Two-Phase flow with phase change in conventional and miniature channels

Prof. Manmohan Pandey
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 05
The Separated Flow Model (contd.)

Welcome back to the course on Two-Phase flow with phase change and conventional and miniature channels. We were discussing the modelling of two-phase flow in that we have discussed the homogeneous model and then the separated flow model with Lockhart Martinelli correlation.

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Modelling of Two-Phase Flow
– The Separated Flow Model
(continued)

Today we will continue the discussion of Separated Flow Model with another correlation Martinelli and Nelson correlation and after that we will solve some numerical examples with Lockhart Martinelli correlation and Martinelli and Nelson correlation.

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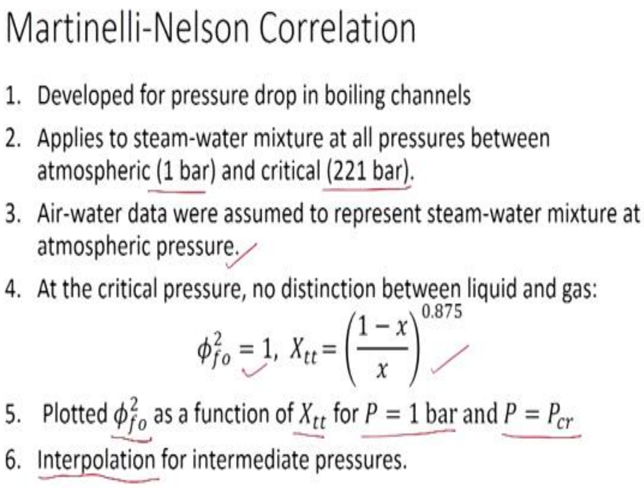

Martinelli and Nelson correlation was obtained from Lockhart Martinelli correlation and it was developed for pressure drop in boiling channels. It applies to steam water mixture at all pressures between the atmospheric pressure and the critical pressure which is approximately 221 bar. And, this is of very much practical use because steam water mixture is used in many industrial applications like boilers and the boiling conditions as well as condensing conditions.

The air water data of Lockhart and Martinelli, it was assumed to represent steam water mixture at atmospheric pressure and at the critical pressure we know that there is no distinction between liquid and gas.

$$\phi_{fo}^2 = 1, \quad X_{tt} = \left(\frac{1-x}{x}\right)^{0.875}$$

Lockhart and Martinelli assumed turbulent-turbulent regime which is exist in most of the industrial applications involvement conventional channels. As we will see in the numerical examples in case of conventional channels with realistic values we will get both phases as turbulent. So, then they plotted phi f o square as a function of X tt for P equal to 1 bar and P equal to P critical and then for intermediate pressures they interpolated. Now how did they get phi f o square?

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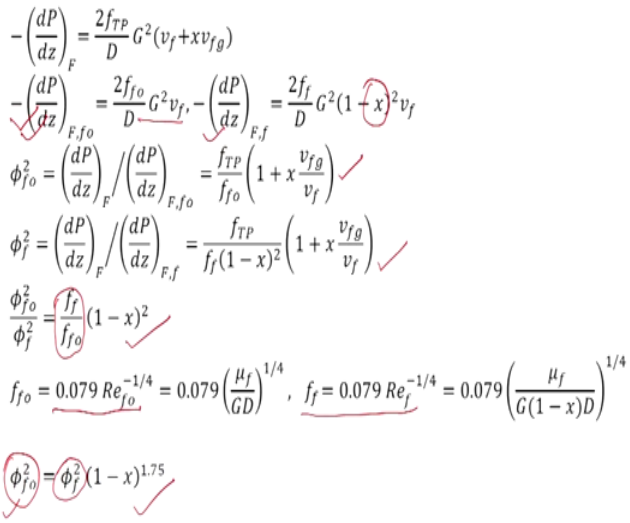

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D}G^2\{(v\}_f + xv_{fg})$$

$$-\left(\frac{dP}{dz}\right)_{F,fo} = \frac{2f_{fo}}{D}G^2v_f, -\left(\frac{dP}{dz}\right)_{F,f} = \frac{2f_f}{D}G^2(1-x)^2v_f$$

$$\phi_{fo}^2 = \left(\frac{dP}{dz}\right)_F \Big/ \left(\frac{dP}{dz}\right)_{F,fo} = \frac{f_{TP}}{f_{fo}}\left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$\phi_f^2 = \left(\frac{dP}{dz}\right)_F \Big/ \left(\frac{dP}{dz}\right)_{F,f} = \frac{f_{TP}}{f_f(1-x)^2}\left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$\frac{\phi_{fo}^2}{\phi_f^2} = \frac{f_f}{f_{fo}}(1-x)^2$$

$$f_{fo} = 0.079\,Re_{fo}^{-1/4} = 0.079\left(\frac{\mu_f}{GD}\right)^{1/4}, \quad f_f = 0.079\,Re_f^{-1/4} = 0.079\left(\frac{\mu_f}{G(1-x)D}\right)^{1/4}$$

$$\phi_{fo}^2 = \phi_f^2\,(1-x)^{1.75}$$

So, Lockhart Martinelli correlation gives phi f square and Martinelli Nelson have used phi f o square because phi f o square is more convenient in case of boiling or condensing channels because the local quality is difficult to know, but the overall flow rate is known. As you can see in phi f o square in the expression for in dP by d z f f o when we want to evaluate there is no x here, but in this dP by d z is f coma f in this there is local quality is invert. So, to calculate this quantity we will need to know local quality whereas, to calculate this pressure gradient we do not need any local quality.

So, this pressure gradient is easier to evaluate in case of boiling channels and therefore, it is more convenient to use phi f o square. So, how do we obtain phi f o square from phi f square? For that we can derive a relation phi f o square we have derived this earlier; phi f o square is given by this and phi f square is given by this expression and then if we take the ratio then we get this relation between phi f o square and phi f square. And, now we need this ratio of the friction factors.

So, since it is turbulent-turbulent regime using that we get f f o and f f and then we take the ratio and then we get this expression. So, they got phi f o square for atmospheric pressure as well as critical pressure and then by interpolation they got phi f o square for intermediate pressures.

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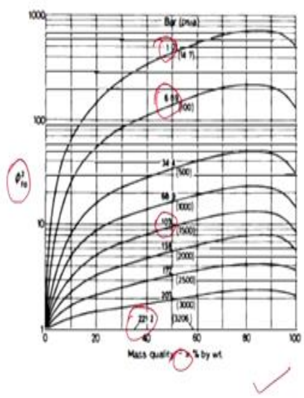

Steam quality %, by wt.	Pressure, bar (psia) 1.01 (14.7)	6.89 (100)	34.4 (500)	68.9 (1000)	103 (1500)	138 (2000)	172 (2500)	207 (3000)	221.2 (3206)
1	5.6	3.5	1.8	1.6	1.35	1.2	1.1	1.05	1.00
5	30	15	5.3	3.6	2.4	1.75	1.43	1.17	1.00
10	69	28	8.9	5.4	3.4	2.45	1.75	1.30	1.00
20	150	56	16.2	8.6	5.1	3.25	2.19	1.51	1.00
30	245	83	23.0	11.6	6.8	4.04	2.62	1.68	1.00
40	350	115	29.2	14.4	8.4	4.82	3.02	1.83	1.00
50	450	145	34.9	17.0	9.9	5.59	3.38	1.97	1.00
60	545	174	40.0	19.4	11.1	6.34	3.70	2.10	1.00
70	625	199	44.6	21.4	12.1	7.05	3.96	2.23	1.00
80	685	216	48.6	22.9	12.8	7.70	4.15	2.35	1.00
90	720	210	48.0	22.3	13.0	7.95	4.20	2.38	1.00
100	525	130	30.0	15.0	8.6	5.90	3.70	2.15	1.00

This is the graph given by Martinelli and Nelson and here we have on the horizontal axis we have mass quality on the vertical axis we have phi f o square and these are for different pressures, this is for atmospheric pressure and then we have higher pressure and still higher pressures up to very high pressures. Here we have the same data in the form of tables. So, by knowing the quality and the pressure we can get phi f o square.

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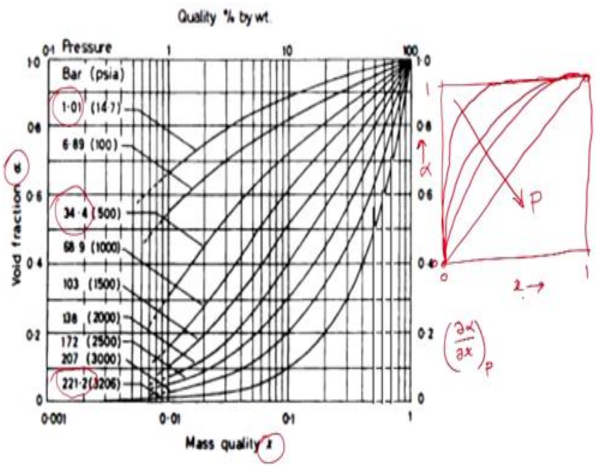

Martinelli and Nelson also plotted the void fraction alpha; again in a similar method they got alpha for atmospheric pressure using Lockhart Martinelli correlation and for critical pressure there is no distinction between the phases. So, and then by interpolation they got for the intermediate pressures.

So, we have mass quality on the horizontal axis, void fraction on the vertical axis, the horizontal scale is logarithmic, it is a semi log plot and we have alpha for different pressures; so, for horizontal for atmospheric pressure and higher pressures and very high pressures. Actually we should sketch the plot of alpha versus x on a linear scale to see how it varies. It will be like this; 0 1 and here it is 0 1 here we have alpha at x equal to 0 alpha is equal to 0 and at x equal to 1 alpha is equal to 1, but it does not vary like that does not vary linearly.

At critical pressure we get a linear graph, but at lower pressures we get this type of variation. So, for low values of x as x increases alpha increases very fast the slope is high dou by alpha by dou by x at constant pressure and this is the slope of the curve and this is very high for low values of x and for higher values of x the slope decreases and then for very high values of x the slope is very low, but for very high values even for not so high values of x alpha becomes very close to 1 ok.

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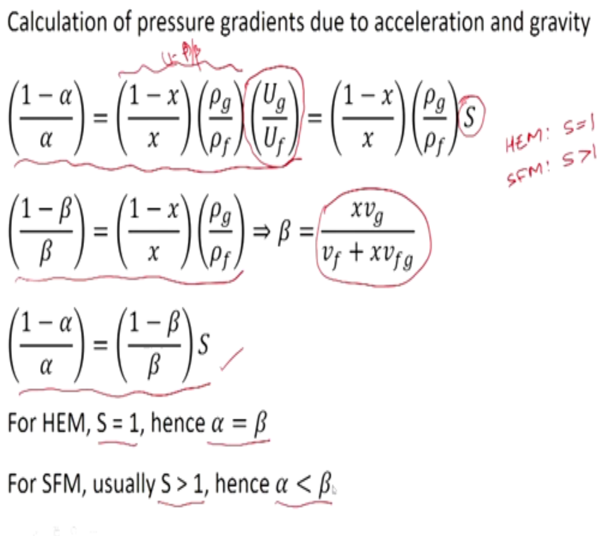

Calculation of pressure gradients due to acceleration and gravity:

$$\left(\frac{1-\alpha}{\alpha}\right)=\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_f}\right)\left(\frac{U_g}{U_f}\right)=\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_f}\right)S$$

$$\left(\frac{1-\beta}{\beta}\right)=\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_f}\right)\Rightarrow\beta=\frac{xv_g}{v_f+xv_{fg}}$$

$$\left(\frac{1-\alpha}{\alpha}\right)=\left(\frac{1-\beta}{\beta}\right)S$$

$HEM\!: S=1;\quad hence\ \alpha=\beta$

$SFM\!: S>1;\quad hence\ \alpha<\beta$

So, now how to calculate pressure gradient due to acceleration and gravity? For pressure gradient due to acceleration and gravity we need the void fraction and its partial derivatives. The void fraction alpha is given by this: the fundamental void quality relation which we have derived before in the first lecture and this quantity the ratio of the velocities of the phases is called the slip ratio S and usually S is greater than 1, for HEM we assume that both phases move with the same velocity.

So, S is equal to 1 and then separated flow model we account for the difference of velocities and S is usually greater than 1 in horizontal flow as well as vertical upward flow or gas will move faster and liquid will move slower. So, slip ratio will be greater than 1. Now if we put S equal to 1 then the void fraction that we get is the volumetric quality and 1 minus beta upon beta is equal to this expression and if we solve for beta we get this expression, from this we can calculate beta.

And here this expression this is 1 minus beta upon beta. So, we get 1 minus alpha upon alpha is equal to 1 minus beta upon beta and to S. For homogeneous model S equal to 1, so, we get alpha equal to beta and for separated flow model S is greater than 1. So, alpha should be less than beta.

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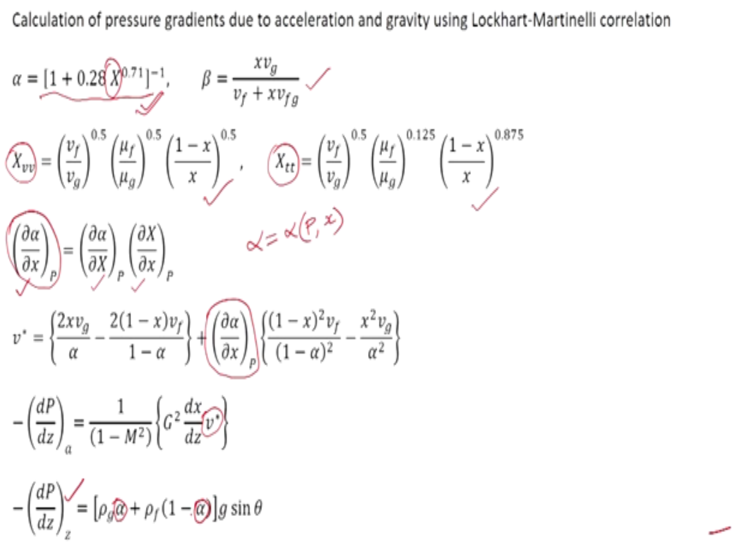

Calculation of pressure gradients due to acceleration and gravity using Lockhart-Martinelli correlation:

$$\alpha = [1 + 0.28X^{0.71}]^{-1}, \quad \beta = \frac{xv_g}{v_f + xv_{fg}}$$

$$X_{vv} = \left(\frac{v_f}{v_g}\right)^{0.5}\left(\frac{\mu_f}{\mu_g}\right)^{0.5}\left(\frac{1-x}{x}\right)^{0.5}, \quad X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5}\left(\frac{\mu_f}{\mu_g}\right)^{0.125}\left(\frac{1-x}{x}\right)^{0.875}$$

$$\left(\frac{\partial\alpha}{\partial x}\right)_P = \left(\frac{\partial\alpha}{\partial X}\right)_P\left(\frac{\partial X}{\partial x}\right)_P$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{(1-\alpha)}\right\} + \left(\frac{\delta\alpha}{\delta x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$$-\left(\frac{dP}{dz}\right)_a = \left(\frac{1}{1-M^2}\right)\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g\ sin\theta$$

Now, if we use the Lockhart Martinelli correlation then we can find alpha from the graphs given by Lockhart Martinelli or we can use a correlation for example, the Butterworth correlation. This was obtained by fitting the graphs of Lockhart and Martinelli and beta can be obtained from this expression and then we should check whether alpha is less than beta or not. Now depending on the flow regime we can find capital X, to capital X the Martinelli parameter Xvv is equal to this and X tt is given by this expression as we know and then by substituting the appropriate correlation here we can express alpha in terms of properties and the quality.

So, we get alpha as a function of pressure and quality and then we need this partial derivative dou by alpha by dou by x at constant pressure. So, this as I have just explained by finding the slope of the graph of alpha versus x for a constant pressure we can get the value of this partial derivative or if we are not using graph, but using a correlation like this correlation then we differentiate it after substituting for the Martinelli parameter we differentiate with respect to quality and by using the chain rule by multiplying these 2 derivatives we get the this derivative.

And then we substitute it here and get v star and then using the value of v star here we get the pressure gradient due to acceleration; M square has to be evaluated for that there is a long and complicated expression. So, in this course we will calculate M square from homogeneous model and use the same as M square here also as a rough estimate, but for a more accurate calculation M square has to be obtained using the expression for separated flow model. Now, this alpha in the pressure gradient due to gravity, this alpha can be substituted and then we can get the pressure gradient due to gravity ok.

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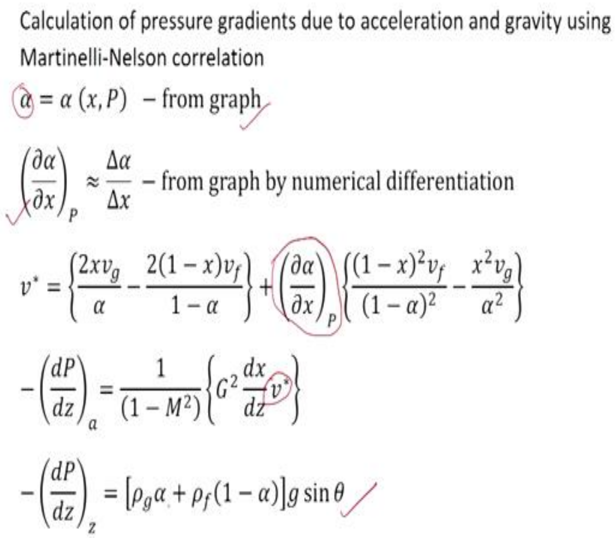

Calculation of pressure gradients due to acceleration and gravity using Lockhart-Martinelli correlation:

$$\alpha = \alpha(x, P) - from\ graph$$

$$\left(\frac{\partial\alpha}{\partial x}\right)_P \approx \frac{\Delta\alpha}{\Delta x} - \ from\ graph\ by\ numerical\ differentiation$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{(1-\alpha)}\right\} + \left(\frac{\delta\alpha}{\delta x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$$-\left(\frac{dP}{dz}\right)_a = \left(\frac{1}{1-M^2}\right)\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$-\left(\frac{dP}{dz}\right)_z = [\rho_g \alpha + \rho_f (1-\alpha)] g\ sin\theta$$

Now, if we use Martinelli Nelson correlation then we can get alpha from graph for a given quality and pressure and then by finding the slope of the graph we can get this partial derivative by taking 2 nearby values of the quality and corresponding values of the void fraction and then dividing we can get this partial derivative and then substitute it here and calculate v star then substitute v star here. And, calculate the pressure gradient due to acceleration the pressure gradient due to gravity can be calculated by using the void fraction obtained from the graph.

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Modelling of Two-Phase Flow – The Separated Flow Model (numerical examples using the Lockhart-Martinelli correlation)

Now, let us consider some numerical examples using Lockhart Martinelli correlation.

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Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 10 cm

$G = 100\ \mathrm{kg/m^2 s}$, x(0)=0, $q'' = 20\ \mathrm{kW/m^2}$

To find the pressure gradient at z = 5 cm

Solution:

Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}\ \mathrm{Pa.s}, \mu_g = 12.26 \times 10^{-6}\ \mathrm{Pa.s}, h_{fg} = 2257.45\ \mathrm{kJ/kg}$

$v_f = 1.043 \times 10^{-3}\ \mathrm{m^3/kg}, v_g = 1.6939\ \mathrm{m^3/kg}, v_{fg} = 1.693\ \mathrm{m^3/kg}$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.443\ \mathrm{m^{-1}}, x(5\mathrm{cm}) = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782 - 1.6939}{1000} = -1.57 \times 10^{-5} \mathrm{m^3 kg^{-1} Pa^{-1}}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1, 1 - \mathrm{M}^2 \approx 1, (1 - \mathrm{M}^2)^{-1} \approx 1$$

$$Re_f = \frac{G(1-x)D}{\mu_f} = 691 \Rightarrow \text{Laminar flow} \qquad Re_g = \frac{GxD}{\mu_g} = 360 \Rightarrow \text{Laminar flow}$$

$$f_f = 16/Re_f = 0.023 \qquad f_g = 16/Re_g = 0.044$$

Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 10 cm

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$$Re_f = \frac{G(1-x)D}{\mu_f} = 691 \Rightarrow Laminar\ flow \qquad Re_g = \frac{GxD}{\mu_g} = 360 \Rightarrow Laminar\ flow$$

$f_f = 16/Re_f = 0.023$ $\qquad$ $f_g = 16/Re_g = 0.044$

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$$X_{vv} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.5} = 0.793$$

$\alpha = [1 + 0.28\, X^{0.71}]^{-1} = 0.808,\ \ \beta = 0.973$ $\qquad \alpha < \beta$

$$\phi_g^2 = 1 + C\,X + X^2 = 5.59\ (\mathrm{C} = 5)$$

$$\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2} = 8.9$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_f}{D} G^2 (1-x)^2 v_f\, \phi_f^2 = 2.05\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_g}{D} G^2 x^2 v_g\, \phi_g^2 = 2.05\ kPa/m$$

Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)} \left\{ G^2 \frac{dx}{dz} v^* \right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{ \frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha} \right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\}$$

$$X_{vv} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.5} = 0.793$$

$$\phi_g^2 = 1 + C\,X + X^2 = 5.59\ (C = 5)$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_g}{D} G^2 x^2 v_g\, \phi_g^2 = 2.034\ kPa/m$$

Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

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$$\alpha = [1 + 0.28\,X^{0.71}]^{-1} \quad X_{vv} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.5}$$

$$\alpha = \left[1 + 0.28\left\{\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.5}\right\}^{0.71}\right]^{-1}$$

$$\frac{\partial \alpha}{\partial x} = -\alpha^{-2}\left[0.28\frac{d}{dx}\{X\}^{0.71}\right] = -\alpha^{-2}\left[0.28 \times 0.71 X^{-0.29}\frac{\partial X}{\partial x}\right]$$

$$\frac{\partial X}{\partial x} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \frac{d}{dx}\left\{\left(\frac{1-x}{x}\right)^{0.5}\right\} = 0.5\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{-0.5} \left(\frac{-1}{x^2}\right)$$

$$\therefore \frac{\partial \alpha}{\partial x} = 5.97$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.236\ m^3/kg$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 1.045\ \text{kPa/m}$$

$$\alpha = [1 + 0.28\, X^{0.71}]^{-1} = 0.808, \quad X_{vv} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.5}$$

$$\alpha = \left[1 + 0.28 \left\{\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.5}\right\}^{0.71}\right]^{-1}$$

$$\frac{\partial \alpha}{\partial x} = -\alpha^{-2}\left[0.28 \frac{d}{dx}\{X\}^{0.71}\right] = -\alpha^{-2}\left[0.28 \times 0.71 X^{-0.29}\frac{\partial X}{\partial x}\right]$$

$$\frac{\partial X}{\partial x} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \frac{d}{dx}\left\{\left(\frac{1-x}{x}\right)^{0.5}\right\} = 0.5\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{-0.5} \left(\frac{-1}{x^2}\right)$$

$$\therefore \frac{\partial \alpha}{\partial x} = 5.97$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.236\ \mathrm{m}^3/kg$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 1.045\ kPa/m$$

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$$-\left(\frac{dP}{dz}\right)_F = 2.054\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 1.045\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0$$

$$-\left(\frac{dP}{dz}\right) = 2.054 + 1.045 + 0 = \underline{3.10}\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = 2.034\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 1.045\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0$$

$$-\left(\frac{dP}{dz}\right) = 2.034 + 1.045 + 0 = 3.08\ kPa/m$$

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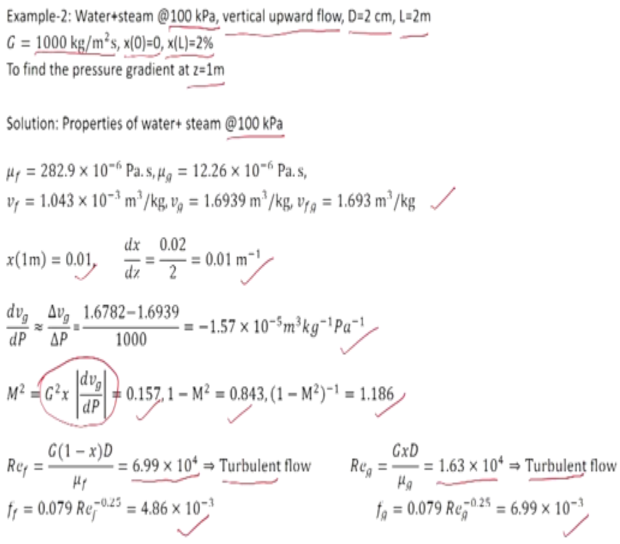


Example-2: Water+steam @100 kPa, vertical upward flow, D=2 cm, L=2m

$G = 1000\ kg/\mathrm{m}^2 s$, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m

Solution: Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}\ Pa.s,\ \mu_g = 12.26 \times 10^{-6}\ Pa.s,$

$v_f = 1.043 \times 10^{-3}\ \mathrm{m}^3/kg,\ v_g = 1.6939\ \mathrm{m}^3/kg, v_{fg} = 1.693\ \mathrm{m}^3/kg$

$$x(1\mathrm{m}) = 0.01,\ \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \mathrm{m}^{-1}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782\text{-}1.6939}{1000} = -1.57 \times 10^{-5} m^3 kg^{-1} Pa^{-1}$$

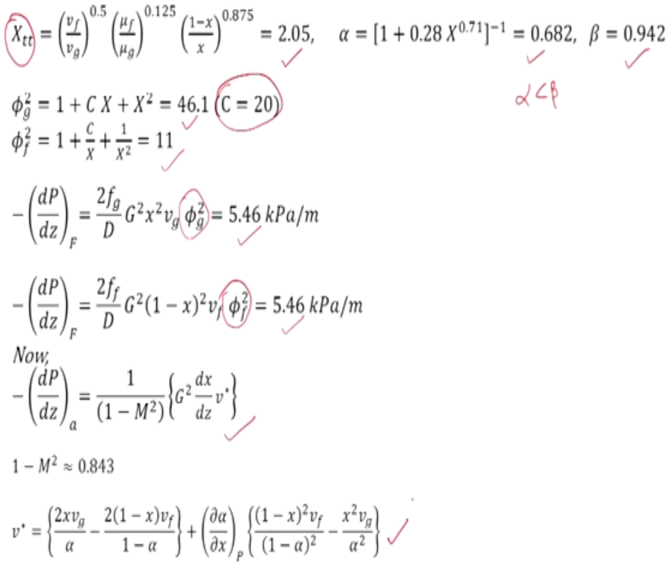
$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 0.157,\ 1 - \mathrm{M}^2 = 0.843,\ (1 - \mathrm{M}^2)^{-1} = 1.186$$

$$Re_f = \frac{G(1-x)D}{\mu_f} = 6.99 \times 10^5 \Rightarrow Turbulent\ flow$$

$$Re_g = \frac{GxD}{\mu_g} = 1.63 \times 10^4 \Rightarrow Turbulent\ flow$$

$$f_f = 0.079\ Re_f^{-0.25} = 2.73 \times 10^{-3} \qquad f_g = 0.079\ Re_g^{-0.25} = 6.99 \times 10^{-3}$$

(Refer Slide Time: 27:50)



$$X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875} = 2.05$$

$$\phi_g^2 = 1 + C\,X + X^2 = 46.2\ (C = 20)$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_g}{D} G^2 x^2 v_g\, \phi_g^2 = 5.47\ kPa/m$$

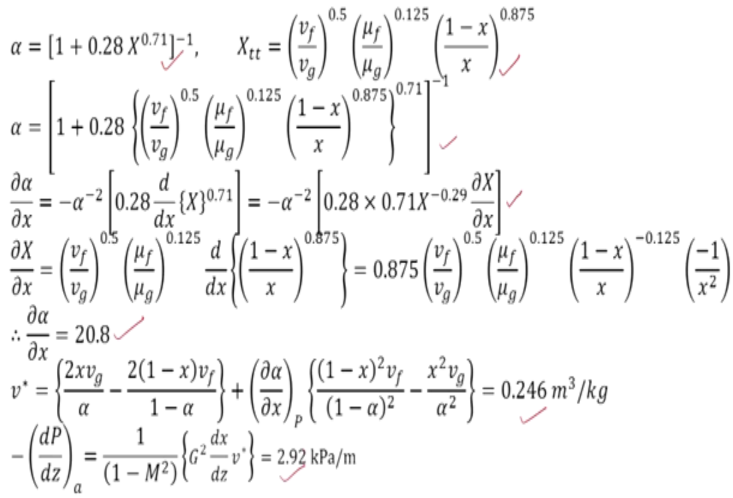
Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 0.843$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

(Refer Slide Time: 29:21)



$$\alpha = [1 + 0.28\, X^{0.71}]^{-1},\ \ X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875}$$

$$\alpha = \left[1 + 0.28 \left\{\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875}\right\}^{0.71}\right]^{-1}$$

$$\frac{\partial\alpha}{\partial x} = -\alpha^{-2}\left[0.28\frac{d}{dx}\{X\}^{0.71}\right] = -\alpha^{-2}\left[0.28 \times 0.71 X^{-0.29}\frac{\partial X}{\partial x}\right]$$

$$\frac{\partial X}{\partial x} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \frac{d}{dx}\left\{\left(\frac{1-x}{x}\right)^{0.875}\right\}$$
$$= 0.875\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{-0.125} \left(\frac{-1}{x^2}\right)$$

$$\therefore \frac{\partial\alpha}{\partial x} = 20.8$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.246\ \mathrm{m}^3/kg$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 2.92\ kPa/m$$

(Refer Slide Time: 30:21)

$$-\left(\frac{dP}{dz}\right)_F = 5.46\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 2.92\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 2.99\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 5.46 + 2.92 + 2.99 = 11.37\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = 5.46\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 2.92\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 2.99\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 5.46 + 2.92 + 2.99 = 11.37\ kPa/m$$

(Refer Slide Time: 31:18)

Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=2m

$G = 1000\ \mathrm{kg/m^2 s}$, x(0)=0, x(L)=2%

To find the pressure gradient at z=1m

Solution:

Properties of water+ steam @10 MPa

$\mu_f = 81.80 \times 10^{-6}\ \mathrm{Pa.s}, \mu_g = 20.27 \times 10^{-6}\ \mathrm{Pa.s},$

$v_f = 1.453 \times 10^{-3}\ \frac{\mathrm{m^3}}{\mathrm{kg}}, v_g = 1.803 \times 10^{-2}\ \mathrm{m^3/kg}, v_{fg} = 0.01658\ \mathrm{m^3/kg}$

$x(1\mathrm{m}) = 0.01, \quad \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \mathrm{m^{-1}}$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781 - 0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1, 1 - \mathrm{M}^2 \approx 1, (1 - \mathrm{M}^2)^{-1} \approx 1$$

$Re_f = \frac{G(1-x)D}{\mu_f} = 2.42 \times 10^5 \Rightarrow$ Turbulent flow

$f_f = 0.079\, Re_f^{-0.25} = 3.56 \times 10^{-3}$

$Re_g = \frac{GxD}{\mu_g} = 9866 \Rightarrow$ Turbulent flow

$f_g = 0.079\, Re_g^{-0.25} = 7.93 \times 10^{-3}$

Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=2m

$G = 1000\ kg/\mathrm{m}^2 s$, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m

Solution: Properties of water+ steam @10 MPa

$\mu_f = 81.80 \times 10^{-6}\ Pa.s,\ \mu_g = 20.27 \times 10^{-6}\ Pa.s,$

$v_f = 1.453 \times 10^{-3} \frac{\mathrm{m}^3}{\mathrm{kg}},\ v_g = 1.803 \times 10^{-2}\ \mathrm{m}^3/kg,\ v_{fg} = 0.01658\ \mathrm{m}^3/kg$

$$x(1\mathrm{m}) = 0.01,\ \ \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \mathrm{m}^{-1}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781\text{-}0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1 - \mathrm{M}^2)^{-1} \approx 1$$

$$Re_f = \frac{G(1-x)D}{\mu_f} = 2.42 \times 10^5 \Rightarrow Turbulent\ flow$$

$$Re_g = \frac{GxD}{\mu_g} = 9866 \Rightarrow Turbulent\ flow$$

$f_f = 0.079\ Re_f^{-0.25} = 3.56 \times 10^{-3}$ $\qquad$ $f_g = 0.079\ Re_g^{-0.25} = 7.93 \times 10^{-3}$

(Refer Slide Time: 33:20)

$$X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875} = 18.8,$$

$$\alpha = [1 + 0.28\, X^{0.71}]^{-1} = 0.308 \qquad \beta = 0.114 \qquad \alpha > \beta$$

$$\phi_g^2 = 1 + C\,X + X^2 = 730.44\ (C = 20)$$

$$\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2} = 2.06$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_f}{D} G^2 (1-x)^2 v_f\, \phi_f^2 = 1.047\, kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_g}{D} G^2 x^2 v_g\, \phi_g^2 = 1.047\, kPa/m$$

Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$$X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875} = 18.8$$

$$\alpha = [1 + 0.28\, X^{0.71}]^{-1} = 0.308 \quad \beta = 0.114 \;\rightarrow\; \alpha > \beta$$

$$\phi_g^2 = 1 + C\,X + X^2 = 730.44\ (C = 20)$$

$$\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2} = 2.06$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{\mathrm{f}}}{D} G^2 (1-x)^2 v_{\mathrm{f}}\, \phi_{\mathrm{f}}^2 = 1.047\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_g}{D} G^2 x^2 v_g\, \phi_g^2 = 1.047\ kPa/m$$

Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

(Refer Slide Time: 35:16)

$$\alpha = [1 + 0.28\, X^{0.71}]^{-1} X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875}$$

$$\alpha = \left[1 + 0.28 \left\{\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875}\right\}^{0.71}\right]^{-1}$$

$$\frac{\partial \alpha}{\partial x} = -\alpha^{-2}\left[0.28 \frac{d}{dx}\{X\}^{0.71}\right] = -\alpha^{-2}\left[0.28 \times 0.71 X^{-0.29} \frac{\partial X}{\partial x}\right]$$

$$\frac{\partial X}{\partial x} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \frac{d}{dx}\left\{\left(\frac{1-x}{x}\right)^{0.875}\right\} = 0.875 \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{-0.125} \left(\frac{-1}{x^2}\right)$$

$$\therefore \frac{\partial \alpha}{\partial x} = 257$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.756\ m^3/kg$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 7.56\ \mathrm{kPa/m}$$

$$\alpha = [1 + 0.28\, X^{0.71}]^{-1},\ \ X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875}$$

$$\alpha = \left[1 + 0.28 \left\{\left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875}\right\}^{0.71}\right]^{-1}$$

$$\frac{\partial \alpha}{\partial x} = -\alpha^{-2}\left[0.28 \frac{d}{dx}\{X\}^{0.71}\right] = -\alpha^{-2}\left[0.28 \times 0.71 X^{-0.29} \frac{\partial X}{\partial x}\right]$$

$$\frac{\partial X}{\partial x} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \frac{d}{dx}\left\{\left(\frac{1-x}{x}\right)^{0.875}\right\}$$

$$= 0.875 \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{-0.125} \left(\frac{-1}{x^2}\right)$$

$$\therefore \frac{\partial \alpha}{\partial x} = 257$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.756\ \mathrm{m}^3/kg$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 7.56\ kPa/m$$

(Refer Slide Time: 36:10)

$$-\left(\frac{dP}{dz}\right)_F = 1.047\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 7.56\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = [\rho_g \alpha + \rho_f(1-\alpha)] g \sin\theta = 4.83\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 1.047 + 7.56 + 4.83 = 13.44\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = 1.047\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 7.56\ kPa/m$$

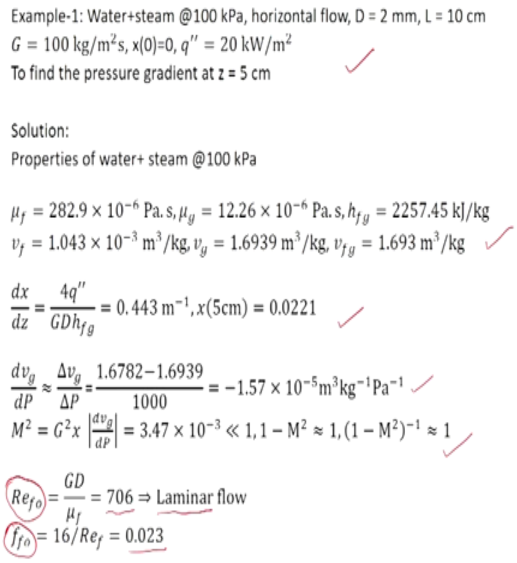
$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 4.83\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 1.047 + 7.56 + 4.83 = 13.44\ kPa/m$$

(Refer Slide Time: 36:48)

Modelling of Two-Phase Flow
– The Separated Flow Model
(numerical examples using the
Martinelli-Nelson correlation)

(Refer Slide Time: 36:50)



Example-1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 10 cm

$G = 100\ kg/\mathrm{m}^2 s$, x(0)=0, $q'' = 20\ kW/\mathrm{m}^2$. To find the pressure gradient at z = 5 cm

Solution: Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}\ Pa.s,\ \mu_g = 12.26 \times 10^{-6}\ Pa.s,\ h_{fg} = 2257.45\ kJ/kg$

$v_f = 1.043 \times 10^{-3}\ \mathrm{m}^3/kg,\ v_g = 1.6939\ \mathrm{m}^3/kg,\ v_{fg} = 1.693\ \mathrm{m}^3/kg$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.\ 443\ \mathrm{m}^{-1},\ x(5\mathrm{cm}) = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782 - 1.6939}{1000} = -1.57 \times 10^{-5} \mathrm{m}^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1, \quad 1 - M^2 \approx 1, \quad (1 - M^2)^{-1} \approx 1$$

$$Re_f = \frac{GD}{\mu_f} = 706 \Rightarrow Laminar\ flow$$

$$f_{fo} = 16/Re_f = 0.023$$

(Refer Slide Time: 38:22)

$$\alpha \approx 0.73,\ \beta = 0.973$$
$$\phi_{fo}^2 = 12.98$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{fo}}{D} G^2 v_f \phi_{fo}^2 = 3.06\ kPa/m$$

Now,
$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_p \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

$\alpha \approx 0.73, \beta = 0.973$

$\phi_{fo}^2 = 12.98$

$$-\left(\frac{dp}{dz}\right)_F = \frac{2f_{fo}}{D} G^2 v_f \phi_{fo}^2 = 3.06 kPa/m$$

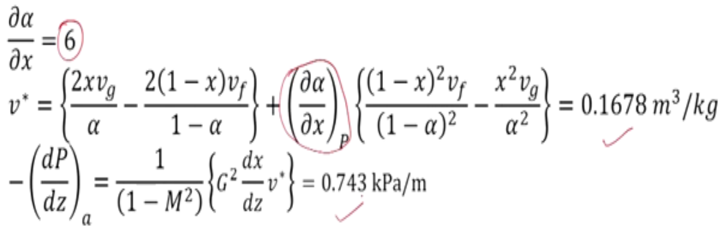
Now,

$$-\left(\frac{dp}{dz}\right)_a = \frac{1}{1 - M^2} \left\{G^2 \frac{dx}{dz} v^*\right\}$$

$1 - M^2 \approx 1$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

(Refer Slide Time: 39:27)

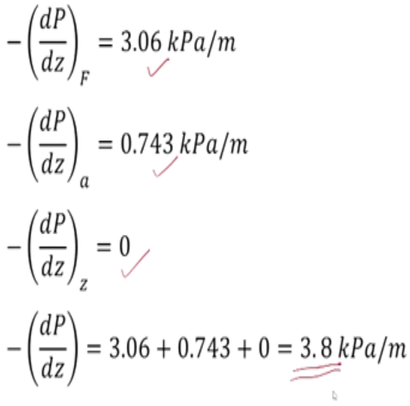


$$\frac{\partial \alpha}{\partial x} = 6$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.1678\, m/s$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1 - M^2)} \left\{G^2 \frac{dx}{dz} v^*\right\} = 0.743 kPa/m$$

(Refer Slide Time: 40:19)



$$-\left(\frac{dP}{dz}\right)_F = 3.06\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 0.743\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0$$

$$-\left(\frac{dP}{dz}\right) = 3.06 + 0.743 + 0 = 3.8\ kPa/m$$

(Refer Slide Time: 40:45)

Example-2: Water+steam @100 kPa, vertical upward flow, D=2 cm, L=2m

$G = 1000\ kg/\mathrm{m}^2 s$, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m

Solution: Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}\ Pa.s,\ \mu_g = 12.26 \times 10^{-6}\ Pa.s,$

$v_f = 1.043 \times 10^{-3}\ \mathrm{m}^3/kg,\ v_g = 1.6939\ \mathrm{m}^3/kg,\ v_{fg} = 1.693\ \mathrm{m}^3/kg$

$$x(1\mathrm{m}) = 0.01,\ \ \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \mathrm{m}^{-1}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782\text{-}1.6939}{1000} = -1.57 \times 10^{-5} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 0.157,\ 1 - \mathrm{M}^2 = 0.843,\ (1 - \mathrm{M}^2)^{-1} = 1.186$$

$$Re_{f\mathrm{o}} = \frac{GD}{\mu_f} = 706 \Rightarrow Laminar\ flow$$

$$f_{f\mathrm{o}} = 16/Re_f = 0.023$$

(Refer Slide Time: 41:38)

$$\alpha \approx 0.67, \quad \beta = 0.943$$

$$\phi_{fo}^2 = 5.6$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{fo}}{D} G^2 v_f \phi_{fo}^2 = 0.1322\, kPa/m$$

$$\frac{\partial \alpha}{\partial x} = 6$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.098\, m^3/kg$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\} = 1.167 \text{ kPa/m}$$

$$\alpha \approx 0.67, \beta = 0.943$$

$$\phi_{fo}^2 = 5.6$$

$$-\left(\frac{dp}{dz}\right)_F = \frac{2f_{fo}}{D} G^2 v_f \phi_{fo}^2 = 0.1322\, kPa/m$$

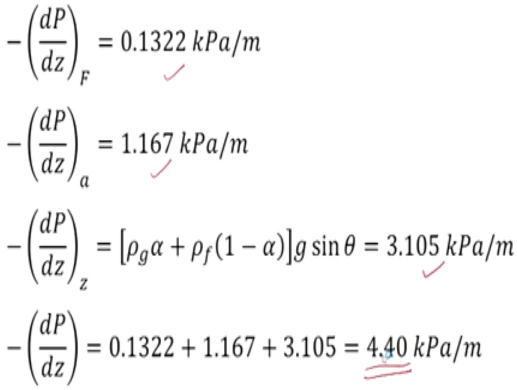
Now,

$$\frac{\partial \alpha}{\partial x} = 6$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.098 \text{ m/s}$$

$$-\left(\frac{dp}{dz}\right)_{\mathrm{a}} = \frac{1}{1-M^2}\left\{G^2 \frac{dx}{dz} v^*\right\} = 1.167 \text{ kPa/m}$$

(Refer Slide Time: 42:56)



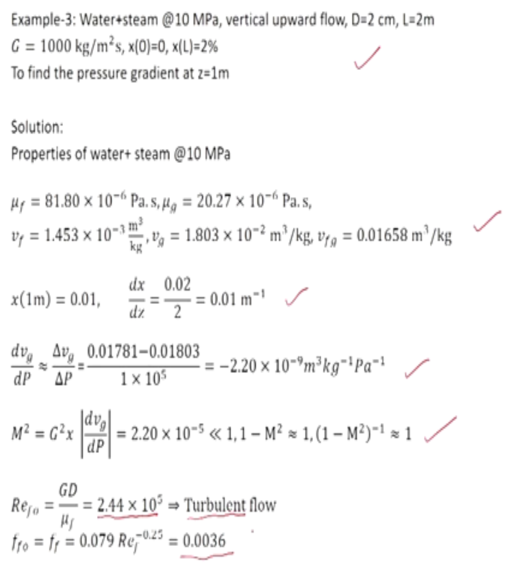
$$-\left(\frac{dP}{dz}\right)_F = 0.1322\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 1.167\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 3.105\ \mathrm{kPa/m}$$

$$-\left(\frac{dP}{dz}\right) = 0.1322 + 1.167 + 3.105 = 4.4\ kPa/m$$

(Refer Slide Time: 43:26)



Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=2m

$G = 1000\ kg/\mathrm{m}^2 s$, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m

Solution: Properties of water+ steam @10 MPa

$\mu_f = 81.80 \times 10^{-6}\ Pa.s,\ \mu_g = 20.27 \times 10^{-6}\ Pa.s,$

$v_f = 1.453 \times 10^{-3} \frac{\mathrm{m}^3}{\mathrm{kg}},\ v_g = 1.803 \times 10^{-2}\ \mathrm{m}^3/kg,\ v_{fg} = 0.01658\ \mathrm{m}^3/kg$

$x(1\mathrm{m}) = 0.01,\ \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \mathrm{m}^{-1}$

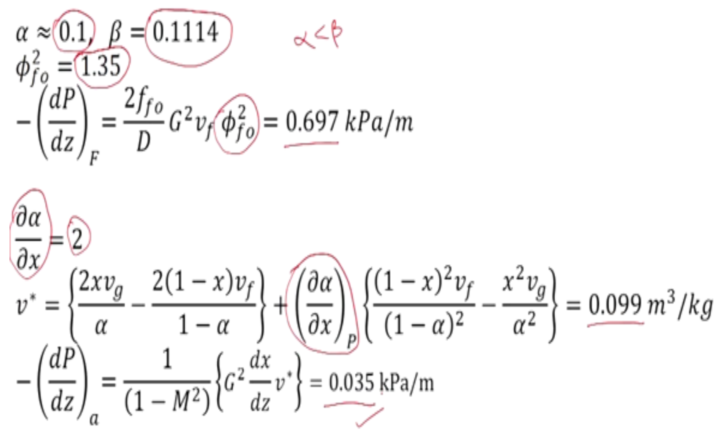
$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781\text{-}0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1 - \mathrm{M}^2)^{-1} \approx 1$$

$$Re_{fo} = \frac{GD}{\mu_f} = 2.44 \times 10^5 \Rightarrow \text{Turbulent } flow$$

$$f_{fo} = 0.079 Re_f^{-0.25} = 0.036$$

(Refer Slide Time: 44:23)



$\alpha \approx 0.1, \beta = 0.114$

$\phi_{fo}^2 = 1.35$

$$-\left(\frac{dp}{dz}\right)_F = \frac{2f_{fo}}{D} G^2 v_f \phi_{fo}^2 = 0.697 \ kPa/m$$

Now,

$$\frac{\partial \alpha}{\partial x} = 2$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\} = 0.099 \ \mathrm{m/s}$$

$$-\left(\frac{dp}{dz}\right)_{\mathrm{a}} = \frac{1}{1-M^2}\left\{G^2 \frac{dx}{dz} v^*\right\} = 0.035 \ \mathrm{kPa/m}$$

(Refer Slide Time: 46:48)

$$X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875} = 18.8,$$

$$\alpha = [1 + 0.28\, X^{0.71}]^{-1} = 0.308, \quad \beta = 0.114$$

$$\phi_g^2 = 1 + C\,X + X^2 = 730.44\ (C = 20)$$

$$\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2} = 2.06$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_f}{D} G^2 (1-x)^2 v_f\, \phi_f^2 = 1.047\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_g}{D} G^2 x^2 v_g\, \phi_g^2 = 1.047\ kPa/m$$

Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

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$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_g}{D} G^2 x^2 v_g\, \phi_g^2 = 1.047\ kPa/m$$

Now,

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{(1-M^2)}\left\{G^2 \frac{dx}{dz} v^*\right\}$$

$$1 - M^2 \approx 1$$

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

(Refer Slide Time: 46:59)

$$-\left(\frac{dP}{dz}\right)_F = 0.697\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 0.035\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 6.12\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 0.697 + 0.035 + 6.12 = 6.85\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_F = 0.697 kPa/m$$

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$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = 6.12 \mathrm{kPa/m}$$

$$-\left(\frac{dP}{dz}\right) = 0.697 + 0.035 + 6.12 = 6.85\ kPa/m$$