**Mathematical Modeling of Manufacturing Processes**
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Lecture - 23
Solidification in Welding-1

Today we will discuss about one of the subsection of welding module that solidification and microstructure which is one of the important aspects relevant to the welding process and that actually decides the final microstructure and that can be link with the mechanical properties of a welded structure. We will try to go through the basics of the solidification mode or what are the types of the nucleation normally happens in during solidification specific to the welding process.

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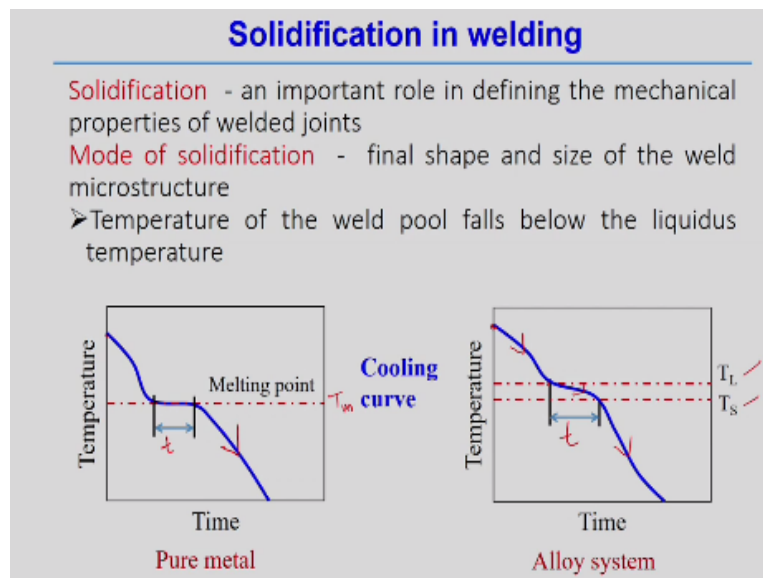
So first we try to understand how the time temperature diagram of a molten material looks like for example this if we look into this figure first figure. So this is the time temperature diagram specific for a pure metal. So you know it started with the super heated temperature and a very high temperature that means it is a liquid metal and then it comes to that at the melting point temperature.

So this is actually melting point temperature. So solidification starts when the temperature reduces below the melting point temperature, but since there is a change of the phase from liquid phase to solid phase so that this time gap will be there and that time gap that represents one straight line that is this indicates there is a phase change by a specified time. So phase

change means it comes to from liquid phase to solid phase and then after that gradually it comes to the ambient temperature.

So this actually the solidification normally relevant to that part when there is a change of the phase from liquid phase to solid phase. So for pure metal this is the typical time temperature diagram, but if we look into that alloy system in this case the change of the phase actually happens within range of the temperature. So that range of the temperature between the solidus temperature and liquidus temperature.

So we see this is the liquidus temperature and this is the solidus temperature so that was as compared to the pure metal it was a one constant temperature so that constant temperature only phase transformation happens, but in case of alloy system there is range of temperature (()) (03:01) phase transformation actually happens. It starts with the some superheated point maybe temperature very high initial and then comes to that exactly the liquidus temperature.

And then it transforms to the gradually temperature decreases to solidus temperature solidification happens between the liquidus and solidus temperature and further it comes back to the ambient temperature. So this is here actually this solidification happens between this time gap and between this 2 TL and TS temperature. So this range of the solidus and liquidus temperature are different for different types of alloy system.

The main difference between pure metal and the alloy system is like that in pure metal or solidification normally happens we say it is at constant temperature, but in case of alloy system it normally happens over a range of the temperature. So these are the all typical time versus temperature diagram in specific to a particular material when there is a change of the phase.

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Solidification in welding

Cooling curve for binary alloy - Completely soluble
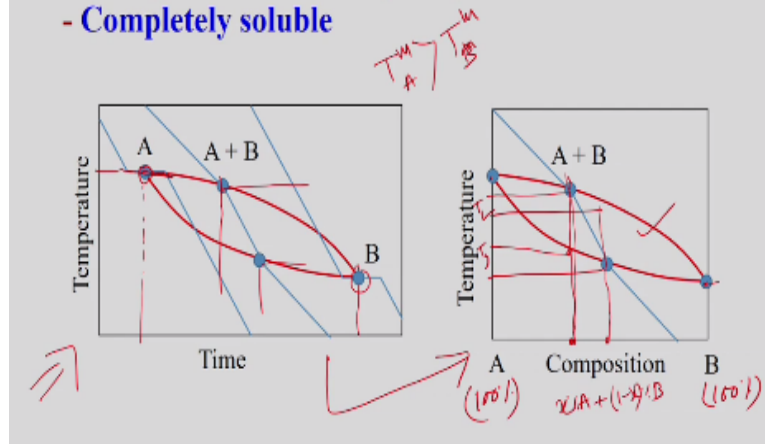
Now we will try to look into that how the just analyze of binary system that means assuming the alloy having the 2 components only say components A and components B and they are mixing this and they are making some kind of the alloy system of course the assumption is that they are completely soluble with respect to each other. So normally if we look into the time temperature diagram for an alloy system it is like that with respect to time.

So there is a change of the phase between solidus and liquidus temperature happens at particular point and of course this point, but at this point it corresponds to that only 100% composition A that means only one of the material single material. So single material having the single point over which the solidification happens and then similarly in case of B also that B is also pure metal.

At this point so there is 100% composition of the B material, but then B material from transition from liquid to solid phase normally happens on single point temperature, but if you say in between it is at this point it is a mixture of A and B. So for mixture of A and B that actually the solidification happens over a range of the temperature. So between the solidus and liquidus temperature.

So now that means that temperature range this actually depends on what is the composition of the alloy system, that span the range between solidus and liquid temperature that span actually decided by what is the composition of A and composition B is mixing. For example, we can assume that at this point there is 50% A and 50% B make an alloy system. So for this alloy system the solidification actually happens over a range of the temperature.

So this is the corresponding temperature solidus and you can say this is the liquidus temperature, but in case of pure metal it happens one single point temperature similarly pure metal B temperature and of course in this case if we say the temperature melting temperature for A definitely melting point temperature for B. So in that case this is point A and this point B and if you draw the diagram maybe typical looks like this kind of profile.

Now this conversation is exactly we can say the time temperature diagram how it is coming in case of an alloy system, but now if you try to convert it, try to represents this solidification behavior or phase transformation from liquid phase to the solid phase during the cooling period. In that case if you look into the temperature versus composition. So temperature versus composition diagram we will be looking like that of course in case it is a binary system having mixing of the A and B.

But in this cases there is we are not considering the time scale rather we are raising the x axis as a composition and y axis as a temperature that scale if we look into that this 2 alloy system. So A corresponds to that composition means it is a 100% A at this point so one single melting point temperature. Similarly, at this point it is a 100% B in a single material then in that case we will be getting the solidification.

Then phase transformation from solid phase to liquid phase and liquid phase to solid phase this is the single point temperature, but if you look in between say this point. So there is the temperature for this composition the temperature here is the liquidus temperature and here it corresponds to the solidus temperature and this corresponds to the liquidus temperature. So it is a particular say x%A+1-x% of B. So that is the mixture of this 2.

So in that case we are getting the solidification happens between the range of the temperature between I think liquidus temperature and here it is a solidus temperature. Similarly, this is corresponding this composition in this case there is a percentage of B actually increasing and percentage of the A is decreasing. So that is in different alloy system. In this cases the corresponding solidus and liquidus temperature will be different.

So that by looking into that composition and this binary phase diagram composition versus temperature we can get the information what is the solidus and liquidus for a particular

material and what is the range of the temperature over which the solidification actually happens. So this diagrams are useful and for a particular alloy system the standard diagram are also available, but we need to utilize this diagram to do the further calculation.

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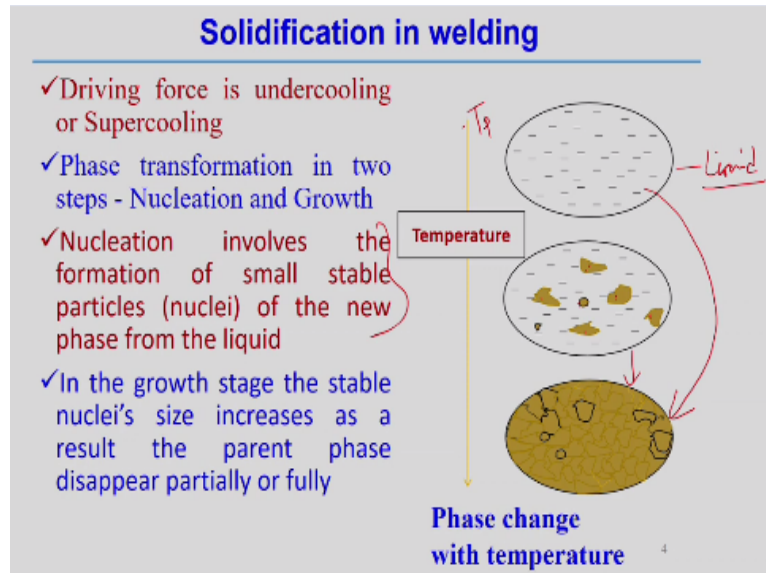
Now how solidification happens in this case if you see that in solidification that we start with the very that it is a initial phase it is a liquid phase. So suppose this is a peak temperature that very high temperature which is above the melting point temperature all metal maybe in the liquid phase and there is a decrement of the temperature. So certain point within the liquid phase there is a nucleation actually happens.

So nucleation starts and subsequently when the nucleation try to grow the reduction of the temperature then it becomes the solidified structure. So that means there is a change of the phase from liquid phase to solid phase in between the phases at the start of the nucleation actually and then subsequently growth from the nucleus and then we can get the one solidified structure.

So here actually there is a change of the phase occurs with respect to a temperature, but this phase change maybe in case of pure system, pure material all this in case of pure metal normally happens at the one single point temperature, but practically it may be associated with some kind of undercooling, but in case of alloy system actually this phase transformation with respect to temperature happens over a range of the temperature between solidus and liquidus temperature.

And of course in that range of the temperature normally coexistence of the liquid phase as well as the mixture of the solid phase and this zone normally called the mushy zone in case of any welded structure when you try to analyze the cooling of the molten pool that starts from the peak temperature to the ambient try to reach the ambient temperature. So then solidification is actually associated with the 2 phenomena one is the nucleation.

And the second is the growth of this nucleus, but nucleation involves the formation of the stable particles of the new phase from the liquid. Actually the nucleation start at particular point it maybe it start homogeneously throughout the domain or there maybe the heterogeneous nucleation that heterogeneous, but in case of heterogeneous nucleation some substrate should be present there to start the nucleation process.

So this one aspect is the solidification which is related to the nucleation and what are the nucleation mechanisms and what are the driving forces for the nucleation process and second part is that growth stage how the growth phenomena normally happens starting from the nucleation of a particular material. So growth phenomena, the stable nucleus size actually increases.

And of course after certain point of time this growth of a particular grain actually becomes competitive with respect to each other and then it creates the different sizes of the grains or maybe uniform sizes of the grains depending upon the driving mechanism, type of the material all matters and temperature gradient present everything matter relevant to what maybe the growth mechanism normally happens after the nucleation. So first we will look into that nucleation.

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Solidification in welding

Progress of phase transformation:
Nucleation and Growth

✓ Homogeneous Nucleation
✓ Heterogeneous Nucleation

Homogeneous Nucleation
Formation of a critically sized
solid from the liquid

Driving force: high undercooling
without an external interface

Undercooling
Evolution of latent
heat after undercooling

So progress of phase transformation nucleation and growth in general, but what are the different types of the nucleation actually happens during the solidification process. First one is the homogenous nucleation and normally we can say the heterogeneous nucleation, but what are the driving force for the homogenous nucleation and what maybe in case of heterogeneous nucleation.

If we look into the homogenous nucleation. So in this case the critical size of the nucleus form in the form of the solids. So that means from the liquid phase this is a directly from the liquid phase to the formation of the solid phase with certain critical size. So that critical size is important here because the nucleation theory tells that some amount of the energy barrier is required to start the nucleation process.

And of course to minimum amount of the energy or maybe we can say the optimum amount of the energy is required to start the nucleation process such that that nucleation will be sustainable in nature. So that critical size we will try to look into the different theory, but see one is the driving forces for the nucleation normally we say the homogeneous nucleation process that is the degree of undercooling in a particular metal system.

So we already discussed that if we look into the solidification we normally assume this is the change of temperature not change of temperature in case of pure metal over there is fixed temperature the solidification of liquid phase to solid phase transformation actually happens, but practically it is associated maybe some amount of the undercooling. Undercooling is completely local phenomena.

And probably during the evolution of the latent heat how it happens latent heat means the latent is actually associated with the phase transformation from liquid phase to solid phase we must be associated with some amount of latent heat. but how this latent heat actually extract it of that system because of the extraction it may possible in some localized area during the solidification that it can reach up to certain amount of the undercooling.

That means liquid phase it is a even below the melting point temperature, but material can be in the liquid phase. So that degree of undercooling is a measurable quantity and maybe it is associated with different types of the material, but this high degree of undercooling actually promotes the homogeneous nucleation process otherwise also if degree of undercooling is very less then that actually promote the heterogeneous nucleation process.

So we say the high undercooling for relevant to the homogeneous nucleation process, but without any external interior that means that actually in case of heterogeneous nucleation process the external interiors interface that actually acts as to initiate the nucleation process, but that is relevant to the heterogeneous nucleation process. We will discuss both of this nucleation process here also.

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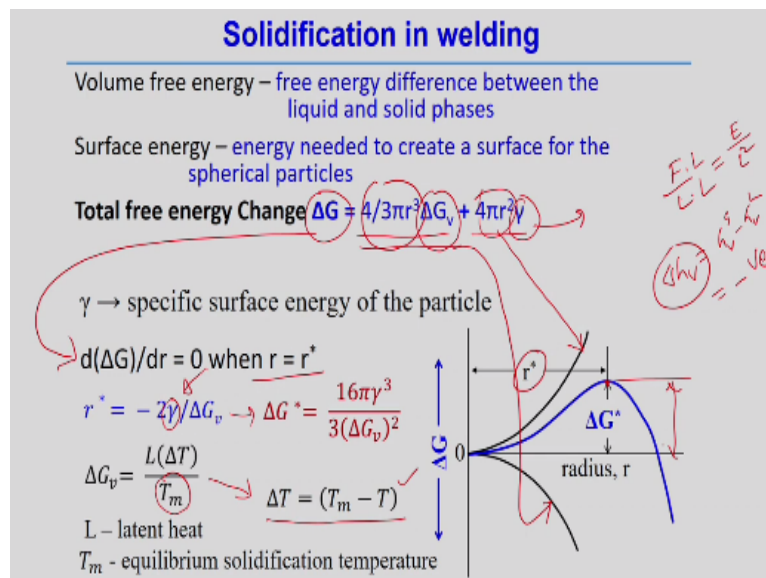
But to start the nucleation process we can look into that the volume free energy actually volume free energy is required to differentiate between the liquid and solid phase. So that normally we measure with respect to per unit volume and there is a surface energy changes which also required that is called the energy needed to create surface for kind of spherical

particles.

So how it is associated with that so when there is a change of the phase solidification happens from liquid phase to solid phase. It is associated to change of the free energy change that is volume free energy with respect to volume distributed with reference to the volume and second one is the surface energy. For example, total volumetric energy change is associated with the 2 component.

One is the surface energy change and another is the volume free energy change since but why it is associated with a volume free energy and surface energy change because if we assume that the same phase change from liquid phase to solid phase and in this cases we are assuming when it becomes the solid phase it is a kind of spherical particles we are assuming that.

So size is decided by the radius of the sphere and the shape is we assume it is in a spherical shape. So when completely liquid phase (()) (18:30) what is the total energy change during this change of the phase we can calculate that volume free energy delta Gv is defined per unit volume, but what is the total volume of a sphere particle are that is 4/3 pi r cube. So this is total free energy change which is associated to the welding a particular volume of a sphere.

Second one is the surface energy change so if gamma is defined the surface energy or gamma can be defined in other way also surface energy which is equivalent to the surface tension also. Surface tension normally measure force per unit length. Now energy we multiply by both the side so this becomes energy per L square that means energy per unit area. So this is defined surface energy per unit area.

And this is the total surface area of this particle. Now if we look into that so this surface energy is actually this graphically we try to represent what is the total delta G what is the total free energy change we can estimate it is a positive and this is continuously increasing this is total energy, but if we look into that volume free energy it represent by this curve. It is a negative quantity because actually delta Gv volume free energy change.

It is changing from the phase from liquid phase to solid phase. So Gv corresponds to the solid-Gv corresponds to the liquid actually this is the negative quantity we measure because it

is change of the phase from the liquid phase to solid phase. So this becomes negative quantity that is why it is drawn in the negative side. So it means that both energy actually play the role and if we look into combining these 2 delta G plot it.

And actually it reach some optimum point then after that there is a gradual decrement of this delta G. So from here we can estimate that this optimum point corresponds to critical size of the nucleus so that represented by r* here. So r* is the critical size of the nucleus that means if when there is a change of the phase from liquid phase to solid phase what should be the minimum size of the particles.

In that sense here we can say minimum size of the nucleus that will sustainable and after that this nucleus we will be able to grow further that means minimum size of the nucleus a critical size of the nucleus should be r* and at the same time corresponding to the r* what is the amount of the total volume free energy that energy barrier is actually this amount of the energy barrier is actually required that amount of the energy barrier is required to start the nucleation process.

So we can estimate also what way we can estimate that we see the typical nature of this volume free energy as well as the surface energy plot we assuming as a particular size of the nucleus and then if you plot it and then getting the different trend such that there must be some amount of the optimum point we will be getting that optimum point actually indicates the critical amount of the energy is required to start the nucleation.

So what is the critical size mathematically how we can estimate this things. We simply do delta G the derivative of that such that when derivative of delta G with respect to size r=0 that indicates they are able to find out what is the optimum point and that=0 corresponds to when r=r*. So if you do that we can find out that r* is critical size of the nucleus=-2 gamma/delta Gv.

Since delta Gv is actually negative quantity and we can find out what is the critical size of the nucleus and what is the corresponding amount of the barrier of the energy required to start the nucleation process. So that we can estimate all this things we can estimate the amount of the energy even mathematically. So it is relevant to that so that here what is the surface energy the critical size of the nucleus defines specific surface energy.

And of course what is the volume free energy also corresponds to this particular nucleation process or particular what is the types of the material based on that this value actually varies and we can estimate this things, but further delta Gv can be calculated in the simple way that delta Gv is basically is associated with the latent heat of a particular material and what is the delta T delta T here is the degree of undercooling.

So degree of undercooling and the latent heat and this actually happens over the melting point temperature Tm so that latent heat what is the change of the temperature and what is the Tm is the here we can say the melting point temperature we can say that equilibrium temperature over which normally solidification happens. So Tm equilibrium solidification temperature which is corresponds to the melting point temperature and then delta T is normally Tm/Tm-T.

So Tm-T means we can say that if this is correspond T and this is correspond Tm so this difference you can say is the delta T. So delta T can be measured in terms of that delta Gv can be estimate and only thing in terms of the latent heat and if all letters are available we can estimate what is the critical size of the nucleus and what is the energy barrier to start the nucleation process and corresponds to the homogeneous nucleation process.

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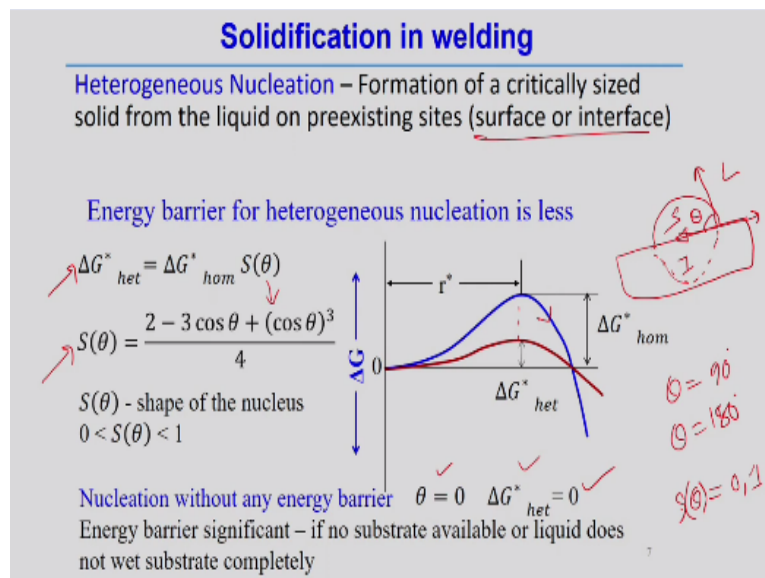
But similar kind of phenomena can be explained if we look into that heterogeneous nucleation process. So that starts with the formation of the critically size nucleus similar concept we can use here also, but heterogeneous nucleation start actually start over a surface or interface or in presence of any nucleating agents that sides or substrate actually to start to

initiate the nucleation process.

So in heterogeneous nucleation process if we do the similar kind of phenomena if we start it start with some substrate material nucleation process starts. Suppose this is the size of the nucleus and if you look into what is the difference surface tension acts between these 3 medium point. So suppose this is a liquid phase, this is solid phase and this is the for example interface, interface means is the substrate material over which the nucleation actually start.

So if we make the balance the interface surface energy or between this 2 different medium for example between the solid nucleus and liquid phase and between the liquid phase and the interface phase and between the solid phase and the interface phase if we look into all this aspect then we can find out that the energy barrier for heterogeneous nucleation is corresponds to the homogeneous nucleation, but it is a function of S theta.

Actually theta can be defined like this, this angle can be defined as a theta so then this is it is a function of explicitly as a function of S theta. So there is a physical interpretation of S theta for example if theta=90 degree for example theta=90 degree for example theta=90 degree in this case then we can start with the nucleation process assuming the half sphere and of course if theta=180 degree.

We can start that theta=180 degree it means it is a complete sphere that means energy barrier maybe which is equal to the energy barrier for homogenous nucleation process. So here we can see that so by looking into that S theta so actually theta actually vary from 0 to 1 sorry S theta actually varies from 0 to 1 and it is at in between and depending upon this thing when S theta=1.

Then it is a heterogeneous nucleation energy barrier is equivalent to the homogeneous nucleation process, but if we try to represent this things that this corresponds to the homogeneous nucleation process the energy barrier required in this cases, but in case of heterogeneous it is obvious that the maximum of the heterogeneous nucleus can be equivalent to the homogeneous nucleation process.

But since S theta it varies between 0 to 1 so actually the heterogeneous nucleation process the energy barrier is less than that of the homogenous nucleation process and we can estimate this

thing, but if you know what is the functional value of S theta. So but other point is that critical size of the nucleus is the same theoretically as compared for both the homogeneous nucleation process as well as the heterogeneous nucleation process/

And since actually the expression from the volume free energy and surface energy from what way we estimate that what is the critical size of the nucleus. Similar calculation we can do here also, but all this calculation is explicitly is independent of the theta term here that is why critical size of the nucleus both the cases is same, but energy barrier in case of heterogeneous nucleation is less as compared to the homogenous nucleation process.

So if you look into specific cases for example if theta=0 and then delta G* heterogeneous=0 it corresponds to the nucleation process without any energy barrier, but at which condition it happens probably any energy barrier is significant if no substrate material is (()) (29:45) heterogeneous nucleation starts or liquid does not wet the substrate completely. If we say in other way also if liquid molten pool wet completely to the substrate material.

Then probably this is the condition satisfy in that case almost negligible amount of the energy barrier is required to start the heterogeneous nucleation process. So these are the mathematical sense through which we can relate what is the different nucleation theory, where it is applicable and how we can calculate the critical size of the nucleus and what is the energy barrier it required to start the nucleation process.

Now we come to that solidification in welding process. We start with the liquid of uniform compositions rarely uniform of course most of the cases what is the uniform composition in the liquid phase that may not be there in the after the solidification. So after the solidification they can it is possible to make the mixture of the 2 different phases in the solidified structure it is quite possible.

But is not necessary that before solidification it can be on the one single phase only the liquid phase although it can be alloy system.
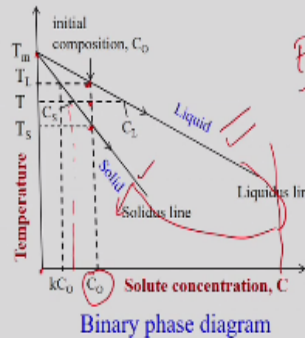
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Binary phase diagram

So redistribution normally happens of the atoms well after the during the solidification process. So we need to understand that how composition of a specific alloy system after redistribution what happens, what are the different phases present there. Now redistribution of the solute for example in this cases we are discussing the redistribution of the solute atoms. We can take one more example.

We normally create the steel assuming the binary alloy system it is a composition iron and carbon. So iron and carbon certain percentage of the carbon actually I think maximum (()) (31:55) carbon can be mix with a iron and if we look into the phase diagram and we see that iron and carbon maybe the liquid stage they can on a single phase and this normally happens at a very low percentage of the carbon.

But once the liquid phase change into the solid phase of the solidification we can say the different kind of the structure and that different kind of the structure it can be the binary structure. Here we see it can be in a liquid single phase and when it solidify it can be 2 different phases. Therefore, it is a carbon percentage we are representing is a solid concentration here.

So it all depends on the solute concentration it depends on the specific alloy system. So what happens when there is a solidification of a particular concentration. For example, we start with this C0. So C0 for example C0 is corresponds to the composition iron+ say 0.1% of carbon. Suppose this is the composition so solute concentration is only 0.1% of curve, but if we start with this things.

And then at this particular temperature this corresponds to the particular temperature this is the solid concentration Cs in the solid phase and the liquid phase the solute concentration becomes CL. So that means during the transition from one particular point it is from completely liquid phase when we try to solid phase and as a function of temperature the solute concentration may vary depending upon the temperature.

But once it comes to this point then solid concentration becomes C0 so that means this and microstructure depends on that. So that means when there is a solidification happens from this change of the temperature liquid to the solid phases there maybe the redistribution of the solid concentration may happen during this process and that is very much obvious if we look into that binary phase diagram.

If you see this is the binary phase diagram part of the binary phase diagram we have shown here y axis on the temperature and x axis on the solute concentration and of course this indicates the melting point temperature of the pure metal. So at this point it is a completely so in this case it is a 100% iron if we assume that. So one single melting point temperature, but when you keep on adding the increasing the carbon percentage.

Then there is a change of the solidus and liquidus temperature and at the same time there is a change of the solute concentration. So it depends the redistribution of the solute concentration actually depends on the thermodynanmics it is we can explain better using the phase diagram and of course kinetics that missed diffusion, undercooling, fluid flow all this phenomena during the solidification process all actually impact on the redistribution of the solid.

And we can get completely different structure, but if we understand the structure we need to understand the solidification behavior of a particular alloy system and mathematical sense what way you are representing the alloy system it is already explained this thing, but still I am repeating the same thing to make more clearly to understand this phenomena. So it corresponds to we have represent instead of curve represent the one straight line.

And part of the binary phase diagram we are showing here liquid phase that means this is above which the liquid phase exist and below which this line is the solid phase exist. So that is surrounded by the actually liquidus temperature and solidus temperature, but as a function

of the solute concentration. Solid concentration changes for example this is the solidus temperature and this corresponds to the liquidus temperature.

Solute concentration is C0 if we change the solute concentration there is a change of the solidus and liquidus temperature, but this zone is actually between the solidus and liquidus temperature. So by looking into this diagram we will try to explain the solidification behavior.

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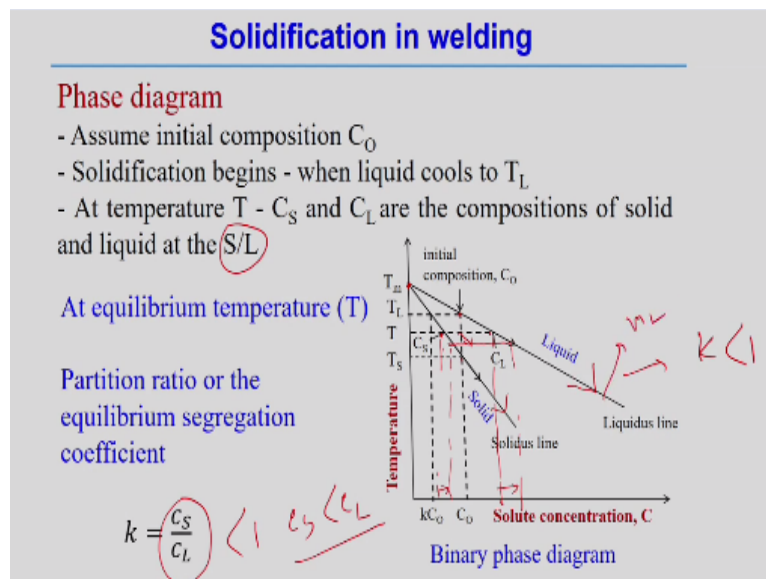
Now phase diagram so we assume the initial composition is the C0 particular position the solidification begins when we start at this point and ends at this point that there is a change of the phase at temperature T we see at a particular temperature T corresponds to this Cs is the and CL are the composition of the solid and liquid at the solid liquid interface. So SL means solid liquid interface.

So this is the composition, but at particular temperature the composition at the solid phase and composition concentration at the liquid phase and it changes. So we assume that if solidification happens at an equilibrium temperature T which is in between the solidus and liquidus temperature. We can define another parameter also that in terms of the K, K can the partition ratio or the equilibrium segregation coefficient which is defined by this two CS and CL.

The ratio of the CS and CL can be defined in the partition ratio that means composition of the solid/ composition of the liquid phase. Now what are the information to understand the phase

diagram, what are the information we can find out from this phase diagram. First is that solute content of the liquid is defined by CL is continues to rise during the solidification by lowering the solidification temperature.

So this thing we can look into the diagram. So by lowering the solidification temperature suppose this is the solute concentration C. So lowering the solidification temperature for example the solidification assume this happens at this temperature. So when you are lowering the solidification temperature so it shifts this temperature to other lowering the temperature so correspondence there is a Cs this point now this is increment to this corresponding to this point. Similarly, CL also increases which was initially this point. So there is a increment of the Cs and increment and increment of the Cs.

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So that we represent in this way that solute content of the liquid actually rises increases if we lower the solidification temperature. The second one of course once the solid grows from the liquid phase and its solute content also continues to rise that also we explained this 2 points that there is a continues increment of the Cs and CL if we lower down the solidification temperature.

If k<1 then Cs and CL both increases as temperature T of the solid liquid interface drops during the solidification. So that means if k=<1 if k<1 that means it is a<1. So Cs<CL that is always obvious Cs then concentration in the solid phase is less than that of the concentration of the liquid phase because depends on the solidus and liquidus line this slope is actually decreasing slope or we can say the negative slope.

So that is why if k<1 then Cs CL always be< so corresponds to this diagram k should be always<1. If the binary phase diagram can be of opposite sides opposite that means this solidus and liquidus line not exactly the decreasing slope if it is increasing slope in that case this phenomena can be different or it is just vice-versa of this k<1 in this case. So k<1 increases both increases as the temperature of the T (()) (40:20)

So it is always that k<1 in this cases Cs and CL always increases if temperature drops that means solidification happens in the lower temperature. Slope of the liquidus line say for example mL is<0 and k<1 and vice versa of course slope is it is obvious we have seen the condition here is the m<0 one is negative slope and corresponds to k<1. So mL is negative corresponds to slope and k<1 corresponds to the diagram.

Similarly, liquidus line is we assume it is straight line if we assume the liquidus line is a straight line a temperature T on the liquidus can be expressed in that way. So that it corresponds to that Y mx+c. So m is the slope c is the intercept. So this is equivalent to this any point we can represent the temperature of course if we takes care of the proper the sign convention positive, negative accordingly.

That means C is the intercept here so it start with the melting point so corresponds to this diagram. So this is the melting point this melting point so we can say this is the intercept of the y axis and these are the slopes, slopes represents the mL and corresponding to at any point we can represent the temperature at any point on the liquidus line. This if we assume the liquidus line is a straight line.

So that all kind of information we can get from this binary phase diagram which may be useful to estimate further.
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Solidification in welding

Complete diffusion in solid and liquid

- f_s and f_L - the fractions of solid and liquid
- Area $C_s f_s$ and $C_L f_L$ amounts of solute in the solid and liquid
- Area $C_0(f_s + f_L)$ represents the amount of solute in the liquid before solidification
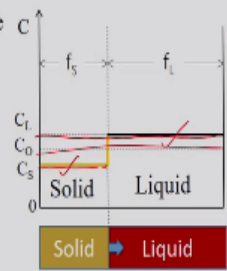
$$C_s f_s + C_L f_L = C_0(f_s + f_L) = C_0$$

- Lever rule
  $$f_L = \frac{C_0 - C_S}{C_L - C_S} \quad \text{and} \quad f_S = \frac{C_L - C_0}{C_L - C_S}$$
- Composition of liquid $C_L = \dfrac{C_0}{f_L + k(1 - f_L)}$

Conservation of solute

$$f_S + f_L = 1$$

$f_s = 0.2$
$f_L = 0.8$

$C_S < C_L$

$k = \dfrac{C_S}{C_L}$

Solid → Liquid

Total volume of material

Equilibrium solidification

Now I will look into that complete diffusion in the solid and liquid we are assuming that this is a diffusion actually happens it is a solid and liquid diffusion in the solid and liquid phase such that we can estimate what is the solid fraction and liquid fraction in the solid the liquid fraction and solid fraction from the conservation of the solute. So we try to look into this try to look into this diagram.

Suppose this is the solid phase and this is a liquid phase and coexist and that coexistence actually happens between the solidus and liquidus temperature and there is chances that there is a continuous certain at any particular temperature solidification temperature this is the critical situation so this fraction is a solid fraction or solid part and this is the liquid phase. If we look into that the fraction actually varies from 0 to 1 solid fraction liquid fraction.

So here fL is the fraction of the solid and liquid such that conservation of solute say that it should be always fs and fL will be 1. So if we say that fL=0 becomes a 100% all are in the solid phase if fs=0 we can say that all are in the liquid phase. So that mean conservation so at any particular point we can see that fs=0.2 then there should be fL should be 0.8 because this integrates that in the mixture between when there is a solid and liquid coexist that 20% is a solid fraction 20% is a solid and remaining 80% is a liquid phase.

So like that now at particular solidification temperature if we assume the solid fraction liquid fraction, but how we can estimate this fs and fL the solid fraction liquid fraction so if we look into that Cs fs actually represents the amount of the solute within the solid. Basically Cs represent the solute part the amount of the solute concentration, but when you represent the

fraction with respect to the total volume.

Then fs that represents the total solute concentration amounts of the solute at the solid part. Similarly, CL and fL represents the amount of the solute in the liquid part, but if we start with the initial concentration with the C0 so C0*fs+ fL at a a particular point that represents the amount of the solute in the liquid before the solidification. So we can look into this diagram also so if we observe that at the solid part the Cs<1 so Cs is always<CL that is very much obvious from the phase diagram.

So now particular Cs is corresponds to the solid and CL it corresponds to the liquid phase and Cs corresponds to the solid phase and C0 is between if we see we can look back the diagram also Cs in between sorry C0 is the in between and Cs is<C0 and CL>C0 and of course in this case is CL>Cs and C0 is basically<Cs>CL in between. So similar thing at a particular temperature we are trying to represents what is the solid phase and liquid phase exist during the mixture.

So that Cs and CL and in between the C0 so that we make a balance so in that C0*fs+ fL that means concentration equivalent to what is the total fraction and corresponds to the initial concentration should be equal to any particular temperature solidification temperature this is the fraction of Cs and fs and CL and fL corresponds to that. So of course fs and fL from the conservation of the solute because we are assuming the complete diffusion in solid and liquid phase.

From here we can find out this is equal to the C0 because fs+ fL=1. So once we have this equation we can apply the lever rule and we can find out that fL in terms of the solute concentration C0 Cs CL and Cs so all this C0 Cs CL values can be found out from the binary phase diagram. Similarly, we can estimate the fs also so solid fraction and liquid fraction we can estimate and using all this relation we can find out what is the composition of the liquid if we introduce the k because k is already defined I think Cs/CL.

So from here using this value of k we can find out in terms of other parameter for example C0 is the initial solute concentration and then of a liquid fraction and of course if we define the k, k is the partition ratio then we can find out what is the composition of the liquid CL and what is the composition of the solid phase that we can easily estimate by using this diagram

at equilibrium solidification.

That means solidification happens assuming the solidification happens at certain equilibrium temperature which is normally in between the solidus and liquidus temperature.