**Two-Phase flow with phase change in conventional and miniature channels**
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**Lecture – 04**
**The Separated Flow Model**

Welcome back to the course on Two-Phase flow with phase change in conventional and miniature channels. We are discussing the modeling of two phase flow and last time we have discussed the homogeneous model.

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Modelling of Two-Phase Flow
– The Separated Flow Model

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Separated Flow Model

- Two phases flow separately and interact with each other
- Velocities of the two phases can be different
- Thermal equilibrium between phases
- Properties of the two phases evaluated separately
- Correlations for two-phase frictional multiplier
- Correlations for void fraction or slip ratio

Following are its assumptions:

- Two phases flow separately and interact with each other.
- Velocities of the two phases can be different but there will be thermal equilibrium between phases. This means if the two phases are liquid and gas phases of the same substance (water and steam) then both will be at the saturation temperature corresponding to the local pressure.
- Properties of the two phases will be evaluated separately
- Correlations for two-phase frictional multiplier will be used.
- Correlations for void fraction or slip ratio will be used. Void fraction is required to calculate the gravitational and acceleration pressure gradients.

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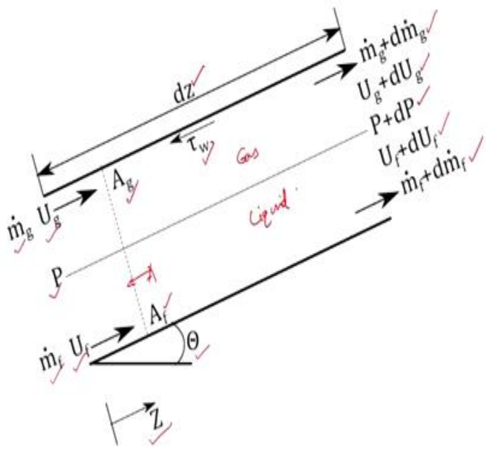

So, consider a two phase flow flowing through a channel (figure 1). So, as explained in earlier lecture, z coordinate is the axial coordinate; the angle with the horizontal is theta. The channel cross section area is A and the cross sectional area occupied by the liquid phases $A_f$ and the area occupied by the gas phases $A_g$. The mass flow rate of the gas is $\dot{m}_g$, velocity of the gas phase is $U_g$, mass flow rate of the liquid phase is $\dot{m}_f$, velocity of liquid phase is $U_f$ and the pressure at the inlet is P, the length of the element is dz. The pressure at the outlet is P+dP, velocities are $U_g$+$dU_g$ and $U_f$+$dU_f$ and mass flow rates are $\dot{m}_g + d\dot{m}_g$ and $\dot{m}_f + d\dot{m}_f$. The wall shear stress is $\tau_w$ which

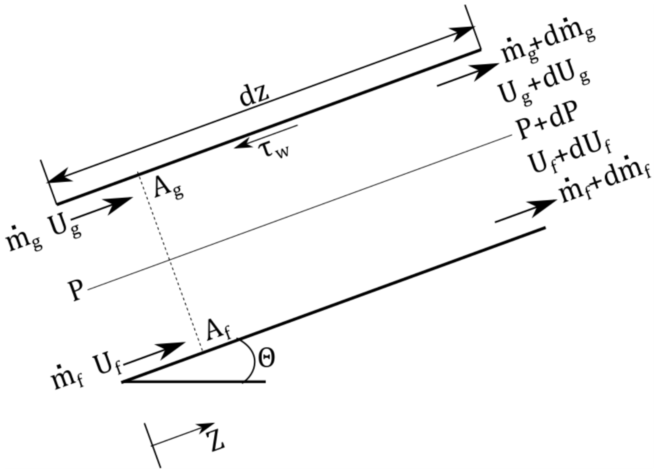


Figure 1: Two phase mixture flowing through a pipe

we will divide into two parts $\tau_{gw}$ and $\tau_{fw}$, also as mentioned before the interactions between the two phases are taken into account in this model.

So, there will be wall shear stress at the interface, also I should mention that here the two phases are shown separately. Gas phase is here, liquid phase is here, but this does not mean that this model is only for stratified flow; it is applicable for other types of like flow patterns also; like annular flow. This model is better for the flow patterns in which the phases are separated, but it need not necessarily be stratified flow. So, this is just a representation; it is a schematic diagram.

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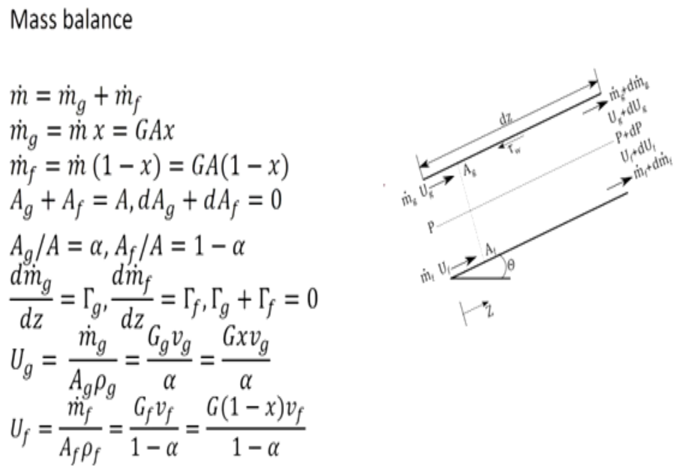


So, now consider the mass balance:

$$\dot{m} = \dot{m}_g + \dot{m}_f$$

$$\dot{m}_g = \dot{m}\, x = GAx$$

$$\dot{m}_f = \dot{m}\,(1-x) = GA(1-x)$$

$$A_g + A_f = A,\ dA_g + dA_f = 0$$

$$A_g/A = \alpha, A_f/A = 1-\alpha$$

$$\frac{d\dot{m}_g}{dz} = \Gamma_g, \frac{d\dot{m}_f}{dz} = \Gamma_f, \Gamma_g + \Gamma_f = 0$$

$$U_g = \frac{\dot{m}_g}{A_g\rho_g} = \frac{G_g v_g}{\alpha} = \frac{Gxv_g}{\alpha}$$

$$U_f = \frac{\dot{m}_f}{A_f\rho_f} = \frac{G_f v_f}{1-\alpha} = \frac{G(1-x)v_f}{1-\alpha}$$

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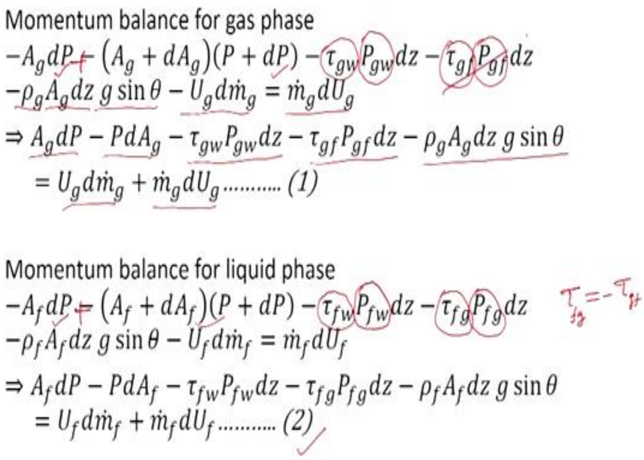


Momentum balance for gas phase:

$$-A_g dP + (A_g + dA_g)(P + dP) - \tau_{gw}P_{gw}dz - \tau_{gf}P_{gf}dz - \rho_g A_g dz\, g\sin\theta - U_g d\dot{m}_g = \dot{m}_g dU_g$$

$$\Rightarrow A_g dP - PdA_g - \tau_{gw}P_{gw}dz - \tau_{gf}P_{gf}dz - \rho_g A_g dz\, g\sin\theta = U_g d\dot{m}_g + \dot{m}_g dU_g \ldots\ldots\ldots \text{(1)}$$

Momentum balance for liquid phase:

$$-A_f dP + (A_f + dA_f)(P + dP) - \tau_{fw}P_{fw}dz - \tau_{fg}P_{fg}dz - \rho_f A_f dz\, g\sin\theta - U_f d\dot{m}_f = \dot{m}_f dU_f$$

$$\Rightarrow A_f dP - PdA_f - \tau_{fw}P_{fw}dz - \tau_{fg}P_{fg}dz - \rho_f A_f dz\, g\sin\theta = U_f d\dot{m}_f + \dot{m}_f dU_f \ldots\ldots\ldots.. (2)$$

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Adding eqs. (1) and (2)

$$(A_g + A_f)dP - P\left(dA_g + dA_f\right) - (\tau_{gw}P_{gw} + \tau_{fw}P_{fw})dz$$
$$-(\tau_{gf} + \tau_{fg})P_{gf}dz - (\rho_g A_g + \rho_f A_f)dz\, g\sin\theta$$
$$= U_g d\dot{m}_g + \dot{m}_g dU_g + U_f\, d\dot{m}_f + \dot{m}_f dU_f$$
$$\Rightarrow AdP - (\tau_{gw}P_{gw} + \tau_{fw}P_{fw})dz - [\rho_g\alpha + \rho_f(1-\alpha)]Adz\, g\sin\theta$$
$$= d(\dot{m}_g U_g + \dot{m}_f U_f) \ldots\ldots\ldots. (3)$$
$$-\frac{dP}{dz} - \frac{1}{A}(\tau_{gw}P_{gw} + \tau_{fw}P_{fw}) - [\rho_g\alpha + \rho_f(1-\alpha)]\, g\sin\theta = \frac{1}{A}\frac{d}{dz}(\dot{m}_g U_g + \dot{m}_f U_f)$$
$$-\frac{dP}{dz} = \frac{1}{A}(\tau_{gw}P_{gw} + \tau_{fw}P_{fw}) + \frac{1}{A}\frac{d}{dz}\left(\dot{m}_g U_g + \dot{m}_f U_f\right) + [\rho_g\alpha + \rho_f(1-\alpha)]\, g\sin\theta$$

Adding equations (1) and (2):

$$(A_g + A_f)dP - P\left(dA_g + dA_f\right) - (\tau_{gw}P_{gw} + \tau_{fw}P_{fw})dz$$
$$-(\tau_{gf} + \tau_{fg})P_{gf}dz - (\rho_g A_g + \rho_f A_f)dz\, g\sin\theta = U_g d\dot{m}_g + \dot{m}_g dU_g + U_f d\dot{m}_f + \dot{m}_f dU_f$$

$$\Rightarrow AdP - (\tau_{gw}P_{gw} + \tau_{fw}P_{fw})dz - [\rho_g\alpha + \rho_f(1-\alpha)]Adz\, g\sin\theta = d\left(\dot{m}_g U_g + \dot{m}_f U_f\right) \ldots\ldots\ldots.. (3)$$

$$-\frac{dP}{dz} - \frac{1}{A}(\tau_{gw}P_{gw} + \tau_{fw}P_{fw}) - [\rho_g\alpha + \rho_f(1-\alpha)]\; g\sin\theta = \frac{1}{A}\frac{d}{dz}(\dot{m}_g U_g + \dot{m}_f U_f)$$

$$-\frac{dP}{dz} = \left\{\frac{1}{A}(\tau_{gw}P_{gw} + \tau_{fw}P_{fw})\right\} + \left\{\frac{1}{A}\frac{d}{dz}\left(\dot{m}_g U_g + \dot{m}_f U_f\right)\right\} + \left[\left\{\rho_g\alpha + \rho_f(1-\alpha)\right\}g\sin\theta\right]$$

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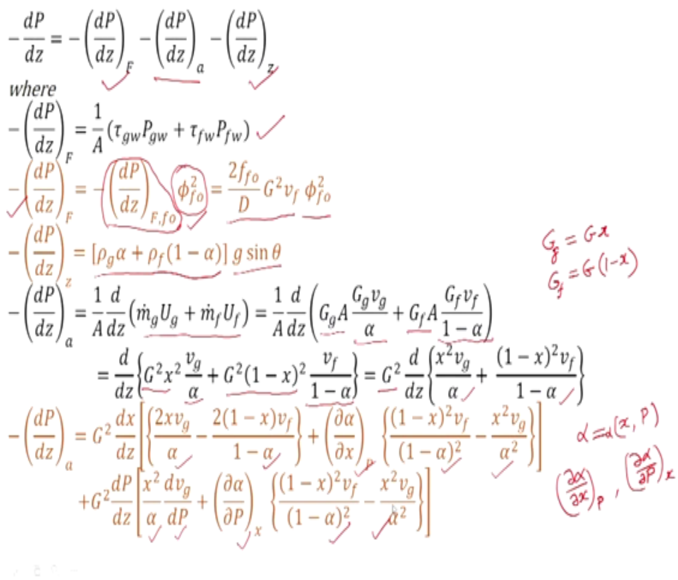

$$-\frac{dP}{dz} = -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

Where:

$$-\left(\frac{dP}{dz}\right)_F = \frac{1}{A}\left(\tau_{gw}P_{gw} + \tau_{fw}P_{fw}\right) = -\left(\frac{dP}{dz}\right)_{F,fo} \phi_{fo}^2 = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2$$

$$-\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{1}{A}\frac{d}{dz}\left(\dot{m}_g U_g + \dot{m}_f U_f\right) = \frac{1}{A}\frac{d}{dz}\left(G_g A \frac{G_g v_g}{\alpha} + G_f A \frac{G_f v_f}{1-\alpha}\right)$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{d}{dz}\left\{G^2 x^2 \frac{v_g}{\alpha} + G^2 (1-x)^2 \frac{v_f}{1-\alpha}\right\} = G^2 \frac{d}{dz}\left\{\frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha}\right\}$$

$$-\left(\frac{dP}{dz}\right)_a = G^2 \frac{dx}{dz}\left[\left\{\frac{2x v_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right] + G^2 \frac{dP}{dz}\left[\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right]$$

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$$-\frac{dP}{dz} = -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

$$-\frac{dP}{dz} = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 + G^2 \frac{dx}{dz}\left[\left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right]$$

$$\underbrace{+G^2 \frac{dP}{dz}\left[\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right]}_{M^2} + [\rho_g\alpha + \rho_f(1-\alpha)]\, g\sin\theta$$

$$-\frac{dP}{dz}(1-M^2) = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2$$

$$+G^2 \frac{dx}{dz}\left[\left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right] + [\rho_g\alpha + \rho_f(1-\alpha)]\, g\sin\theta$$

where

$$M^2 = -G^2\left[\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right] = G^2\left|\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right|$$

Now,

$$-\frac{dP}{dz} = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 + G^2 \frac{dx}{dz}\left[\left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right]$$

$$+G^2 \frac{dP}{dz}\left[\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right] + \left[\rho_g\alpha + \rho_f(1-\alpha)\right] g\sin\theta$$

$$\Rightarrow -\frac{dP}{dz}(1-M^2) = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 + G^2 \frac{dx}{dz}\left[\left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial\alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right]$$

$$+\left[\rho_g\alpha + \rho_f(1-\alpha)\right] g\sin\theta$$

where

$$M^2 = -G^2\left[\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right] = G^2\left|\frac{x^2}{\alpha}\frac{dv_g}{dP} + \left(\frac{\partial\alpha}{\partial P}\right)_x \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}\right|$$

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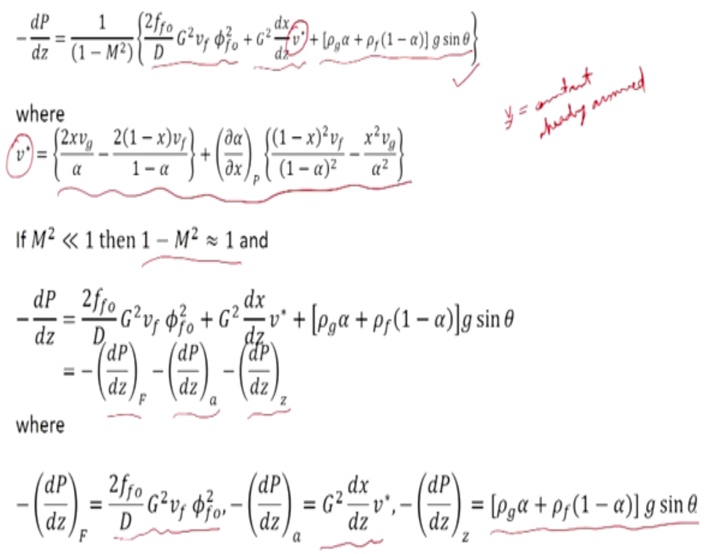


$$-\frac{dP}{dz} = \frac{1}{(1-M^2)}\left\{\frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 + G^2 \frac{dx}{dz} v^* + \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta\right\}$$

where

$$v^* = \left\{\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha}\right\} + \left(\frac{\partial \alpha}{\partial x}\right)_P \left\{\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2}\right\}$$

If $M^2 \ll 1$ then $1 - M^2 \approx 1$ and:

$$-\frac{dP}{dz} = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2 + G^2 \frac{dx}{dz} v^* + \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta = -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

where

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{fo}}{D} G^2 v_f\, \phi_{fo}^2, -\left(\frac{dP}{dz}\right)_a = G^2 \frac{dx}{dz} v^*, -\left(\frac{dP}{dz}\right)_z = \left[\rho_g \alpha + \rho_f (1-\alpha)\right] g \sin\theta$$

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Choked flow

$$-\frac{dP}{dz} \rightarrow \infty \text{ when } 1 - M^2 \rightarrow 0 \text{ that is } M^2 \rightarrow 1$$

$$\Rightarrow G_{\max}^2 \left| \frac{x^2}{\alpha} \frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right| = 1$$

$$\Rightarrow G_{\max} = \left( \left| \frac{x^2}{\alpha} \frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right| \right)^{-\frac{1}{2}}$$

$$\Rightarrow \dot{m}_{\max} = A \left( \left| \frac{x^2}{\alpha} \frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right| \right)^{-\frac{1}{2}}$$

This is the maximum mass flow rate that can flow through the channel.


Choked flow:

$$-\frac{dP}{dz} \rightarrow \infty \text{ when } 1 - M^2 \rightarrow 0 \text{ that is } M^2 \rightarrow 1$$

$$\Rightarrow G_{\max}^2 \left| \frac{x^2}{\alpha} \frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right| = 1$$

$$\Rightarrow G_{\max} = \left( \left| \frac{x^2}{\alpha} \frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right| \right)^{-\frac{1}{2}}$$

$$\Rightarrow \dot{m}_{\max} = A \left( \left| \frac{x^2}{\alpha} \frac{dv_g}{dP} + \left(\frac{\partial \alpha}{\partial P}\right)_x \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right| \right)^{-\frac{1}{2}}$$

This is the maximum mass flow rate that can flow through the channel.

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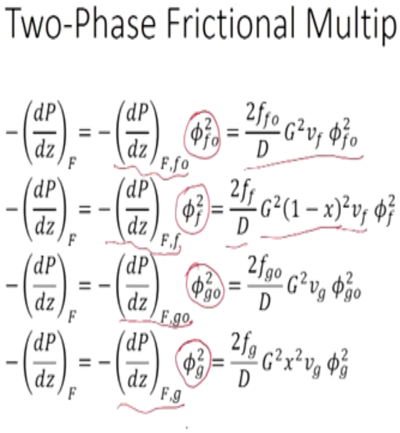


Now, there is the question of two phase frictional multipliers which are defined as follows:

$$-\left(\frac{dP}{dz}\right)_F = -\left(\frac{dP}{dz}\right)_{F,fo} \phi_{fo}^2 = \frac{2f_{fo}}{D} G^2 v_f \, \phi_{fo}^2$$

$$-\left(\frac{dP}{dz}\right)_F = -\left(\frac{dP}{dz}\right)_{F,f} \phi_f^2 = \frac{2f_f}{D} G^2 (1-x)^2 v_f \, \phi_f^2$$

$$-\left(\frac{dP}{dz}\right)_F = -\left(\frac{dP}{dz}\right)_{F,go} \phi_{go}^2 = \frac{2f_{go}}{D} G^2 v_g \, \phi_{go}^2$$

$$-\left(\frac{dP}{dz}\right)_F = -\left(\frac{dP}{dz}\right)_{F,g} \phi_g^2 = \frac{2f_g}{D} G^2 x^2 v_g \, \phi_g^2$$

As discussed in the earlier lecture, the subscript fo and go denotes the hypothetical frictional pressure gradient assuming all the fluid is in liquid and gas phase respectively. Similarly, the subscript f and g means only liquid or gas is flowing.

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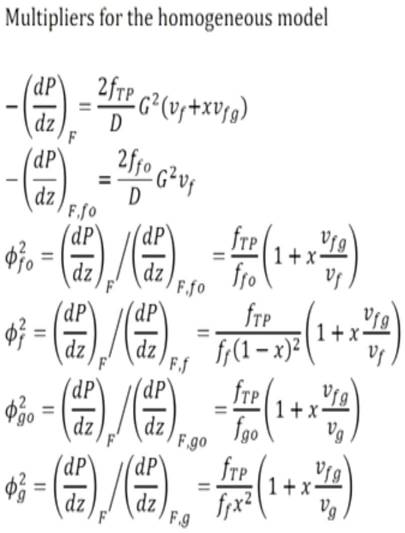


Multipliers for the homogeneous model

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D}G^2(v_f + xv_{fg})\ and$$

$$-\left(\frac{dP}{dz}\right)_{F,fo} = \frac{2f_{fo}}{D}G^2v_f$$

$$\therefore,\ \phi_{fo}^2 = \left(\frac{dP}{dz}\right)_F \Big/ \left(\frac{dP}{dz}\right)_{F,fo} = \frac{f_{TP}}{f_{fo}}\left(1 + x\frac{v_{fg}}{v_f}\right);$$

$$\phi_f^2 = \left(\frac{dP}{dz}\right)_F \Big/ \left(\frac{dP}{dz}\right)_{F,f} = \frac{f_{TP}}{f_f(1-x)^2}\left(1 + x\frac{v_{fg}}{v_f}\right);$$

$$\phi_{go}^2 = \left(\frac{dP}{dz}\right)_F \Big/ \left(\frac{dP}{dz}\right)_{F,go} = \frac{f_{TP}}{f_{go}}\left(1 + x\frac{v_{fg}}{v_g}\right);\ and$$

$$\phi_g^2 = \left(\frac{dP}{dz}\right)_F \Big/ \left(\frac{dP}{dz}\right)_{F,g} = \frac{f_{TP}}{f_f x^2}\left(1 + x\frac{v_{fg}}{v_g}\right)$$

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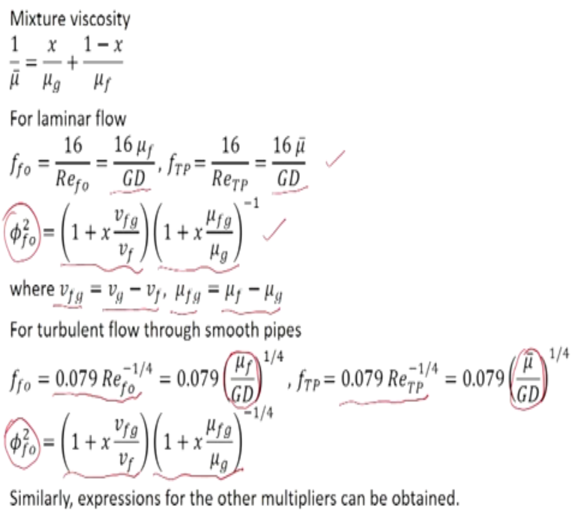


Mixture viscosity

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} (McAdamsCorrelation)$$

For laminar flow

$$f_{fo} = \frac{16}{Re_{fo}} = \frac{16\,\mu_f}{GD},\ f_{TP} = \frac{16}{Re_{TP}} = \frac{16\,\bar{\mu}}{GD}$$

$$\phi_{fo}^2 = \left(1 + x\frac{v_{fg}}{v_f}\right)\left(1 + x\frac{\mu_{fg}}{\mu_g}\right)^{-1}$$

where $v_{fg} = v_g - v_f,\ \ \mu_{fg} = \mu_f - \mu_g$

For turbulent flow through smooth pipes

$$f_{fo} = 0.079\,Re_{fo}^{-1/4} = 0.079\left(\frac{\mu_f}{GD}\right)^{1/4},\ f_{TP} = 0.079\,Re_{TP}^{-1/4} = 0.079\left(\frac{\bar{\mu}}{GD}\right)^{1/4}$$

$$\phi_{fo}^2 = \left(1 + x\frac{v_{fg}}{v_f}\right)\left(1 + x\frac{\mu_{fg}}{\mu_g}\right)^{-1/4}$$

Similarly, expressions for the other multipliers can be obtained.

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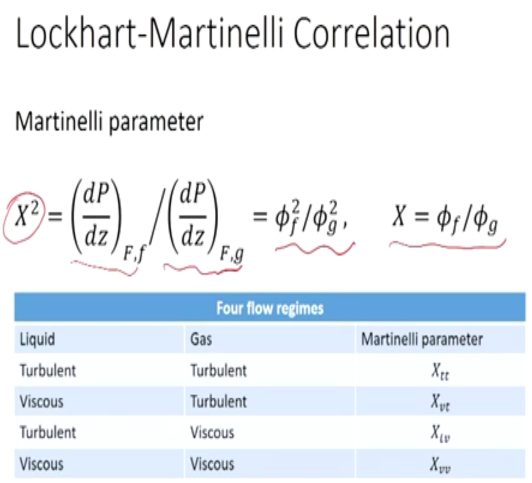


Now, the question is how to find the frictional multipliers for Martinelli; for separated flow model? For that there are correlations; the classic correlation is by Lockhart and Martinelli; in this a new parameter is defined which is called Martinelli parameter and it is denoted by capital X.

$$X^2 = \left(\frac{dP}{dz}\right)_{F,f} \Big/ \left(\frac{dP}{dz}\right)_{F,g} = \phi_f^2/\phi_g^2 \Rightarrow X = \phi_f/\phi_g$$

| Four flow regimes | | |
|---|---|---|
| Liquid | Gas | Martinelli parameter |
| Turbulent | Turbulent | $X_{tt}$ |
| Viscous | Turbulent | $X_{vt}$ |
| Turbulent | Viscous | $X_{tv}$ |
| Viscous | Viscous | $X_{vv}$ |

Now, the question is how to estimate X for that correlations are required. Lockhart and Martinelli considered four flow regimes (as shown in the above table); these are based on the Reynolds numbers of the liquid phase and the gas phase. If the liquid phase is in the turbulent regime and the gas phase is also in the turbulent regime, then it is called turbulent- turbulent; turbulent-

turbulent regime and the corresponding Martinelli parameter is denoted by $X_{tt}$. If the liquid phase is laminar or viscous and the gas phase is turbulent; then it is called viscous turbulent regime and is denoted by $X_{vt}$. Commonly, in conventional channels the regime is turbulent-turbulent; both the phases are turbulent, but in miniature channels commonly it is viscous-viscous; both phases are laminar.

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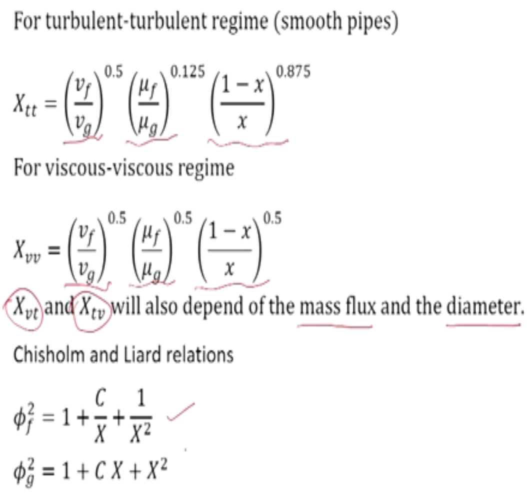


For turbulent-turbulent regime (smooth pipes)

$$X_{tt} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.125} \left(\frac{1-x}{x}\right)^{0.875}$$

For viscous-viscous regime

$$X_{vv} = \left(\frac{v_f}{v_g}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.5} \left(\frac{1-x}{x}\right)^{0.5}$$

$X_{vt}$ and $X_{tv}$ will also depend of the mass flux and the diameter.

Now, to correlate the frictional multipliers; there are relations given by Chisholm and Liard. Originally, Lockhart and Martinelli had given graphs (figure 2), then later on Chisholm and Liard gave these relations:

$$\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$$

$$\phi_g^2 = 1 + C\,X + X^2$$

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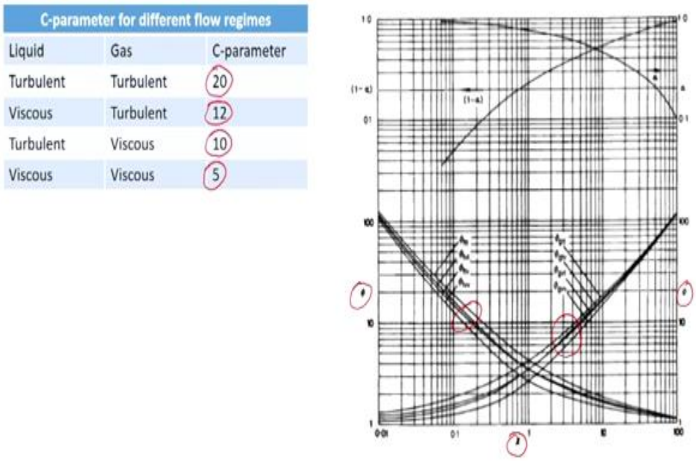


| C-parameter for different flow regimes | | |
|---|---|---|
| Liquid | Gas | C-parameter |
| Turbulent | Turbulent | 20 |
| Viscous | Turbulent | 12 |
| Turbulent | Viscous | 10 |
| Viscous | Viscous | 5 |

For the turbulent-turbulent regime; the value of C is 20, for viscous turbulent regime it is 12, for turbulent viscous regime it is 10 and for viscous-viscous regime; it is 5. So, using these four values of C; we can find these $\phi_f^2$ and $\phi_g^2$ for all the four flow regimes as functions of the Martinelli parameter X.

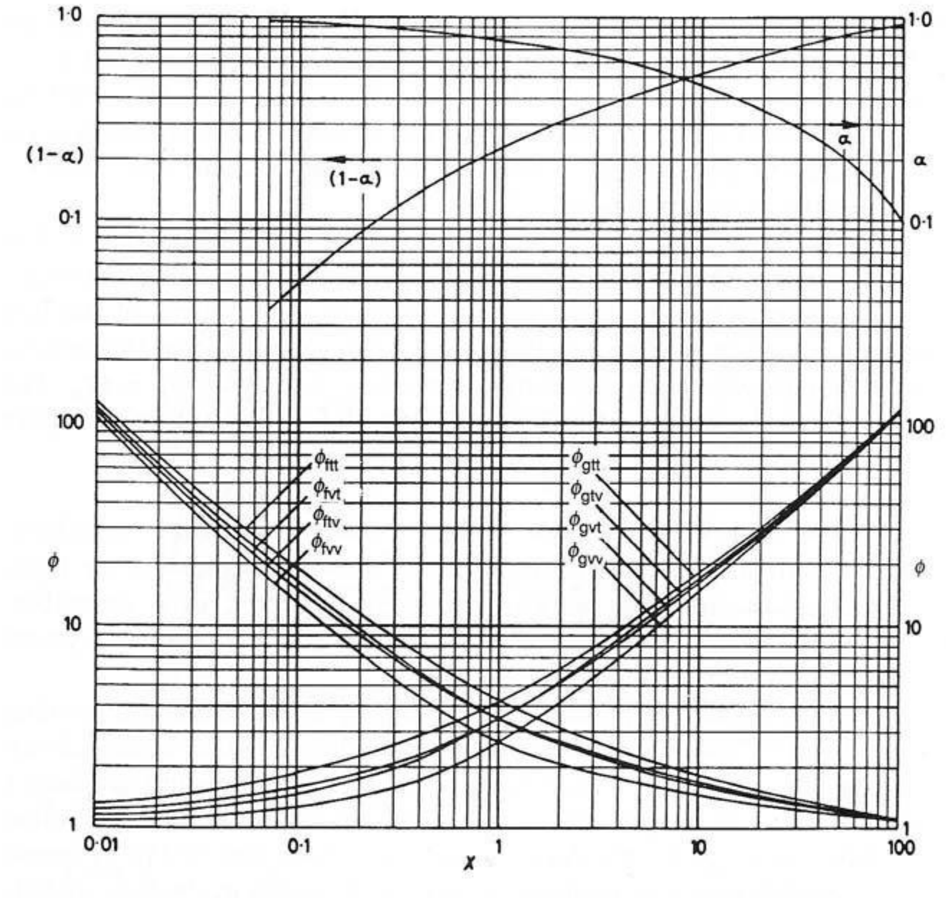


Figure 2: Lockhart-Martinelli Correlation (Image source: internet)

The original graphs given by Lockhart and Martinelli (figure 2) as shown here; the frictional multipliers phi have plotted as functions of Martinelli parameter capital X. Here also it is ϕ; both scales are logarithmic here; to further four flow regimes we have different graphs, four different graphs for four flow regimes. And so here also we have four graphs for $\phi_g$ for the four flow regimes. So, from these graphs for any of these four flow regimes; we can find $\phi_f$ or $\phi_g$. Then the upper graph is far the void fraction α which is also a function of the Martinelli parameter X.

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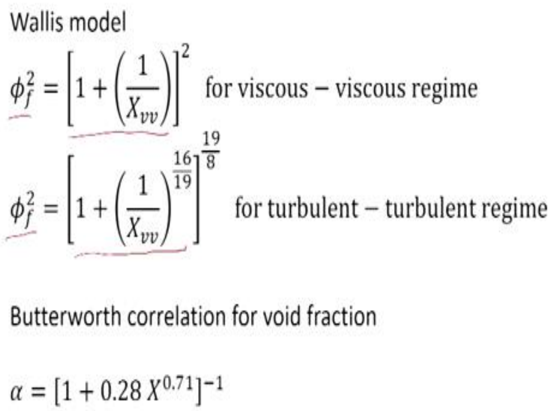


There is also an analytical expression given by Wallis:

$$\phi_f^2 = \left[1 + \left(\frac{1}{X_{vv}}\right)\right]^2 \quad \text{for viscous} - \text{viscous regime}$$

$$\phi_f^2 = \left[1 + \left(\frac{1}{X_{vv}}\right)^{\frac{16}{19}}\right]^{\frac{19}{8}} \quad \text{for turbulent} - \text{turbulent regime}$$

For the void fraction Butterworth gave a relation:

$$\alpha = [1 + 0.28\, X^{0.71}]^{-1}$$

So, you can either use the graphs originally given by Lockhart and Martinelli; these graphs for the frictional multiplier and the void fraction or you can use the Wallis expressions for $\phi_f^2$ and Butterworth’s relation for void fraction or alternately for the frictional multiplier; you can use these Chisholm and Liard relations also; using the appropriate value of the C parameter.