Mathematical Modeling of Manufacturing Processes
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Lecture - 28
Cooling and Solidification at Different Casting Processes

Hello everybody. Now, I will discuss the another module that is specific to the casting processes and powder metallurgy.

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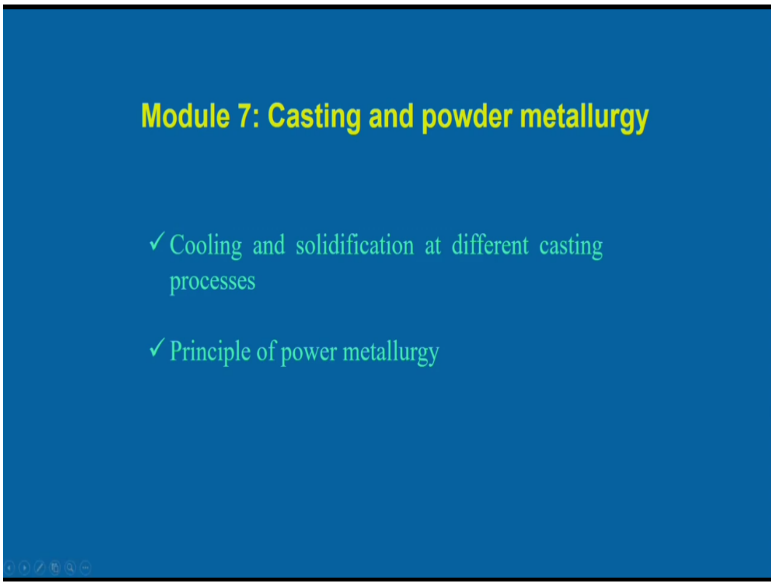

Module 7: Casting and powder metallurgy

✓ Cooling and solidification at different casting processes

✓ Principle of power metallurgy

So, these are the two different topics will try to cover in this module. First, in casting process will try to look into that what are the cooling and the solidification for different casting processes, mainly we will try to get some overall general idea, mainly the solidification processes but we may not be very specific discussion of the different types of the casting processes or different types of the modeling and all this phenomena.

Will try to focus on that the mathematical aspects, the simple calculation in terms of the heat done by using casting, the heat transfer effect during the casting processes and of course in the next part we will try to look into the principle of the powder metallurgy in details and their aspects, different modeling aspects or maybe we can say the analysis aspect we will try to look into that.

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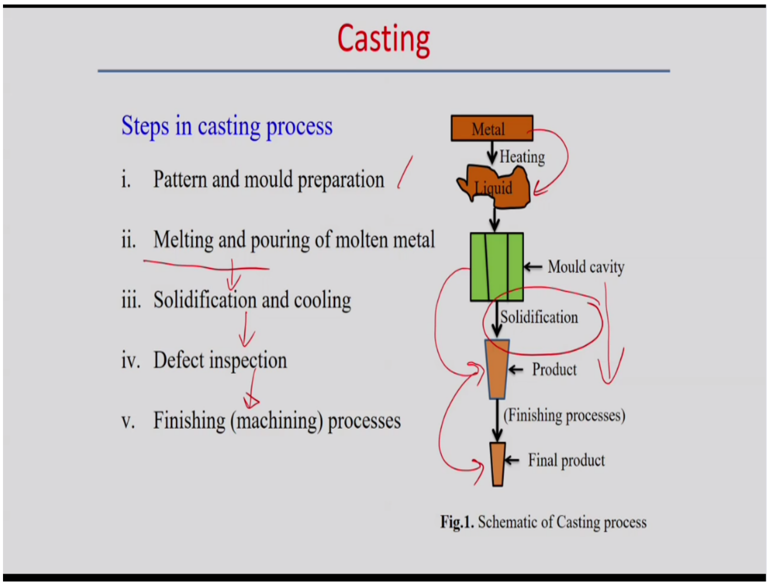

Casting

Steps in casting process

i. Pattern and mould preparation

ii. Melting and pouring of molten metal

iii. Solidification and cooling

iv. Defect inspection

v. Finishing (machining) processes

Fig.1. Schematic of Casting process

So, it is a well-known fact that casting process it is an almost standardized process but first we are looking into the casting process but specific to metallic materials. So, in first we need to melt the molten, we create the molten material and then the metal by heating and change the phase in the liquid phase and then that liquid molten material we then we put in the mould cavity so that mould cavity is the desired shape which we are looking for after solidification of a particular component.

And then once the mould cavity takes some time to solidify to get the shape of the product which we are looking for but of course some more allowances would be given to the product such that after getting the product, the actual product can come or final product can come after finishing of the solidified component. This is the very various schematic or simple representation of the casting process.

But there are lot of technological difficulties, lot of analysis involved during this process specifically we are looking for the solidification in this stage, so mould cavity, how solidification happens and we get the final product and what are the difficulties, how the heat transfer basically happens during the solidification process, will try to look into that aspects.

So, steps in casting in general we can say first we prepare the pattern and after preparing the pattern, we prepare the mould and this thing mould preparation I am not going into much detail how mould can be prepared, different types all these things, that is not required at this stage. So, after mould preparation, then within that cavity melting of the material happens and then pouring of the molten material to the cavity.

And then solidification happens and then subsequently allowed to cooling of this material that come back to the ambient temperature and then after that inspection happens. If there is any kind of defects exists within these structures, if there is defects, then we can analyze the same to overcome that such kind of the defects and finally the product finishing is required of the product, mainly the machining processes to get the desired surface finish we are looking for in the final product.

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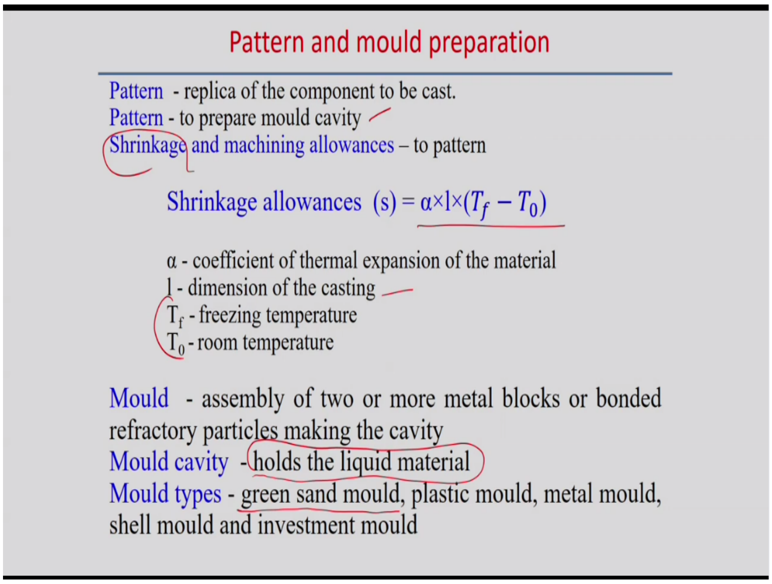

Pattern and mould preparation

Pattern - replica of the component to be cast.
Pattern - to prepare mould cavity
Shrinkage and machining allowances – to pattern

Shrinkage allowances $(s) = \alpha \times l \times (T_f - T_0)$

α - coefficient of thermal expansion of the material
l - dimension of the casting
T_f - freezing temperature
T_0 - room temperature

Mould - assembly of two or more metal blocks or bonded refractory particles making the cavity
Mould cavity - holds the liquid material
Mould types - green sand mould, plastic mould, metal mould, shell mould and investment mould

So, now first we look into that pattern and mould preparation and overall these things. So, pattern is basically the replica of the component to be cast. So, this has to be made according to the size, volume of the component we are looking for. So, this creates the cavity, mould cavity according to the shape of the pattern, shape and size of the pattern and that mould cavity actually filled by the molten material.

But of course, when you try to look into the making of the pattern, we need to provide some kind of the shrinkage and machining allowances, basically at the final step some finishing operation or machining operation is required in a cast product, so we need some extra allowances, so extra dimension to give that such that even after finishing this thing after machining the component, cast component we can get the desired exact dimension we are looking for.

And of course, we need to provide some kind of the shrinkage allowance because most of the cases when there is a change of the volume, the liquid molten metal solidify this thing, there may be the, it is associated some sort of some even it is very small but amount of the change of the volume. So, that aspect we need to consider when we try to design the pattern.

Shrinkage allowance simply we can estimate in that way that alpha l and the temperature basically freezing temperature and ambient temperature, the temperature difference and alpha is the coefficients of the thermal expansion of a particular material, divide material to material, l is the dimension of the casting and these two are the temperature. So, accordingly we can get some idea beforehand before designing the pattern we get what may be the shrinkage allowance can provide of a particular pattern.

Then, mould is simply the assembly of the two or more metal blocks or bonded refractory particles that actually creates the cavity, basically we can say the hold the cavity such that this cavity should be hold by the liquid molten material, liquid material. So, that mould cavity is actually filled by the liquid material. So, there are different types of the mould. Mould which housing that liquid component during the casting process.

So, there are different types of the material can be used, different methodologies can be used, it can be greens sand moulding, plastic mould, basically metal mould, shell mould and the investment mould. These are the different types of the mould material can be used and depending upon the which material we are supposed to casting, based on that we can choose the different types of the mould during the casting process.

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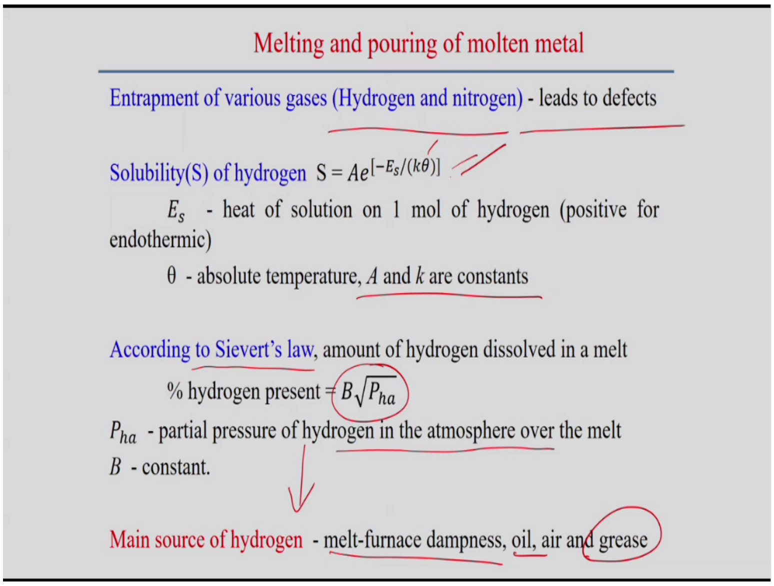

Melting and pouring of molten metal

Entrapment of various gases (Hydrogen and nitrogen) - leads to defects

Solubility(S) of hydrogen $S = Ae^{[-E_s/(k\theta)]}$

E_s - heat of solution on 1 mol of hydrogen (positive for endothermic)

θ - absolute temperature, A and k are constants

According to Sievert's law, amount of hydrogen dissolved in a melt

% hydrogen present = $B\sqrt{P_{ha}}$

P_{ha} - partial pressure of hydrogen in the atmosphere over the melt

B - constant.

Main source of hydrogen - melt-furnace dampness, oil, air and grease

Now, first step of the casting is we need to melt the material and then that molten material have to pour into the mould cavity, so but there may be a lot of difficulties of or maybe some issues associated on the melting of the material. So, that we can analyze in that aspect that for example when there is a melting of the material, there is a chances to the entrapment of the various gases can dissolve within the molten material.

And that actually because this specifically the hydrogen and nitrogen, so that chances of the entrapment of the gases increases with increase in the temperature. So, basically molten material having the maximum chances to entrapment of the hydrogen and nitrogen. So, we have to be very careful when you are melting the material because it finally leads to the some kind of the casting defects.

Then, we analyze these things based on the solubility of the hydrogen. The solubility of the hydrogen can be represented like this equation. So, here we can see that Es is the heat of solution for one mol of the hydrogen, it can be positive for the endothermic reaction. So, accordingly if you see this equation that solubility of a hydrogen, it is actually varying the heat of solution.

And of course, theta is the absolute temperature, it depends on the temperature and A or k are the constants, so that it is actually varying in the negative exponential way that entrapment of this thing that of the hydrogen, solubility of the hydrogen in a cast structure but at the same time it is using the Sievert’s law, the hydrogen dissolved in a molten material or percentage of hydrogen dissolved can be represented in terms of the partial pressure.

If you see that Pha actually partial pressure of the hydrogen in the atmosphere over the melt with respect to the melt material and B is a constant. So, percentage of hydrogen present in a molten material that depends on the partial pressure of this particular component. So, that is the one serious problem in case of during the casting process, we only try to melt the material, there is a lot of chances to entrapment of mainly the hydrogen this thing.

We can roughly estimate what is the amount of the hydrogen by using these two equations, the solubility of the hydrogen and using the Sievert’s law we can roughly estimate the entrapment of the hydrogen during the casting or during the melting of a material which is about to cast but what may be the sources of this hydrogen, it can melt-furnace dampness from there, that can be the source, oil using air can be the source of the hydrogen.

And even the contact it is in the container having the grease, that kind of oily things that actually the main source of the hydrogen and that actually takes parts of during the melting process. You have to be very careful when you try to melt the molten material for a casting process.

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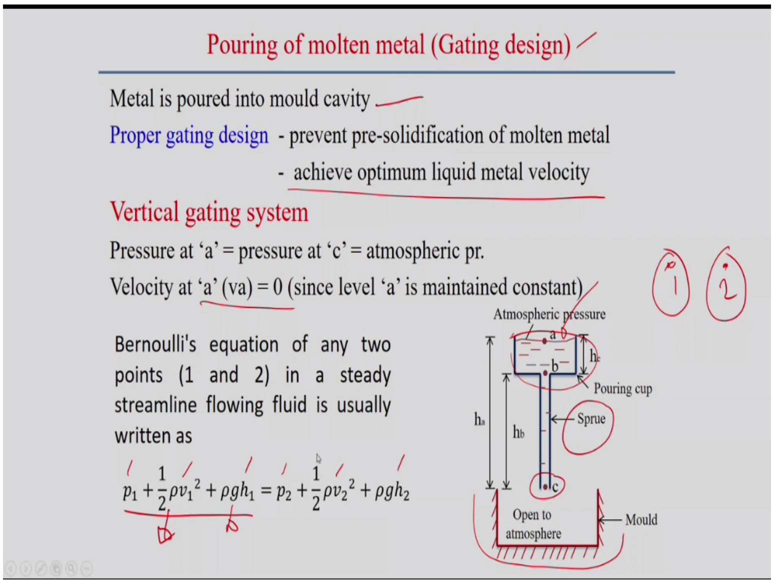

Pouring of molten metal (Gating design)

Metal is poured into mould cavity

Proper gating design - prevent pre-solidification of molten metal

- achieve optimum liquid metal velocity

Vertical gating system

Pressure at ‘a’ = pressure at ‘c’ = atmospheric pr.

Velocity at ‘a’ (va) = 0 (since level ‘a’ is maintained constant)

Bernoulli's equation of any two points (1 and 2) in a steady streamline flowing fluid is usually written as

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Now, next step is that after melting then we need to pour the molten material in the mould cavity but what we can these things that is better described by the is called the gating design. So, basically how to design the gate, gating system during the casting process, so the main purpose is that how the molten metal is poured into the mould cavity, (()) (10:48) if you see this specific schematic figure of this, so this is suppose this is the mould okay some this mould is open to atmosphere.

And then there is a container kind of reservoir, we can pour the molten material here and then molten material passes through the sprue and then it falls on the mould cavity but this is also necessary to design this gating system which is called actually vertical gating system, so that what should be the size of the reservoir, what should be the size of the sprue, dimension of the sprue, shape, size.

We can analyze these things and we can make some conclusion that what may be the typical shape of this pouring cup or pouring basin or what should be the size of the sprue in this case.

So, let us look into we try to analyze these things because proper gating designs prevent the pre-solidification of the molten material. So, objective is to even with we cannot directly put the molten material into the mould cavity rather we need to follow certain gating system such that molten material actually reach to the mould cavity.

It should not solidify before reaching the mould cavity. To look into that purpose, then optimum gating or proper gating design is actually required and of course other part is that how quickly we transfer the metal to the mould cavity so such that it is also necessary to optimize the liquid molten velocity and that in that purpose it is necessary to design the size of the sprue and dimension or maybe what should be the diameter that actually is contact with the mould cavity.

So, what we can do these things, let us look into this aspect, so first say at point a there is a kind of reservoir, so there is a it maintains that height, reservoir height as a constant and at point c, there also atmospheric pressure exist at the same time at point also atmospheric pressure exist because these things open to atmosphere but at the velocity at a can be we consider as 0 because we maintain the level as a constant.

So, such that velocity at point a will be the 0, molten material velocity at point 0 but to analyze these things we need to know Bernoulli's equation and that Bernoulli's equation we can make the energy balance at the steady state during a fluid flow that can be written between the two points, so respect to in a fluid flow between the two points, point 1 and 2 if we apply the Bernoulli's equation then what is the energy head exist due to the pressure.

Then, this is the kinetic energy due to the flow of the material or liquid molten metal, static due to the liquid head, there must also hold some amount of the energy and then that should be the balance at the steady state, so energy balance at point 1 and at point 2 should be equal. So, in that way and of course we only have to look into the dimension of this equation, normally we represent per unit mass.

We can apply the equations in that way also, so here you can see that h is the height of the liquid at point 1 of course with respect to some reference, h2 also height of the liquid at point 2 but with respect to the reference value, velocity v1 at point 1, velocity v2 at point 2, what is the absolute pressure at point p1, absolute pressure at point 2. So, this should be balanced, of course in this equation when there is a fluid flow from point 1 to point 2 or in that direction.

So, we are assuming there is no loss of the energy in this case and of course it follows the steady state situation.

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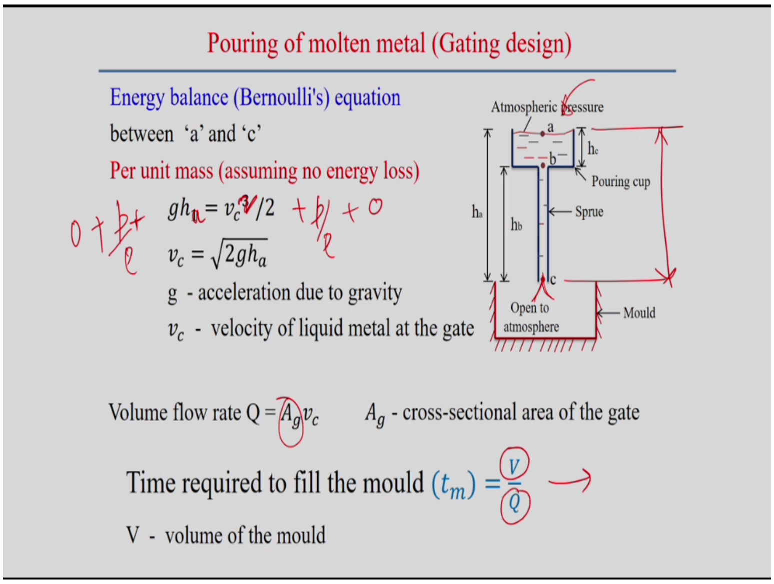

Pouring of molten metal (Gating design)

Energy balance (Bernoulli's) equation

between 'a' and 'c'

Per unit mass (assuming no energy loss)

$$0 + \cancel{p}/\rho + gh_a = v_c^2/2 + \cancel{p}/\rho + 0$$

$$v_c = \sqrt{2gh_a}$$

g - acceleration due to gravity

v_c - velocity of liquid metal at the gate

Volume flow rate Q = $A_g v_c$ A_g - cross-sectional area of the gate

Time required to fill the mould $(t_m) = \frac{V}{Q}$

V - volume of the mould

Now, we will try to look into this Bernoulli's equation in our problem, so for example here is a situation that pour the molten material on the reservoir but at the same time we need to maintain at the constant head that label of the point a is made as a constant and then liquid metal flow here and of course you see it is obvious that at point a also exist atmospheric pressure, at point c also atmospheric pressure.

Now, between point a and c if we apply the Bernoulli's equation, then we can found out that since both the cases both side the pressure same so absolute because both are atmospheric pressure cancel each other okay and of course with respect to the reference point c, the static head we can see gh that term and of course this side also the static head equal to 0 because the c point we take as a reference point.

And of course we make the balance gh, so it should be p/rho, it should be p/rho. So, now we reach if we apply the Bernoulli's equation between point a and c we can reach this equation gh1+vc square/2, vc is actually the velocity at point c but velocity at point a=0, so velocity at point a=0, so kinetic energy=0 this side. Now, if you look into that follow this equation here gh1=it should be ha if we replace the height between a and c.

The height between a and c is the ha we define here. So, from here we can estimate what is the velocity vc=root over twice gh, g is the acceleration due to gravity and vc is the velocity of liquid metal at the gate. So, we can get this equation then we will be able to know what is the velocity of molten metal at the point c. Now, what is the volume flow rate that actually passes through this sprue at point c.

So, that what is the cross-section, Ag is the cross-section area and v is the volume, so per unit time what is the amount of the volume passes through at point, so that is the volume flow rate we can estimate from here. Now, we can easily estimate what is the time required to fill the mould cavity. So, if we know what is the volume of the mould cavity, if we know volume of the mould cavity and this is the volume flow rate basically per unit time what is the volume is filling with the mould cavity and that ratio we can estimate.

This is the time required to fill the mould, so this is the typical time required, we can roughly estimate in case of open vertical, basically it is called vertical gating system because directly the molten metal passes vertically to the mould cavity and of course it may have some kind of other practical problem because if you directly put the molten material to the mould cavity there may be the erosion of the mould may also happen.

So, sometimes we can look into the horizontal gating system such that horizontal gating system smoothly transfer of the molten metal to the mould cavity is possible.

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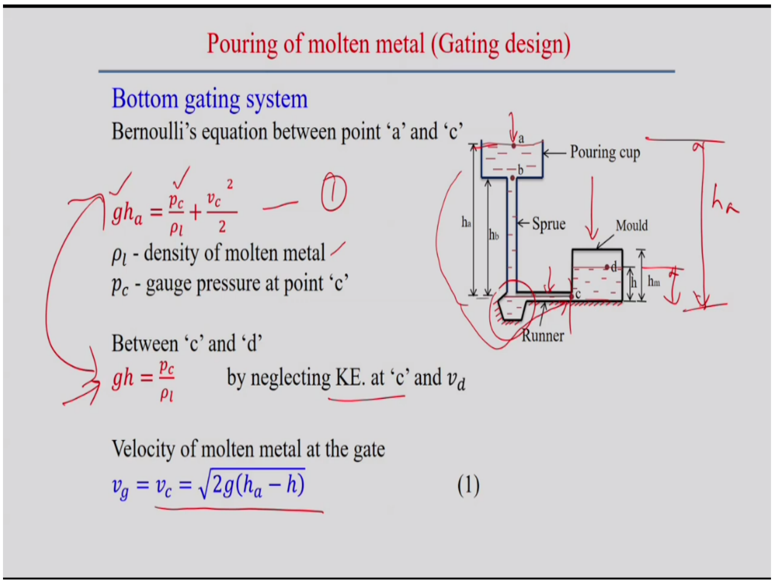

Pouring of molten metal (Gating design)

Bottom gating system

Bernoulli's equation between point 'a' and 'c'

$$gh_a = \frac{p_c}{\rho_l} + \frac{v_c^2}{2} \quad \text{— ①}$$

ρ_l - density of molten metal

p_c - gauge pressure at point 'c'

Between 'c' and 'd'

$gh = \frac{p_c}{\rho_l}$ by neglecting KE. at 'c' and v_d

Velocity of molten metal at the gate

$$v_g = v_c = \sqrt{2g(h_a - h)} \qquad (1)$$

So, that is also called the bottom gating system because instead of directly putting the molten metal to the mould cavity you can pass the molten metal to the this thing with the proper designing and then we pass the molten material through the bottom gating system and of course this part is some practical purpose, we can put some kind of reservoir kind of this small reservoir kind of this things, molten material pour this thing there such that entrapment of the lighter impurities can be taken care of by this kind of design.

So, now in similar way we can apply the Bernoulli's equation between point a and c we can estimate what is the velocity specific to the bottom gating system. So, if we put the Bernoulli's equation between point a and c, you can see that gh and here is the pressure absolute atmospheric sorry gauge pressure at point c and vc is the velocity at point c but in this case although here is the atmospheric pressure but at the point c, it is not necessary it is within the mould cavity.

So, the pressure can be different from the atmospheric pressure. So, rho l is the density of the molten material and this is the gauge pressure at point c, so between point this is the one equation we can get and then between point c and d also we can put the Bernoulli's equation in this cases by neglecting the kinetic energy at c. So, in this case we can neglect the kinetic energy at c and of course the velocity kinetic energy due to the velocity at point d.

So, in that case between point c and d, we can see this equation, we got this equation from the Bernoulli’s energy balance equation but point is that why we should neglect the kinetic energy at point c because in this case there may be the molten material flow suddenly exposed via large container basically mould cavity is very big we assume that thing. So, in that cases when we expect the large container, in that sense we can neglect the kinetic energy at point c.

And of course, this velocity at the point d, it may be the very low as compared to the velocity as c or maybe we can say the velocity through the runner such that in that cases the kinetic energy may be negligible due to the velocity of the molten material at point d. Now, combining these two equations, we can found out that what is the velocity at vc but this velocity we count not the other volume.

That means not the other domain may be this velocity we can assume that velocity is specific to the velocity of the runner. So, in that case, we can estimate this velocity is basically the difference ha-h, so h is the liquid column with respect to this base and the difference liquid column here also, this also the h and this is the liquid column of ha. So, that velocity is basically due to the column height at the point c we can roughly estimate suppose this is the equation number 1.

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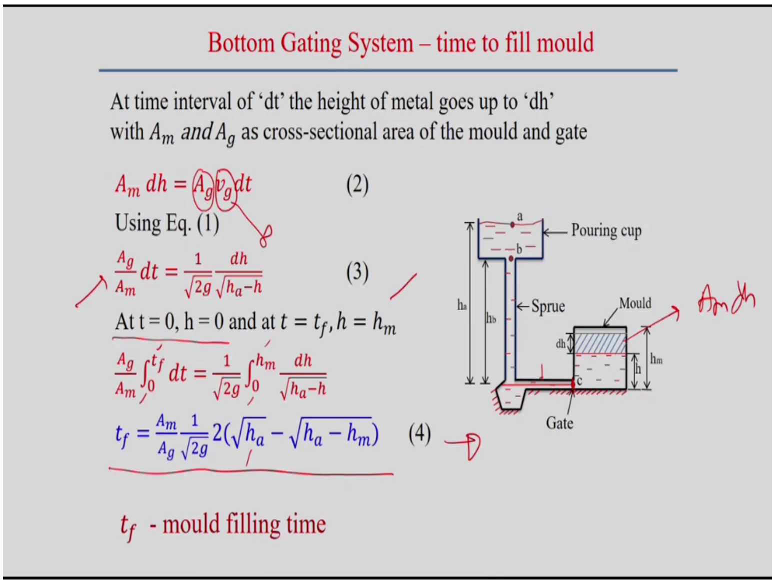

Bottom Gating System – time to fill mould

At time interval of 'dt' the height of metal goes up to 'dh' with A_m and A_g as cross-sectional area of the mould and gate

$$A_m\,dh = A_g v_g dt \quad (2)$$

Using Eq. (1)

$$\frac{A_g}{A_m} dt = \frac{1}{\sqrt{2g}} \frac{dh}{\sqrt{h_a - h}} \quad (3)$$

At t = 0, h = 0 and at $t = t_f, h = h_m$

$$\frac{A_g}{A_m} \int_0^{t_f} dt = \frac{1}{\sqrt{2g}} \int_0^{h_m} \frac{dh}{\sqrt{h_a - h}}$$

$$t_f = \frac{A_m}{A_g} \frac{1}{\sqrt{2g}} 2\left(\sqrt{h_a} - \sqrt{h_a - h_m}\right) \quad (4)$$

t_f - mould filling time

At that time, now we look into that elemental time dt and that elemental time dt, there is a increment of the height on the mould cavity that is dh. Now, we know that Am, Ag we defined the cross-sectional area of the mould as well as the gate, so maybe this was the runner, so at this point what is the cross-sectional area of the gate is I think we assumed as Ag. Now, we need to maintain the continuity equation here.

We can say that Am that means Am is the cross-section area of this Am, so Am*dh this is the volume of the material, this is the volume of the material for elemental time dt, the volume filled by the elemental time dt that is equal to the material supplied to the mould cavity that is cross-section area of the gate, velocity of the gate, liquid molten velocity at the gate and that elemental time dt.

And then if you make it equal these things and of course by using the velocity vg, the velocity at the gate or velocity at the point c, we already estimated the velocity at the point, so we can say this is the velocity of the c, vg put it here velocity, then we reach this kind of equation 3. Now, we put if we do the integration then we knew the limits of the integration what we look into the boundary or condition time conditions here.

At time t=0 assume the h=0, so at time t=0 mould cavity was not filled at all and then once at time t=tf certain time which the height of the liquid molten material at the mould cavity is hm. Of course, in these cases, we assume the cross-section is uniform of the mould cavity. Now, if we do the integration over this limit 0 to tf, then accordingly the height changes from 0 to hm.

If you integrate over these two then we getting the mould filling time tf equal to this in that form. So, here Am, Ag should be defined and of course ha is defined and then if you want to achieve if hm also defined, then we can find out what is the filling time in case of the bottom gating system. So, tf is basically mould filling time. So, here we can see that in case of bottom gating system this is the expression of the filling time.

And which is different from that expression of the filling time in case of the vertical gating system, so that is these two expressions are completely different in these cases.

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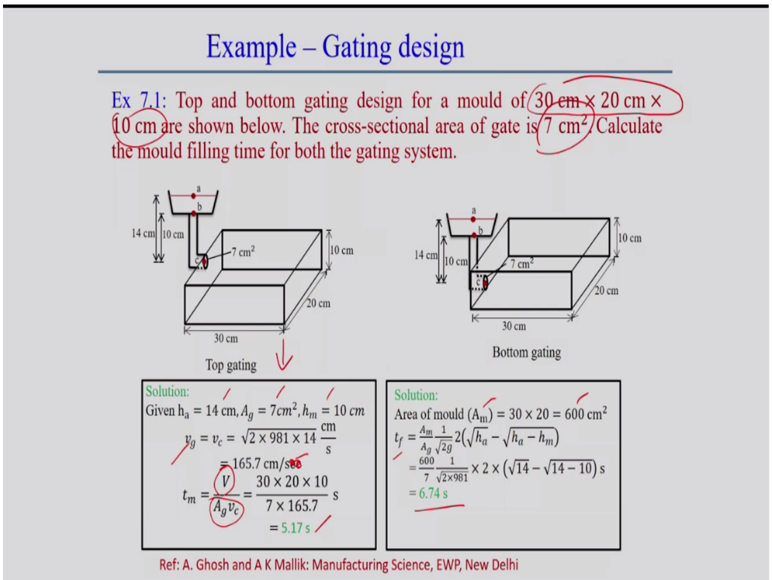

Example – Gating design

Ex 7.1: Top and bottom gating design for a mould of 30 cm × 20 cm × 10 cm are shown below. The cross-sectional area of gate is 7 cm². Calculate the mould filling time for both the gating system.

Solution:

Given $h_a = 14\ \text{cm}, A_g = 7 cm^2, h_m = 10\ cm$

$$v_g = v_c = \sqrt{2 \times 981 \times 14}\ \frac{\text{cm}}{\text{s}}$$

$$= 165.7\ \text{cm/sec}$$

$$t_m = \frac{V}{A_g v_c} = \frac{30 \times 20 \times 10}{7 \times 165.7}\ \text{s}$$

$$= 5.17\ \text{s}$$

Solution:

Area of mould (A_m) = 30 × 20 = 600 cm²

$$t_f = \frac{A_m}{A_g}\frac{1}{\sqrt{2g}}2\left(\sqrt{h_a} - \sqrt{h_a - h_m}\right)$$

$$= \frac{600}{7}\frac{1}{\sqrt{2 \times 981}} \times 2 \times \left(\sqrt{14} - \sqrt{14 - 10}\right)\ \text{s}$$

$$= 6.74\ \text{s}$$

Ref: A. Ghosh and A K Mallik: Manufacturing Science, EWP, New Delhi

Now, we can look into the example, we can differentiate between these two. For example, here one example you can see that both top and bottom gating system with the mould cavity volume is the same in the both the cases but with one case is one is a top gating system is attached, another case is bottom gating system is attached and of course the dimension of the mould cavity is 30 centimeter by 20 centimeter by 10 centimeter.

And it is shown in the figure also and cross-sectional area of the gate is given 7 centimeter square. Now, we can estimate the mould filling time for the both the gating system. In the first case, we can see all that are given and we can estimate in the first case may be top gating system ha height, molten metal height is given 14 centimeter and then cross-section is given and hm the height of the mould cavity that is also given hm.

So, we can directly put, first we estimate what is the velocity vg root over of twice g*h that all data are given, then we can find out this is the value. So, velocity is 165.7 centimeter per second, from here we can estimate the volume of the mould cavity divided the volume flow rate. That is Ag*vc, so from that we can estimate this is the filling time. Similarly, if we look into the bottom gating system also the same dimension, I think same height, the liquid metal height but position are the gating position are different in this cases.

In bottom gating system, we can estimate all data are given here and we put the tf the mould filling time, we can found out that it is different. So, in case of bottom gating system, the mould filling time is more 6.74 seconds and in other cases the top gating system the mould filling time is less 5.17 seconds, so it is in this case although all dimension I think are same in the both the cases.

But we can see the bottom gating system since it require more time to fill the mould cavity. So, therefore it is important that if we compute in that way the top gating system definitely it takes the less time, so it is more prepared because otherwise if before filling the mould cavity completely then if material solidify then in that cases the design may not be proper, so in that case the top gating system is more preferable than in case of bottom gating system but of course top gating system there is other practical difficulties also there.

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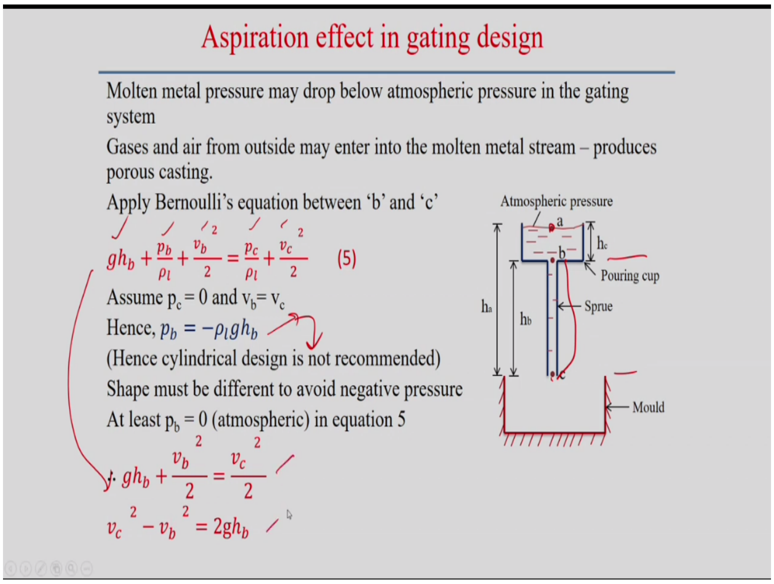

Aspiration effect in gating design

Molten metal pressure may drop below atmospheric pressure in the gating system

Gases and air from outside may enter into the molten metal stream – produces porous casting.

Apply Bernoulli's equation between 'b' and 'c'

$$gh_b + \frac{p_b}{\rho_l} + \frac{v_b^2}{2} = \frac{p_c}{\rho_l} + \frac{v_c^2}{2} \qquad (5)$$

Assume $p_c = 0$ and $v_b = v_c$

Hence, $p_b = -\rho_l g h_b$

(Hence cylindrical design is not recommended)

Shape must be different to avoid negative pressure

At least $p_b = 0$ (atmospheric) in equation 5

$$\therefore gh_b + \frac{v_b^2}{2} = \frac{v_c^2}{2}$$

$$v_c^2 - v_b^2 = 2gh_b$$

Now, if we look into that aspiration effect in gating design may be we estimated, assume the certain point the atmospheric pressure exist but if we look into very carefully the top gating system, the sprue within the sprue it is not necessary that atmospheric pressure exists. We can show simply by mathematical calculation also that there may be the variation of the pressure within this sprue and accordingly we need to design.

It is not necessary sprue can be the uniform cross-section throughout this. So, design must be change but mathematically how you can see that part. So, molten material pressure may drop below the atmospheric pressure in certain part of the gating system and of course if it is below the atmospheric pressure then there the possibility of the gases and air can be entered inside the molten metal if there is a difference exists as compared to the atmospheric pressure and definitely that produces the casting defects.

Now, will try to investigate that how it can be done. So, we have done the Bernoulli's equation between point a and c we have already applied and we can found out the relation and filling time all these things but if you look into that Bernoulli's equation between point b and c, what happens if we look into that, this is the head liquid head, pressure gauge pressure and velocity at point b.

Similarly, pressure at point c, velocity at point c and static pressure will be 0 because we have taken c point as a reference point and now if we assume that pc=0 and of course if we assume the vb=vc. That means the velocity of the liquid metal at point b and velocity of the liquid metal at point c is the same. Then, we can reach the pb calculated that is basically the minus a negative pressure.

So, if there is a negative pressure, it is less than the atmospheric pressure. Then, what will happen? Then, that already discussed the interrupt of the gases or air from outside will come will mix with the molten material. So, therefore uniform cross-section cylindrical design is not recommended based on this calculation because that indicates the negative pressure exists within this.

So, their shape must be different such that we can avoid the negative pressure within the sprue but what we can do these things. So, at least pb=0 atmospheric pressure, that means at least we assume that pc=0. So, at least we can see that pb is the negative pressure but at least we see that pb at least would be the atmospheric pressure in this case. So, therefore in 5 if we put simply pb=0 in the sense that gauge pressure=0 in this case.

The gauge pressure will be measured with reference to the atmospheric pressure. So, in that case we reach, modify this equation and we can get that vc=vb square-twice is equal to VB square minus twice ghb this kind of expression by putting pb=0, that means at least we are equivalent to the atmospheric pressure.

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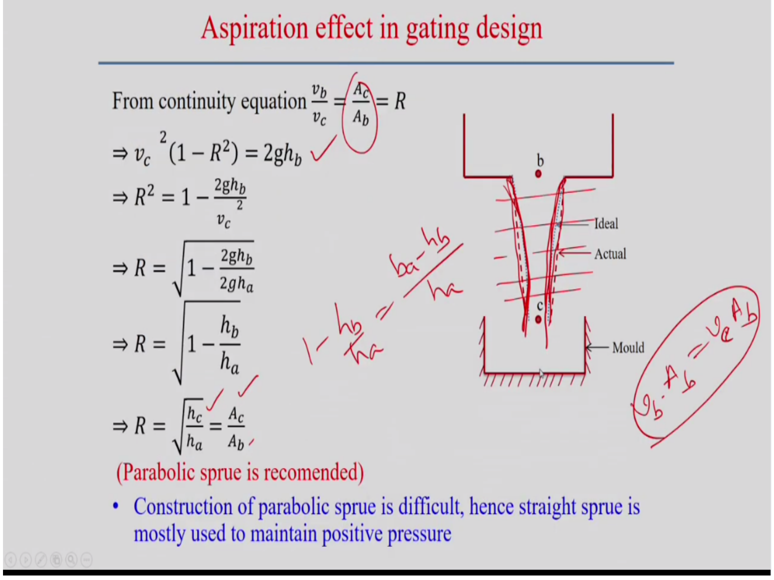

Aspiration effect in gating design

From continuity equation $\frac{v_b}{v_c} = \frac{A_c}{A_b} = R$

$$\Rightarrow v_c^{\,2}(1 - R^2) = 2gh_b \checkmark$$

$$\Rightarrow R^2 = 1 - \frac{2gh_b}{v_c^{\,2}}$$

$$\Rightarrow R = \sqrt{1 - \frac{2gh_b}{2gh_a}}$$

$$\Rightarrow R = \sqrt{1 - \frac{h_b}{h_a}}$$

$$\Rightarrow R = \sqrt{\frac{h_c}{h_a}} = \frac{A_c}{A_b}$$

$1 - \frac{h_b}{h_a} = \frac{h_a - h_b}{h_a}$

$v_b \cdot A_b = v_a A_b$

(Parabolic sprue is recomended)

- Construction of parabolic sprue is difficult, hence straight sprue is mostly used to maintain positive pressure

Now, from continuity equation that continuity equation that at this point between these two points b and c that velocity at vb*the area Ab should be same as vc*Ab. So, this continuity equation should be maintained. That means the volume flow rate at any section it should be the same. So, we can maintain, by default it is necessary to maintain the continuity equation. If we follow the continuity equation is maintained between the point b and c.

Then, this can be expressed vb/vc=the ratio of this cross-section Ac/Ab, suppose this is equal to R. Now, we can look into that vc square that means previous equation if we look into this equation, if we put the vb other term okay vb in terms of R, vb square=R square*vc square then from this equation we can reach the next equation and here we can find out that R square=1-twice ghb/vc.

So, R equal to this so that ratio vb/vc something like that 1-hb/ha and finally we can see that 1-hb/ha=ha-hb/ha so that hb-ha we can see hb-ha sorry ha-hb we can see ha-hb=the height is the pouring basin that we maintain constant that Ac. So, here I put that Ac that Ac/ha. So, Ac/ha itself is a root over hc/ha which is also equal to R. R is basically Ac/Ab, so Ac/Ab this is the ratio.

So, therefore this equation itself indicates some kind of from this equation we can see that it is a kind of parabolic, from this equation we say that parabolic sprue is recommended in this cases. So, this is the ideal shape parabolic from that calculation, parabolic shape that means this kind of shape continuously changing the cross-section is recommended to avoid the negative pressure within this thing.

But of course the construction of the parabolic shape of the sprue is really difficult manufacturing. So, therefore if we follow the kind of state tapered kind of sprue design then of course it maintains positive pressure also. So, then this is the ideal situation parabolic but actual is that kind of tapered sprue normally we design, we form in the gating system to avoid the negative pressure within the system.

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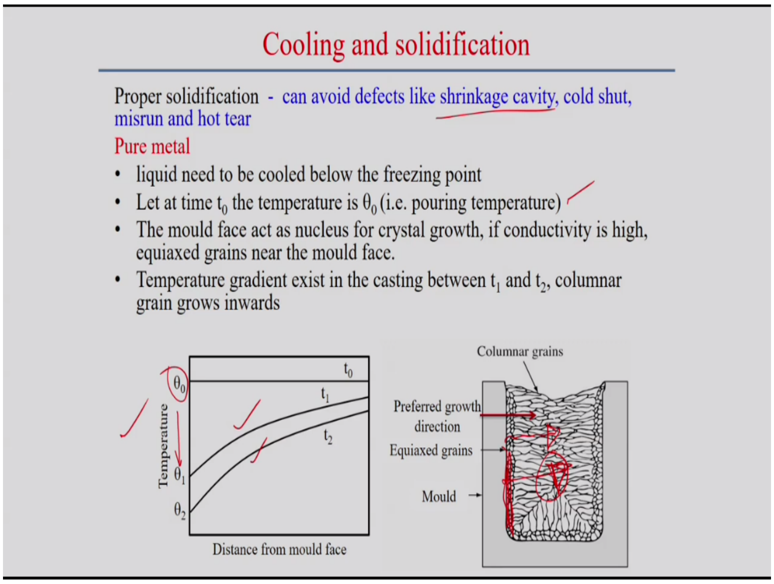

Cooling and solidification

Proper solidification - can avoid defects like shrinkage cavity, cold shut, misrun and hot tear

Pure metal

- liquid need to be cooled below the freezing point
- Let at time t_0 the temperature is θ_0 (i.e. pouring temperature)
- The mould face act as nucleus for crystal growth, if conductivity is high, equiaxed grains near the mould face.
- Temperature gradient exist in the casting between t_1 and t_2, columnar grain grows inwards

Now, we come to that point, the cooling and solidification relevant to the casting process. So, proper solidification is required or we need to understand how solidification happens basically what way the solidification is basically what way the heat is extracted during the casting process, accordingly you can avoid different kind of the defects. For example, proper solidification not done then shrinkage cavity, cold shut, misrun, hot tear.

That kind of defects can also form in these cases, so therefore we need to understand the proper solidification in this casting process but if we look into the pure metal, in this case for pure metal that liquid need to be, to do the solidification liquid need to be cool down below the freezing point temperature. For example, if you look into this graph, the distance from the mould face, so distance from the mould face we can see this is the mould face, distance from the mould face.

So, actually solidification starts at the mould face, so that part solidify first. So, that means extraction of the heat and that heat is carried away through the mould surface through the mould and once this heat extracted then gradually the solidification front actually moves towards the center but when the solidification starts at this point at the interface till at the middle point, the molten material remains in the liquid state.

So, that means to completely solidify detection time, so if we look into this that theta 0 is the melting point temperature freezing temperature we can say and at time t0. Initially, all are in liquid phase temperature but after certain time distance from this in the temperature actually reduces at the mould interface but of course towards the center, the temperature remains towards the melting point towards the melting point temperature.

So, that is why this indicates some nonlinear curve over the time and that with respect to the distance from the face. Of course, this typical temperature distribution exist with some kind of temperature gradient also exist, temperature gradient means the difference of the temperature with respect to distance. So, that actually decided by the slope of this curve.

So, for example at the time t0, the liquid all metal and temperature at least theta 0 temperature that means at least in the melting point temperature, then mould face acts as a nucleus for the crystal growth so solidification starts. If conductivity is very high, so if mould face conductivity is very high, then it normally forms the equiaxed kind of grains at the mould face and then temperature gradient exists in the casting between the t1 and between the time t1 and t2.

Therefore, columnar grains goes towards, basically the columnar kind of the grains forms towards the center if this direction which direction this temperature gradient with the steeped temperature gradient actually exists in that particular direction. So, in that direction, the grain growth occurs columnar grain growth occurs.

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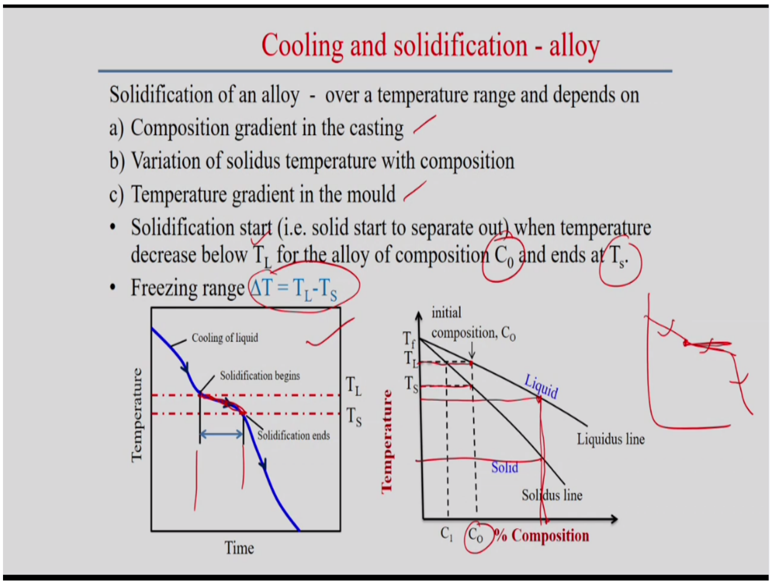

Cooling and solidification - alloy

Solidification of an alloy - over a temperature range and depends on

a) Composition gradient in the casting

b) Variation of solidus temperature with composition

c) Temperature gradient in the mould

- Solidification start (i.e. solid start to separate out) when temperature decrease below T_L for the alloy of composition C_0 and ends at T_s.
- Freezing range $\Delta T = T_L - T_S$

But we analyze this solidification process in case of for example alloy system because pure metal we have the melting point temperature, single melting point temperature but in case of alloy, solidification actually happens over a range of the temperature, normally in between the solidus and liquidus temperature. So, therefore this range of temperature actually depends on the composition gradient in the casting process.

In case of alloy, this becomes complicated, so it depends on the composition gradient within the casting material and of course then variation of the solidus temperature with composition. So, of course the solidus and liquidus temperature that temperature also changes with the variation of the alloy composition. For example, if we look temperature versus composition, here you can see at C0 solid concentration in a particular alloy system.

So, this is the liquidus temperature and this is the solidus temperature. So, solidification in general happens between these two temperatures. For example, in other cases, if the concentration is different, solid concentration is more in the alloy composition, so in these cases, this temperature is corresponding the liquidus temperature and this temperature corresponds to the solidus temperature.

So, that solidus and liquidus temperature information we can easily get from the temperature versus composition on the binary phase diagram. There you can easily found out what is the range in that. So, that means if there is a variation of the solidification temperature range in case of alloy and that intuitively as a function of the composition or the solid concentration of an alloy system.

And of course finally depends on the temperature gradient in the mould that is mainly the driving factor or the solidification mould. So, solidification starts definitely for a particular concentration, solidification starts normally below the TL liquidus temperature and ends at solidus temperature but this solidus temperature and liquidus temperature, it is as a function of the alloy composition C0 for example.

So, therefore freezing range can be represented like that delta T=TL-TS. Here we can see that how solidification in case of alloy but that this graph has been shown with respect to temperature versus time. So, cooling of the liquid start from the superheated condition, then once it reach to the liquidus temperature, then solidification begins and then it starts then it makes a slope that in the time temperature and solidification ends at the solidus temperature.

So, there is a slope exists and in case of pure metal that slope actually in case of alloy system, there is a slope but in case of pure metal actually the single point melting temperature this line remains as a horizontal line but in case of alloy system, it is between solidus and liquidus so it creates kind of the slope is that this thing the gradient is there must exist some kind of the gradient here with respect to temperature versus time diagram.

Solidification ends here, so we can say this is the solidification time in the sense change of the phase from liquid phase to solid phase.

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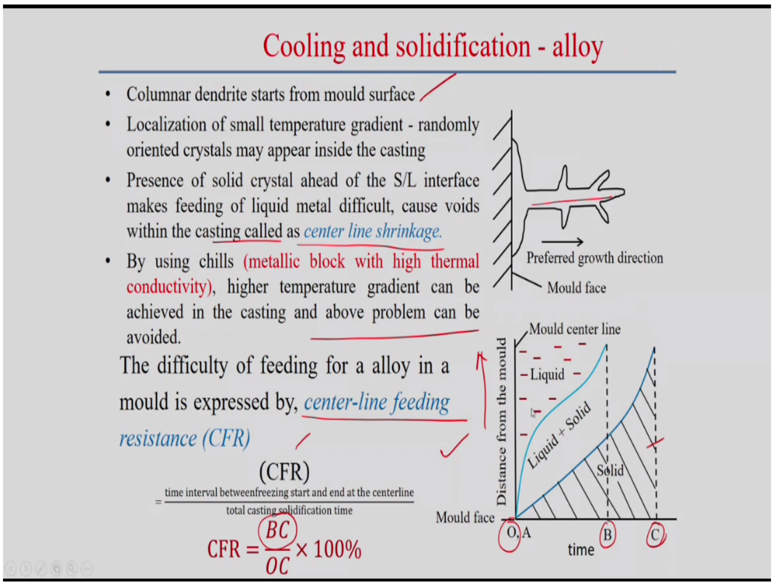

Cooling and solidification - alloy

- Columnar dendrite starts from mould surface
- Localization of small temperature gradient - randomly oriented crystals may appear inside the casting
- Presence of solid crystal ahead of the S/L interface makes feeding of liquid metal difficult, cause voids within the casting called as *center line shrinkage.*
- By using chills (metallic block with high thermal conductivity), higher temperature gradient can be achieved in the casting and above problem can be avoided.

The difficulty of feeding for a alloy in a mould is expressed by, *center-line feeding resistance (CFR)*

$$(CFR) = \frac{\text{time interval between freezing start and end at the centerline}}{\text{total casting solidification time}}$$

$$CFR = \frac{BC}{OC} \times 100\%$$

So, in general we can see that columnar dendritic actually starts from the mould interface and the dendritic form with the secondary and primary dendritic with that dendrite basically growth happens with the preferred growth direction and all depends on the temperature gradient in a particular direction. Of course, some sort of under cooling also responsible for the mode of the solidification but that is not our objective here.

But columnar dendritic starts normally from the mould interface and of course there is some localization of the small temperature gradient may exist because we know that it depends what way the heat transfer actually happens during the solidification process. So, once there is change in the localization of the small temperature gradient randomly create some kind of the crystallization within the casting itself.

So, based on that we can say that it is a columnar dendritic structure or kind of equiaxed structure forms in a casting process but since localization of the temperature gradient may create some kind of the crystallization, so it is also necessary to know that how the presence of the solid crystals is there in, about the presence of the solid crystals in front of the solid liquid interface that makes interface.

And because if that solid liquid that already crystallization starts in some intermediate point, so that may block some interrupt of the liquid that is the difficulty of that liquid to start the solidification. It also depends on what way the temperature gradient actually forms. So, to measure that this voids liquid metal feeding may be difficult in certain point because in that cases to measure these things there is a one terminology can use that casting called the center line shrinkage process.

So, center line shrinkage process is also necessary to define here because during the casting process sometimes we use some kind of the chills, chills is basically some metallic blocks having high temperature, thermal conductivity such that it modify the rate of the heat transfer during the solidification process. So, higher temperature gradient can be achieved by putting the chills in the interface or mould interface, it is possible to do these things.

So, in casting, the above problem can be avoided that means the blocking of the liquid metal by the already solidified metal that that kind of situation can be avoided by using in specifically which is more significant in case of alloy system because here composition also having some role, we have seen that composition also changes the solidus and liquidus temperature.

And of course, it is possible to solute concentration can be different part of a casting system, so that solute content depends in the solute contents changes the solidus and liquidus temperature, so it actually creates this kind of situation problem of the center-line shrinkage. So, that is mostly expressed at the center-line feeding resistance that simply measured in that way or estimated in that way.

Suppose, this is the mould face and distance from the mould face is this direction and this is the liquid face, this is a mixture of the solid and liquid that means between the solidus and liquidus temperature the adjourn and this is the solid part. In this case, the point B is defined, point C is defined and origin O is defined from here. So, center-line feeding resistance can be estimated like that.

Time interval between the freezing start and the end of the center line/the total casting solidification time. So it is as simple as that, it can represent the timing between BC/OC*100% that is into 100 that convert in terms of the percentage. So, this way we can estimate the center-line feeding resistance in casting and solidification process.

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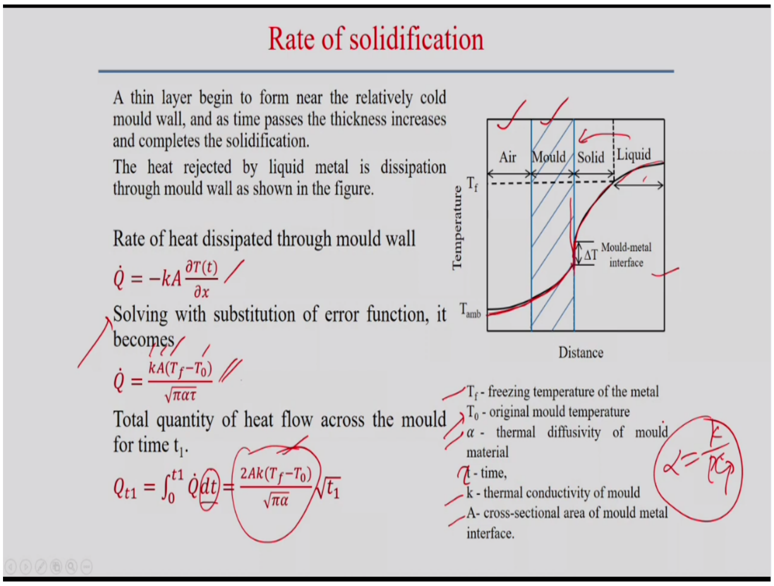

Rate of solidification

A thin layer begin to form near the relatively cold mould wall, and as time passes the thickness increases and completes the solidification.

The heat rejected by liquid metal is dissipation through mould wall as shown in the figure.

Rate of heat dissipated through mould wall

$$\dot{Q} = -kA\frac{\partial T(t)}{\partial x}$$

Solving with substitution of error function, it becomes

$$\dot{Q} = \frac{kA(T_f - T_0)}{\sqrt{\pi\alpha\tau}}$$

Total quantity of heat flow across the mould for time t_1.

$$Q_{t1} = \int_0^{t1}\dot{Q}dt = \frac{2Ak(T_f - T_0)}{\sqrt{\pi\alpha}}\sqrt{t_1}$$

T_f - freezing temperature of the metal

T_0 - original mould temperature

α - thermal diffusivity of mould material

τ - time,

k - thermal conductivity of mould

A- cross-sectional area of mould metal interface.

$\alpha = \frac{k}{\rho c_p}$

Now, we try to estimate the rate of the solidification during the process, we look into that in a particular process. Here we can see the air is there, then upon air is in contact with the mould material and then liquid metal certain part of the liquid metal solidify and this is the typical (()) (47:00) and temperature distribution is like that the liquid metal is above the melting point temperature within the solidified less than the melting point temperature.

Mould interface some temperature difference exists and then there is some variation nonlinear variation of the temperature within the mould and finally it reach to the ambient temperature that delta T temperature actually at the when it creates this is the solid phase, so it creates at this point that is basically there may be change of the phase from liquid phase to the solid phase.

Now, rate of solidification what we can estimate these things. So, a thin layer starts to form and at the interface near the interface of the mould interface relatively cold, so when the liquid metal is in contact with the relatively cold mould interface. So, heat transfer will start from the liquid metal to the mould and then gradually there is a forming of the once that creates the situation less than the melting point temperature that part is solidified.

And that particular rate, the solidification front actually moves towards the liquid molten pool and once all metals change the state from liquid state to the solid phase then solidification completes in this case. So, therefore start with a relatively cold mould wall in contact with the liquid metal and then time passes the thickness in contact with the mould the thickness of the solid part actually increases with respect to time.

But of course the heat rejected by the molten metal should be passes through the mould phase and such that the solidification front creates or moves as a particular velocity. So, now we can estimate from other way also that rate of heat dissipation through the mould wall. So, we assume that we know that mould wall the rate of the heat dissipation can be estimated Q dot=-kA the temperature gradient dT/dx we assume that at a particular time temperature=T.

So, then this is the rate of the heat dissipated through the mould wall, we can estimate these things. Now, the proper boundary conditions and of course we assuming the infinitely long mould wall, this rate of the heat transfer can be estimated in terms of the parameter, that means the initial what is the T0 is the basically original mould temperature and Tf is the freezing temperature of a particular material.

So, we get the freezing temperature of the metal, so in terms of the freezing temperature and temperature of the mould, k is the conductivity and that conductivity should be of the mould, conducted of the mould material, thermal conductor of the mould material and then you can estimate that pi alpha and this tau is basically tau is that actually time here and the A is the cross-section area that we assume the cross-section area.

That cross-section area is in contact with the liquid molten material. So, we can estimate the rate of the heat and of course here and this actually there are steps also, we are not discussing the steps because those steps involved to estimate the error function all these things and finally look into these aspects now. Now, suppose this is the rate of the heat transfer that passes through the mould wall, then we can estimate what is the total amount of the heat actually passes through that mould at a particular time t1.

So, that Q at a particular time t1 what is the amount of the heat actually flow through this mould, that we need to integrate over the elemental time dt. So, Q dot is that basically it indicates the rate of the heat transfer. So, when we multiply with the elemental time and if we integrate the time with respect to particular time t1, we can estimate the total amount of the heat flow up to the time t1 we can estimate this value.

And all these values is well-defined, so once we estimate this value and that is from this properties, thermal alpha is basically thermal diffusivity of the mould material, so alpha is basically k/rho Cp, so all but this we measure in terms of the properties of the mould material.

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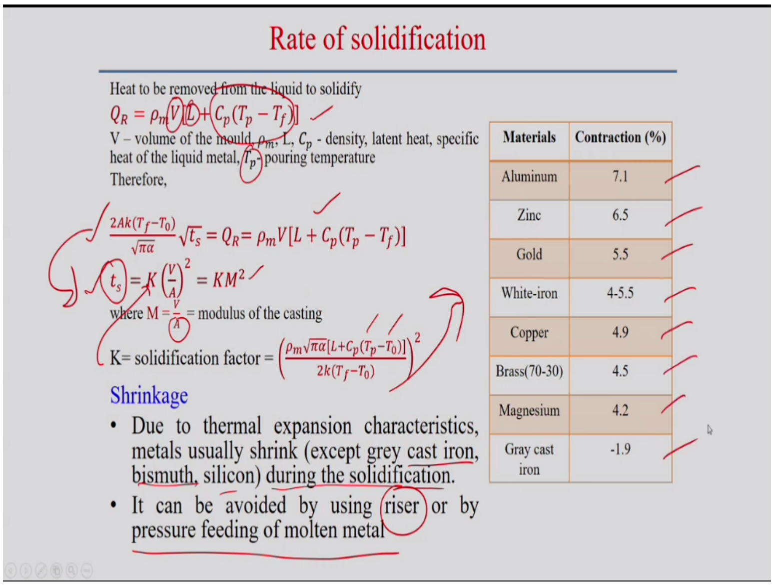

Rate of solidification

Heat to be removed from the liquid to solidify

$Q_R = \rho_m V [L + C_p(T_p - T_f)]$

V – volume of the mould, ρ_m, L, C_p - density, latent heat, specific heat of the liquid metal, T_p- pouring temperature

Therefore,

$$\frac{2Ak(T_f-T_0)}{\sqrt{\pi\alpha}}\sqrt{t_s} = Q_R = \rho_m V[L + C_p(T_p - T_f)]$$

$$t_s = K\left(\frac{V}{A}\right)^2 = KM^2$$

where $M = \frac{V}{A}$ = modulus of the casting

$$K = \text{solidification factor} = \left(\frac{\rho_m\sqrt{\pi\alpha}[L+C_p(T_p-T_0)]}{2k(T_f-T_0)}\right)^2$$

Shrinkage

- Due to thermal expansion characteristics, metals usually shrink (except grey cast iron, bismuth, silicon) during the solidification.
- It can be avoided by using riser or by pressure feeding of molten metal

Materials	Contraction (%)
Aluminum	7.1
Zinc	6.5
Gold	5.5
White-iron	4-5.5
Copper	4.9
Brass(70-30)	4.5
Magnesium	4.2
Gray cast iron	-1.9

Now, what is the heat transfer to the mould material that is should be equal to what is the heat extracted from the liquid molten metal to solidify or to change the liquid metal to the solid metal. So, in that way other we can estimate what is the heat removed from the liquid metal that QR=rho m V is the volume of the mould and rho m Cp specific heat, latent heat all as the properties of the metal here.

And Tp is the pouring temperature, so Tp is normally some super heating temperature above the melting point temperature, Tp is the pouring temperature and Tf is the freezing temperature. So, basically Tp-Tf this is the quantity of the heat this comes due to the super heating of the molten material and then latent heat actually comes to account the change of the phase.

So, that total quantity of the actually heat should be extracted from the molten metal to solidify this. Solidify in the sense just to change the super heating temperature to up to the reaching of the freezing temperature. So, that quantity of the heat can be equal to at a particular suppose there is a heat extraction we already estimated that heat extraction with respect to the time t1 by the mould.

Suppose, solidification time is ts we just simply replace t1 in terms of the ts and make it equal to the both the side what is the heat extracted from the liquid metal and what is the heat passes through the mould make equal cases and we can found out that this and assuming t is the solidification time and that solidification time means that that time is equivalent or can be defined like that change of the phase from liquid phase to solid phase.

But in terms of the temperature from super heating peak temperature to up to the freezing temperature. So, change that kind of that temperature range that is the amount of that temperature range takes the time ts and that is called the solidification time. So, solidification time can be represented like that K*V/A square. So, we can say that K*M square, M is the modulus of the casting.

So, basically V and A is relevant to what is the volume of the casting or volume of the mould cavity or you can say or A is the cross-section of that cross-section which is responsible for the heat transfer. So, that is not cross-section area that is the surface area, which surface area is responsible for that which in contact with the mould material. So, in general we see this the very simple form we can represent the solidification time which is a function of the volume by area ratio.

But K is the solidification factor you can say but K is relevant to all the material properties of the either molten material as well as the mould material and the Tp peak temperature of the system, T0 is the mould temperature already defined and Tf freezing temperature. So, all is well-defined, thermal diffusivity, density, specific heat, latent heat, freezing temperature, peak temperature and molten material temperature so that mould temperature.

All can be defined, we can consider this as a constant term and it depends of a specific casting system, mould material and cast material. So, that we incorporate this is a solidifying factor and we can simplify this equation and this from the solidification time is the ratio and it mainly depends on the ratio of the volume and surface area. Now, point is that in actual process there may be due to the thermal expansion characteristics definitely.

The metals usually shrink except grey cast iron, bismuth and silicon most of the materials actually shrink during the solidification. Then, we need to recover or maybe we need to compensate that amount of the shrinkage volume of the material. So, to compensate the shrinkage of volume of the material normally riser is that is called the riser which riser is designed and attached along with the mould cavity such that this material supply, the shortage material supply can come from the riser.

So, that is the purpose of the riser to feed the molten material which is at least that amount of the shrinkage volume during the solidification process. We can see the different material having different shrinkage volume, contraction, different range we can see also here from this list.

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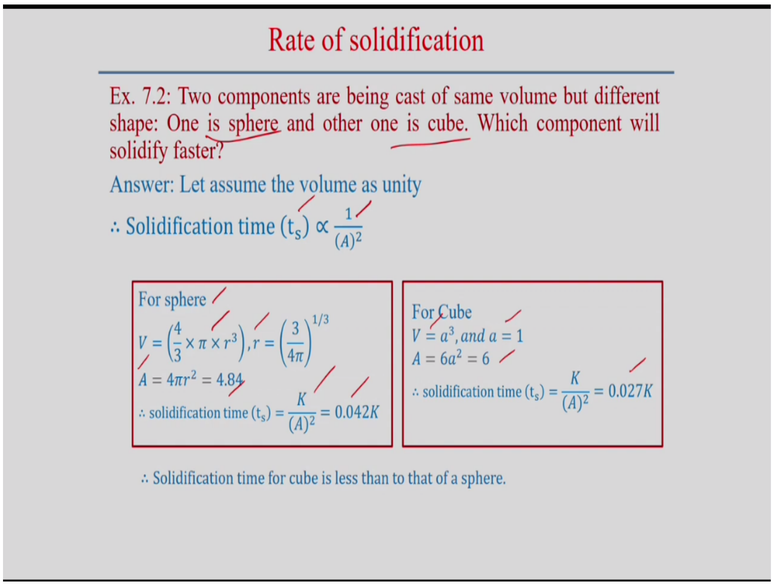

Rate of solidification

Ex. 7.2: Two components are being cast of same volume but different shape: One is sphere and other one is cube. Which component will solidify faster?

Answer: Let assume the volume as unity

$\therefore$ Solidification time $(t_s) \propto \frac{1}{(A)^2}$

For sphere

$V = \left(\frac{4}{3} \times \pi \times r^3\right), r = \left(\frac{3}{4\pi}\right)^{1/3}$

$A = 4\pi r^2 = 4.84$

$\therefore$ solidification time $(t_s) = \frac{K}{(A)^2} = 0.042K$

For Cube

$V = a^3, and\ a = 1$

$A = 6a^2 = 6$

$\therefore$ solidification time $(t_s) = \frac{K}{(A)^2} = 0.027K$

$\therefore$ Solidification time for cube is less than to that of a sphere.

Now, we will try to look into that some simple problem to understand the rate of the solidification. So, for example two components are being cast of the same volume but different shape. Shapes are different, one is the sphere, other is the cube for example. Which component will solidify faster? Let us look into that. So, if we assume, volume are same, we assume the unit volume.

So, therefore solidification time depends on the volume by area ratio. So, you need volume and area, suppose A square so that is proportional to that solidification time. For sphere, we assume V as 1, so from that this is the volume of the sphere and from here we can estimate what is the radius assuming the unit volume. So, once we estimate the radius, we can estimate the surface area.

This is the quantity and solidification time we can find out in terms of the K because K is the constant term that is because system is same, only there is a change of the shape, so K should be the same both the cases, it comes this way. Even if we look into for cube also, this is the volume V=a cube so suppose from here unit volume a should be 1 here, area should be 6 unit solidification time we can find out 0.027K.

So, therefore solidification time for cube is less as compared to the sphere. So, that means it is very obvious that solidification time actually depends on what is the volume of the cavity as well as the surface area which is responsible for the transfer of the heat during this process. So, therefore if there is a change of the shape, so solidification time can be different even if we keep the solidification factor is the same for this cases but solidification time can be different for the different shape.

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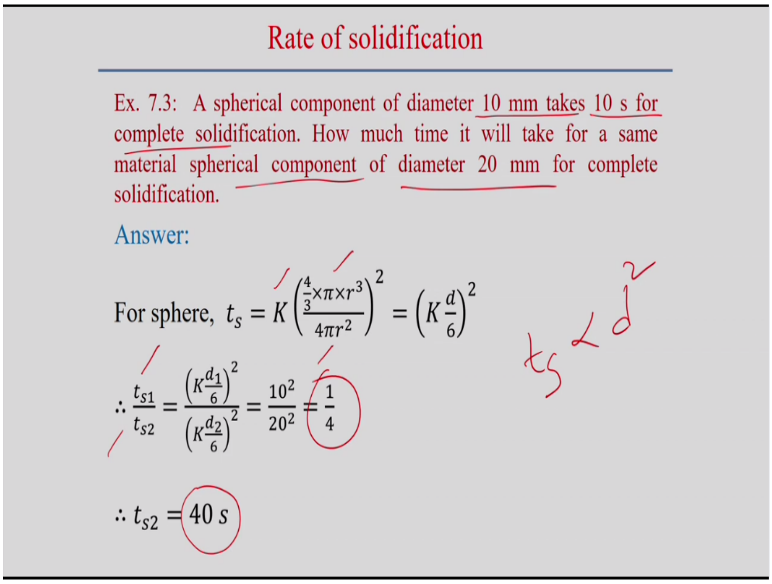

Rate of solidification

Ex. 7.3: A spherical component of diameter 10 mm takes 10 s for complete solidification. How much time it will take for a same material spherical component of diameter 20 mm for complete solidification.

Answer:

For sphere, $t_s = K\left(\frac{\frac{4}{3}\times\pi\times r^3}{4\pi r^2}\right)^2 = \left(K\frac{d}{6}\right)^2$

$t_s \propto d^2$

$\therefore \frac{t_{s1}}{t_{s2}} = \frac{\left(K\frac{d_1}{6}\right)^2}{\left(K\frac{d_2}{6}\right)^2} = \frac{10^2}{20^2} = \boxed{\frac{1}{4}}$

$\therefore t_{s2} = \boxed{40\ s}$

Another example you can look into that, we can look compare although same shape, spherical component diameter 10 second complete solidification but how much time it will take for a same material that means material is the same, same material spherical component of diameter that means sizes are different basically in this cases, one case diameter is 10 millimeter, another case diameter is 20 millimeter but they are having the shape is same, both are spherical in nature.

So, we can estimate the solidification time, we know that is proportional to the volume by area ratio. So, if we estimate the volume by area ratio for a sphere, we can see in terms of and K is the constant here, so it depends on the d/6 square, d is the diameter. So, the solidification time ratio ts1/ts2 it is simply d1 square/d2 square because it is proportional to that so 1/4. So, therefore solidification time for less spherical component having 10 millimeter takes 10 second but the other cases it takes the 40 seconds.

So, in this case, solidification time is actually proportional to the here you can see the proportional to the d square basically, the square of the size, r square or maybe radius square or maybe diameter square.