Mathematical Modelling of Manufacturing Processes
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Lecture - 07
Solid-State Deformation and Residual Stress-2

Hello everybody, so far we have discussed on the elastic model or maybe elasticity problem related to the manufacturing processes.

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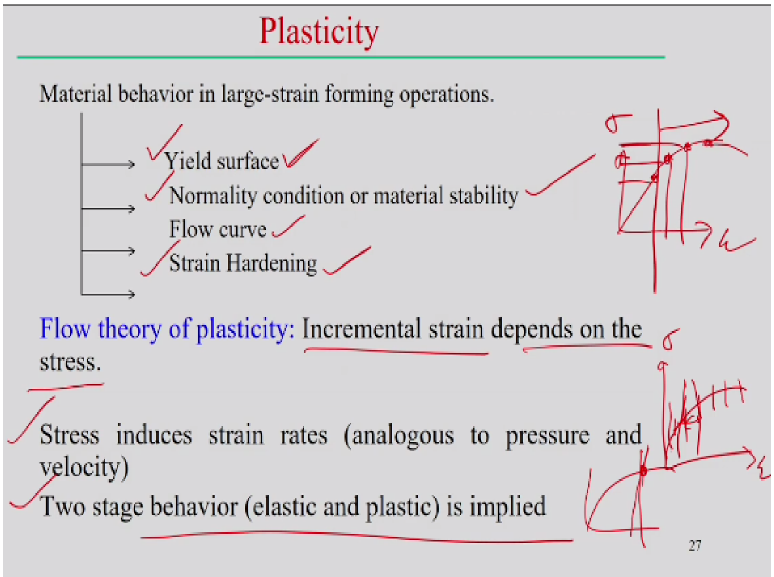

Now we will try to discuss the plastic deformation behaviour and what we can develop the different laws of plasticity, specifically there are more generally we use the von Mises yield condition during the plastic deformation of the particular component. So we will try to explore these things and we try to develop how the von Mises yield conditioned actually comes and whatever you can utilize these things for the development of the any kind of mathematical model.

So plasticity, we know that the data are normally if we conduct one universal tensile testing machine the tensile testing of a specimen we apply the load in particular single direction and then we define this is a yield point particular the yield point is a transition point between elastic deformation and plastic deformation and therefore what we are normally most of the material actually behaves within the elastic limit.

The stress is proportional to the strain and based on that we define the Young's modulus and other material parameters and we utilize all these things in the development of the elastic model, but in actual practical application most of the engineering materials behave like last row plastic material that means in the plastic deformation whatever we can represents the plastic deformation and specifically it is one dimension.

It is very and if you want to develop from the experiment we can easily extract the data for the plastic deformation, but if it is three dimensional situation say in machine component or in any particular component it is subjected to three dimensional stress state of the stress. Then what we can define we can explain the plastic behaviour or plasticity of this particular material.

So therefore it is very important to know and the material behaviour specifically very large strain forming operation so more that means plastic zone is to a some very large extent. In that cases first task is to define the yield surface. What way we can define the yield surface? Yield maybe a point when we apply the unidirectional load, but it may not be a point when it is subjected to some kind of a say more than one load is acting, one direction load is acting.

For example, both sigma x and sigma y is acting and having 2 principal stresses for a particular stress issues and in that cases we can or maybe more than 2 that means 3 dimensional, 3 principal stresses we can evaluate, nonzero principal stresses exist in a particular situation in that cases we can define the yield surface rather than yield point. So therefore first part of the plasticity model or plasticity analysis the how to define the yield surface.

Second point is that to sustain the plastic deformation we need to put some condition that is called normality condition or material stability and that material stability is analogous to the plastic deformation behaviour. So what we can establish the material stability or normality condition we put to sustain the plastic deformation. Then flow curve also, so in plastic deformation then we define the flow curve.

But to basically flow curve behaviour is necessary we try to develop the plasticity model but we will be focusing most on the yield surface and this normality condition and of course once we define the yield surface then what we can incorporate the effect of the strain hardening, because we have seen in the stress-strain diagram this is elastic limit, stress is proportional to the strain.

But beyond that there if you within the plastic zone there is a increment each and every point if you see there is increment of the strain. So that strain as well as the when there is increment of the strain that means plastically deform the material the strength level also increases, the stress level also increases. So that that is because of the material having the strain hardening effect.

So in single point it is well defined the strain hardening effect can be defined but if it is a surface what we can define the strain hardening happening that means how the yield surface evolves during the plastic deformation process then that way so that we can takes care of the strain hardening effect of a particular meter specific to one and plasticity theory. So therefore strain hardening effect will try to look how we can incorporate.

So therefore flow theory of the plasticity means the flow behaviour plastic deformation happens at the so basically we are interested to this zone, plastic deformation zone. So analysis in plasticity model normally done in the incremental mode. So that mean incremental strain, we look into the incremental strain that depends on the stress value. So suppose this is the plastic deformation zone.

So we normally divide in this small component, small incremental strain. So this strain and stress and then that incremental strain and depends on the stress value so we predict because in plastic deformation if you look into that that each and every elemental thing there is change of the slope, so that continuous change of the slope. So that is why the accuracy of the calculation depends on how small we can take this elemental strength.

So that in incremental mode normal stress analysis or plasticity model normally develops using the incremental strain. Now normally we use the stress induces the strain rate which is analogous to the pressure and the velocity. So if there is a pressure difference then only fluid flow and with certain velocity. So here also if stress induces also strain rate and that also stained it also having some influence in the plastic deformation behaviour standard.

So therefore two-stage behaviour elastic and plastic, but it is normally implied in the sense that of course we normally we do the elasto-plastic analysis. So there is an elasto-plastic analysis in most of the manufacturing process. For example, in case of bending process. So in bending process also there must be some amount of the elastic spring back effect.

So we need to incorporate that the elastic spring back effect then only we can get the actual plastic deformation needed to get particular bend angle, that we will discuss in the during the material forming process module there, but of course that elastic spring back effect is basically nothing but the material having some kind of the elastic components.

So that once even if we know already discussed that even if you do in a plastic deformation zone so when you remove the load at a particular point there is some recovery will be there, that recovery because of the metal having the some sort of some amount of the elastic properties. So that is why we can represent the material behaviour in the sense of that either elasto-plastic material kind of this thing or perfectly plastic material or we can say perfectly elastic material.

So based on the stress-strain diagram of a particular material. So that kind of situation we can explain here also.

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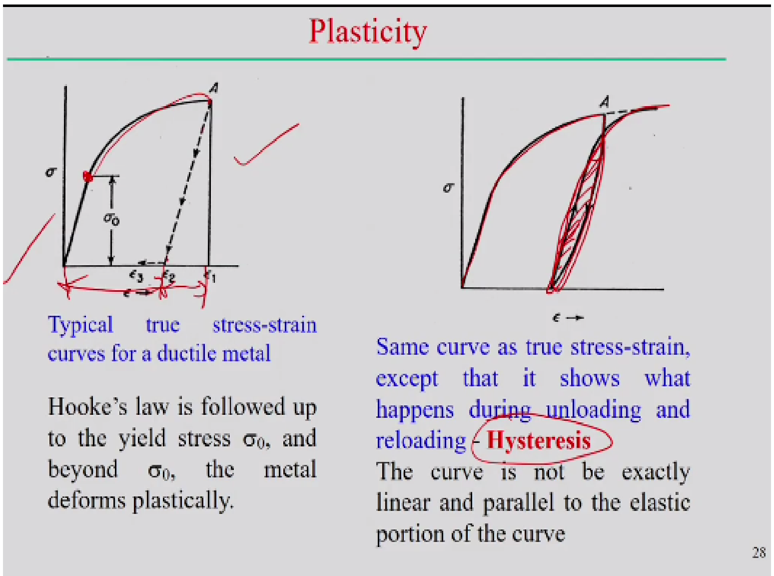

For example, true stress-strain curve for a ductile material. If you see the true stress-strain curve ductile, we have already discussed true stress-strain curve for a ductile material. Here we can see that sigma 0 is the yield stress value here and from here to this is a plastic deformation. Now if there is at point A if we remove the load then it will come back to this point.

So therefore this is the amount is the permanent strain exist within the material because it cross the yield point so permanent strain must exist but this amount strain represents the elastic recovery during this deformation process. So this is typically called elasto-plastic material behaviour or this is typical stress-stain diagram of a particular ductile material and most of the engineering materials actually follows this kind of stress strain behaviour.

Now for example, if this is the situation stress-strain curve is something like that here if removed the load and reload it there is some gap exists. So then that is represented in the stress strain, it happens during the stresses so that we see that if this is representation of the stresses of a particular stress strain curve in typical nature and of course we have already discussed in terms of material properties.

If the area is more that means material is having good damping properties it having the capability of observing the vibratory load during application. So that is why it is a typical behaviour of the material. So that if this curve is not exactly linear and parallel to the elastic portion of the curve. So then some sort of gap is there. So that gap represent some kind of the material properties.

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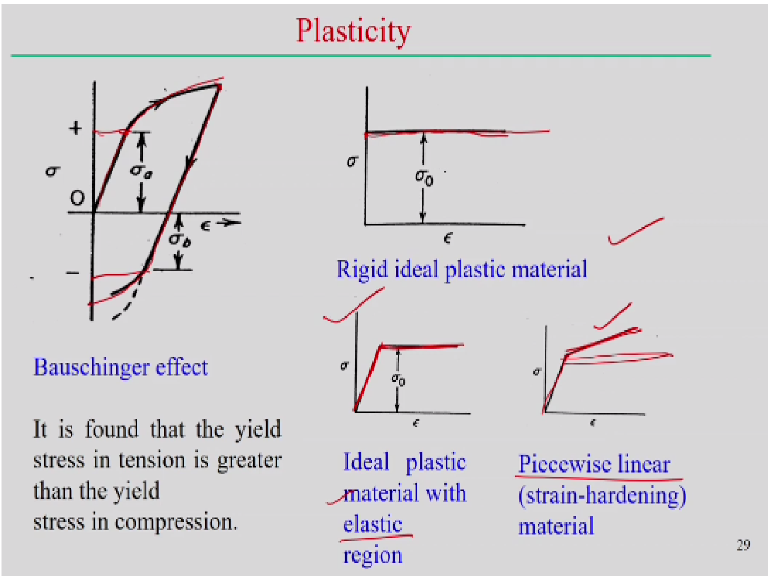

If stress strain behaviour is something like that is a applying the tensile load reach this point and we remove we just reverse the load from tensile to we apply the it is come back to this point remove the load and if we apply the compressive load then it will reach up to this point. So therefore yield point in the tensile load and yield point in the compressive load are different.

So this type of situation arise then we say this is the material is having Bauschinger effect. So if we consider the Bauschinger effect that means if the yield point in tensile load and yield point in compressive load both are same then we can say the material having Bauschinger effect. Similarly, this is the stress-strain diagram of a particular metal this is called a Rigid ideal plastic material.

So that means there is no elastic component here, directly it starts from the plastic deformation here at this point and that there is no strain hardening effect also because this line is parallel to the strain axis. So therefore this type of behaviour material is called the rigid ideal plastic material behaviour and of course you can see that ideal plastic material with the elastic region that is also that means this part is the linear part that is called the elastic part exist within material, but there is no strain hardening effect.

It is simply parallel to the during the plastic deformation zone. So this is called the ideal plastic material with elastic region, this type of behaviour. Of course Piecewise linear that means here linear component is there and we can say another linear part is there, piecewise. We can say also bilinear curve also, but of course it is having some strain hardening effect because this line is not exactly parallel to the strain axis.

It is having some slope with respect to having some slope that is why it is, slope means there are some positive slope exist during the plastic second linear part. It indicates that material having the state hardening effect. So these are the typical material behaviour we can represent by simple stress-strain diagram and having the different nature. So based on that we can develop the material model and we can analyse the deformation behaviour of a particular material.

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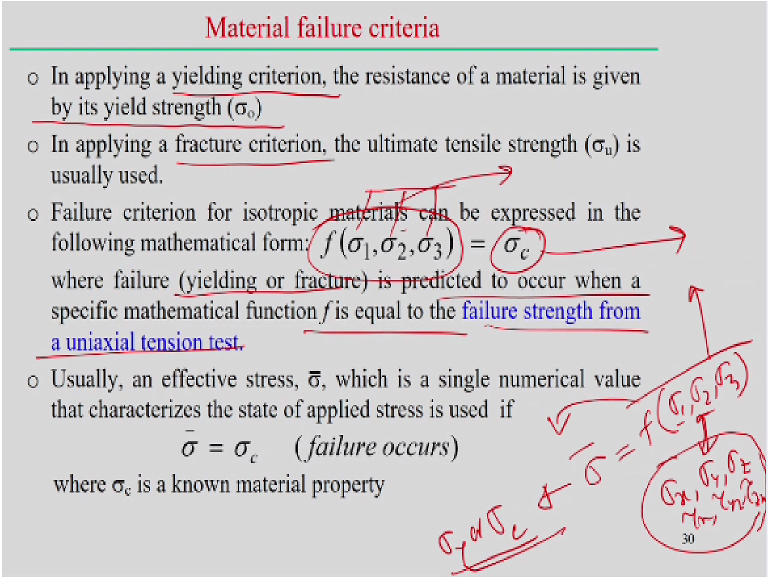

Now why you are interested to know that yield point of a particular situation because in applying the yielding condition the resistance of a material given by the yield strength of course yield criteria we need to find out the yielding criteria that yielding means just starting on the plastic deformation zone. So therefore it is important to know at which point the plastic deformation starts.

That is why we always try to find out what is the yielding condition or yielding criteria for a particular material. Of course even if you define the fracture criteria then we should know what is the ultimate tensile strength of a particular material is normally used. So therefore ultimate tensile strength is normally used to decide the fracture criteria of a particular component.

So that is why yield stress and ultimate tensile strength having some significance and to analyse the failure criteria of a particular material. So therefore failure culture of isotropic material is often expressed in the mathematical form that failure criteria would design the functional form of the failure criteria of sigma 1, sigma 2 and sigma 3. These are the three principal stresses.

So that means we can find out that it is a function of the, so for example, in a particular component it is subjected to some different kind of the loading and combined loading but we represent it in terms of the stress tensor, we represent the 6 components of the stress and 3 components of the normal stress and 3 components of the shear stress. So total 6 components of the shear stress that can be represented in terms of the principal stress component.

That is sigma 1, sigma 2 and sigma 3 so then you even try to decide the failure criteria from the analysis with the external applied load we can find out what are the principal stresses acting for a particular material and then for the same material we have the data for the different values of the for example, sigma C maybe yield strength or maybe ultimate tensile strength.

But that ultimate tensile strength and yield strength is actually defined in a uniaxial tensile testing of a particular specimen or particular sample. So therefore failure criteria either yielding or a fracture we can decides which is predicted to occur on a specific mathematical function f is equal to the failure strength from a uniaxial tension test. So this data are available for a particular material or well defined.

But this data is not defined this is during the process the load is acting to this component and then we can based from there we can find out the principal stresses and we can compare whether it is fracture or yielding happens or not by just comparing the experimental data for the same material. Now usually effective stress that means when we say the effective stress or we can say that effective stress is significant because effective stress can be represent one single numerical value.

So that can be a function of sigma 1, sigma 2 and sigma 3, because sigma 1, sigma 2 and sigma 3 are the principal stresses and these principal stresses are basically defined sigma y, sigma z that at rest tau xy, tau yz, tau zx. So therefore this is the original state of the stress acting for a particular material at a certain situation from there we can finding out the functional form of this sigma.

And sigma 2 in terms of principal stresses and then based on that we decide what may be the functional form of this principal stresses such that it can be represented in terms of the equivalent stress or that is representation of the single value. Then when we represent the single value then we compare this single value stress with respect to the experimental value. It may be either c or sigma y.

So then if we compare this way this is theoretically estimated, we convert it single value and then we compare with the experimental value. Then we decide whether we can fit the criteria whether there is a failure happens or yielding happens or not for a particular material. So this is the normal procedure to find out to compare this thing, the experimental value and the theoretical value.

But of course what may be the functional form of this principle as a function of principles in this functional form is to decide depending upon the different hypothetical processes or some assumptions based on that we can predict the different kind of theory and from that theory we can define the functional form. One of the such theory I will try to explain that what we can define this functional form such that we can decide the failure criteria or yield criteria for a particular material.

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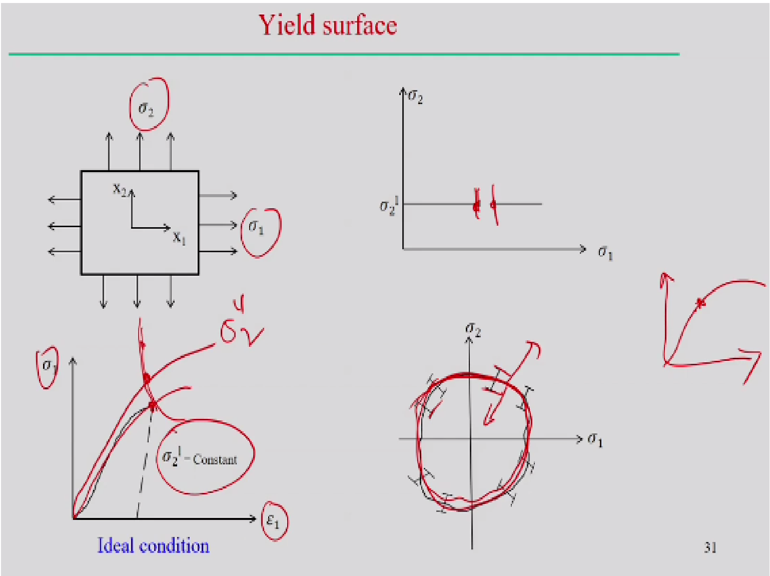

Now before that what we can define the yield surface of a particular material. So it is well defined that yield point of a particular material if we do that uniaxial tensile testing that means material is subjected to only one directional load or one directional stress based on that we can define the yield point, but in practical the material is not subjected to the uniaxial loading condition may be material is most of the cases practically the multi axial loading condition.

And there is a multi-axial loading condition then how we can define the yield surface. So for example, suppose sigma 1 is acting on a particular element at the same time sigma T is also acting. So in this cases what do we define the yield stress value. So practical approach is that we can decide we can keep either sigma 1 or sigma 2 as a constant value. For example, we consider sigma 2 as a constant fixed value.

Then if we conduct the stress percent, if we vary the sigma 1 for a particular sigma 2 we can get a single curve like this. So maybe this is the failure point, ideal condition. Now if we change the sigma 2 value, if we increase or decrease the next level increment or decrement of the sigma 2 value then can we reconduct the tensile testing, then we can get this curve, this similar curve may be different.

And this is the sigma 2, different values of the sigma 2. So in that way at the different value of the sigma 2 we can get the stress versus strain that means sigma 1 versus epsilon 1 a different curve. So therefore it is a different curve and that in each and every different curve there may be some yield point. So that yield point when you join together then it clears the kind of the curve if it is 2-dimensional.

If it is 3-dimensional then that can be represented in the form of a surface. So therefore but this changing the sigma to maybe there is a change, the gap, this is the one yield point, this is another yield point may happens and of course if we try to experimentally define all this thing at this yield surface maybe it is an huge number of experiments just by changing manually the different values of the sigma 2.

And of course if it is 3 dimensional then it becomes more complicated because there are so many combination of a particular because we have to keep this for a fixed value of sigma 2 and sigma 3 we can conduct the experiment for varying the sigma 1. So therefore with this combination the lots of combination is possible and therefore we need to define a lots of experiments to define the yield surface.

So therefore even if we do conduct the experiment or not exactly there is a range actually when we changing sigma 2 dot next value so it is a range of the stress, so suppose if it is a two dimension the sigma 1 and sigma 2 on this stress axis if we plot it this is the curve that represents the yield stress value. So it is within that elastic beyond that plastic deformation. So that way but it is not exactly if we are not fitting exactly curve.

There is a some variation actually happens during this process. What we roughly estimate this can be the closed curve on a stress axis that represents the yield curve when it is subjected to 2-dimensional state of the stress. Similarly, if it is 3-dimensional then we define as a surface.

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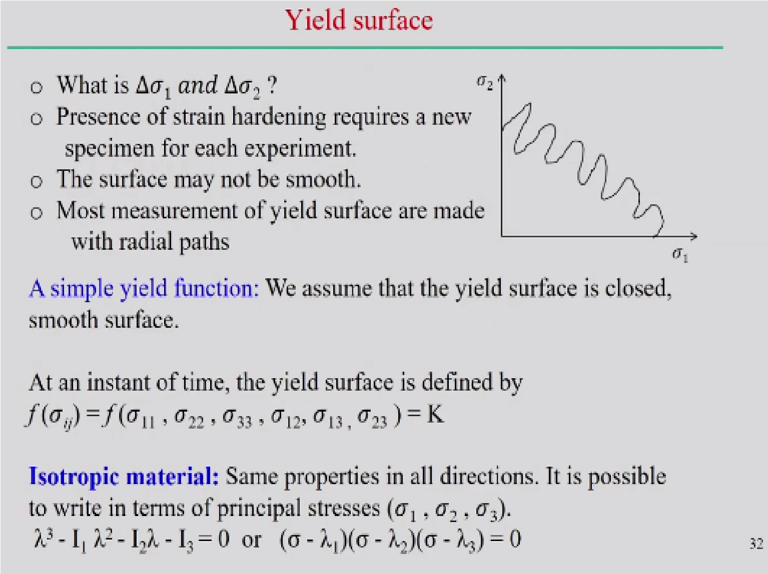

So that but question is that we increase delta sigma 1 and delta sigma 2 to conduct the experiment, but what may be the amount as minimum as possible then we can create the good estimation of the yield surface profile or yield curve, but in that cases we need a lots of experiment. So therefore it actually vary depending upon the delta sigma 1, sigma 2, it is actually in this vary the yield profile.

So we approximate it as a curve so some averaging value. Of course presence of the strain hardening requires the new specimen for each and every specimen therefore since there is a strain hardening effect is also there in elasto-plastic material so therefore we need to conduct more number of experiment and of course the surface may not smooth also is a kind of curvature is there.

So therefore most of the measurement of yield surface are made with the radial path so that means this direction radial path we can measure most of the yield surface. Therefore it is almost impossible to conduct the experiment for different conditions by changing sigma 1, sigma 2 for a particular material. Then people have developed different theoretical analysis of this thing to predict the yield surface for a particular material.

Of course just looking into the analogy of a particular during the experimental process. So we can assume the ill function we assume that yield surface is a closed and smooth surface rather than some wavy surface. Now once we assume the yield surface is a closed smooth surface for three-dimensional state of the stress therefore at particular instant the yield surface can be defined in the functional form.

For example,s f sigma ij is the index form, sigma is the stress tensor, so is having all those components 1, 2, 3, 4, 5, 6, 7, 8, 9; 9 components will be there. So therefore it is a function of assume that sigma 1 1, sigma 2 2, 3 3 that means normal stress and the shear stress components and equal to K is a constant. Now this is the functional form, we assume this is the functional form because all stress component having the contribution toward the yielding of a particular component where it is subjected to 3-dimensional state of the stress.

Now if we assume that isotropic material, so isotropic material means same properties in all direction. If we assume these things so therefore it is possible to write in terms of the principle stresses. So once we assume it is a isotropic properties of a material, isotropic material, then once we assume the isotropic material this functional form is reduced in terms of the principle stresses sigma 1, sigma 2, sigma 3.

So therefore once we as a function of sigma 1, sigma 2, sigma 3 in terms of the principle stresses then we can represent that this is what we can find out the principal stresses then that if you having this stress component then from here we can finding the characteristic equation and from this characteristic equation we can form the cubic equation. This cubic equation is in this form and where I 1, I 2 and I 3 as the stress invariant.

And this lambda is the basically roots of this equation. So these roots of this equations are the principal stresses. This equal to 0 or algebraic form of this equation can be represented in that way sigma - lambda 1, sigma - lambda 2, sigma - lambda 3. So here lambda 1, lambda 2, lambda 3 are the roots of this equation and it's a cubic equation for that. So this is the way to represent this stress component in terms of the principal stress components.

And that we are just simply representing the cubic equation such that the roots of the cubic equation represents the principal stresses of a particular situation. Now stress invariant we have already discussed.

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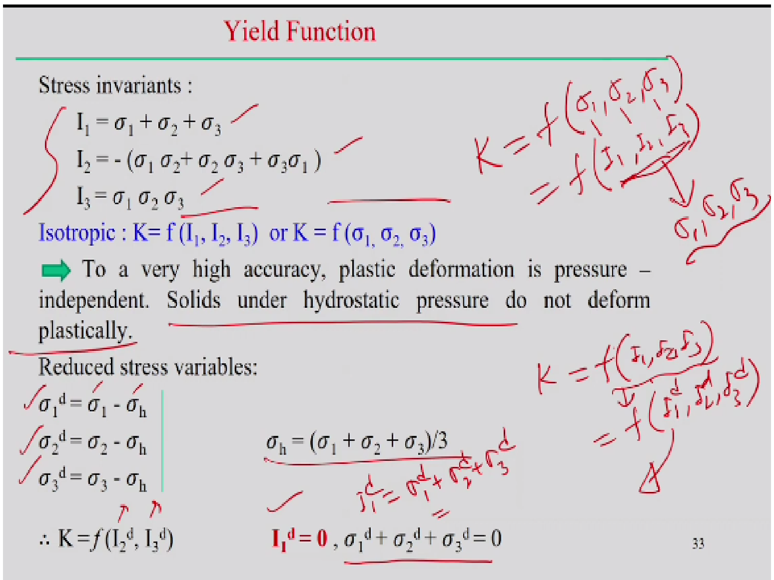

But stress invariant just for I 1 = C in terms of the principal stresses. These are the stress invariants sigma 1, sigma 2, sigma 3. I 2 also sigma 1, sigma 2, sigma 2, sigma 3 + sigma 3 sigma 1 and I 3 is the sigma 1, sigma 2, sigma 3. These are the stress invariant of this cubic equation. Now isotropic material so therefore K functional form is a function of isotropic material we can use with the original stress state to principal stress state.

Function of this, or this sigma 1, sigma 2, sigma 3 as a function of one way or we can say it is a function of sigma f is I 1, I 2, I 3 because I 1, I 2, I 3 actually represented in terms of the or this stress invariance is in terms of the principal stresses sigma 1, sigma 2 and sigma 3. So therefore isotropic material we can represent K the functional form either as a function of stress invariant or K as a function of all the 3 principal stress components.

Now to a very high accuracy plastic deformation we have already discussed that stress components can be decomposed in to 2 part, one is the hydrostatic stress component and the deviatoric stress component and we have already discussed that hydrostatic stress components having no influence on the plastic deformation. So therefore since plastic yielding having no influence and of the hydrostatic stress component.

Therefore we can separate out this stress component original stress component to the deviatoric stress component because the yielding depends on the deviatoric stress component and it is independent of the hydrostatic stress component. So therefore solace under hydrostatic pressure do not deform plastically, the hydrostatic stress having no influence on the plastic deformation.

Therefore we just rule out the hydrostatic stress component from the original stress step. So therefore if you look into only the deviatoric component, so deviatoric component is the sigma 1 is the actual principal stress minus the hydrostatic stress component. Similarly, we represent the all 3 deviatoric stress components in this way such that hydrostatic stress component is the average of all these things that we have already discussed.

Now we can say that K is a function of the stress invariant I 1, I 2 and I 3 and original stress state, but when we convert to into the deviatoric stress component I 1d, I 2d and I 3d from here to here we can see since hydrostatic component having no role on the plastic deformation yielding, so even try to finding out the yield criteria then we can convert it in from the original stress to the deviatoric stress component.

But if you look into deviatoric stress invariant here I 1d. So I 1d we can see the I 1d actually 0. If you find out that I 1 = sigma 1 sigma 2 Sigma 3 and then and in terms of the deviatoric component I 1d is actually I 1d = sigma 1d + sigma 2d + sigma 3d. If we add all this components d, so here we can find out that it becomes actually 0. So then if it is 0 because sigma 1d = 0.

Then we can say the K is a function of only the I 2d and I 3d that means this functional form we can reduce that only to stress invariant and to stress deviatoric stress invariant I 2d and I 3d. So therefore for isotropic material and pressure independent material pressure independent material in the sense that pressure does not influence the yielding of a particular material.

So therefore for isotropic pressure independent materials the functional form can be reduced into this k f I 2d I 3d. Now the plastic response of the materials is often the plastic response we observe most of the engineering materials that there is no Bauschinger effect, that means the yield stress in tensile and yield stress in compressive load are same. So in that sense we can say that material is having no Bauschinger effect.

So since the material having no Bauschinger effect then we can say that in analogy to material having no Bauschinger effect.

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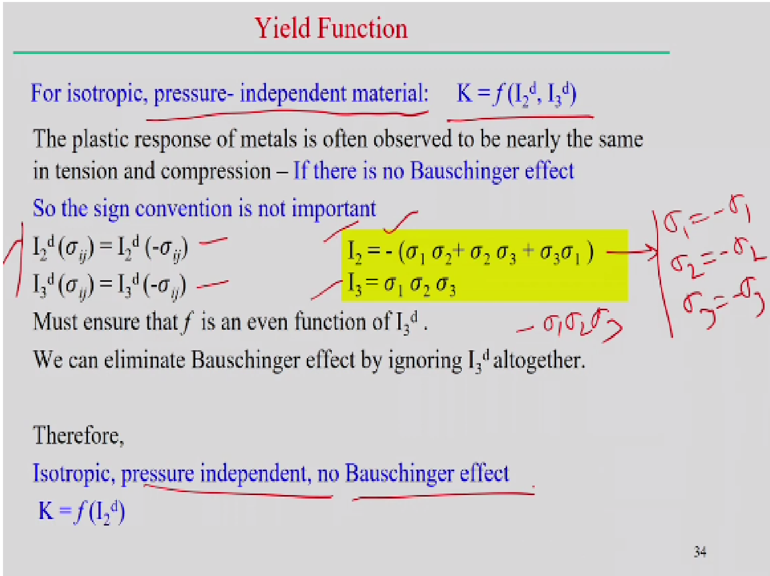

We can say that can be mathematically said that I 2d sigma I3 = I2d - sigma IJ, so this is actually is a even function that I3d sigma I3. So this when you represent this mathematical expression, it actually represents that if these are equal changing the sign the function is the even function. So therefore even function in the sense that since the loading in the tensile and compressive yield stress value.

Compressive yield stress value in the negative sign will seem but same magnitude, the same magnitude but signs are different mathematical we can say the function should be even function. So therefore f.2 indicate no Bauschinger effect that means the functional form should be the even functions. Now it should satisfy these two that indicates mathematically that no Bauschinger effect exists here.

But if we look into that functional from the I2 and I3 or I2d or I3d we can if we see that if we change the sign of the sigma 1, sigma 2 and sigma 3 together then we can see there is no change of the sign in the functional from here. If we replace sigma 1 = - sigma 1 sigma 2 = -sigma 2, sigma 3 = - sigma 3. If we put it here then it becomes the same expression, but if we put here then it changes sign I 3 becomes now - sigma 1 sigma 2 and sigma 3 or same thing for deviatoric component.

Therefore I 3 does not satisfy this conditions that means in mathematical sense this functional form is not even function, it is not satisfying. So therefore we can say for isotropic pressure independent if there is no Bauschinger effect then the functional form can be represent only on the second stress invariant because it is not satisfying, it means that, it is not satisfying mathematically means it is not satisfying the Bauschinger effect.

Therefore we can say I can reduce further the functional form is in the form of only second invariant. Now we can do the further calculation manipulation of the second invariant in that way.

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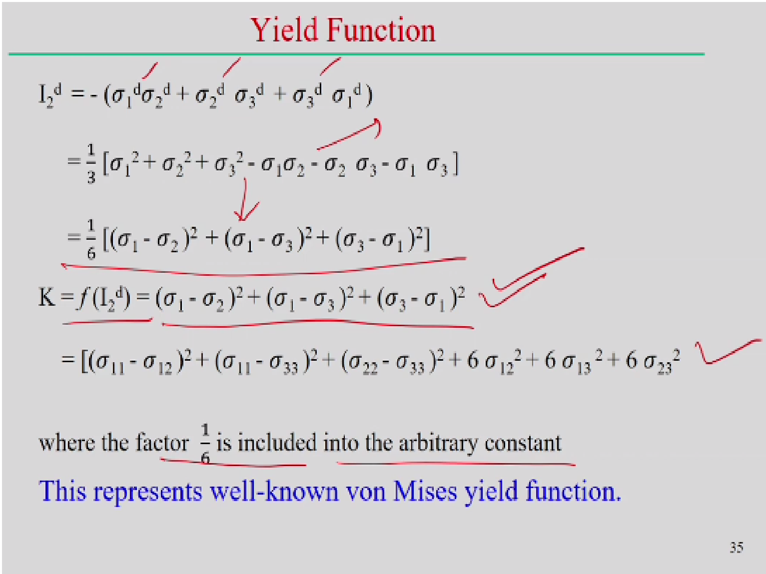

Sigma 1 sigma 2, sigma 2 sigma 3, sigma 3 sigma 1 d, deviatoric component, if we put all these values we can find out in terms of the principle stresses, this is the expression for that and we can convert it in the 1/6 of this is the expression for that, now K becomes finally as function of sigma 2 I 2d as second stress invariant and it is in terms of this, this is the functional form.

Or in terms of the principal stress or this is the in terms of the original initial stress test. So therefore we can see that factor K we can say in this way that it is the functional form but factors 1/6 inducted included in the arbitrary constant. So it is we can just maintaining the constant, but the functional form is this one. So this functional form actually represents the well-known the von Mises yield function.

And this von Mises yield function is one of the most widely used in the plasticity model and we can see that how this functional form actually develops by assuming the several functions this functional from actually valid with the assumptions that material in practically material is isotropic material and the yielding is pressure independent and if there is no Bauschinger effect then only the yield functional form can be represent in this way.

Now we can cross check also that this functional form of the yield surface.

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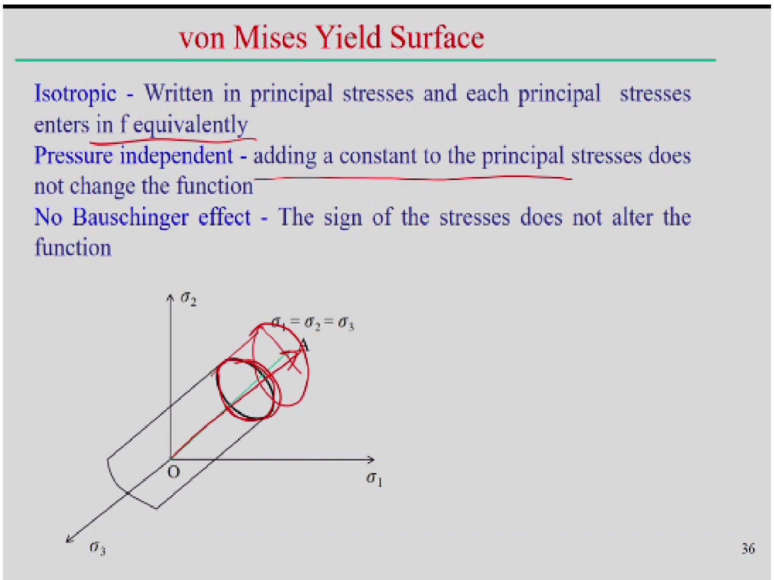

So if you see the isotropic material, isotropic material the properties is independent of the direction therefore written in the principal stresses and each principal stresses enters in the functional form equivalently, in the sense that if you see all components sigma 1 sigma 2, sigma 1 sigma 3 and sigma 2 sigma 1, all enters the equivalent, there is no constant or something like that.

So therefore it is validating the isotropic material properties and the pressure independent means mathematical you can say adding a constant term to the principal stresses does not change the functional form. If we add that constant term in all sigma 1 and = say sigma 1 + some constant term similarly, sigma 2 = sigma 2 + and sigma 3 = sigma 3 + P. If you put add constant term.

And if you put here also then there is no change in the functional form, so that means the yielding actually independent of the pressure and of course the Bauschinger singer effect, the Bauschinger can be proved that if sign of the stresses does not alter the function if you change the sign of the stresses sigma 1 to - sigma 1, sigma 2 to - sigma 2 and sigma 3 to – sigma 3, the functional form remains the same.

That means there is no Bauschinger effect in this expression. So we can represent that this is yielding in graphically, this is the yield surface and this axis represents the sigma 1 = sigma 2 and sigma 3 and this is the yield surface and that yield surface can be here also that means the value actually increasing it is independent of the hydrostatic stress component.

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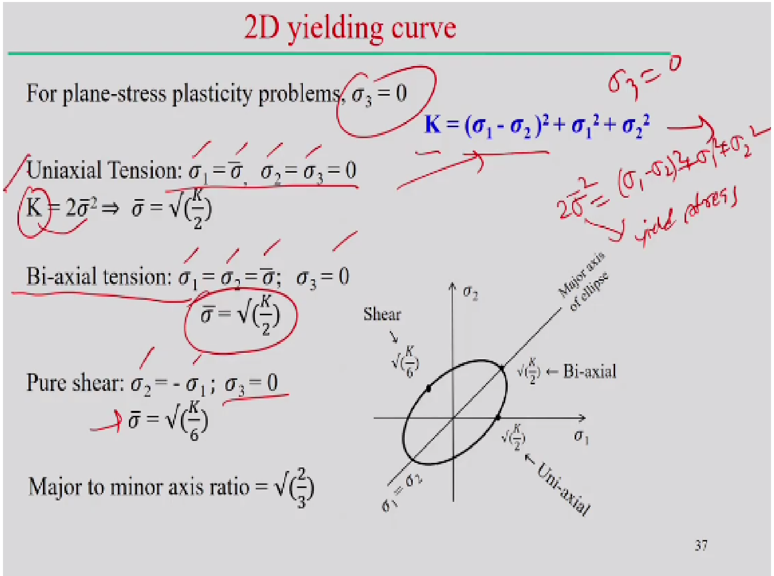

Now two-dimensional that is the from 3 dimensional yield stress components to 2 dimensional curve, we would try to discuss that things for example, plane stress plasticity problem sigma 3 = 0, then what will be the yield surface functional form, their functional form K = sigma 1 - sigma 2 = 0, sigma and – sigma 2 square + sigma 1 – sigma 3, just simply sigma 3 = 0.

This is the yield surface functional form of the yield surface, yield curve for a 2 dimensional stress state. Now we can find out the value of the K also, this is the, K is the this thing that K can be normally decided from the experimental observation. What we can do these things for example, we assume the uniaxial tensile testing, the situation uniaxial testing the sigma 1 = sigma bar.

Say for example, this is the yield stress value, uniaxial tension is sigma 2 sigma 3 = 0, so id we put all these condition, this expression then we can find out that k = twice sigma bar square or sigma bar = root over of K/2. So this is the K value of this thing. So now we find out twice K value sigma bar square = sigma 1 - sigma 2 square + sigma 1 square + sigma 2 square.

So this is the yield surface functional form and this is the situation and we can define the K by simply doing into the uniaxial tensile stressing specimen. So therefore this sigma bar is actually this is the functional form and this sigma bar is the yield stress, normal yield stress of a particular material. Similarly, K can have the different values if you do the biaxial tensile testing.

See in biaxial tension if we see sigma 1 sigma 2 both are same equal to sigma bar and sigma 3 = 0, then we can find out sigma bar = this expression. Similarly, if pure shear condition we can conduct the experiment, pure shear condition the principal, the situation is that sigma 2 = - sigma 1 and sigma 3 = 0 and in this cases we can find out that sigma bar = root over K/6, so therefore the K value or expression of this yields curve can be modified looking into the different type of the observation.

So for example, uniaxial tensile testing, biaxial, pure shear all these different cases are different value. Now we can see if this is the sigma 1 and this is sigma 2 axis and if we put this this is the point represents a uniaxial root over K/2 and this represents the biaxial here the sigma bar = root over of K/2 and of course pure shear condition this is the point and this axis represent the sigma 1 sigma axis.

This is the major axis of an ellipse. So this is the minor axis of an ellipse. So this is the locus of all this point such that minor to major axis = root over 2/3, so this way we can represent the different value of the K and the different functional form of a yield curve and the different situation.

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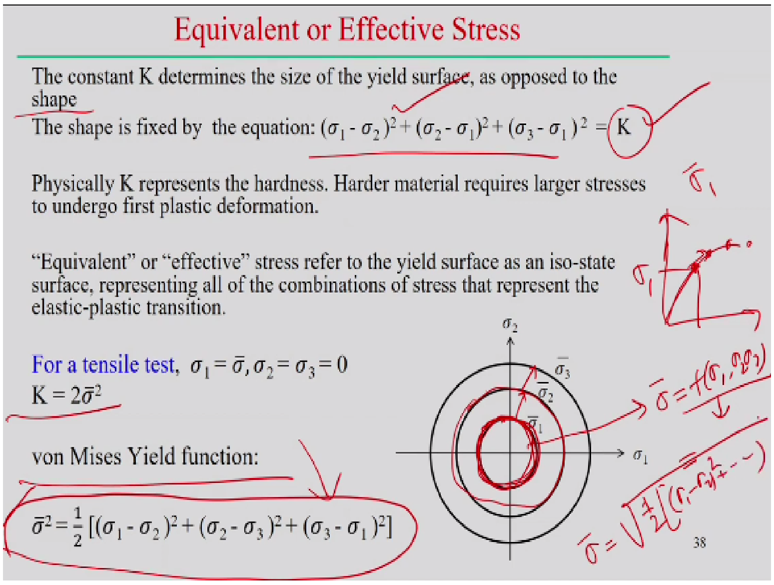

Now equivalent or effective stress, the constant K determines the size of the yield surface definitely the size of the yield surface and that yield surface and if we put the value of the K from the different experimental value that actually decides the yield surface size as opposed to the shape, shape is decided by this functional form, but K define the actually size of the yield surface.

Now K represents the hardness, now if there is a change of the value for example, we can conduct the sigma 1 is the for example, the value of the, from the uniaxial tensile test we can find out the sigma 1 value, but uniaxial stress testing value, this is the yield point, the first yield point, just elastic to plastic zone, sigma 1 for example,. Then with the strain hardening effect it changes.

So therefore if you incorporate the strain hardening effect the yield surface about something like that, it is the first yield point then this is next yield point, next yield point mean this moves from here to here because of the strain hardening effect of a particular material. Similarly, it changes from here to here, from here to here. The strain hardening effect can be incorporated by simply changing the size and that size change is decided by what is the value of K we are using.

So therefore by modifying the value of the K we can incorporate the strain hardening effect. So equivalent or effective stress refers to the yield surface as an iso state surface representing all the combination of the state that represent the elasto-plastic transition. This is iso stress surface. Now this represents the equivalent stress value. So we can say this is a single numerical value.

But that single numerical value is a function of sigma 1, sigma 2 and sigma 3, but if we follow the one measures yield surface condition then it is having functional form of this one. So this called K value or sigma bar, this is called the equivalent stress that means one single value which is a function of all the principal stress components and of course this functional form can change if we follow different kind of the theory also.

For uniaxial tensile testing K already discussed by sigma bar square and von Mises yield function become this one, sigma bar square = 1/2 these things, but this is for uniaxial tensile testing, we can define this value, this is the yield surface and that yield surface in the form of the sigma bar square and of course this Sigma bar if you say what is the Sigma bar value then we can say the root over half, half of all this expression sigma 1 - sigma 2 square.

So this is the equivalent stress if we follow the von Mises yield condition.

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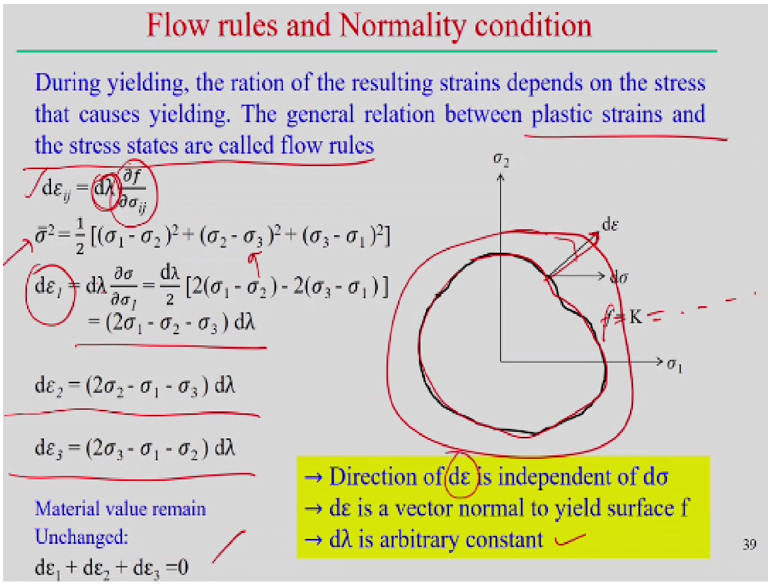

Similarly, flow rules and the normality conditions that also exist during yielding and what way we can define the how the yield surface evolves but the sustainability of the yield surface during the plastic deformation continuous deformation we need to put some kind of the normality conditions in the plasticity model also.

So this yielding the ratio of the resulting strain, now we will try to look it to the strain condition resulting strain depends on the stress that causes the yielding so therefore the relation between the plastic strain and the stress state are called the flow rules, flow rules what we can represents that d epsilon increment of the strain is the plastic multiplied basically d lambda some constant term.

And d lambda is arbitrary constant and df/d sigma z, so therefore the increment of the strain depends on the basically the state of the stress. So suppose this is the yield surface for a particular situation, yield surface, this yield surface represented by f and f = k, that is the K is the functional form of the sigma 1, sigma 2, sigma 3. Now normality condition for this cases can be represented.

What is the increment of the strain particular direction. So this value increment of the strain value that shoud follow this multiplier and this how that d delta F/delta Sigma Iz. Now if we follow this equation sigma bar square = half of this thing we can find out what is the increment of the strain in particular direction d epsilon 1 = d sigma by I think here we can see that delta sigma y.

It is the functional form by sigma 1, so here we can find out the derivative of this one with respect to delta sigma 1. We can find out this is the expression. Similarly, d epsilon 2 = this is the expression d epsilon 3 = this is the expression. Now of course in this cases the material will remain unchanged so therefore the elemental in d epsilon 1 d epsilon 2 and d epsilon 3 should be 0, that can be proved also.

If you try to, there is no change in the material volume during the deformation process. Now this rule says that direction of the d epsilon that which direction the increment of the strength is independent of the d sigma. It is not necessary this should be the same direction. So therefore d epsilon is the vector normal to the yield surface that we will have to define that. The d epsilon is the normal to the yield surface f and d lambda is the arbitrary constant.

So therefore this deformation behaviour or increment of the strain depends on the what are the arbitrary constant we are considering and of course it depends also the at the stress state for a particular situation. This is normally called the normality condition in the plastic deformation situation.

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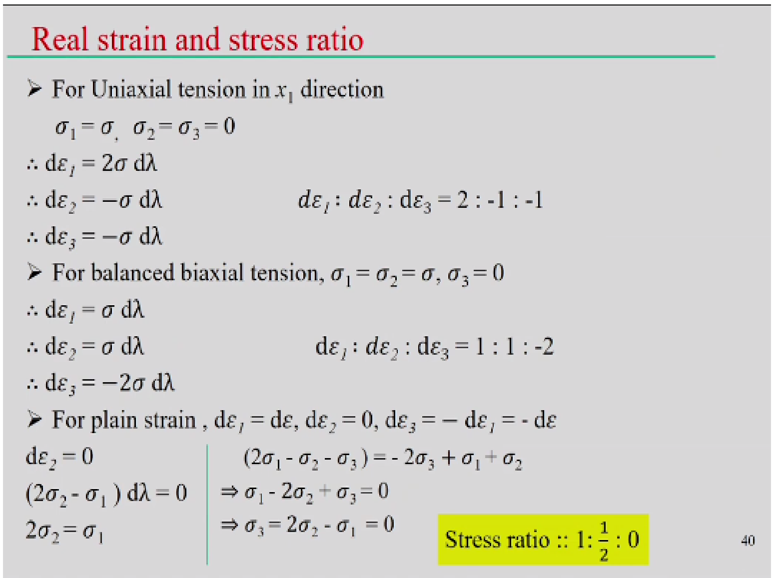

Now we can find out the real stress and strain ratio during the with looking into the normality conditions for uniaxial tensile testing in particular x the uniaxial tensile testing what we can find out this normality condition that x1 direction sigma 1 = sigma for example, sigma 2 thus some stress value sigma 3 = 0. So in this cases d epsilon 1 = this, similarly, d epsilon 2 = this, d epsilon 3 = this for uniaxial tension rating.

So in this case the increment ratio of the incremental strain is 2:-1:-1, similarly, balance biaxial tension sigma 1 and sigma 2 = sigma and sigma 3 = 0. In that situation we can find out epsilon 1 epsilon 2 and the epsilon 3 is like that and the issue is something like that 1:1:-2. So during the deformation so ratio of the different strain components are different for the different deformation weaver and that this ratio actually valid if you follow some kind of the normality condition during the plastic deformation process.

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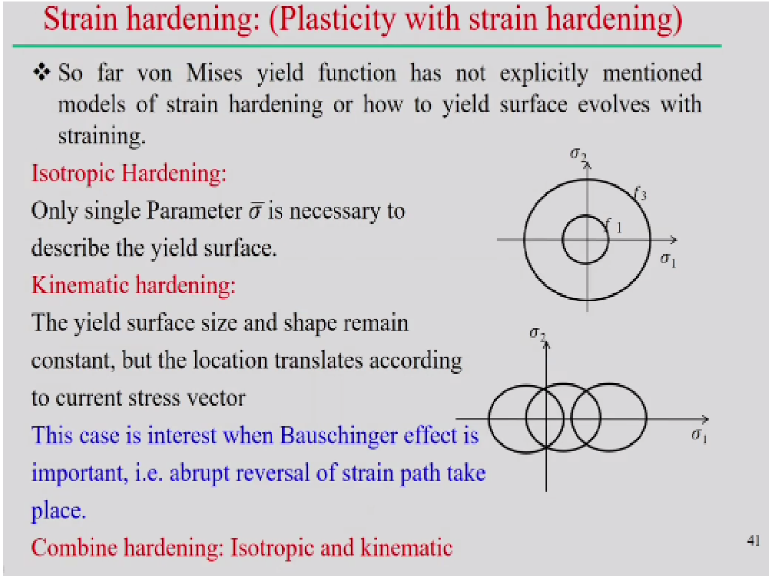

Now strain hardening effect what we can incorporate the strain hardening effect during the plastic deformation behaviour. So strain hardening the plasticity with the strain hardening we can say that von Mises yield function not explicitly mention the models about the strain hardening effect because we say that k = as a functional form of sigma 1 sigma 2 like that, but what a k evolves that it is not well defined.

So if we follow the isotropic hardening for a 3-dimensional stress strain region 3-dimensional plasticity then sigma bar evolves in that way also. So this is the functional from 1 f 1 and then it evolves from here to here and that because of this value or this depends on the strain hardening and this is all equal way it evolves from initial strain yield point to the next yield point due to the strain hardening effect.

Then this is this type of call this, this is called the isotropic hardening. Now kinematic hardening also possible, the yield surface size and shape remains the same, but the location actually translates. So suppose this is the initial yield surface. Now initial yield surface here it evolves next, it actually transfers from one position to another position.

So therefore this kind of hardening effect is called the kinematic hardening and of course this situation arises when Bauschinger effect is important or there is a change of the stress state or abrupt reversals of the strain path takes place, this is the kinematic hardening model and this is isotropic hardening model but in practically most material model material behaviour may follow combining of the isotropic and hardening model.

And it becomes more complicated when we combine all this material behaviour.

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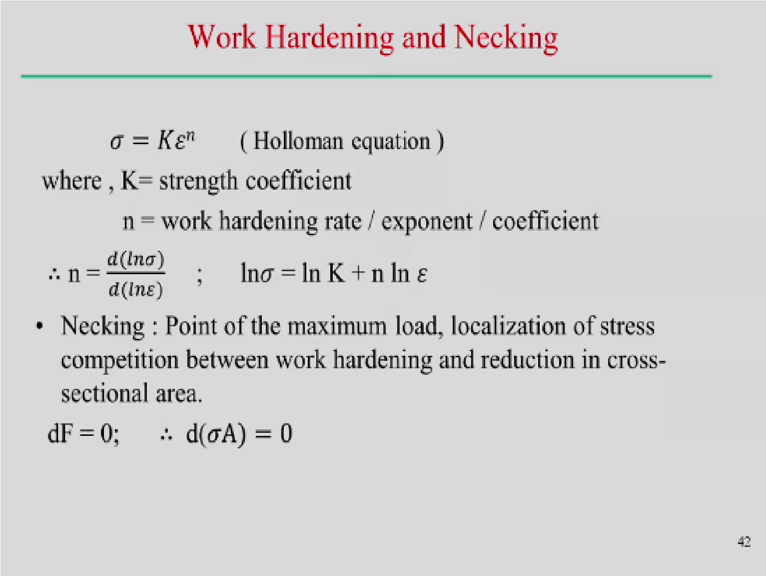

Now we look into the work hardening and effect or necking effect during the process. So we have already discussed that 3-dimensional stress state and in terms of the principle stress and finally we can make at the equivalent stress value and that equivalent stress most of the cases we can say that equivalent state there are single dimensional stress they can in most easiest way or simple way represents the stress strain in terms of this equation.

So where k is the strength coefficients and this is called the Holloman type of the equation that is simply relates the one dimensional stress to the one dimensional strain it may be like that in 3-dimensional situation this is the equivalent stress and this is the equivalent strength and that equivalent stress and equivalence terms related to in this form and a single value.

So therefore K is the strength coefficients and n normally represent the strain hardening effect and work hardening effect. So now in the logarithm if we put in the logarithm scale and sigma = ln K + n ln epsilon then we can find out this is the expression ln sigma = ln k + n + ln epsilon. So therefore in this case this is the equation of most convenient simple way to represent the relate between the stress and strain.

But in actual deformation process of thing if we look in the stress-strain diagram we assume the neglect the elastic component if we replace only the plastic behaviour. So this equation K epsilon to the power n and now up to certain point there is a uniform deformation and once uniform deformation at this point, the necking starts in a deformation of a particular sample. So therefore when the necking starts at this point.

So at the necking point or maximum load actually if we look into engineering stress-strain diagram the necking point the maximum load and localization of the stress, necking means there is a reduction of the cross section and localization of the stress. So therefore competition between the work hardening that means large deformation means it start to work harden more then with strength level also increases.

And at the same time the reduction in the cross section they are competitive with respect to each other, but there is a reduction at this point. So that represents the in load deformation card in engineering stress-strain diagram is the optimum slope. So therefore at this necking condition mathematically we can find out that change of the force = 0, DF = 0, that means slope = 0 at this point.

Now if dF = 0, so therefore f can be represented in the load exist stress into a cross section area. So that is d sigma A = 0, that is the condition of the necking.

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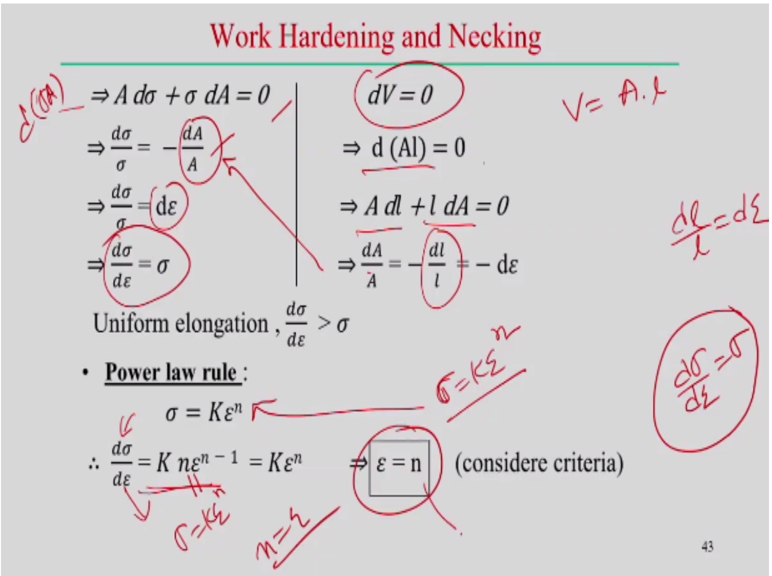

Now d sigma A can be represented this A d sigma or sigma dA = 0 from here we can d sigma/sigma = -dA/A, but dA/A can be represent if you see during the plastic deformation also we can assume that there is no change in the volume. So when no change in the volume means dV = 0. So dV suppose V is the crossing area of a cylindrical component, A cross sectional L = length L.

So therefore d AL= 0 therefore we can decompose into AdL/LdL = 0 from here find out that dA/A = -dL/L. So –dL/L is the elemental strain of a particular component. So that we will put it here and here we can put it this value we can find out this sigma. So d sigma with d epsilon = sigma. So this is the conditions that condition is called d sigma/d epsilon = sigma, this is the condition for the necking for a particular deformation of a particular component.

Now of course this during this necking point it is assumed that it is acting as a equivalent load or single axial, single direction load is acting on this particular component. So now if we assume that stress is related to strength by following the Holloman's type of the equation if we assume the stress strain relation is this. Of course stress and strain relation may be are different equation.

But if we assume this is the relation stressed and then what will be the necking condition. So then if we follow from here d sigma/d epsilon = K, n epsilon n – 1, but we can find out that K which is = c, this is the expression and then, but d sigma/d epsilon = condition = sigma and sigma = K epsilon to the power n, so therefore K epsilon to the power n equal to this. From here we can find out that n = epsilon that means this is called the considere criteria.

And that criteria actually established the necking condition but this criteria comes assuming that stress-strain relation follow the Holloman type of equation, but remember that if stress strain relation is something different then this you may not find this kind of condition. We can start with this expression and in this expression we can assume some stress as a function of strain, but different expression.

If we put this expression here then we can find out the necking condition, but for this particular situation if you follow the Holloman type of equation this is the considered a condition.

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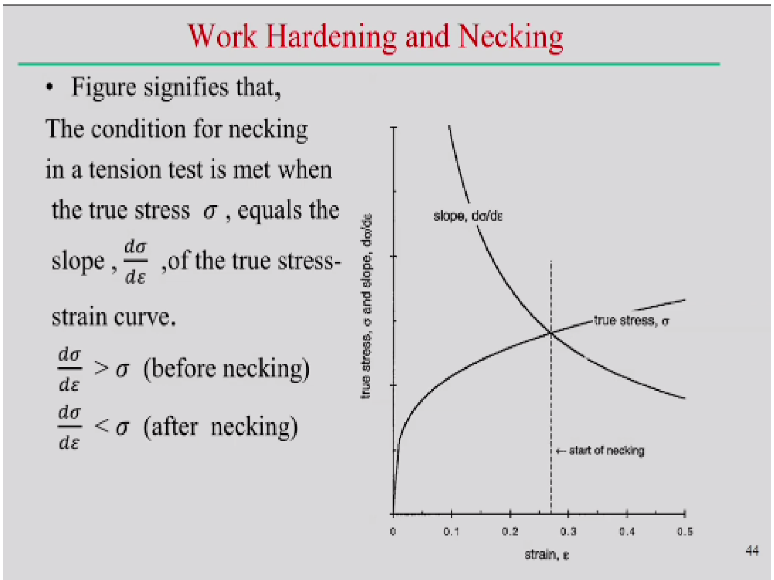

We can easily explain using this graph if we look into that and this is suppose this is the 2 stress strain diagram here and of course this depends on the slope d sigma/d epsilon. Now this if d sigma/d epsilon greater than sigma particular situation if you get this is the sigma curve. So if this slope is greater than this sigma this side this indicates the before necking and this sigma with d epsilon less than sigma.

That means d sigma/sigma is less than sigma that actually indicates this side is the necking situation.

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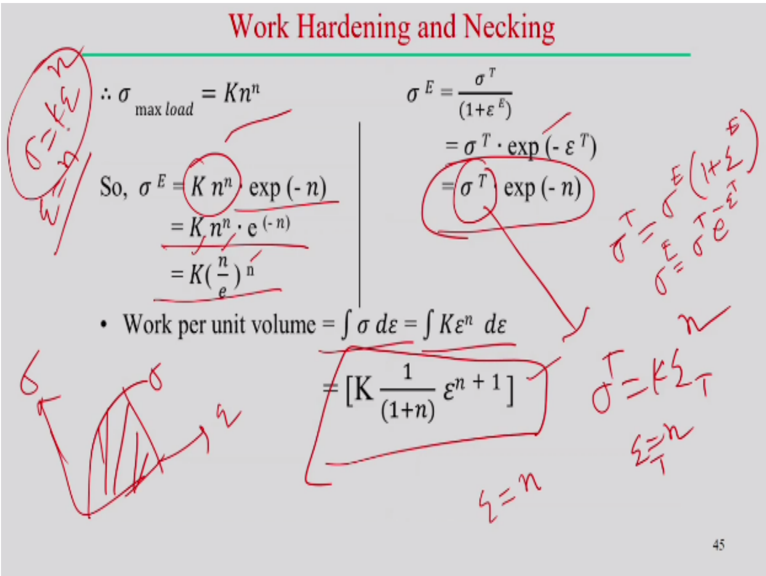

Now we can do further calculation also that work hardening. So maximum load here you can put if you use the Holloman type of the equation, this relation with stress strength. So in these cases the considered condition is this one sigma = n, now what is the maximum load actually that means necking condition the load becomes maximum. So at maximum loading condition stress = K epsilon to the power n then K is this value and epsilon = n.

So k n to the power n, this is the condition and of course we can find out the engineering stress and true strength and because here we can when using this relation sigma = K epsilon to the power n actually we are using the value of the true stress and true strain here. So therefore the relation between the true stress equal to engineering stress into 1 + engineering strength.

This is the relation between the true stress and engineering stress well. So here if we put this value sigma T and epsilon to the power n here our engineering strain = true stress engineering stress = true stress into e to the power – strain. So if we put this well we can find out this is the expression and from here sigma e can also be estimated sigma = sigma T, sigma T actually sigma T = K epsilon T to the power n.

So therefore K n to the power n because here epsilon = n. T = n. So different exponential n. So here we can find out this expression so all in terms of the non material parameter strain coefficient in hardening coefficients and e and n, in terms of that we can find out. Of course within this stress strain diagram we can find out what is the work done per unit volume is the simple relation between it is the stress curve, strain, stress.

So here integration of the stress d epsilon n from here we can find out the expression for that and we can find out this expression work hardening and making phenomena in this particular case. So here you see all this expression is basically on this epsilon to the power n what if we know epsilon = n. So you put epsilon n to the power n + 1. So therefore we can do this kind of sample calculation and if we set the criteria for the necking conditions.

And if we know the relation with the stress and strain we can do all this kind of further calculation. Now even standards sensitivity can also be used most of the analysis in the engineering problem. So like what are you can represent the Holloman type of equation.

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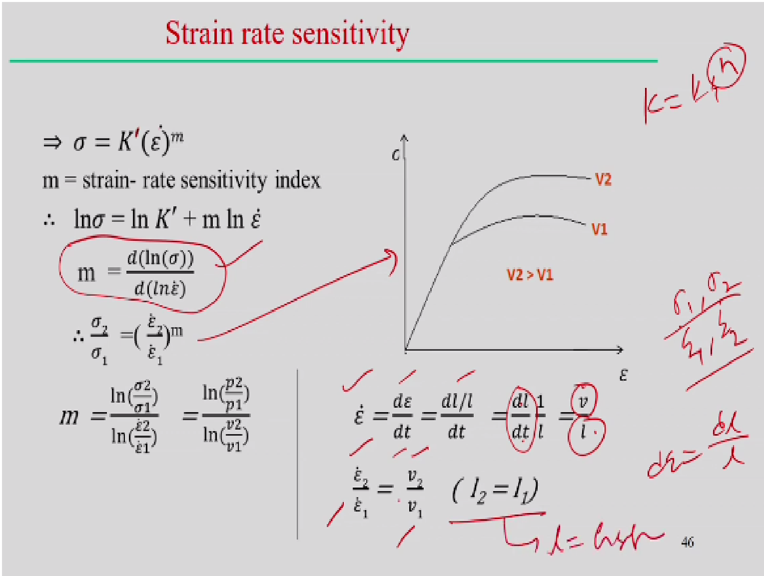

K = k epsilon to the power n, but strain is the strain hardening coefficients but similar we can represent the standard where k dot is the strength coefficients maybe or some constant value and epsilon dot so epsilon dot = the strain rate here to the power m. So M is the standard sensitivity index. Now in logarithm scale we can find out this is the expression and in logarithm scale M is the slope basically.

Slope of the stress versus strain rate curve. So if we find out the slope particular point we can find out what is the m value here. Now if we have due to different situation the different value of the sigma 2 and sigma 1 experimental also we can do different value of the sigma 1 and sigma 2 we can estimate the corresponding strain rate 1 and corresponding strain rate 2 we can estimate the value of M also.

Because this M is the very practical useful parameter for a particular material and experimental we can evaluate. For example, suppose strained can be represented like that d epsilon/dT change of this thing d epsilon can be represented like that d epsilon = elemental length with respect to original length. So dL/L / dT, so dL/dT/1/L dL/dT simply velocity V/L.

Now if we look into actual in a universal tensile testing machine. So here we control the cross head speed so when you conduct the experiment at the 2 different velocity means the 2 different cross head speed and length of the sample remains the same at the 2 different cases. So now once the length of the sample is the same and if you conduct the experiment in 2 different strain rate that means 2 different cross head speed then we can find out the 2 different strain rate value ratio of epsilon 2 to epsilon 1 = V2/V1.

Assuming the L2 that means L = constant. So here this graphical you can represent this V1 and this is the 2 different cross head speed and that ratio can be easily estimated and if you put this value here we can easily find out the value of the standard sensitivity index. So this way we can through simply experiment we can find out the value of the M and then we use this M value for a particular model when there is effect of the strain rate in a particular situation.

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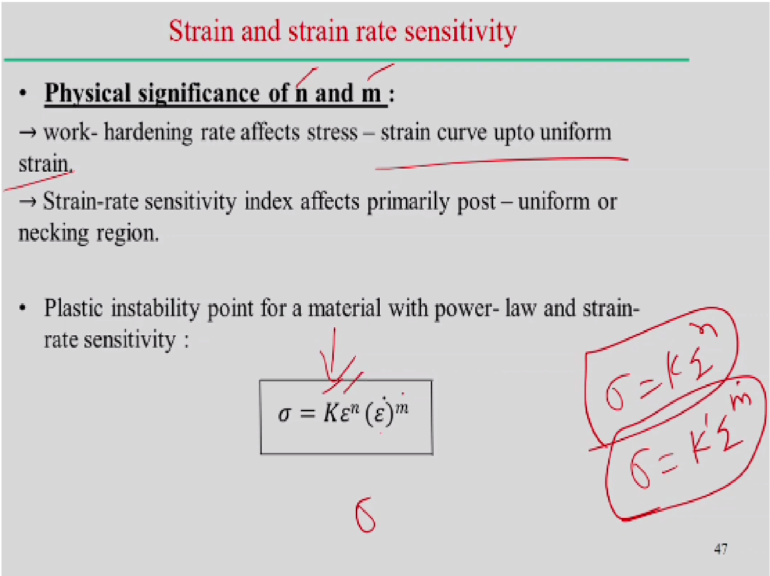

Now of course if you see we have used the 2 different coefficients one is the n work hardening or strain hardening coefficients another is the m, m is the standard sensitivity index and of course physical interpretation of the n and m are differing these cases. So of course the very work hardening effect is only the n up to uniform strain. So basically up to the uniform strain before making the value of n is more important.

Therefore in that cases stress K epsilon to the power n is the most suitable equation there such that there is effect of only the strain hardening, but beyond the ones the necking start in that cases the strain hardening effect is not significant rather strain rate sensitivity is more significant there. So therefore beyond the necking point we represent the stress model is the epsilon to the power m dot is more relevant.

So therefore but actually in some material and depending upon the material deformation behaviour the model can also be done considering the effect of the both strain rate effect as well as the strain hardening effect, so in that cases the expression can be modified like this sigma = K, epsilon to the power n and epsilon dot to the power m. So thank you very much for your kind attention.