Two-Phase flow with phase change in conventional and miniature channels

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Lecture - 01
Introduction and Notation

Welcome to the course on Two Phase flow with phase change in conventional and miniature channels. I am Manmohan Pandey, professor of Mechanical Engineering, at Indian Institute of Technology, Guwahati.

So, what is two phase flow and what is two phase flow with phase change? What are conventional channels and miniature channels and that we will discuss today and then we will introduce notation for two phase flow. So, what is two phase flow? As the name suggests, when two phases are flowing together it is two phase flow. Matter commonly exists in three phases; solid, liquid and gas.

So, if any two phases are flowing together it should be called two phase flow. For example, solid and liquid flow commonly, they occurs in slurries and if it is solid and gas flow, that is also two phase flow. It is found in pneumatic transport and fluidized beds in the industrial applications. Then there is liquid and gas flow, this is quite common in industrial applications, when a liquid and a gas are flowing together for example, air and water flowing together in a pipe.

In this course, we will be concerned mostly with liquid gas flow. Also, when there are two immiscible liquids flowing together, that is also sometimes referred to as two phase flow. For example, oil and water flowing together in a pipe that is also called two phase flow. Although, strictly speaking it is not a two phase flow, but a two component single phase flow. So, as I said earlier, in this course we will be mainly concerned with the flow of liquid and gas and the two phase flow can be with or without phase change.

For example, if it is air and water flow, then air cannot become water and water cannot become air. So, it will remain two phase flow without phase change, but if it is steam water mixture flowing in a pipe, then steam can condense into water or water can evaporate into steam. So, therefore, it will be two phase flow with phase change or at least there is a possibility of phase change.

In this course, we will be concerned with both two phase flow, without phase change and with phase change also. There is a classification of conventional and miniature channels based on the different physics of the flow. In small channels, different effects are important, in conventional channels, different effects are important. So, therefore, the smaller channels, two phase flow in smaller channels has to be studied in a different way. There is a characteristic length called Laplace length scale, which is defined as follows.

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$\lambda_L = \sqrt{\dfrac{\sigma}{g \Delta\rho}}$ Laplace length scale

$\Delta\rho = \rho_L - \rho_G$

$D_H \gtrsim \lambda_L$ Conventional channel

$D_H \lesssim \lambda_L$ Miniature channel

$Bd = \dfrac{g \Delta\rho D_H^2}{\sigma}$ Bond number

$= D_H^2 / \lambda_L^2$

$\sqrt{Bd} = D_H / \lambda_L$

$\sqrt{Bd} \leq 0.3$ Minichannel

Laplace length scale: $\lambda_L = \sqrt{\frac{\sigma}{g\,\Delta\rho}}$ $(Where\ \Delta\rho = \rho_L - \rho_G)$

$$D_H \geq \lambda_L \quad Conventional\ channel$$
$$D_H \leq \lambda_L \quad Miniature\ channel$$

Where D_H : Hydraulic diameter of the channel

Here, σ is the surface tension, g is the acceleration due to gravity and $\Delta\rho$ is the density difference. So, this $\Delta\rho$ this is equal to the density of liquid minus the density of gas. Usually, ρ_L is much larger than ρ_G in two phase flow mixtures. Actually, this represents the relative strength of the surface tension effect with respect to the buoyancy effect. If the hydraulic diameter (D_H) is larger than the Laplace length scale, then it is called conventional channel and otherwise, it is called miniature channel.

There is a non dimensional number called bond number (B_d). It is defined as:

Bond number: $B_d = \frac{g\Delta\rho D_H^2}{\sigma} = \frac{D_H^2}{\lambda_L^2}$

$$\sqrt{B_d} = \frac{D_H}{\lambda_L}$$

$\sqrt{B_d} \leq 0.3 \quad Minichannel$

So, there is a classification in which if bond number is less than or equal to 0.3, then it is called minichannel. What does it mean is that if bond number is small, then it means that the relative strength of the buoyancy effect is less than that of the surface tension effect and buoyancy effect becomes negligible, if the square root of bond number is less than or equal to 0.3, so in that case stratified flow becomes impossible. We will discuss later different types of flow regimes, stratified flow is that in a horizontal pipe, when that liquid stays in the lower part of the pipe and a gas stays in the upper part of the pipe and the two phases are segregated. So, that type of flow is not possible, if the channel is very small and the bond number square root of that is less than 0.3. Sometimes instead of the bond number or the Laplace length scale, an absolute classification is used:

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Microchannels $D_H = 10 - 100\ \mu m$

Minichannels $D_H = 100 - 1000\ \mu m$

Mesochannels $D_H = 1 - 3\ mm$

Conventional channels $D_H \geq 3\ mm$

Microchannels: $D_H = 10 - 100\ \mu m$

Minichannels: $D_H = 100 - 1000\ \mu m$

Mesochannels: $D_H = 1 - 3\ mm$

Conventional channels: $D_H \geq 3\ mm$

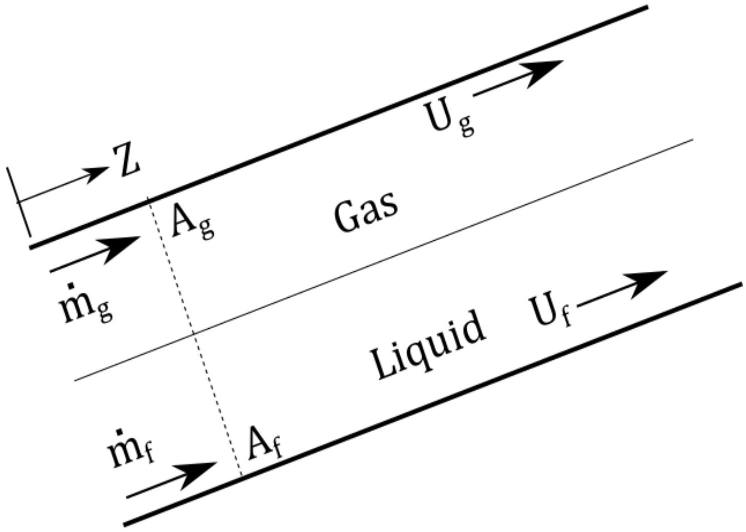

Figure 1: Two phase mixture flowing through a pipe

Figure 1 shows a two phase mixture is flowing through a pipe and there is liquid and gas, here, suffix g and suffix f will be used for the gas and the liquid phase respectively. Suppose, the area is A_g and A_f, the area of gas and liquid respectively. Actually, the phases will not always be segregated like this. There will be different types of flow patterns, but this is just a schematic. So, the total area of the cross section occupied by the gas phase is A_g and the total cross sectional area occupied by the liquid phase is A_f, but the shapes of these areas can be different in different flow patterns.

Then there is a quantity known as **void fraction**, which is denoted by α,

$$\alpha = \frac{A_g}{A}; \quad 1 - \alpha = \frac{A_f}{A}; \quad A_g + A_f = A$$

A is the total area of cross section and the velocities of the phases will be denoted by U_g and U_f, $\dot{m}$ is the mass flow rate, h is the enthalpy and Q is the total volumetric flow rate.

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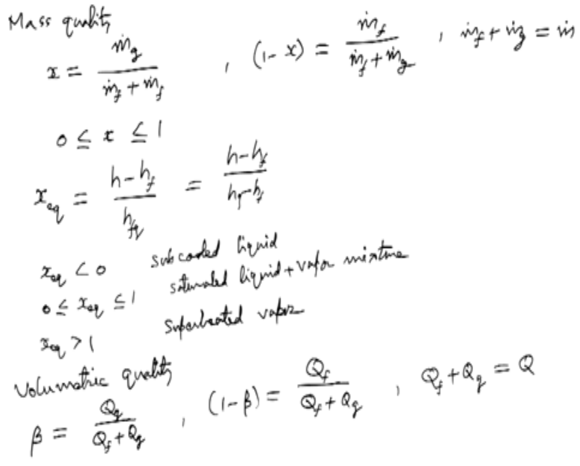

Mass quality *x*:

$$x = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_f}; \quad 1 - x = \frac{\dot{m}_f}{\dot{m}_g + \dot{m}_f}; \quad \dot{m}_g + \dot{m}_f = \dot{m};\ (0 \le x \le 1)$$

Thermodynamic equilibrium quality x_{eq}:

$$x_{eq} = \frac{h - h_f}{h_g - h_f} = \frac{h - h_f}{h_{fg}}$$

$x_{eq} < 0$ subcooled liquid

$0 \le x_{eq} \le 1$ $saturated$ liquid + $vapour\ mixture$

$x_{eq} \ge 1$ $saturated\ vapour$

Volumetric quality β:

$$\beta = \frac{Q_g}{Q_f + Q_g}; \quad 1 - \beta = \frac{Q_f}{Q_f + Q_g}; \quad Q_f + Q_g = Q$$

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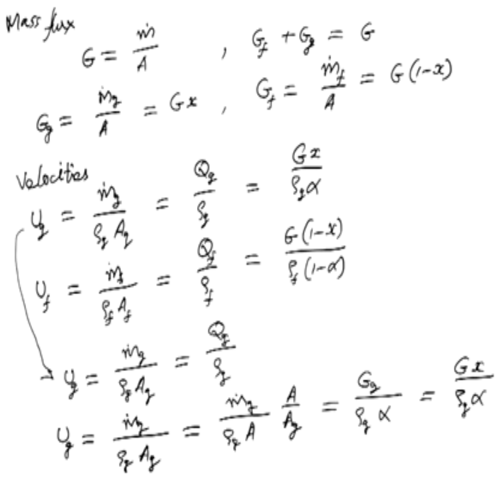

Mass flux G:

$$G = \frac{\dot{m}}{A}; \quad G_g = \frac{\dot{m}_g}{A} = Gx; \qquad G_f = \frac{\dot{m}_f}{A} = G(1-x); \quad G_f + G_g = G$$

$$G_g = \frac{\dot{m}_g}{A} = \frac{Q_g \rho_g}{A} = j_g \rho_g$$

Velocities U:

$$U_g = \frac{\dot{m}_g}{\rho_g A_g} = \frac{Q_g}{A_g} = \frac{Gx}{\rho_g \alpha}; \ \ and \ U_f = \frac{\dot{m}_f}{\rho_f A_f} = \frac{Q_f}{A_f} = \frac{G(1-x)}{\rho_f (1-\alpha)}$$

Derivation of U_g:

$$U_g = \frac{\dot{m}_g}{\rho_g A_g} = \frac{Q_g}{A}\frac{A}{A_g} = \frac{G_g}{\rho_g \alpha} = \frac{Gx}{\rho_g \alpha}$$

Similarly, relation for U_f can be derived.

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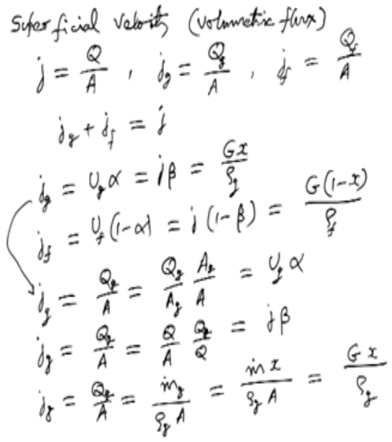

Superficial velocity/volumetric flux j:

$$j = \frac{Q}{A}; \quad j_g = \frac{Q_g}{A}; \quad j_f = \frac{Q_f}{A}; j_g + j_f = j$$

$$j_g = U_g\alpha = j\beta = \frac{Gx}{\rho_g}; \quad j_f = U_f(1-\alpha) = j(1-\beta) = \frac{G(1-x)}{\rho_f}$$

Derivation for j_g:

$$j_g = \frac{Q_g}{A} = \frac{Q_g}{A_g}\frac{A_g}{A} = U_g\alpha; \quad j_g = \frac{Q_g}{A} = \frac{Q}{A}\frac{Q_g}{Q} = j\beta; \quad j_g = \frac{Q_g}{A} = \frac{\dot{m}_g}{\rho_g A} = \frac{\dot{m}x}{\rho_g A} = \frac{Gx}{\rho_g}$$

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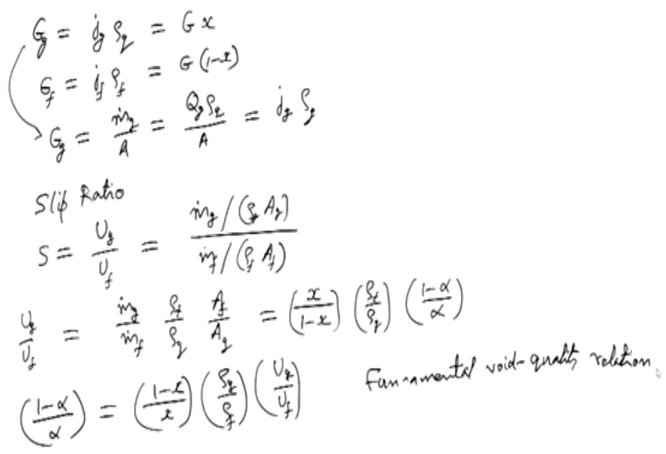

Slip ratio S:

$$S = \frac{U_g}{U_f} = \left(\frac{\dot{m}_g}{\rho_g A_g}\right) \Big/ \left(\frac{\dot{m}_f}{\rho_f A_f}\right)$$

$$\frac{U_g}{U_f} = \frac{\dot{m}_g}{\dot{m}_f}\frac{\rho_f}{\rho_g}\frac{A_f}{A_g} = \left(\frac{x}{1-x}\right)\left(\frac{\rho_f}{\rho_g}\right)\left(\frac{1-\alpha}{\alpha}\right)$$

$$\left(\frac{1-\alpha}{\alpha}\right) = \left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_f}\right)\left(\frac{U_g}{U_f}\right) \rightarrow \text{Fundamental void quality relation}$$

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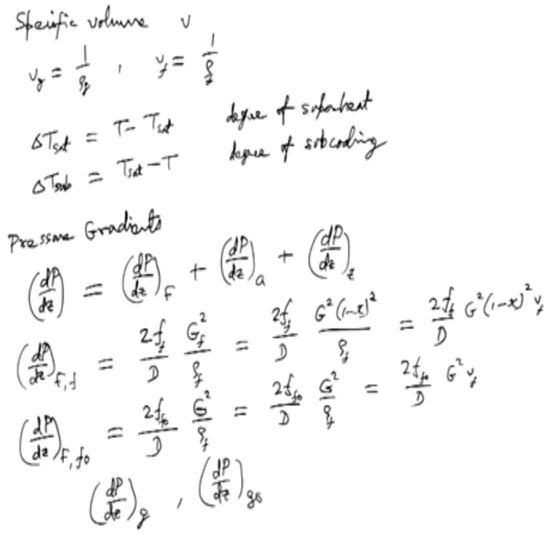

Specific volume:

$$v_g = \frac{1}{\rho_g}; \quad v_f = \frac{1}{\rho_f}$$

Degree of superheat:

$$\Delta T_{sat} = T - T_{sat} \text{ (Degree of superheat)}$$

$$\Delta T_{sub} = T_{sat} - T \text{ (Degree of subcooling)}$$

Pressure gradients:

$$\left(\frac{dP}{dz}\right) = \left(\frac{dP}{dz}\right)_F + \left(\frac{dP}{dz}\right)_a + \left(\frac{dP}{dz}\right)_z$$

$$\left(\frac{dP}{dz}\right)_{F,f} = \frac{2f_f}{D}\frac{G_f^2}{\rho_f} = \frac{2f_f}{D}\frac{G^2(1-x)^2}{\rho_f} = \frac{2f_f}{D}G^2(1-x)^2 v_f$$

$$\left(\frac{dP}{dz}\right)_{F,fo} = \frac{2f_{fo}}{D}\frac{G^2}{\rho_f} = \frac{2f_f}{D}G^2 v_f$$

Suffix F, a, and z represents frictional, acceleration and gravitation respectively. 'F,f' represents liquid ignoring the gas phase while 'F,fo' represents pure liquid flowing through the channel.

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$\left(\frac{dP}{dz}\right)_{fF}$ pressure gradient due to friction in liquid phase

$\left(\frac{dP}{dz}\right)_{gF}$ " " " gas "

$$Re_f = \frac{G_f D}{\mu_f} = \frac{G(1-x)D}{\mu_f}$$

$$Re_{fo} = \frac{GD}{\mu_f}$$

$$Re_g = \frac{GxD}{\mu_g}, \quad Re_{go} = \frac{GD}{\mu_g}$$

$$f = f(Re) = \begin{cases} \frac{16}{Re} & \text{for laminar flow} \\ 0.079\, Re^{-1/4} & \text{for turbulent flow through smooth pipes} \end{cases}$$

Reynolds Number:

$$Re_f = \frac{G_f D}{\mu_f} = \frac{G(1-x)D}{\mu_f}; \quad Re_{fo} = \frac{GD}{\mu_f}$$

$$Re_g = \frac{GxD}{\mu_g}; \quad Re_{go} = \frac{GD}{\mu_g}$$

$$f = f(Re) = \begin{cases} \frac{16}{Re} \text{ for laminar flow} \\ 0.079 Re^{-\frac{1}{4}} \text{ for turbulent flow through smooth pipes (Blausius relation)} \end{cases}$$

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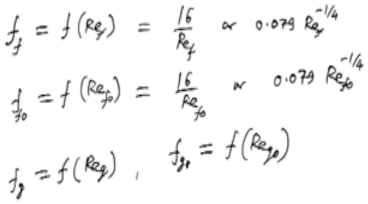

Thus:

$$\mathrm{f_f} = \mathrm{f(Re_f)} = \frac{16}{\mathrm{Re_f}} \quad \text{or} \quad 0.079\mathrm{Re_f}^{-1/4}$$

$$f_{fo} = f(Re_{fo}) = \frac{16}{Re_{fo}} \quad or \quad 0.079 Re_{fo}^{-1/4}$$

$$f_g = f(Re_g), \quad f_{go} = f(Re_{go})$$