Two-Phase flow with phase change in conventional and miniature channels
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Lecture – 03
Modelling of Two-Phase Flow
-The Homogeneous Model

Welcome back to the course on Two-Phase flow with phase change in conventional and miniature channels. Today, we will discuss the Modelling of Two-Phase Flow and in particular the homogeneous model. Modelling of two-phase flow is important to calculate pressure drop in two-phase flow and to design two-phase flow equipment.

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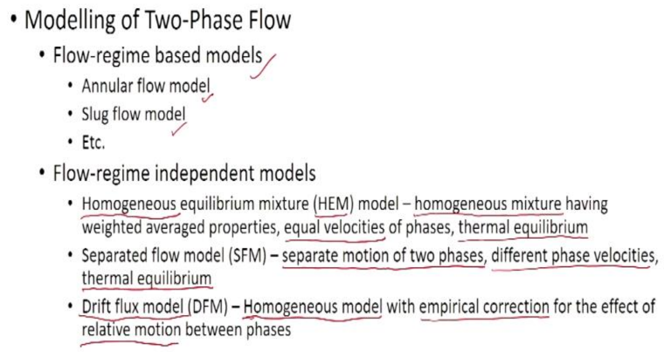

There are different types of models of two-phase flow. There are flow regime based models. These are for specific types of flow regimes. If we know the flow regime then we can use the model for that particular flow regime. For example, there are models for annular flow; there are models for slug flow and similarly, there are specific models of different types of flow regimes.

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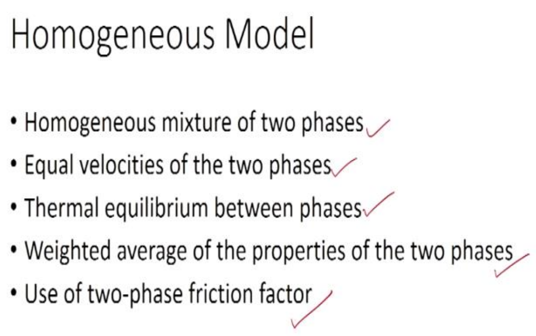

But suppose we do not know the flow regime then there are some models which can be used regardless of flow regimes. The simplest of them is the Homogeneous equilibrium mixture model or in short it is called HEM model or popularly known as homogeneous model. In this the two-phases are assume to be a homogeneous mixture. And the properties of the mixture are calculated by weighted averaging of the properties of the two-phases. It is assumed that the velocities of the two-phases are the same.

Actually, the phase velocities are usually different, but for simplicity in this model, it is assumed that both phases are moving together with the same velocity and it is also assumed that there is thermal equilibrium between the phases; that means, both phases are at the same temperature. If both phases are liquid and gas phases of the same substance; for example, water and steam, then both of them will be at the saturation temperature at the local pressure.

So, therefore, the properties of the liquid and gas phases can be calculated by knowing the pressure. Then, there is a separated flow model which is comparatively general. In this the separate motion of the two-phases is considered and the balance equations for the two-phases are written separately and solved. The phase velocity is are taken as different, but thermal equilibrium is assumed ok. Then there is a drift flux model.

In this there is a concept of drift flux which we will discuss later and using that the relative motion between the phases is accounted for. But this combines the simplicity of the homogeneous model, but also takes into account the relative motion of the phases. The homogeneous model is used, but there are empirical correlations for the effect of the relative motion between the phases. So, first we will discuss the homogeneous model.

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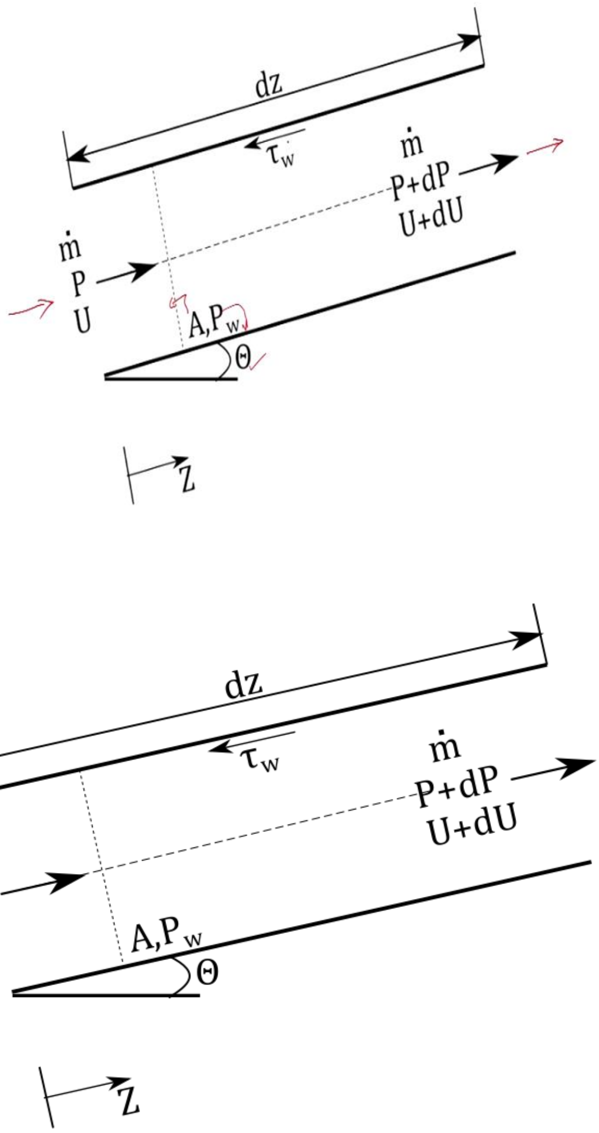

Figure 1: Two phase mixture flowing through a pipe

In the homogeneous model, the two fluid is assumed to be a homogeneous mixture of the two-phases and the velocities of the two-phases are assumed to be the same thermal equilibrium is assumed; that means, the same temperature and the properties are calculated by weighted averaging and there is a two-phase friction factor which is used to calculate the friction and pressure gradient.

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So, suppose a two-phase mixture is flowing through a channel (figure 1) and suppose the channel is at an angle theta with the horizontal. The inlet a mass flow rate is m dot and we will consider steady state. So, inlet and outlet mass flow rates are the same. The inlet pressure is P and outlet pressure is P+dP. We are considering a small element infinitesimal element of the length dz. The axial coordinate is denoted by Z. The inlet velocity is U outlet velocity is U+dU. As I have already mentioned the velocities of the of both phases assumed to be the same. The cross sectional area is A that is this and the wetted perimeter is denoted by P_w the wall shear stress is τ_w.

$$U_g = U_f = U$$

$$\bar{v} = v_f + x v_{fg},\ \bar{\rho} = 1/\bar{v}$$

$$\dot{m} = \bar{\rho} U A,\ G = \bar{\rho} U = U/\bar{v}$$

Mass balance:

$$\dot{m} = \dot{m}_g + \dot{m}_f$$

$$\dot{m}_g = \dot{m}\, x = GAx$$

$$\dot{m}_f = \dot{m}\,(1-x) = GA(1-x)$$

Momentum balance:

$$P\,A - (P + dP)A - \tau_w P_w dz - \bar{\rho} g \sin\theta\, A dz = GA(U + dU) - GAU$$

$$-A\, dP - \tau_w P_w dz - \bar{\rho} g \sin\theta\, A dz = GA\, dU$$

$$-\frac{dP}{dz} = \frac{\tau_w P_w}{A} + G\frac{dU}{dz} + \bar{\rho} g \sin\theta$$

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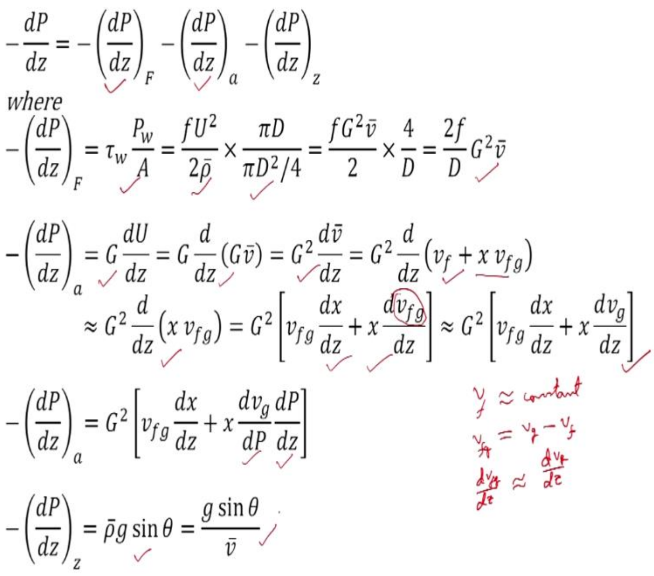

$$-\frac{dP}{dz} = -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

where

$$-\left(\frac{dP}{dz}\right)_F = \tau_w \frac{P_w}{A} = \frac{fU^2}{2\bar{\rho}} \times \frac{\pi D}{\pi D^2/4} = \frac{fG^2\bar{v}}{2} \times \frac{4}{D} = \frac{2f}{D} G^2\bar{v}$$

$$-\left(\frac{dP}{dz}\right)_a = G\frac{dU}{dz} = G\frac{d}{dz}(G\bar{v}) = G^2\frac{d\bar{v}}{dz} = G^2\frac{d}{dz}\left(v_f + x\, v_{fg}\right)$$

$$\approx G^2\frac{d}{dz}\left(x\, v_{fg}\right) = G^2\left[v_{fg}\frac{dx}{dz} + x\frac{dv_g}{dz}\right] \approx G^2\left[v_{fg}\frac{dx}{dz} + x\frac{dv_g}{dz}\right]$$

$$-\left(\frac{dP}{dz}\right)_a = G^2\left[v_{fg}\frac{dx}{dz} + x\frac{dv_g}{dP}\frac{dP}{dz}\right]$$

$$-\left(\frac{dP}{dz}\right)_z = \bar{\rho} g \sin\theta = \frac{g\sin\theta}{\bar{v}}$$

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$$-\frac{dP}{dz} = -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

$$= \frac{2f}{D}G^2\bar{v} + G^2\left[v_{fg}\frac{dx}{dz} + x\frac{dv_g}{dP}\frac{dP}{dz}\right] + \frac{g\sin\theta}{\bar{v}}$$

$$-\frac{dP}{dz}\left[1 + G^2x\frac{dv_g}{dP}\right] = \frac{2f}{D}G^2\bar{v} + G^2v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}$$

$$-\frac{dP}{dz} = \left[\frac{2f}{D}G^2\bar{v} + G^2v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}\right] \Big/ \left[1 + G^2x\frac{dv_g}{dP}\right] \quad \leftarrow -M^2$$

$$-\frac{dP}{dz} = \frac{1}{1-M^2}\left[\frac{2f}{D}G^2\bar{v} + G^2v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}\right]$$

Now,

$$-\frac{dP}{dz} = -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

$$= \frac{2f}{D}G^2\bar{v} + G^2\left[v_{fg}\frac{dx}{dz} + x\frac{dv_g}{dP}\frac{dP}{dz}\right] + \frac{g\sin\theta}{\bar{v}}$$

$$-\frac{dP}{dz}\left[1 + G^2x\frac{dv_g}{dP}\right] = \frac{2f}{D}G^2\bar{v} + G^2v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}$$

$$-\frac{dP}{dz} = \left[\frac{2f}{D}G^2\bar{v} + G^2v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}\right] \Big/ \left[1 + G^2x\frac{dv_g}{dP}\right]$$

$$-\frac{dP}{dz} = \frac{1}{1-M^2}\left[\frac{2f}{D}G^2\bar{v} + G^2v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}\right]$$

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where

$$M^2 = -G^2x\frac{dv_g}{dP} = G^2x\left|\frac{dv_g}{dP}\right|$$

If $M^2 \ll 1$ then $1 - M^2 \approx 1$ and

$$-\frac{dP}{dz} = \frac{2f}{D}G^2\bar{v} + G^2v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{\bar{v}}$$

$$= -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

where

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f}{D}G^2\bar{v}, -\left(\frac{dP}{dz}\right)_a = G^2v_{fg}\frac{dx}{dz}, -\left(\frac{dP}{dz}\right)_z = \frac{g\sin\theta}{\bar{v}}$$

where

$$M^2 = -G^2 x \frac{dv_g}{dP} = G^2 x \left|\frac{dv_g}{dP}\right|$$

If $M^2 \ll 1$ then $1 - M^2 \approx 1$ and

$$-\frac{dP}{dz} = \frac{2f}{D} G^2 \bar{v} + G^2 v_{fg} \frac{dx}{dz} + \frac{g \sin\theta}{\bar{v}}$$

$$= -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_a - \left(\frac{dP}{dz}\right)_z$$

where

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f}{D} G^2 \bar{v}, -\left(\frac{dP}{dz}\right)_a = G^2 v_{fg} \frac{dx}{dz}, -\left(\frac{dP}{dz}\right)_z = \frac{g \sin\theta}{\bar{v}}$$

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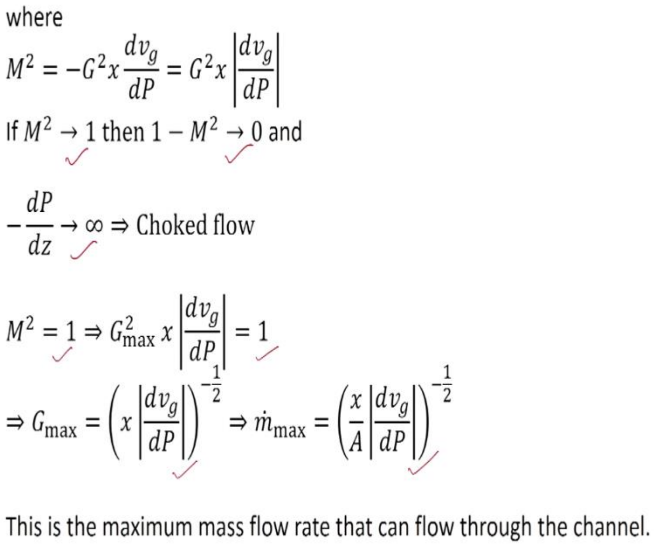

where

$$M^2 = -G^2 x \frac{dv_g}{dP} = G^2 x \left|\frac{dv_g}{dP}\right|$$

If $M^2 \to 1$ then $1 - M^2 \to 0$ and

$$-\frac{dP}{dz} \to \infty \Rightarrow \text{Choked flow}$$

$$M^2 = 1 \Rightarrow G_{max}^2\, x \left|\frac{dv_g}{dP}\right| = 1$$

$$\Rightarrow G_{max} = \left(x \left|\frac{dv_g}{dP}\right|\right)^{-\frac{1}{2}} \Rightarrow \dot{m}_{max} = \left(\frac{x}{A} \left|\frac{dv_g}{dP}\right|\right)^{-\frac{1}{2}}$$

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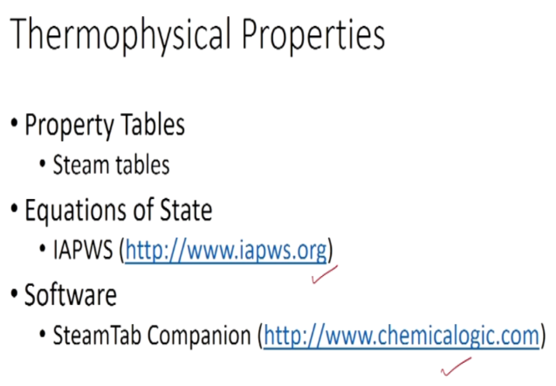

This is the maximum mass flow rate that can flow through the channel. What does this mean? It means that if you go on increasing the inlet pressure or go on reducing the outlet pressure, then the pressure drop will increase; pressure gradient can increase in definitely, but the mass flow rate or the mass flux will not increase beyond a certain value. This is called Choked flow.

Now, we will consider some numerical examples, but before that we will discuss how to calculate properties of fluids. So, there are property tables available for various fluids. Here, we will consider the properties of steam water mixture. So, properties of steam water mixture can be obtained from steam tables and there are equations of state for water and steam, there is an international association for properties of water and steam and its website is http://www.iapws.org.

The equations of state can be downloaded from here and there is software called steam tab companion which is a free software and it is based on a IAPWS as equations of state which are very accurate. Let us look at that.

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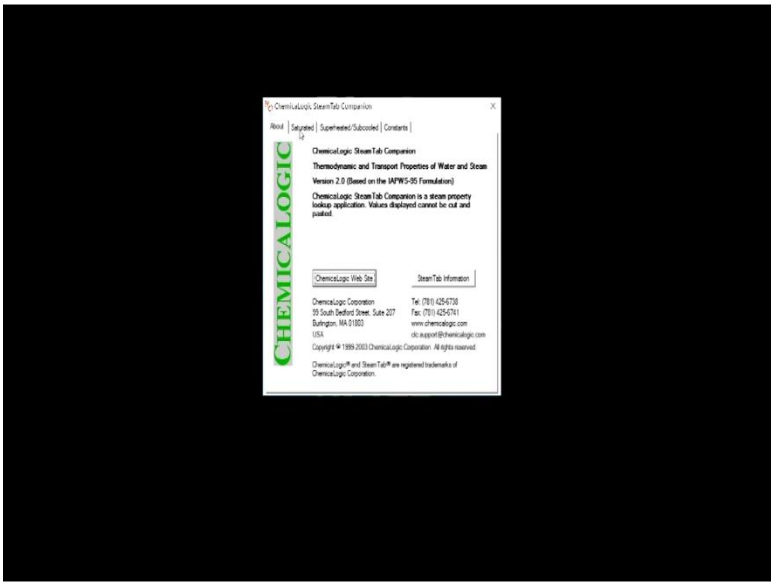

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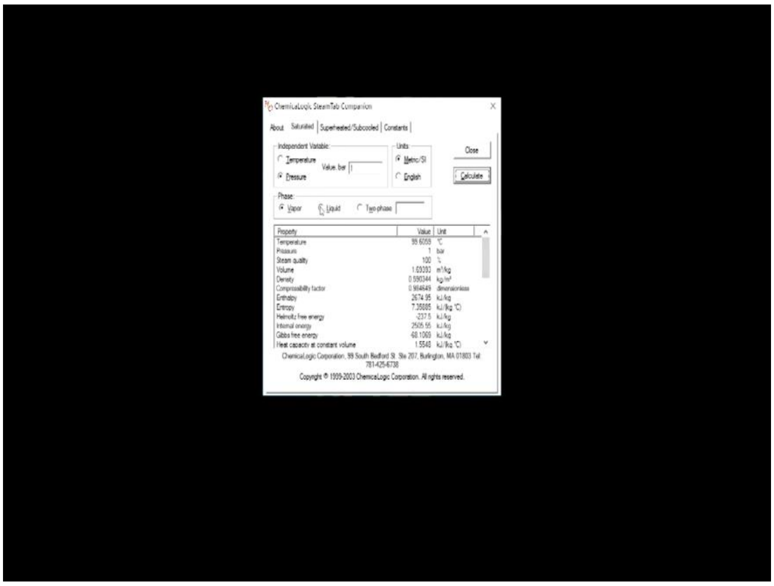

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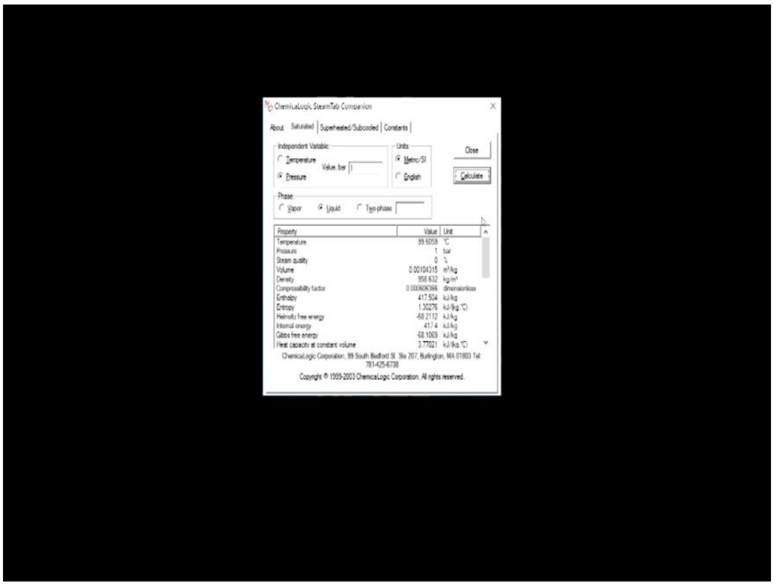

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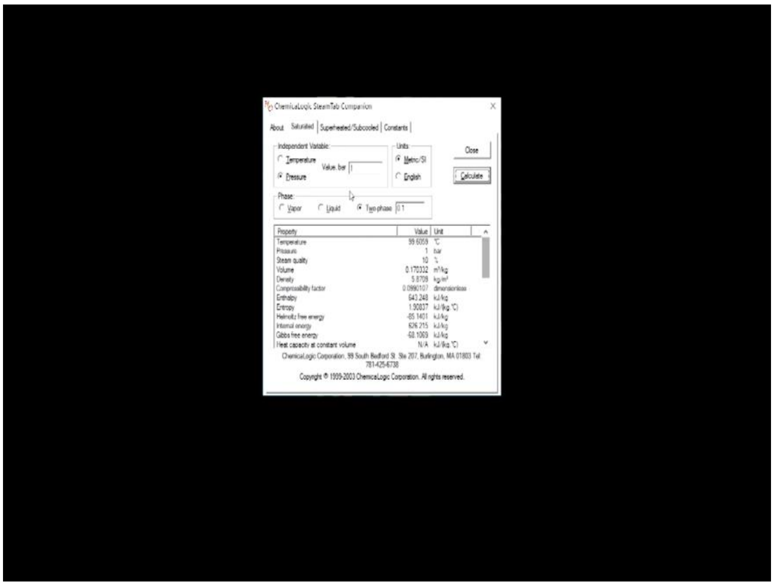

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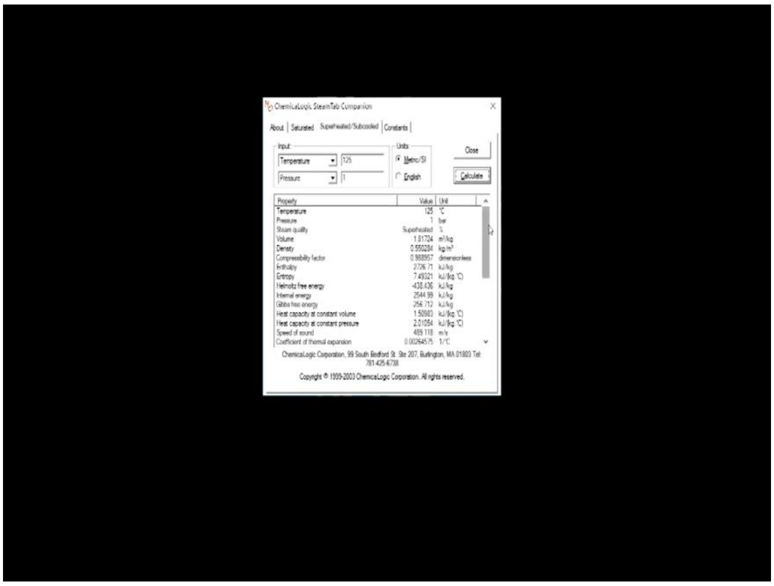

Figure 2: Screen shot of steam tab companion

Figure 2 shows the interface of steam tab companion.

And if you want to calculate the properties of saturated water or a steam, then you give input either temperature or pressure. So, let us give pressure in bar; 1 bar and press the calculate button. So, here it gives the saturation temperature is 99.6 degree C and various properties are given.

Similarly, if you select liquid; then, it will give properties of saturated liquid at that pressure. If we select two-phase and give a quality. Let us say 0.1; 10 percent quality and click the calculate button, then it gives the properties of the mixture. Then, superheated and sub cooled properties can also be calculated.

So, suppose we want sub cooled liquid water at 1 bar pressure and 25 degrees C. Here, it is giving; it is giving the condition as sub cooled and it is giving the properties of the sub cooled water at 1 bar and 25 degree C. Now, suppose we want to calculate the properties of superheated steam at 1 bar and 125 degree C; now, it says that it is superheated and gives the properties of super heated steam ok.

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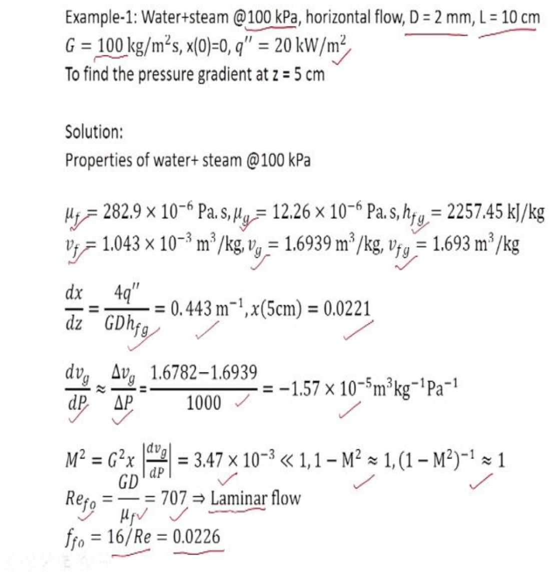

Numerical problem 1: Water+steam @100 kPa, horizontal flow, D = 2 mm, L = 10 cm, $G = 100\ \mathrm{kg/m^2s}$, x(0)=0, $q'' = 20\ \mathrm{kW/m^2}$. To find the pressure gradient at z = 5 cm.

Solution: Properties of water+ steam @100 kPa

$$\mu_f = 282.9 \times 10^{-6}\ \mathrm{Pa.s},\ \mu_g = 12.26 \times 10^{-6}\ \mathrm{Pa.s},\ h_{fg} = 2257.45\ \mathrm{kJ/kg}$$

$$v_f = 1.043 \times 10^{-3}\ \mathrm{m^3/kg},\ v_g = 1.6939\ \mathrm{m^3/kg},\ v_{fg} = 1.693\ \mathrm{m^3/kg}$$

$$\frac{dx}{dz} = \frac{4q''}{GDh_{fg}} = 0.\ 443\ \mathrm{m^{-1}},\ x(5\mathrm{cm}) = 0.0221$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782\text{-}1.6939}{1000} = -1.57 \times 10^{-5}\mathrm{m^3kg^{-1}Pa^{-1}}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 3.47 \times 10^{-3} \ll 1,\ 1 - \mathrm{M}^2 \approx 1,\ (1 - \mathrm{M}^2)^{-1} \approx 1$$

$$Re_{fo} = \frac{GD}{\mu_f} = 707 \Rightarrow \text{Laminar flow}$$

$$f_{fo} = 16/Re = 0.0226$$

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$$\bar{v} = v_f + x\, v_{fg} = 0.0180\; m^3/kg$$

$$f_{TP} \approx f_{fo} = 0.0226$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} \;= 4.07\; kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = G^2 v_{fg} \left(\frac{dx}{dz}\right) = 7.50\; kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left(\frac{g \sin\theta}{\bar{v}}\right) = 0$$

$$-\left(\frac{dP}{dz}\right) = 4.07 + 7.50 + 0 = 11.57\; kPa/m$$

$$G_{\max} = \left(x \left|\frac{dv_g}{dP}\right|\right)^{-\frac{1}{2}} = 1984\ \mathrm{kg/m^2s}$$

$$\bar{v} = v_f + x\, v_{fg} = 0.0180\; m^3/kg$$

$$f_{TP} \approx f_{fo} = 0.0226$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} \;= 4.07\; kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = G^2 v_{fg} \left(\frac{dx}{dz}\right) = 7.50\; kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left(\frac{g \sin\theta}{\bar{v}}\right) = 0$$

$$-\left(\frac{dP}{dz}\right) = 4.07 + 7.50 + 0 = 11.57\; kPa/m$$

$$G_{\max} = \left(x \left|\frac{dv_g}{dP}\right|\right)^{-\frac{1}{2}} = 1984\ \mathrm{kg/m^2s}$$

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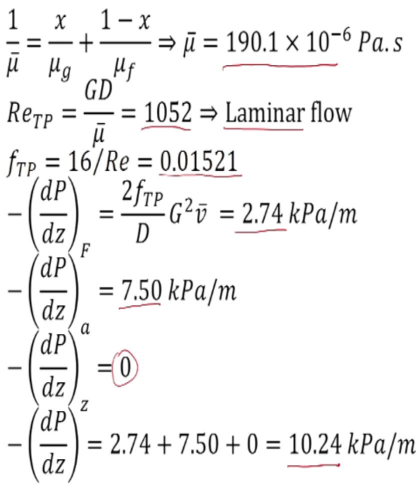

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s\ (McAdams\ correlation)$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 1052 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.01521$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} = 2.74\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 7.50\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0$$

$$-\left(\frac{dP}{dz}\right) = 2.74 + 7.50 + 0 = 10.24\ kPa/m$$

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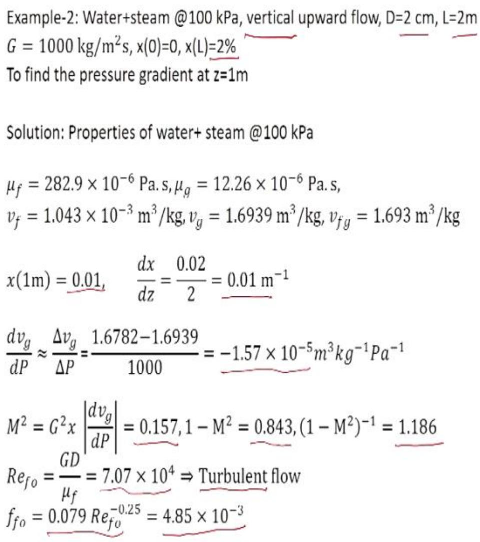

Numerical Problem 2: Water+steam @100 kPa, vertical upward flow, D=2 cm, L=2m, $G = 1000$ kg/m^{2}s, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m.

Solution: Properties of water+ steam @100 kPa

$\mu_f = 282.9 \times 10^{-6}$ Pa. s, $\mu_g = 12.26 \times 10^{-6}$ Pa. s,

$v_f = 1.043 \times 10^{-3}$ m^3/kg, $v_g = 1.6939$ m^3/kg, $v_{fg} = 1.693$ m^3/kg

$$x(1\text{m}) = 0.01, \quad \frac{dx}{dz} = \frac{0.02}{2} = 0.01 \text{ m}^{-1}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{1.6782\text{-}1.6939}{1000} = -1.57 \times 10^{-5} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 0.157,\ 1 - \text{M}^2 = 0.843,\ (1 - \text{M}^2)^{-1} = 1.186$$

$$Re_{fo} = \frac{GD}{\mu_f} = 7.07 \times 10^4 \Rightarrow \text{Turbulent flow}$$

$$f_{fo} = 0.079\, Re_{fo}^{-0.25} = 4.85 \times 10^{-3}$$

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$$\bar{v} = v_f + x\, v_{fg} = 0.0180\, m^3/kg$$

$$f_{TP} \approx f_{fo} = 4.85 \times 10^{-3}$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2 f_{TP}}{D} G^2 \bar{v}\,(1 - M^2)^{-1} = 10.36\, kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = G^2 v_{fg} \left(\frac{dx}{dz}\right)(1 - M^2)^{-1} = 20.1\, kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left(\frac{g \sin\theta}{\bar{v}}\right)(1 - M^2)^{-1} = 0.646\, kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 10.36 + 20.1 + 0.646 = 31.1\, kPa/m$$

$$G_{\max} = \left(x \left|\frac{dv_g}{dP}\right|\right)^{-\frac{1}{2}} = 2949\ \mathrm{kg/m^2 s}$$

$$\bar{v} = v_f + x\, v_{fg} = 0.0180\, m^3/kg$$

$$f_{TP} \approx f_{fo} = 4.85 \times 10^{-3}$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2 f_{TP}}{D} G^2 \bar{v}\,(1 - M^2)^{-1} = 10.36\, kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = G^2 v_{fg} \left(\frac{dx}{dz}\right)(1 - M^2)^{-1} = 20.1\, kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left(\frac{g \sin\theta}{\bar{v}}\right)(1 - M^2)^{-1} = 0.646\, kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 10.36 + 20.1 + 0.646 = 31.1\, kPa/m$$

$$G_{\max} = \left(x \left|\frac{dv_g}{dP}\right|\right)^{-\frac{1}{2}} = 2949\ \mathrm{kg/m^2 s}$$

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 190.1 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 1052 \Rightarrow \text{Laminar flow}$$

$$f_{TP} = 16/Re = 0.01521$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} = 2.74\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 7.50\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0$$

$$-\left(\frac{dP}{dz}\right) = 2.74 + 7.50 + 0 = 10.24\ kPa/m$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 232 \times 10^{-6}\ Pa.s\ (McAdams\ correlation)$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 8.62 \times 10^4 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 4.61 \times 10^{-3}$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v}\ (1 - M^2)^{-1} = 9.85\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 20.1\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 0.646\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 9.85 + 20.1 + 0.646 = 30.6\ kPa/m$$

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Example-3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=2m
G = 1000 kg/m^{2}s, x(0)=0, x(L)=2%
To find the pressure gradient at z=1m

Solution:
Properties of water+ steam @10 MPa

$\mu_f = 81.80 \times 10^{-6}$ Pa. s, $\mu_g = 20.27 \times 10^{-6}$ Pa. s,
$v_f = 1.453 \times 10^{-3} \frac{m^3}{kg}, v_g = 1.803 \times 10^{-2}$ m^3/kg, $v_{fg} = 0.01658$ m^3/kg

$x(1m) = 0.01, \quad \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ m^{-1}$

$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781-0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$

$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1, 1 - M^2 \approx 1, (1 - M^2)^{-1} \approx 1$
$Re_{fo} = \frac{GD}{\mu_f} = 2.44 \times 10^5 \Rightarrow$ Turbulent flow
$f_{fo} = 0.079\ Re_{fo}^{-0.25} = 3.55 \times 10^{-3}$

Numerical Problem 3: Water+steam @10 MPa, vertical upward flow, D=2 cm, L=2m, G = 1000 kg/m^{2}s, x(0)=0, x(L)=2%. To find the pressure gradient at z=1m.

Solution: Properties of water+ steam @10 MPa

$$\mu_f = 81.80 \times 10^{-6}\ \text{Pa. s},\ \mu_g = 20.27 \times 10^{-6}\ \text{Pa. s},$$

$$v_f = 1.453 \times 10^{-3} \frac{\text{m}^3}{\text{kg}},\ v_g = 1.803 \times 10^{-2}\ \text{m}^3/\text{kg},\ v_{fg} = 0.01658\ \text{m}^3/\text{kg}$$

$$x(1\text{m}) = 0.01,\quad \frac{dx}{dz} = \frac{0.02}{2} = 0.01\ \text{m}^{-1}$$

$$\frac{dv_g}{dP} \approx \frac{\Delta v_g}{\Delta P} = \frac{0.01781\text{-}0.01803}{1 \times 10^5} = -2.20 \times 10^{-9} m^3 kg^{-1} Pa^{-1}$$

$$M^2 = G^2 x \left|\frac{dv_g}{dP}\right| = 2.20 \times 10^{-5} \ll 1,\ 1 - \text{M}^2 \approx 1,\ (1 - \text{M}^2)^{-1} \approx 1$$

$$Re_{fo} = \frac{GD}{\mu_f} = 2.44 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{fo} = 0.079\ Re_{fo}^{-0.25} = 3.55 \times 10^{-3}$$

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$$\bar{v} = v_f + x\, v_{fg} = 1.619 \times 10^{-3}\ m^3/kg$$

$$f_{TP} \approx f_{fo} = 3.55 \times 10^{-3}$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2 f_{TP}}{D} G^2 \bar{v} = 0.575\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = G^2 v_{fg} \left(\frac{dx}{dz}\right) = 0.166\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = \left(\frac{g \sin\theta}{\bar{v}}\right) = 6.05\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 0.575 + 0.166 + 6.05 = 6.79\ kPa/m$$

$$G_{\max} = \left(x \left|\frac{dv_g}{dP}\right|\right)^{-\frac{1}{2}} = 2.13 \times 10^5\ \mathrm{kg/m^2 s}$$

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$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 79.4 \times 10^{-6}\ Pa.s$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 2.52 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 3.53 \times 10^{-3}$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} = 0.572\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 0.166\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 6.05\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 0.572 + 0.166 + 6.05 = 6.79\ kPa/m$$

$$\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \Rightarrow \bar{\mu} = 79.4 \times 10^{-6}\ Pa.s\ (McAdams\ correlation)$$

$$Re_{TP} = \frac{GD}{\bar{\mu}} = 2.52 \times 10^5 \Rightarrow \text{Turbulent flow}$$

$$f_{TP} = 0.079\ Re_{TP}^{-0.25} = 3.53 \times 10^{-3}$$

$$-\left(\frac{dP}{dz}\right)_F = \frac{2f_{TP}}{D} G^2 \bar{v} = 0.572\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_a = 0.166\ kPa/m$$

$$-\left(\frac{dP}{dz}\right)_z = 6.05\ kPa/m$$

$$-\left(\frac{dP}{dz}\right) = 0.572 + 0.166 + 6.05 = 6.79\ kPa/m$$