j g is equal to angular bracket alpha j plus angular bracket alpha times U g minus j.

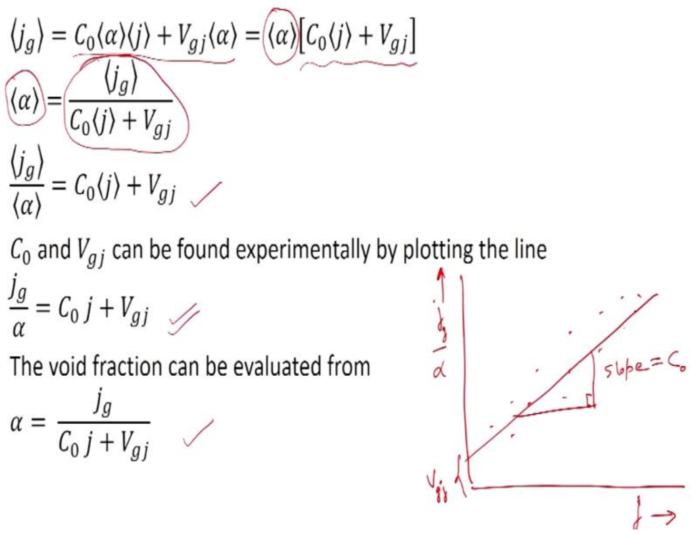
Now, we divide and multiply all the terms by alpha j. A first term we multiply by angular bracket alpha times angular bracket j and also divide by angular bracket alpha and angular bracket j and the second term we multiply by angular bracket alpha and divide by angular bracket alpha. And now this quantity we define a C 0 and this quantity here we called it V gj. So, now, we have this equation angular bracket j g is equal to C 0 angular bracket alpha, angular bracket j plus V gj angular bracket alpha now these two new parameters which we have defined they are as follows.

C 0 is called the two phase distribution coefficient or concentration coefficient and it accounts for global or overall slip due to flow area averaging. C 0 is equal to area average of alpha times j upon area average alpha into area average j and as we have discussed area average of the product of two functions is not equal to the product of the area averages of the two functions, it can happen sometimes only if the two functions vary in a similar manner.

If the void fraction profile or the concentration profile and the velocity profile are similar, then the numerator and the denominator will be equal and in that case C naught will be equal to 1; otherwise if there void fraction profile or the concentration profile and the velocity profile are not similar then C 0 will be different from 1. So, therefore, C 0 accounts for the global effect of the slip between the two phases that is due to that this similarity of the concentration profile and velocity profile and this effect comes due to flow area averaging.

Now, the second parameter V gj it is called the gas drift velocity and it represents the local slip. As we can see here V gj involves the related local relative velocity U g minus j and therefore, it represents the effect of local slip that is the relative velocity between the phases at every point on the cross section and then it is averaged over the cross section.

(Refer Slide Time: 11:45)



$$\langle j_g \rangle = C_0 \langle \alpha \rangle \langle j \rangle + V_{gj} \langle \alpha \rangle = \langle \alpha \rangle \left[C_0 \langle j \rangle + V_{gj} \right]$$

$$\langle \alpha \rangle = \frac{\langle j_g \rangle}{C_0 \langle j \rangle + V_{gj}}$$

$$\frac{\langle j_g \rangle}{\langle \alpha \rangle} = C_0 \langle j \rangle + V_{gj}$$

C_0 and V_{gj} can be found experimentally by plotting the line

$$\frac{j_g}{\alpha} = C_0\, j + V_{gj}$$

The void fraction can be evaluated from

$$\alpha = \frac{j_g}{C_0\, j + V_{gj}}$$

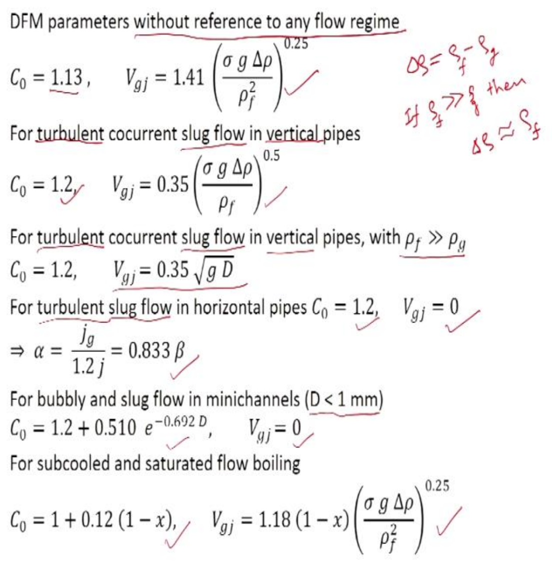
So, now j g is equal to C naught alpha j plus V gj alpha. So, now we take this angular bracket alpha common and we have C naught angular bracket j plus V gj and then we can express angular bracket alpha as angular bracket j g upon angular C naught angular bracket j plus V gj. This is the expression for the void fraction that we have obtained.

We can rewrite this expression in this form also j g upon alpha is equal to C naught angular bracket j plus V gj. Now for simplicity of notation we can get rid of angular brackets and rewrite all the expressions without the angular brackets, but it is understood that we are dealing with one dimensional modeling and therefore, area averaging has been done.

So, j g by alpha is equal to C naught to j plus V gj this is the expression we are having; and alpha can be expressed as j g upon C naught g plus V gj. So, if we can some how obtain C naught and V gj, we should be able to obtain alpha the void fraction from this expression. So, how to obtain C naught and V gj for that consider this equation. If we plot j on the x axis and j g upon alpha from the y axis, then and we plot the data point obtained experimentally and then fit a line.

So, then this y intercept will be equal to V gj and the slope of this line will be equal to C naught. So, this is the way correlations for C naught and V gj have been obtained, researchers have done experiments collected data and then after plotting graph, they have obtained V gj and C naught. Some researchers have used data containing different flow regimes whereas, some others researchers have used data for some specific flow regimes. So, if the data for different flow regimes has been used, then the resulting correlation will be applicable regardless of the flow regime. If the data has been used for a particular flow regime, then that correlation obtained from that data will be applicable for that particular flow regime.

(Refer Slide Time: 15:49)



DFM parameters without reference to any flow regime

$$C_0 = 1.13\,,\ \ V_{gj} = 1.41\left(\frac{\sigma\, g\, \Delta\rho}{\rho_f^2}\right)^{0.25}$$

For turbulent cocurrent slug flow in vertical pipes

$$C_0 = 1.2,\ \ V_{gj} = 0.35\left(\frac{\sigma\, g\, \Delta\rho}{\rho_f}\right)^{0.5}$$

For turbulent cocurrent slug flow in vertical pipes, with $\rho_f \gg \rho_g$

$$C_0 = 1.2, \qquad V_{gj} = 0.35\sqrt{g\, D}$$

For turbulent slug flow in horizontal pipes $C_0 = 1.2, \quad V_{gj} = 0$

$$\Rightarrow\ \alpha = \frac{j_g}{1.2\, j} = 0.833\, \beta$$

For bubbly and slug flow in minichannels (D < 1 mm)

$$C_0 = 1.2 + 0.510\ \ e^{-0.692\, D}, \qquad V_{gj} = 0$$

For subcooled and saturated flow boiling

$$C_0 = 1 + 0.12\,(1 - x),\ \ V_{gj} = 1.18\,(1 - x)\left(\frac{\sigma\, g\, \Delta\rho}{\rho_f^2}\right)^{0.25}$$

So, different correlations have been given for C naught and v gj here I am listing only a few of them. There is a correlation which is without reference to any particular flow regime and this gives C naught as 1.13 and V gj is given by 1.41 sigma g delta rho upon rho f square whole raise to 0.25. Here as we know delta rho is the density difference rho f minus rho g and in many cases the density of the liquid phase is much greater than the density of the gas phase.

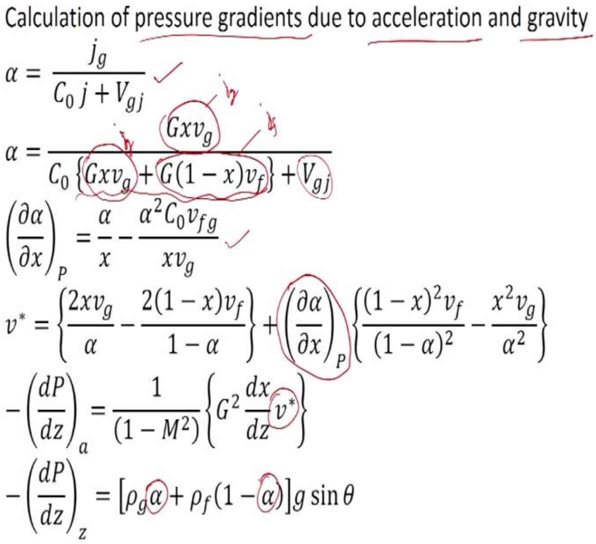
So, then delta rho is approximately equal to rho f only and if this approximation is used, then the numerator becomes the rho f and 1 rho f will get canceled with the rho f in the denominator. Then there are some other correlations which are far some particular flow regime. For turbulent concurrent slug flow in vertical pipes we have this correlation C naught is equal to 1.2 V gj is equal to 0.35 sigma g delta rho upon rho f raise to 0.5. And if it is the same type of flow turbulent slug flow in vertical pipes, but the density of the liquid phase is much larger than the density of the gas phase then as we have discussed delta rho will be equal to rho and rho f rho f will get canceled.

So, in that case we get V gj is equal to 0.35 root of g D. For turbulent flow slug flow in horizontal pipes we have C naught equal to 1.2 and V gj is equal to 0. So, if we go back to this expression and we put here V gj equal to 0 then we get C naught upon this will be equal to j g upon C naught j, but j g by j is nothing, but beta. So, we get beta upon C naught, ok.

So, in this case we get alpha is equal to 0.833 beta. So, bubbly and slug flow in many channels which is less than 1 and diameter, there is a correlation C naught equal to 1.2 plus 0.510 e raise to minus 0.692 D and V gj is equal to 0.

For subcooled and saturated flow boiling there is another correlation which involves the quality x C naught is equal to 1 plus 0.12 1 minus x and V gj is equal to 1.18 1 minus x sigma g delta rho upon rho f square whole raised to 0.25. These are a few correlations for drift flux model there are many more and you can use whichever is most suitable for your problem.

(Refer Slide Time: 19:55)



Calculation of pressure gradients due to acceleration and gravity

$$\alpha = \frac{j_g}{C_0 j + V_{gj}}$$

$$\alpha = \frac{Gxv_g}{C_0\{Gcv_g + G(1-x)v_f\} + V_{gj}}$$

$$\left(\frac{\partial\alpha}{\partial x}\right)_P = \frac{\alpha}{x} - \frac{\alpha^2 C_0 v_{fg}}{xv_g}$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$-\left(\frac{dP}{dz}\right)_z = [\rho_g \alpha + \rho_f(1-\alpha)] g \sin\theta$$

Now, how to calculate pressure gradients due to acceleration and gravity? For pressure gradient due to friction we use the same expression as in the homogeneous model, because the frictional pressure gradient does not involve with the void fraction alpha. But the acceleration and gravity pressure gradients will be different if we use the drift flux model and these will involve the effect of relative motion between the phases. The alpha is calculated from this expression and if we substitute for j then we get this j is equal to Gx v g plus G 1 minus x v f; this one this is j g and this is j f ok.

This is also j g if we differentiate in this expression with respect to quality x at constant pressure, that constant pressure v g and v f will be constant and C naught is constant and V gj let us look at the expressions here, the expressions for V gj involve rho f rho g and sigma. So, these properties if the pressure is constant and we are considering thermal equilibrium. So, it is saturated liquid vapor mixture. So, if the pressure is constant the temperature will also be constant that it will be equal to the saturation temperature and therefore, the surface tension also will be constant and rho f and rho j will be constant.

So, therefore, V gj will be constant. So, considering V gj v g and v fs constant we differentiate alpha with respect to x and after some algebra we get this expression, alpha upon x minus alpha square C naught v fg upon x vg. Now this expression is used to calculate v star and then this value of v star is used to calculate the pressure gradient due to acceleration and the value of alpha obtained from the drift flux model is used here to calculate the pressure gradient due to gravity.

(Refer Slide Time: 23:13)

Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 10 cm

$G = 100\ kg/m^2 s$, x(0)=0, $q'' = 20\ kW/m^2$

To find the pressure gradient at z = 5 cm

Solution:

Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}$ Pa. s, $\mu_g = 12.26 \times 10^{-6}$ Pa. s, $h_{fg} = 2257.45$ kJ/kg

$v_f = 1.043 \times 10^{-3}\ m^3/kg$, $v_g = 1.6939\ m^3/kg$, $v_{fg} = 1.693\ m^3/kg$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.443\ m^{-1}, x(5cm) = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782-1.6939}{1000} = -1.57 \times 10^{-5} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1, 1 - M^2 \approx 1, (1 - M^2)^{-1} \approx 1$$

$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s$ $\qquad Re_{TP} = \frac{GD}{\bar{\mu}} = 1052 \Rightarrow$ Laminar flow

$f_{TP} = 16/Re = 0.01521$

Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 10 cm

$G = 100\ \text{kg/m}^2\text{s}$, x(0)=0, $q'' = 20\ \text{kW/m}^2$. To find the pressure gradient at z = 5 cm

Solution: Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}\ \text{Pa.s},\ \mu_g = 12.26 \times 10^{-6}\ \text{Pa.s},\ h_{fg} = 2257.45\ \text{kJ/kg}$

$v_f = 1.043 \times 10^{-3}\ \text{m}^3/\text{kg},\ v_g = 1.6939\ \text{m}^3/\text{kg},\ v_{fg} = 1.693\ \text{m}^3/\text{kg}$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.\ 443\ \text{m}^{-1},\ x(5\text{cm}) = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782 - 1.6939}{1000} = -1.57 \times 10^{-5} \text{m}^3\text{kg}^{-1}\text{Pa}^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1,\ 1 - \text{M}^2 \approx 1,\ (1 - \text{M}^2)^{-1} \approx 1$$

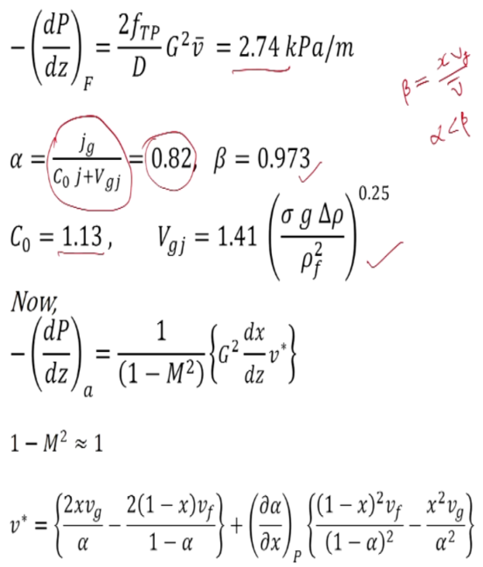
$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s \qquad Re_{TP} = \frac{GD}{\bar{\mu}} = 1052 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.01521$$

Now, we will consider some numerical examples based on the drift flux model. Again we consider the first example with the same data as we have considered before for the other models, water steam mixture at a 100 kilo Pascal, horizontal flow 2 mm diameter channel and length 10 centimeter, mass flux 100 kg per meter square second. In let quality 0, drift flux 20 kilo Watt per meter square and we have to find the pressure gradient at the midpoint z is equal to 5 centimeter the properties of water steam at 100 kilo Pascal. At the same as before, d x by dz is already calculated before 0.0221, dx by dz is 0.443 per meter and x is equal to 0.221.

And dv g by dp is calculated using the numerical differentiation as minus 1.57 into 10 raise to minus 5 s i units and then we calculate m square which is 3.47 into 10 raise to minus 3 and this is much less than 1. And therefore, 1 minus M square is a approximately equal to 1 and 1 upon 1 minus M square is also approximately equal to 1. Then we calculate the mixture viscosity and then the two phase Reynolds number, and it is laminar flow therefore, we use the laminar flow correlation 16 upon Re and got this friction factor and then using this friction factor we get the frictional pressure gradient.

(Refer Slide Time: 25:23)



$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D}G^2\bar{v} = 2.74\ kPa/m$$

$$\alpha = \frac{j_g}{C_0\ j+V_{gj}} = 0.82,\quad \beta = 0.973$$

$$C_0 = 1.13,\ \ V_{gj} = 1.41\left(\frac{\sigma\ g\ \Delta\rho}{\rho_f^2}\right)^{0.25}$$

Now,

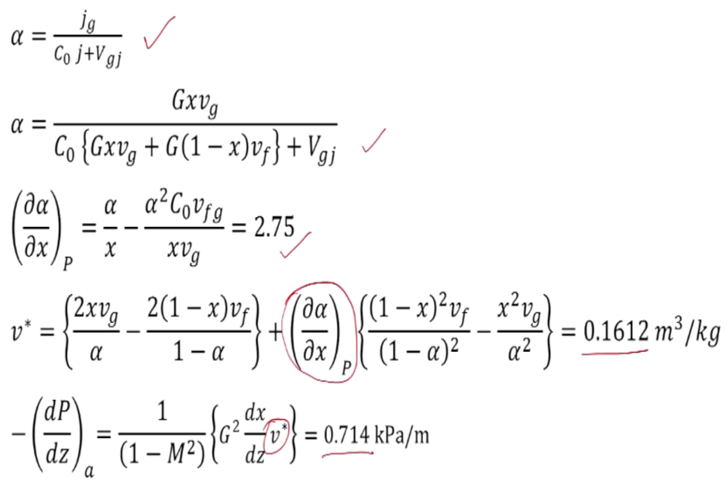
$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2\frac{dx}{dz}v^*\right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P\left\{\frac{(1-x)^2v_f}{(1-\alpha)^2} - \frac{x^2v_g}{\alpha^2}\right\}$$

Now alpha the void fraction is obtained from this expression j g upon C naught j plus Vg j, and C naught here is equal to 1.13 and V gj is calculated using this expression and after substituting we get alpha is equal to 0.82 and beta is equal to x vg upon v bar. So, beta is calculated is 0.973 and we note that here alpha is less than beta irrespected. Now using this alpha we calculate the pressure gradient due to acceleration and for that we will have to calculate v star first.

(Refer Slide Time: 26:51)



$$\alpha = \frac{j_g}{C_0\, j + V_{gj}}$$

$$\alpha = \frac{Gxv_g}{C_0\left\{Gxv_g + G(1-x)v_f\right\} + V_{gj}}$$

$$\frac{\partial\alpha}{\partial x} = \frac{\alpha}{x} - \frac{\alpha^2 C_0 v_{fg}}{xv_g} = 2.75$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.1612\ m^3/s$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 0.714\ \text{kPa/m}$$

So, alpha is given by this and after substituting we get this and then after differentiating and substituting the values we get 2.75, the slope of the alpha x curve at constant pressure is 2.75.

Now, using this value of the slope we get v star equal to 0.1612 meter cube per kg and then using this value of v star we get the acceleration pressure gradient as 0.714 kilo Pascal per meter.

(Refer Slide Time: 27:39)

$$-\left(\frac{dP}{dz}\right)_F = 2.74\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 0.714\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0$$

$$-\left(\frac{dP}{dz}\right) = 5.85 + 0.714 + 0 = 3.45\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = 2.74\ kPa/m$$

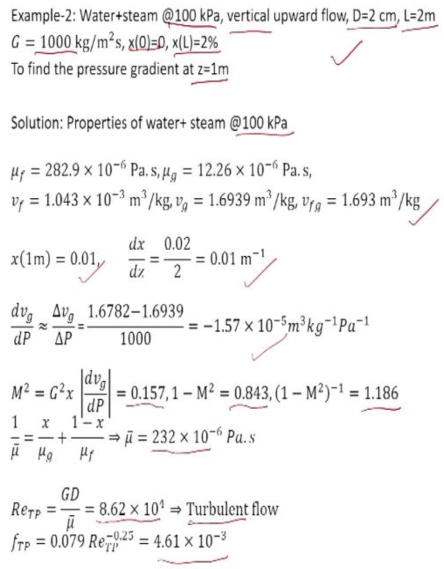
$$-\left(\frac{dP}{dz}\right)_a = 0.714\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0$$

$$-\left(\frac{dP}{dz}\right) = 2.74 + 0.714 + 0 = 3.45\ kPa/m$$

So, now here are the results frictional pressure gradient 2.74 kilo Pascal per meter acceleration pressure gradient 0.714 kilo Pascal per meter and the gravitational pressure gradient is 0 because it is a horizontal pipe, and we now add all 3 pressure gradients and get the total pressure gradient as 3.45 kilo Pascal per meter.

(Refer Slide Time: 28:09)



Example-2: Water+steam @100 kPa, vertical upward flow, D=2 cm, L=2m

$G = 1000\ \mathrm{kg/m^2 s}$, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m

Solution: Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}\ \mathrm{Pa.s},\ \mu_g = 12.26 \times 10^{-6}\ \mathrm{Pa.s},$

$v_f = 1.043 \times 10^{-3}\ \mathrm{m^3/kg},\ v_g = 1.6939\ \mathrm{m^3/kg},\ v_{fg} = 1.693\ \mathrm{m^3/kg}$

$$x(1\mathrm{m}) = 0.01,\ \ \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \mathrm{m^{-1}}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782 - 1.6939}{1000} = -1.57 \times 10^{-5} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 0.157,\ 1 - \mathrm{M}^2 = 0.843,\ (1 - \mathrm{M}^2)^{-1} = 1.186$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 232 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 8.62 \times 10^4 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 4.61 \times 10^{-3}$$

Now, consider the second example with the same data as before 100 kilo Pascal pressure vertical upward flow, diameter 2 centimeter, length 2 meter, mass flux 1000 units, inlet quality 0, out let quality 2 percent and we have to find the pressure gradient at the midpoint. So, property is at 100 kilo Pascal are same as before their quality at the midpoint is 1 percent d x by d z is 0.01 per meter and dv g by dP is minus 1.157 into 10 raise to minus 5 s i units and M square is equal to 0.157 which is not negligible 1 minus M square is equal to 0.843 and 1 up on 1 minus M square is 1.186.

The average viscosity of the mixture is calculated as this and then the two phase Reynolds number is 8.62 into 10 raise to 4. So, it is turbulent flow and then using the turbulent flow correlation the two phase friction factor is 4.61 into 10 raise to minus 3 and using this frictional pressure gradient is obtained is 9.85 kilo Pascal per meter.

(Refer Slide Time: 29:49)

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} \left(1 - M^2\right)^{-1} = 9.85\ kPa/m$$

$$\alpha = \frac{j_g}{C_0\, j + V_{gj}} = 0.82, \quad \beta = 0.943$$

Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1 - M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 0.843$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_p \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} \left(1 - M^2\right)^{-1} = 9.85\ kPa/m$$

$$\alpha = \frac{j_g}{C_0\, j + V_{gj}} = 0.82, \quad \beta = 0.943$$

Now,

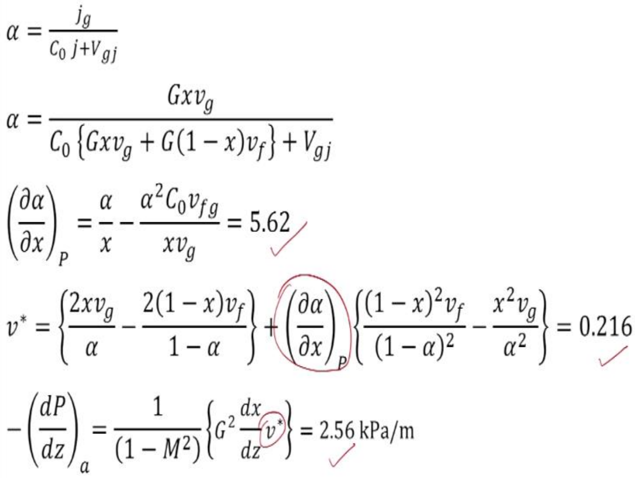
$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1 - M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 0.843$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

Now alpha is obtained by using this expression and it is 0.82 and beta is equal to 0.943. The acceleration pressure gradient is obtained as using this expression and v star is given by this expression.

(Refer Slide Time: 30:31)



$$\alpha = \frac{j_g}{C_0\, j + V_{gj}}$$

$$\alpha = \frac{Gxv_g}{C_0\left\{Gxv_g + G(1-x)v_f\right\} + V_{gj}}$$

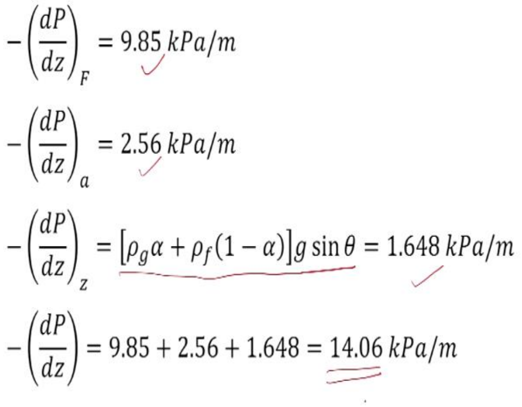
$$\frac{\partial \alpha}{\partial x} = \frac{\alpha}{x} - \frac{\alpha^2 C_0 v_{fg}}{xv_g} = 5.62$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.216$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 2.56 \text{ kPa/m}$$

So, alpha we find the slope of the alpha x square and it is 5.62 and then substitute the slope here and get v star as 0.216 and then substitute v star here and get the acceleration pressure gradient as 2.56 kilo Pascal per meter.

(Refer Slide Time: 30:59)



$$-\left(\frac{dP}{dz}\right)_F = 9.85\ kPa/m$$

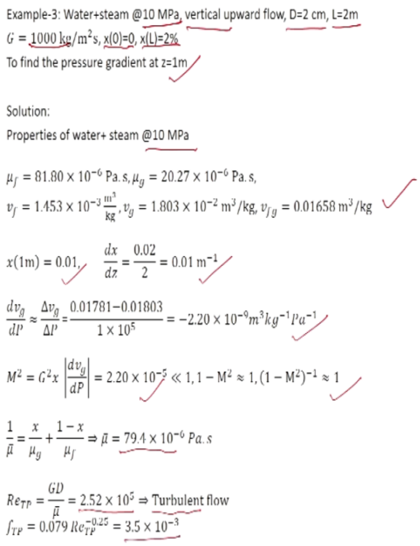
$$-\left(\frac{dP}{dz}\right)_a = 2.56\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g\alpha + \rho_f(1-\alpha)\right]g\sin\theta = 1.648\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 9.85 + 2.56 + 1.648 = 14.06\ kPa/m$$

So, now the results are the following, the fictional pressure gradient is 9.85 kilo Pascal per meter, acceleration pressure gradient is 2.56 kilo Pascal per meter and for the gravitational pressure gradient we use this expression and we get 1.648 kilo Pascal per meter and adding all this pressure gradients we get the total pressure gradient as 14.06 kilo Pascal per meter.

(Refer Slide Time: 31:35)



Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=2m

$G = 1000\ \text{kg/m}^2\text{s}$, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m

Solution: Properties of water+ steam @10 MPa

$\mu_f = 81.80 \times 10^{-6}\ \text{Pa.s},\ \mu_g = 20.27 \times 10^{-6}\ \text{Pa.s},$

$v_f = 1.453 \times 10^{-3} \frac{\text{m}^3}{\text{kg}},\ v_g = 1.803 \times 10^{-2}\ \text{m}^3/\text{kg},\ v_{fg} = 0.01658\ \text{m}^3/\text{kg}$

$x(1\text{m}) = 0.01,\ \ \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \text{m}^{-1}$

$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781 - 0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$

$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1,\ 1 - \text{M}^2 \approx 1,\ (1 - \text{M}^2)^{-1} \approx 1$

$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 79.4 \times 10^{-6}\ Pa.s$

$Re_{TP} = \frac{GD}{\bar{\mu}} = 2.52 \times 10^5 \Rightarrow \text{Turbulent flow}$

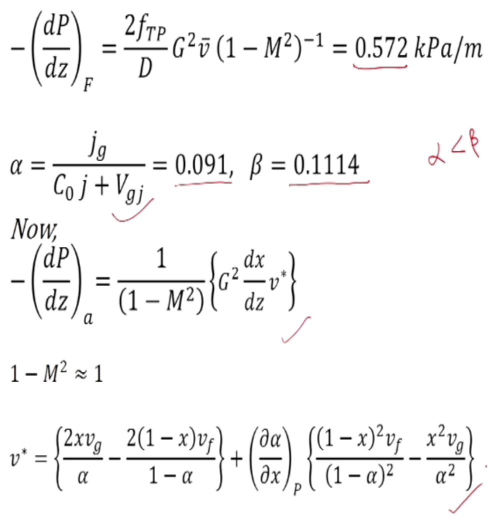
$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 3.5 \times 10^{-3}$

Now the third example data same as before 10 mega Pascal pressure vertical upward flow, diameter 2 centimeter length 2 meter, mass flux 1000 kg per meter square second, inlet quality 0, outlet quality 2 percent pressure gradient has to be found at the midpoint. At 10 mega Pascal these are the properties at the midpoint the quality is 1 percent and dx by dz is equal to 0.01 per meter dv g by dP is of the order of the 10 raise to minus 9 m square is of the order of 10 raise to minus 5 negligible.

So, 1 upon 1 minus M square is approximately equal to 1, the mean viscosity of the mixture is 79.4 into 10 raise to minus 6 Pascal second, the Reynolds numbers is of the order of 10 raise to 5. So, it is turbulent flow.

And using the turbulent flow correlation we get the two phase friction factor 3.5 into 10 raise to minus 3.

(Refer Slide Time: 32:51)



$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} \, (1 - M^2)^{-1} = 0.572\ kPa/m$$

$$\alpha = \frac{j_g}{C_0\, j + V_{gj}} = 0.091, \quad \beta = 0.1114$$

Now,

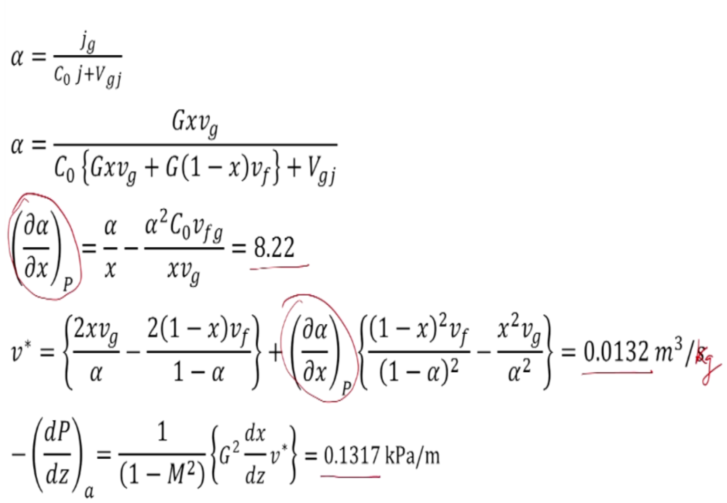
$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1 - M^2)} \left\{ G^2 \frac{dx}{dz} v^* \right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

Now, using this, we get the frictional pressure gradient as 0.572 kilo Pascal per meter, alpha using this expression we get 0.091 and beta is equal to 0.1114. So, here also alpha is less than beta as expected. And for the frictional pressure gradient this is the expression it involves v star.

(Refer Slide Time: 33:37)



$$\alpha = \frac{j_g}{C_0\, j + V_{gj}}$$

$$\alpha = \frac{Gxv_g}{C_0\,\{Gxv_g + G(1-x)v_f\} + V_{gj}}$$

$$\frac{\partial\alpha}{\partial x} = \frac{\alpha}{x} - \frac{\alpha^2 C_0 v_{fg}}{xv_g} = 8.22$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.0132\ m^3/s$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 0.1317\ \text{kPa/m}$$

And now we get this the slope of the alpha x square 8.22 and use the slope to calculate v star which is 0.0132 meter cube per kg and the acceleration pressure gradient is calculated as 0.1317 kilo Pascal per meter.

(Refer Slide Time: 34:11)

$$-\left(\frac{dP}{dz}\right)_F = 0.572\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 0.1317\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 6.18\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 0.572 + 0.1317 + 6.18 = 6.88\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = 0.572\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 0.1317\ kPa/m$$

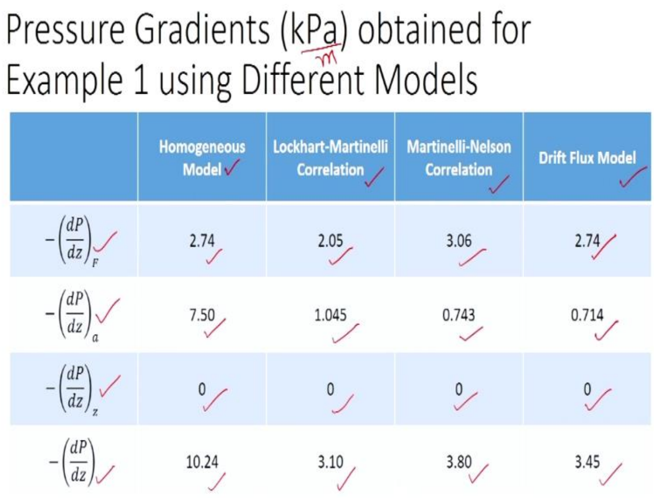
$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 6.18\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 0.572 + 0.1317 + 6.18 = 6.88\ kPa/m$$

So, the frictional pressure gradient is 0.572 kilo Pascal per meter, acceleration pressure gradient is 0.1317 kilo Pascal per meter and the gravitational pressure gradient is calculated using this expression and it is 6.18 kilo Pascal per meter. Now we add all these pressure gradients and get the total pressure gradient as 6.88 kilopascal per meter.

So, now let us summarize the results of all the numerical examples.

(Refer Slide Time: 34:57)



Pressure Gradients (kPa) obtained for Example 1 using Different Models

	Homogeneous Model	Lockhart-Martinelli Correlation	Martinelli-Nelson Correlation	Drift Flux Model
$-\left(\frac{dP}{dz}\right)_F$	2.74	2.05	3.06	2.74
$-\left(\frac{dP}{dz}\right)_a$	7.50	1.045	0.743	0.714
$-\left(\frac{dP}{dz}\right)_z$	0	0	0	0
$-\left(\frac{dP}{dz}\right)$	10.24	3.10	3.80	3.45

Pressure Gradients (kPa) obtained for Example 1 using Different Models

	Homogeneous Model	Lockhart-Martinelli Correlation	Martinelli - Nelson Correlation	Drift Flux Model
$-\left(\frac{dP}{dz}\right)_F$	2.74	2.05	3.06	2.74
$-\left(\frac{dP}{dz}\right)_a$	7.5	1.045	0.743	0.714
$-\left(\frac{dP}{dz}\right)_z$	0	0	0	0
$-\left(\frac{dP}{dz}\right)$	10.24	3.10	3.80	3.45

Here the results of example 1 are shown in tabular form. The frictional pressure gradient, acceleration pressure gradient, gravitational pressure gradient and the total pressure gradient using homogeneous model, Lockhart-Martinelli correlation, Martinelli-Nelson correlation and the Drift Flux model. The frictional pressure the gradient from homogeneous model is 2.74 kilo Pascal per meter, ok. It is 2.74 kilo Pascal per meter and you from Lockhart-Martinelli it is 2.05 kilo Pascal per meter, Martinelli Nelson give 3.06 kilo Pascal per meter, the Drift Flux model and that we use the same expression as that for homogeneous model.

So, it is the same 2.74 kilo Pascal per meter. The acceleration pressure gradient from homogeneous model is 7.50 kilo Pascal per meter and Lockhart Martinelli give 1.045 Martinelli Nelson give 0.743 and Drift Flux model give 0.714. Here the homogeneous model give very high value of the acceleration pressure gradient that is because alpha was highly overestimated using homogeneous model.

The gravitational pressure gradient is 0 for all the model because it is the horizontal pipe and the total pressure gradients are 10.24 3.10 3.80 3.45. The total pressure gradient using homogeneous model is high mainly because of the pressure gradient due to acceleration.

(Refer Slide Time: 37:25)

Pressure Gradients (kPa) obtained for Example 2 using Different Models

	Homogeneous Model	Lockhart-Martinelli Correlation	Martinelli-Nelson Correlation	Drift Flux Model
$-\left(\frac{dP}{dz}\right)_F$	9.85	5.46	0.1322	9.85
$-\left(\frac{dP}{dz}\right)_a$	20.1	2.92	1.167	2.56
$-\left(\frac{dP}{dz}\right)_z$	0.646	2.99	3.105	1.648
$-\left(\frac{dP}{dz}\right)$	30.6	11.37	4.40	14.06

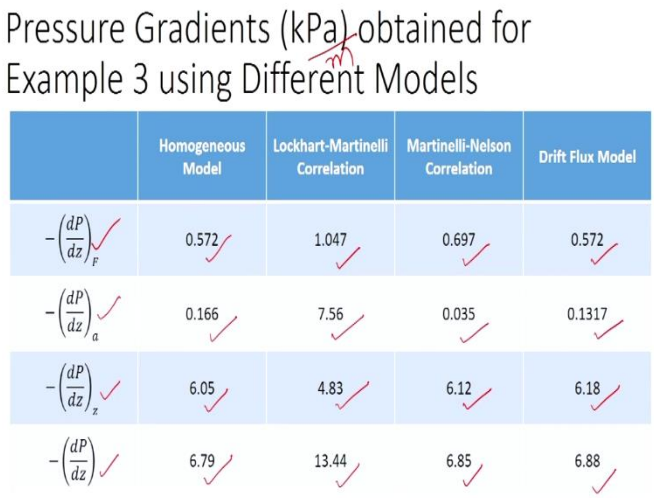
Pressure Gradients (kPa) obtained for Example 2 using Different Models

	Homogeneous Model	Lockhart-Martinelli Correlation	Martinelli - Nelson Correlation	Drift Flux Model
$-\left(\frac{dP}{dz}\right)_F$	9.85	5.46	0.1322	9.85
$-\left(\frac{dP}{dz}\right)_a$	20.1	2.92	1.167	2.56
$-\left(\frac{dP}{dz}\right)_z$	0.646	2.99	3.105	1.648
$-\left(\frac{dP}{dz}\right)$	30.6	11.37	4.40	14.06

In the second example here it is kilo Pascal per meter and homogeneous model give the frictional pressure gradient as 9.85 kilo Pascal per meter Lockhart-Martinelli give 5.46, Martinelli-Nelson give a very low value 0.1322, Drift Flux same as homogenous 9.85. The acceleration pressure gradient from homogeneous model of 20.1 kilo Pascal per meter, Lockhart-Martinelli 2.92, Martinelli-Nelson 1.167 and Drift Flux same as Drift Flux 2.56.

The gravitational pressure gradient for homogeneous it is 0.646 kilo Pascal per meter Lockhart-Martinelli to 2.99, Martinelli=Nelson 3.105 and Drift Flux 1.648. The total pressure gradient from homogeneous model is 30.6 kilo Pascal per meter, Lockhart-Martinelli 11.37, Martinelli-Nelson 4.40 and Drift Flux 14.06. Here also we see that the acceleration pressure gradient from homogeneous model is very high compared to the other models and the frictional pressure gradient given by the Martinelli-Nelson correlation as very low compared to the other models.

(Refer Slide Time: 39:29)



Pressure Gradients (kPa) obtained for Example 3 using Different Models

	Homogeneous Model	**Lockhart-Martinelli Correlation**	**Martinelli - Nelson Correlation**	**Drift Flux Model**
$-\left(\frac{dP}{dz}\right)_F$	0.572	1.047	0.697	0.572
$-\left(\frac{dP}{dz}\right)_{\mathrm{a}}$	0.166	7.56	0.035	0.1317
$-\left(\frac{dP}{dz}\right)_{\mathrm{z}}$	6.05	4.83	6.12	6.18
$-\left(\frac{dP}{dz}\right)$	6.79	13.44	6.85	6.88

The results of the example 3, frictional pressure gradient, homogenous model give 0.572 kilo Pascal per meter, Lockhart-Martinelli 1.047, Martinelli-Nelson 0.697 and Drift Flux 0.572. Pressure gradient due to acceleration, homogenous 0.166, Lockhart-Martinelli 7.56, Martinelli-Nelson 0.035 and Drift Flux 0.3117; pressure gradient due to gravity homogenous 6.05, Lockhart-Martinelli 4.83, Matinalli-Nelson 6.12 and Drift Flux 6.18.

The total pressure gradient from homogenous model is 6.79 kilo Pascal per meter, Lockhart-Martinelli 13.44, Martinelli-Nelson 6.85 and Drift Flux 6.88. In this example we see that Lockhart-Martinelli model gives very high value of the acceleration pressure gradient, where is Martinelli-Nelson correlation gives a very low value of acceleration pressure gradient.

As I mention before these numerical examples are only to illustrate the methods how to use the models, how to calculate the pressure gradients using different models and these are not meant to be used for design purposes and there may be errors in the calculations, because calculations are very complicated. So, you may do your own calculations and check.

Thank you.