Mathematical Modeling of Manufacturing Processes

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Lecture - 22
Heat Transfer and Material Flow

So far we have discussed different types of the heat source or basically in mathematical form what way we can represent the different type of heat sources corresponding to the different welding processes, for example the representation of the heat source, laser and arc welding should be different but that heat sources we discussed that is distributed heat source but using this distributed heat source normally we solve the heat conduction equation to find out the temperature distribution in a solution domain specifically during the welding processes.

But that in this case definitely there is a need to find out what is the governing equation to find out the temperature distribution in a solution domain. So, here I have written that governing equation that is basically you can say that 3-dimensional heat conduction equation and we try to solve this heat conduction equation to finding out the temperature distribution.

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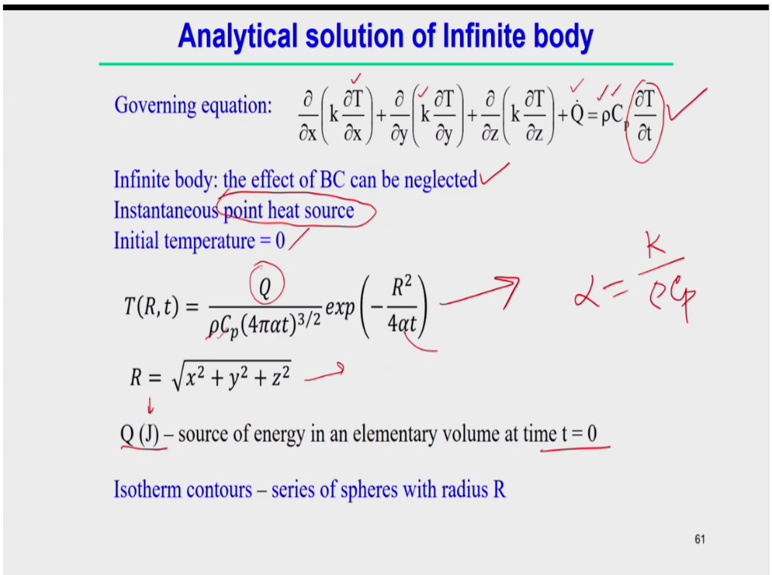

Here, T is the temperature variable T, in case the thermal conductivity to see and Q dot is the terminology internal heat generation and rho is the density, Cp is the specific heat. If you see the right hand side, there is a temperature variation with respect to time. So, the tangent variation of temperature we incorporate in these equations but tactfully need to utilize this governing equation to finding out the solution.

Of course, there is a need of some kind of boundary conditions if we try to solve this equation this differential equation. So, there is a two approaches, the two ways we can solve this equation maybe we can find out what is the analytically what is the temperature distribution in the domain or we can do the numerical simulation or numerical solution of this equation using some mathematical like finite element, finite volume or maybe finite difference methods.

But of course will not exclusively discuss how to implement all this finite element methods or maybe how to utilize using this different finite element or finite volume based packages, commercial packages that is not our target but will try to understand physically that this equation is can be solved in analytically but the limitation of the analytical solution is that there is a need of lot of assumptions.

And that may not practically satisfy when we try to compare the actual welding process but still analytical solution give some idea what maybe the typical temperature distribution and specifically in these cases in the welding processes, so let us look into the analytical solution of this equation with so many assumptions. So, first we assume that if we try to find out the analytical solution and with that we assume this is an infinite body.

That means the infinite body infinitely long body, so in that case probably not necessary to or maybe you can neglect the effect of the boundary conditions, so boundary condition effect neglected in case of infinite body. Second assumption is that instantaneous point heat source. So, we have already talked about the point heat source but here we assume that there is an existence of the point heat source.

That means energy release over a point having initial temperature equal to 0. So, with this assumptions and if you look into this governing equation and if we solve it still will be getting this kind of solution, so solution means the temperature distribution as a function of the space variable as well as the time variable.

Here, if you see we just look into this temperature distribution equation T as a function of R and t, t indicates the time and R is basically the radial distance with respect to the origin 000, so at which point instantaneously the heat energy Q is basically applied. So, Q is the applied energy that we can normally say this is a point heat source and then we get this temperature distribution in terms of other parameters, the properties, thermal properties of the material rho Cp but here we use the terminology alpha, alpha is basically thermal diffusivity.

That can be defined alpha=k/density*Cp, so that is expression for alpha, so in terms of all these material parameters or material properties and thus in space time this temperature distribution we can found out assuming some instantaneous point heat source. Here, clearly defined Q actually which is the unit of this energy is in terms of joule actually that is the source of energy in an elementary volume at time T=0.

That is the representation of the Q and if you look into the solution quality that means nature of the solution, we can see that it actually indicates the series of spheres with radius R and of course in this case if you see that R is in terms of x, y and z, that means will be able to get the temperature distribution in say 3-dimensional form by assuming instantaneous point heat source.

And of course this type of heat source we are getting the temperature distribution in the 3-dimensional form. So, further if we look into that suppose the heat source representation can be different also.

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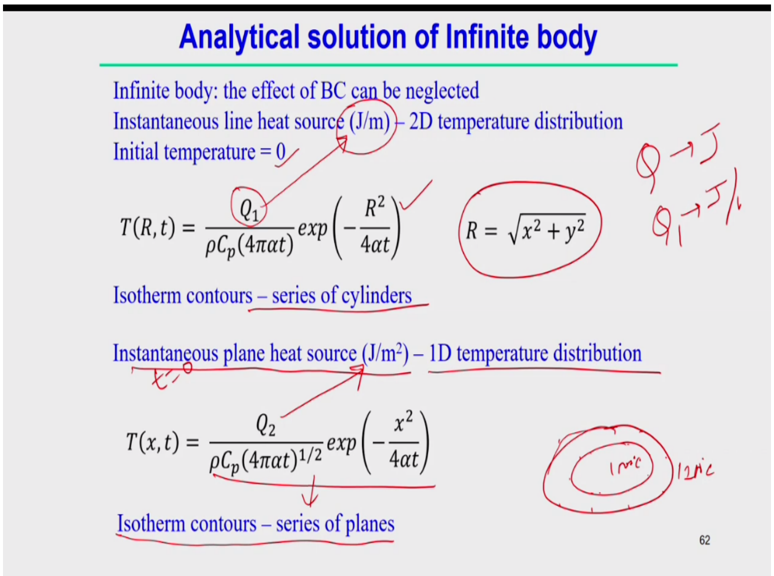

For example, the second one may be infinite body that means we are assuming the infinite body, definitely the effect of boundary condition can be neglected in case of the infinite body but if we assume that heat source representation as a line heat source that means the joule per unit length that means joule per meter that can be the unit of this line heat source. So, in this case will be able to get the two-dimensional temperature distribution and the similar assumption the initial temperature is basically 0.

And we can get this kind of solution that means distribution of the temperature as a function of Q1; Q1 is basically that instantaneous line heat source which is represented by the energy per unit length and that is why it is different from the Q, Q was the only energy point heat source, Q1 corresponds to the point heat source and the unit was joule but in this case the Q1 corresponds to the joule per energy per unit length.

And we are getting this kind of equation and also alpha is the similar thermal diffusivity and all parameters are defined and if you observe that R is basically that root over x square+y square that means its R is the radial distance measured with respect to the origin but if we assume origin=0, 0 then we are getting the isotherm contours is basically this equation represents a series of cylinder.

So, in fact we are basically getting in this case that two dimensional temperature distribution, so two-dimensional temperature distribution if there is a line heat source that means energy over a particular line if that line heat source we represent along the z direction, so there is no variation of the temperature along the z direction rather the variation of temperature will be getting in the xy plane.

So that is why we are getting this kind of solution but there may be the possibility of another type of solution if we say that instantaneous plane heat source. Instantaneous means at T=0 if we apply the energy, supplied the energy per unit area, if that is a representation of the heat source then will be getting the one dimensional temperature distribution. So, similar way we can find out that Q2 is basically representation of the energy supplied.

But that is per unit area and getting the temperature distribution of this typical nature all the similar kind of the expression but that in this case alpha and this only the distribution and whether it is two dimensional or three dimensional that actually varies. So, this equation represent it is a series of isotherm contours is basically series of planes, so we understand the isotherm contours at constant the same temperature how it looks like.

So, series of planes it can be something like that, this is called the isotherm contours such that it at any point the temperature is the same, so this is the another isotherm contour such that any point if you say the temperature can be constant. For example, in this case, temperature if it is 1200 degree centigrade may be this corresponds to 1000 degree centigrade that means that any point the temperature remains the same, that is called the isotherm contour.

So, if we observe that it is possible to solve analytically the governing equation means the heat conduction equation to obtain the temperature distribution in the different three-dimensional, two-dimensional or in the one-dimensional form but if you see that what way you are giving the energy input to the domain accordingly there is a restriction of getting the temperature distribution.

For example, in case of point heat source, we are getting the three-dimensional temperature distribution, in case of line heat source we are getting the two-dimensional heat source, in case of plane heat source basically you are getting the one-dimensional temperature distribution. So, all this temperature distribution varies depending upon what way you are representing the heat source.

So, that is the typical analytical solution. We see the further but of course in this case if you see all these cases the temperature distribution we are getting as a function of time but we have not incorporated the effect of the velocity. That means normally practically the welding happens with some linear velocity in particular direction but the linear velocity effect is not consider all these solutions.

So, probably this solution is more suitable in kind of spot welding to get some initial idea about what may be the typical nature of the temperature distribution physically in case of welding process.

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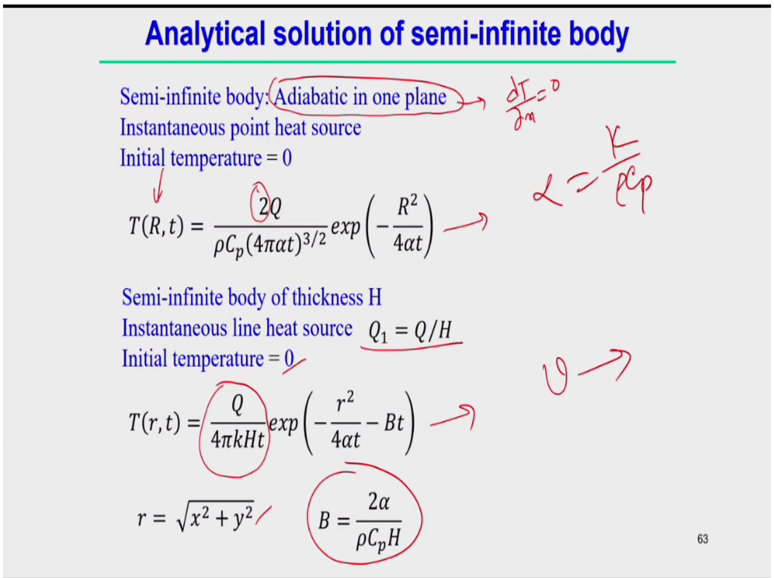

But further development also happens to improve the analytical solution with some other assumptions or of course we can say with some other limitations as well. So, now look into that the semi-infinite body. That means which comes from the infinite to semi-infinite body so that means one finite dimension we consider in the domain and based on that we can found some solution also.

But if we assume the semi-infinite body, on that case that particular plane the boundary condition is like that adiabatic condition. That means adiabatic condition is the basically temperature gradient is basically 0. The temperature gradient 0 means there may not be any heat transfer normal to that surface if we put that kind of boundary conditions and if we assume that instantaneous point heat source.

That means representation of the heat source over a point and initial temperature=0, then we reach the temperature distribution in this form but it is different from the very initial point heat source analysis that it is I think the 2 factors, 2 comes here that is the only difference but this is the solution in case of semi-infinite body as compared to the infinite body. So, alpha is well defined that this k/rho Cp the thermal diffusivity and all parameters relevant to the different material properties and we can get this kind of solution.

Further improvement if we assume the semi-infinite body of thickness H, so if you restrict the thickness dimension which is finite dimension having H in that case, instantaneous line heat source can be represented like that Q1=Q/H that means the energy supplied in terms of the energy per unit length but of course this unit length in this case the dimension is actually defined.

And with the initial temperature and we can get the temperature distribution in this form. Here of course in this form means this is the variable which is also function and then exponentially some value also the variable here r, r=root over x square+y square. That means it represents that we are getting in this case the temperature distribution in the two-dimensional form.

That two-dimensional where r is actually measured with respect to the origin and that is why it is root over x square+y square and B is some other parameter which is related to the other material properties alpha and the finite dimensional H and specific heat and density. So, this is the temperature distribution that we see the two-dimensional temperature distribution the expression is actually different with respect to or with reference to the completely infinite body.

So, that means in semi-infinite body, the solution quality actually improved or we can say that in practical problem it is more realistic as compared to assuming the infinite body. Infinite body means in all three directions x, y and z we assume the dimension is infinitely long so but that is not the practical situation. Practical situation is probably all the finite dimension we normally consider.

But if we consider all finite dimensions, probably in that case it is very difficult to find out the solution analytically. So, that is why these assumptions actually with this the assumptions is made because to get that certain kind of solution that is without this assumption it may not be possible to get this solution or finding out the solution means temperature distribution analytically.

So, we will see further involvement of course in this case also we discuss either infinite body or semi-infinite body but we are not considering the effect of the weld velocity also. There is nowhere it is the velocity means which links what speed the heat source is moving, we have not considered with this solution.

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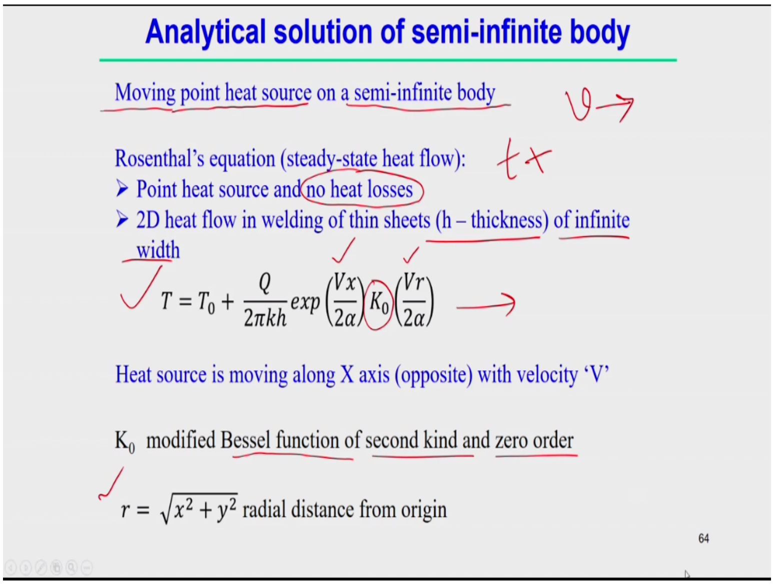

Now, further development with this analytical solution of a semi-infinite body but in this case we consider the moving point heat source. So, it is a point heat source but it is moving we consider and the body is a semi-infinite body. So, if we assume these things and this is a weld velocity, it moves particular directions and with this all these assumptions actually the analytical solution with this condition steady state is initiated by the scientist Rosenthal and we normally see this as a Rosenthal's equations.

It is simply how we can estimate the temperature distribution in steady state heat process, steady state equation. So, steady state equation means the time component is basically there is no time component, so temperature of solution does not vary with respect to time but of course the assumptions to finding out this steady state equation and first assumption was the point heat source already discussed and of course in these cases there is no heat loss.

But practically in the welding problem definitely there may be convective and radiative heat loss from the surface also happens but in this case we are assuming there is no heat losses and if you see the two-dimensional heat flow in the welding of the thin sheets say for example two-dimensional heat flow and we assume it is a thickness having very thin sheet, probably it is realistic of very thin sheets.

Because normally the temperature gradient along the thickness direction is normally very less, if thickness is less, which is a quite reasonable assumptions, so finite thickness but we assume the infinite width so one finite along the thickness dimension but infinite in other direction. Based on that we get two-dimensional steady state temperature distribution can be represented by this equation.

If you see that in this equation, the V actually is the velocity, here the velocity components, so moving velocity components. So, in this case, this solution is more realistic than the previous one because in this case at least we can incorporate the effect of the welding velocity but of course the solution depends on this K0 which is defined by the modified Bessel's function of the second kind and zero order.

So, that with the corresponding expression of the K0, we can find out the different types of the solution of temperature distribution and of course r is the radial distance from the origin and of course the origin means here is the origin which defined at 0, 0 point and then based on that we are getting this kind of steady state temperature distribution.

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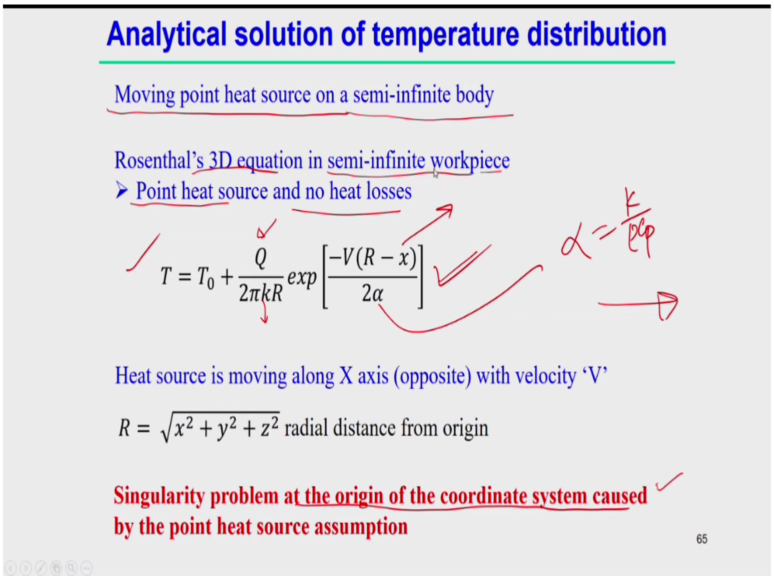

So, further this also modified that moving point heat source of a semi-infinite body but in this case will try to get three-dimensional temperature distribution. Three-dimensional equation corresponds to the semi-infinite workpiece and of course assuming the point heat source and no heat loss. With this assumptions, it is possible to get the temperature distribution like this if you get this equation that T=T0+Q/twice pi kR.

See K is the thermal conductivity and Q is the point heat source that means energy supplied to the domain. Heat source is actually moving along the x-axis, actually with positive x-axis if you define heat source actually moves in the opposite directions based on that we get this kind of solution. Alpha is defined thermal diffusivity and of course if we see the solution of this temperature distribution R is the radial distance from the origin.

It is defined in terms of x, y and z coordinate and x is basically here the x term introduces here and that actually incorporate which direction actually heat source moves. For example, if heat source moves along y direction, then x should be replaced by y and accordingly the temperature distribution can be rewritten in that form. So, it depends on that but one limitation of this kind of solution is that singularity problem at the origin.

Because we cannot estimate the temperature distribution at the origin from this expression, probably we are using the point heat source and that point heat source is at this point heat source because of this point heat source probably at that point it is not possible to get the temperature distribution, that means the singularity problem at the origin of the co-ordinate system exist and caused by the point heat source assumption.

So, that is the one limitation but apart other points we can get this temperature distribution and actually this is the most widely used the analytical solution relevant to the welding processes because this analytical solution moves actually towards the realistic welding situation because if you look into the actual welding process, first thing is that we assume that in the realistic the dimension of the workpiece is finite even in x, y, z thickness all dimension in the finite dimension first thing.

Second thing is that the heat source moves in a particular welding velocity that is in most of the cases we use if it is not spot welding process, so by these assumptions we get at least approximately the solution of the temperature but if you want to get the more accurate solution probably we may not get the kind of analytical expression or analytical expression of the temperature distribution in specific to the welding process.

Definitely in that case, we need to move to solve this equation governing equation numerically with proper boundary conditions. So, this we assume the simple solution Rosenthal’s three-dimensional solution and of course the all assumptions and limitations are already defined but still we can use to get some working knowledge of temperature distribution in a specific welding situation.

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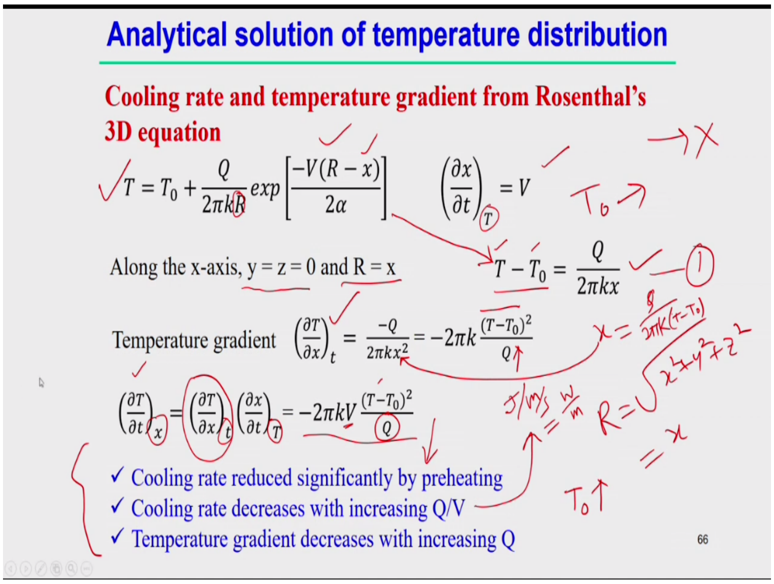

We can explore further this equation, how to utilize this equation in other problem. For example, can you estimate the cooling rate and temperature gradient from the Rosenthal's 3D equations? And of course, we try to look into that how it can be done. So, first but we need to know what cooling rate and temperature gradient is something significant to predict certain action the microstructural phenomena.

That is why always we need to know what is temperature distribution and how cooling rate can be estimated or what is the temperature gradient exist at particular point even temperature gradient cooling rate may be useful to see the solidification behaviour in a welded structure. So, from that point of view, it is important to know what is the cooling rate temperature distribution but by using this temperature distribution or may be from the Rosenthal's 3D solution, how we can find out the cooling rate and temperature gradient.

So, we stick with this equation and then first we can find out that we are assuming that heat source moves along x-axis. So, that is why x will come here this expression. Second part is that now the derivative dx/dt definitely it indicates the velocity and keeping T as a constant also del x/del t at constant temperature that represents the velocity V mathematically. Now, if you see along the x-axis.

So, if you look into along the x-axis, definitely y and z=0 in this case and if you put R=root over of x square+y square+z square. So, along x-axis, it is quite obvious that y=0 and z=0 if you put it then R becomes x. So, then R becomes x. Then, how we modify the temperature, then T-T0 from this equation=Q/twice pi kx because R will be replaced by x because R=x along the x-axis.

And of course here if we put R=x then this becomes 0 then exponential 0 that means that equal to 1. So, we are getting this expression. So, for example this we are getting expression 1 T-T0. Now, how we can estimate the temperature gradient, so temperature gradient del T/del x, x is the variable here, so if you do simple derivative this expression, then we can get from equation 1 –Q/twice pi kx square.

So, we just modify this –Q/twice pi kx square, pi k we just replaced x in terms T/T0, from here also we can express x=Q/twice pi k T-T0. So, we just simply put here replace x here and then we are getting this kind of expression in terms of Q, pi, k and T-T0. So, this is the temperature gradient this thing. Now, we see the cooling rate; cooling rate means temperature change with respect to time.

So, del T/del t and fixed on space, then this can be represented del T/del x and del x/del t but the time constant, temperature constant and if you put this value because del T/del x this represents simply the velocity vector, so we put the velocity V and del x/del t that means temperature gradient we have already estimated and we have put it and we are getting this expression.

So, we are getting this expression in terms of that thermal conductivity, welding velocity and variable temperature T-T0, T0 is the of course we forgot T0 is basically the initial reference temperature, normally T0 is the ambient temperature and Q is the energy supplied to the domain. It is a representation of the point heat source, so from this expression we can make certain conclusion that cooling rate reduced significantly by preheating.

So, if you look into the magnitude, so if there is a preheating, preheating means if T0 can be initial the reference temperature, so if T0 is very high, so this actually the difference actually decreases, so this decreases and if this decreases so overall cooling rate actually reduces. So, in practically when you try to look, do the welding process also that if we preheat, suddenly if we raise the temperature before start of the welding process for the workpiece certain temperature.

So, that actually reduces the temperature difference what is the maximum temperature or what is a preheat temperature, that difference actually reduces. Once that difference reduces that indicates the overall cooling rate can be reduced significantly if there is a preheating. If there is no preheating then cooling rate will be as will be more in that case. Now, if you look into the other way also that del T/del t and cooling rate decreases if Q/V.

Q/V is the representation also, from this Q/V=energy input per unit length basically. So, Q=joule, V=say meter per second so this indicates joule per second watt per meter. So, that means power input basically cooling rate decreases with increasing the Q/V. So, power input or unit length actually decreases with increasing that means if there is more heat input to the domain that actually also reduces the cooling rate.

So that is we can make the conclusion from this expression and other way also temperature gradient decreases with increasing Q. If we see the temperature gradient, here is the temperature gradient and we see that this is temperature this thing. So, if there is increasing of the Q, increasing of the Q actually decreases the temperature gradient. So, this kind of conclusion we can say by simply estimating the cooling rate and temperature.

And based on from the Rosenthal solution and we can make some conclusion that which cases if there is a preheating or if there is no preheating and which cases we can expect cooling rate high or cooling is low because that actually influence the different microstructural transformation.

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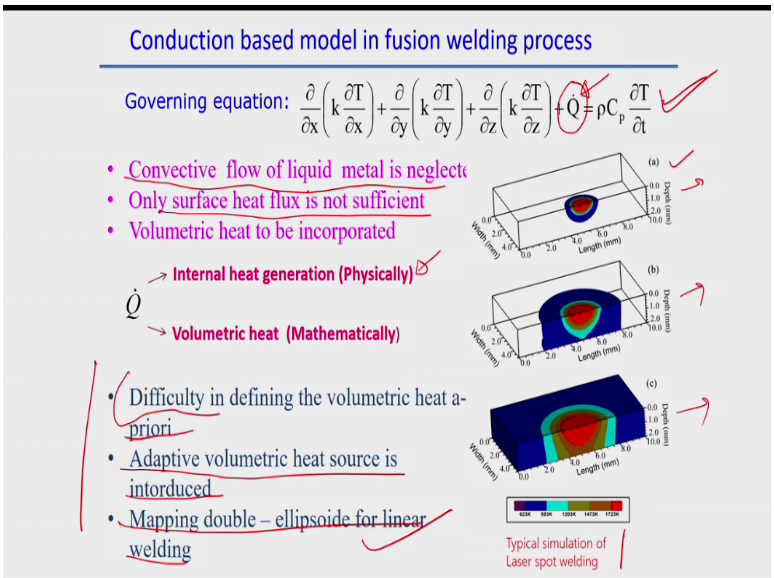

So, apart from this temperature solution, we can look into the other aspect. This we just so analytical temperature distribution, now we look into that. It is possible to find out the temperature distribution numerically but how we can do that numerical temperature distribution. So which take with we use the same governing heat conduction equations but solution may be in terms of the numerical solution.

So, in this case if we say that with this numerical solution, we can get the temperature distribution, so I think if you try to look into the numerical solution that replaces that more realistic welding phenomena, that means we use the heat flux that means distributed heat flux and then we represent the distributed heat flux in the different mathematical modelling of the heat source.

And then finally will get the temperature distribution and in this case of course we must consider what is the heat losses from the boundary. That means proper boundary condition can be incorporated if we try to get the solution numerically. So, I think normally practically there may not be any assumptions or limitations which normally we found out in case of analytical solution.

So, that kind of limitation can we overcome in numerical solution process, so let us look into that one cases that how numerical solution can be done. So, first we look into this, this is the governing equation, there is the heat conduction equations and if you see there is an internal heat generation term and of course along with that there is a boundary conditions also. Boundary condition is as simple as that heat losses from the surface we consider and that losses from the surface is due to the convection and radiation heat losses.

So, that we consider but this solution one assumption is there, the convective flow of the liquid metal. Basically, we are not considering the fluid flow phenomena of inside the small weld pool, so that we neglect and of course in this case or other things is the surface heat flux, sometimes is not sufficient. So, represents if we look into only the surface flux heat source model, then may not sufficient specifically to laser welding processes.

So, in this case, we need to incorporate the kind of the volumetric heat source models. So, volumetric heat source model we have already discussed but this volumetric heat source model how we incorporate in the governing equation, which simply incorporate the volumetric heat source term or that is the source term. Here through this Q dot term in the governing equations.

But internally heat generation term is the instinct part of this governing equation that is in the physical domain but volumetric heat source is basically mathematical domain but we incorporate this calculation through this physically internal heat generation term in the heat conduction equations. Then, we can find out that this is the solution, so this solution actually shows the temperature distribution in the lasers, temperature simulation of a laser spot welding process.

So that in all these cases at different time, so it is very small time, there is such just weld pool form and red color actually indicates the fusion zone but if the time increases that weld pool size actually increases and it takes at a certain time, there is a growth of the weld pool happens over the time. So, that kind of simulation phenomena it is possible to very accurately estimate, we can do just if we follow any kind of the finite element based simulation or finite volume based simulation or finite difference based simulation okay.

But this already we discussed that in relevant to the different type how we represent the heat source that definitely difficult in defining the volumetric heat source, that is the one limitation, although we define the volumetric heat source but this volumetric heat source you need to define the parameters beforehand but that is not the case in case of if we consider the adaptive volumetric heat source is introduced.

So, we have already discussed what is the difference between the conventional heat source model, volumetric heat source model to adaptive volumetric heat source model and of course in this cases we consider the ellipsoidal heat source model but since it is a stationary heat source we are using that means spot welding but if it is a moving heat source, definitely we can map the double ellipsoidal model in case of the linear welding process.

So, that there is lots of things basically relevant to the modeling approach or this thing and of course using some kind of the commercial packages, it is a well-shaped numerical solution of the heat conduction equation and application particular to the welding processes but of course it is not very convenient to look into these things using the conventional type of the heat source that means that limitation.

So, in that limitation means prior definition of the model parameters of the heat source, so this can be overcome if we look into kind of the adaptive volumetric heat source model.

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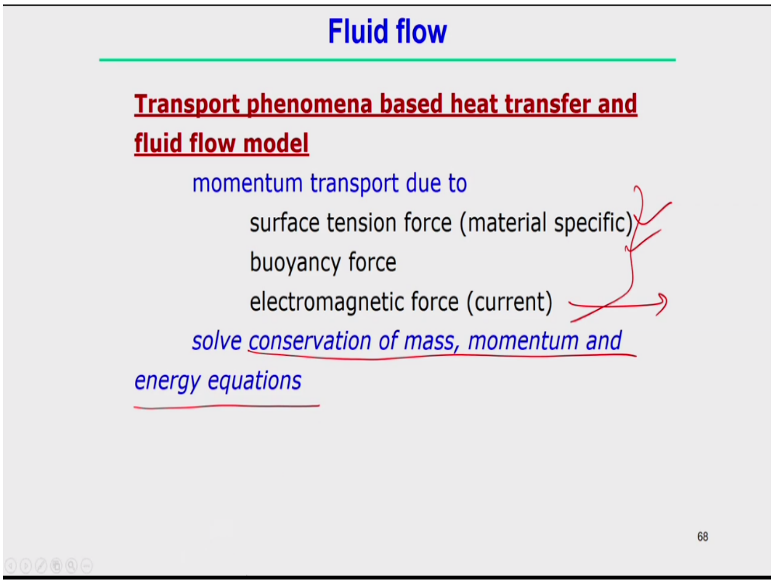

Now, we will shift to the fluid flow analysis basically transport phenomena based heat transfer and fluid flow model in case of welding processes. So, of course most of the cases we found out that we get the temperature distribution and we just solve the heat conduction equation and we get the temperature distribution but in that case the assumptions was do we were assuming the stationary molten pool?

We are not assuming the movement of the liquid molten pool, so but if we look into the actual process welding process there must be some movement of the welding molten pool will be there but how we can capture this movement or momentum happens within the molten pool, in that case we need to consider the transport phenomena based model and in that case of course we need to incorporate the effect of the fluid flow.

Of course, there is a practical importance of the fluid flow in certain application in welding process, will discuss later on but if you look into the fluid flow in this case we need to combine the momentum transport due to these 3 driving forces; one is the surface tension force and definitely material specific, other is the buoyancy force, electromagnetic force. Of course, in arc welding process there is a current flow, so we need to consider the electromagnetic force.

But in case of laser welding process, first two are the driving force, we normally do not consider the electromagnetic forces because in case of laser but from the modelling aspect this case we need to solve the conservation of the mass, momentum and energy equation to get temperature distribution as well as the fluid flow field within the molten weld pool. So, how we can do in case of welding process?

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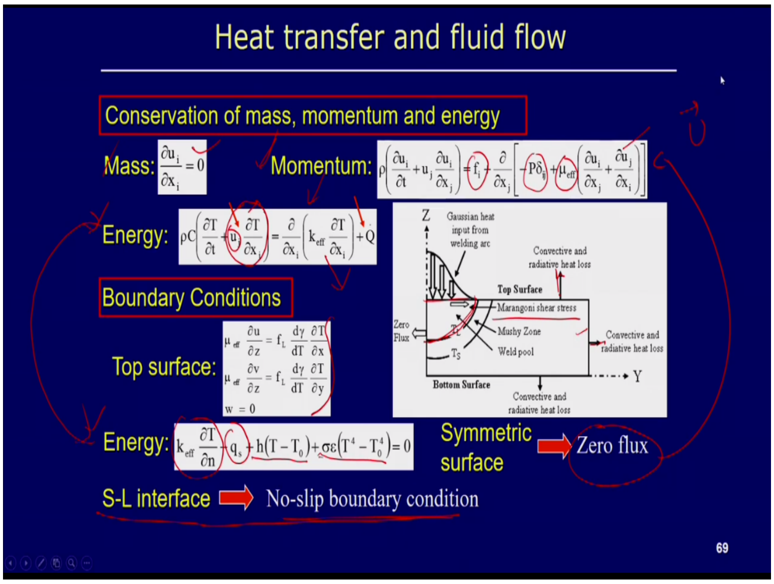

We just look into this, just will see how the boundary interaction on the governing equation will normally follow in case of heat transfer and fluid flow model. So, conservation of mass, momentum, energy; so this is the conservation and of course in this case is we are assuming the incompressible flow in this case and of course there is no density differences and this momentum equation will see the momentum that u is the velocity field while the molten material f is the driving force.

And of course P is the pressure and effective viscosity that means since fluid flow that the property viscous property we need to that material property is need to define here and here also the velocity gradient also we need to consider and that actually comes from the standard momentum equation. So, rather we said we need to look into the Navier-Stokes equation to get the velocity field.

And then once this momentum equation we need to solve and of course at boundary conditions but before that momentum then we need to solve the energy equation also. If we solve the energy equation, will get the temperature distribution but if you see that energy equation this term is actually different as compared to the only heat conduction equation but in heat conduction equation, this term was absent.

But when you incorporate the fluid flow, the convective flow of the liquid metal is important, that actually influence the energy transport. So, that is why we incorporate this velocity field and that is relevant to the temperature gradient and that we incorporate these things in the energy equations. When we consider combined effect of the both heat transfer and fluid flow, so then if you solve it then will be from this equation we getting the temperature distribution.

But here also to get the temperature distribution, we need to know the velocity field also. So, then it depends how we are coupling the momentum and energy equation, based on that we can get the different solution algorithm, solution study is also there to get the velocity field as well as temperature field. Now, before apart from this governing equation, you need to know what are the boundary interactions.

So, boundary interaction if we see the top surface, top surface we basically that we balancing the shear stress. So, tau equal to you know that velocity gradient, effective viscosity which is shear stress is basically balanced by the surface tension force on the top surface and because surface tension force normally acts on the surface because on that there is interaction of the arc with the workpiece material but at the same time there is interaction of the shielding gas with the workpiece material.

So, top surface only surface that surface tension force normal acts that is we generally say this is a Marangoni shear stress is acting on the top surface. So, that is balanced by the shear stress so in that form will getting the top surface boundary conditions and of course it corresponds to the momentum equations and then symmetric surface definitely there will be zero flux that means gradient will be 0 always.

Zero flux means the gradient of the variable will be 0 on the symmetric surface and see this surface is defined by this solid liquid interface that liquidus temperature and this is the solidus temperature and between that zone is the mushy zone and of course from the surface if you see the convective and radiative heat loss from the other surfaces and the bottom surfaces it can be conductivity to heat loss or maybe contact with some other (()) (40:38).

And the top surface, the representation of the heat source assuming the Gaussian input from the welding arc. So, that zero flux is the solid liquid interface, zero is the other type of boundary condition that corresponds to the momentum equations and energy transfer, the boundary condition of the energy balance. That means what is the heat input from the surface and what is output that we are making output means this is the heat flux from the surface to the surface interaction.

So, that we can consider input to the domain, this output from that is heat loss by convection and this term is equal to heat loss/radiation and the first term is basically effective heat transport, conductive heat transport on the surface. So, that makes energy balance, so that represents in terms of the energy equation. So, that means this corresponds to the boundary condition of the energy equation.

And this we can solve it with the help of the boundary conditions, we can find out the temperature distribution and of course momentum equation, mass conservation equation and this n-slip boundary condition, zero flux. Of course, all this using this boundary condition, from here we can get the velocity field. So, that means velocity field in the forms particular note point we get the 3 components of the velocity components within the domain.

But of course, we get the velocity within the molten pool only but temperature distribution we can get within the molten pool as well as the whole domain. So, these are the, equations can be solved numerically to get the temperature distribution as well as the velocity distribution specific to the welding processes.

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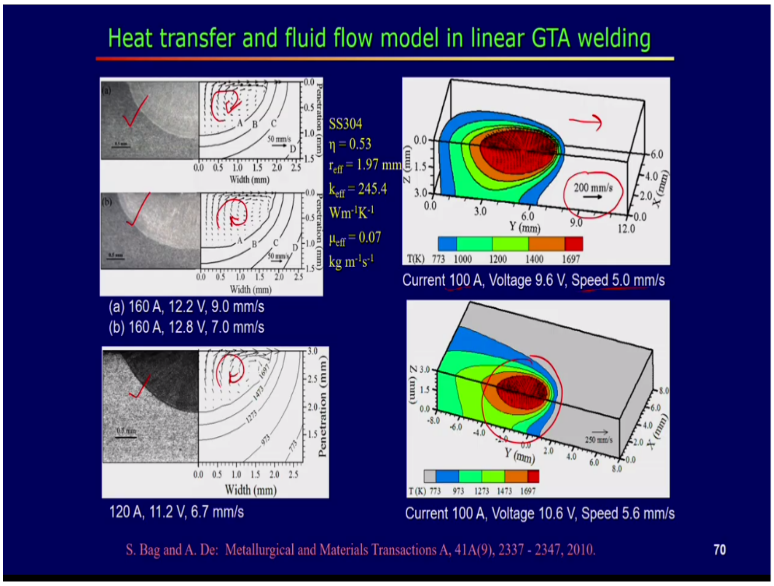

So, we can see that we can see some results also. Here is the heat transfer and fluid flow in the linear GTA welding process. That here the weld moves in particular directions and certain velocity we can see the velocity vector from center, it moves from center to outward periphery in that directions and here also we can get the some kind of profile. So, this corresponds to the welding current 100 amps, voltage 9.6 volt and welding speed 5.0 millimeter per seconds.

And we can get temperature distribution as well as the velocity field by solving the conservation of mass momentum equations. So, similar kind of velocity field we can found out, this is the domain but this here if we see 200 millimeter is the maximum magnitude of the velocity within the small weld pool. So, this is showing it is more clear, so the velocity vector pattern is something like that.

Here also pattern is something like that, here also pattern is something like that. Actually, this pattern depends on the how the surface tension models has been done in these cases as specifically if there is no surface active elements and then liquid molten metal actually flow from center to outward periphery, so that is why we are getting this kind of profile and right hand side is the actual experimental data which we can match with the experimental and the numerical simulation that can be done in this case in the velocity field.

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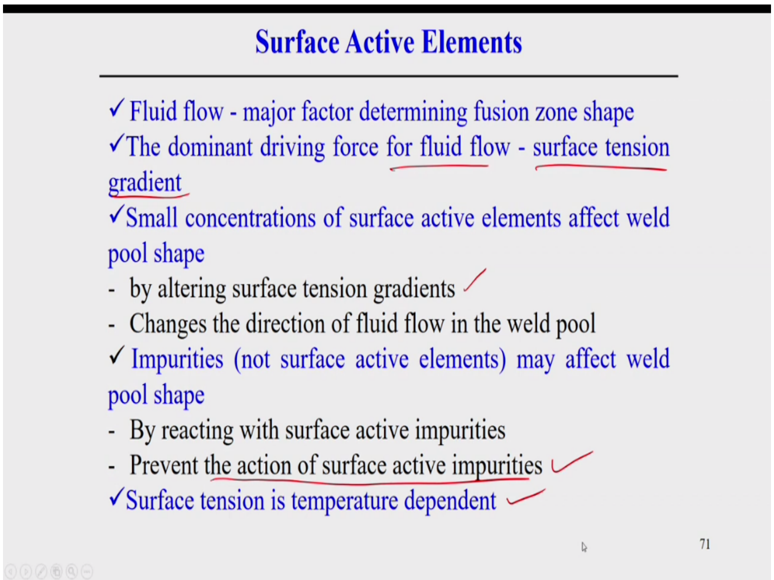

Now, we come to the other aspect which is relevant to the fluid flow phenomena in the welding process because here the fluid flow phenomena actually normally happens is more influenced by the presence of any kind of the surface active elements. So, fluid flow that actually but question is the why we should consider the fluid flow, if we consider the fluid flow first thing is that it actually modify the temperature distributions.

And of course more precise weld pool dimension, prediction is possible if we incorporate the effect of the material flow within the weld pool but more specific if there is effect of the surface active elements that can be better explained by consideration of the effect of the fluid flow. So, dominant driving forces in case of fluid flow is basically surface tension gradient. So, we need to know the surface tension gradient but it is a material specific.

For example, the small concentration of the surface active elements affects the weld pool shape. So, very small percentage surface active elements presents for sulphur or oxygen that actually modify the weld pool shape as compared to the material having no surface active elements. So, but mathematically do simply by modifying the surface tension gradients but if we look into the impurities, impurities are different from the surface active elements.

Because impurities is not the surface active elements but impurities can also affect the weld pool shape because it may react with the surface active impurities and sometimes it may prevent the action of the surface active impurities. So, presence of impurities sometimes suppress the effect of the surface active elements. So, therefore we need to know very specifically the material content basically elements content due to the welding process.

That is more important to identify whether there is effect in surface active elements or not or of course other part is if we consider the surface active elements and if we try to model the surface tension, then it is basically temperature dependent, so it depends on the temperature variation.

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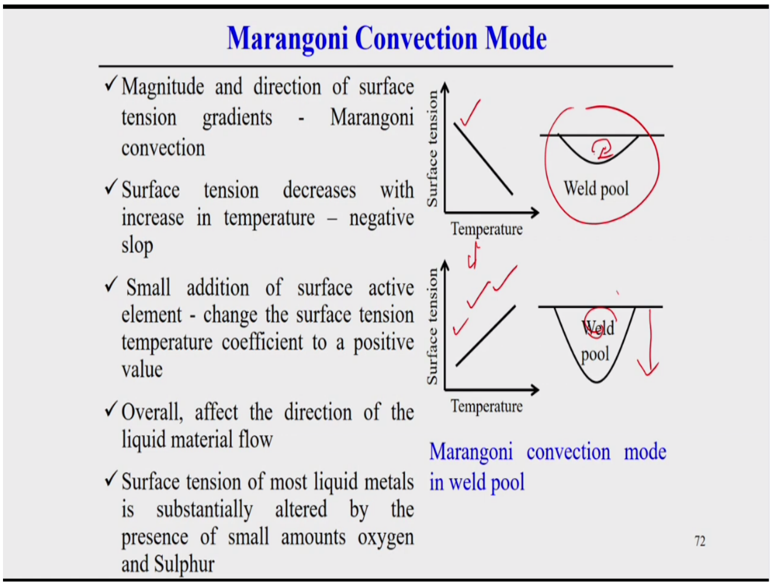

So, based on that there is a different type of the surface tension models, if you look in this is a surface tension model, negative temperature gradient, negative surface tension coefficients. So, weld pool we can expect this kind of the weld pool. If it is positive this way surface tension that it is completely reversed with respect to the first case. So, second one is mainly this kind of profile we can observe if there is a presence of the surface active agents within the weld pool.

And it actually modify the weld pool in this form, so here if you see the depth of penetration is relatively low and width is more because fluid flow from center to outward periphery but the surface tension gradient is this way then there is a reverse of the material flow. So, that actually increases the penetration but narrow the weld width and this is the normal model we consider.

And all this variation of the surface tension with respect to temperature varied practically if there is a presence of the, whether there is surface active elements is actively working or not but practically how we can incorporate the different surface active elements in the welding process.

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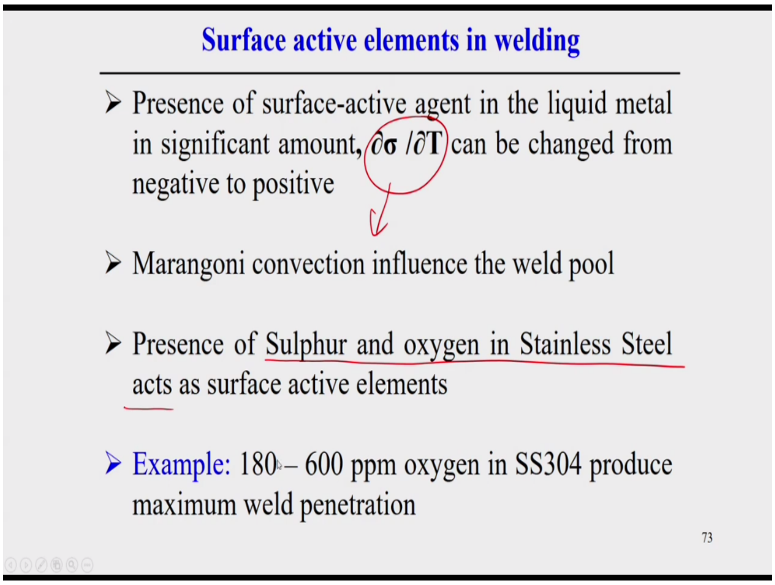

See, of course definitely we look into this mathematical parameter that means how surface tension actually changes with respect to time sorry with respect to temperature and that influence the convective flow liquid metal whether it will flow from center to outward or from outward to center in that directions but practically presence of sulphur and oxygen in stainless steel basically acts as a surface active elements.

Small quantity for example 180 to 600 ppm oxygen present in the stainless steel that actually that small amount of the surface active elements for example oxygen in this case is completely change the weld pool shape but that is better explained in terms first we look into how the surface tension models can be done in presence of the surface active elements, then we need to consider the effect of the fluid flow.

And then after analyzing the fluid flow based on the surface tension model we can look how there is a change of the shape of the weld pool in presence of the surface active elements. Actually, the fluid flow can explain this phenomena of the surface active elements, so in this case only heat conduction model not able to predict the presence of the surface active elements in welding process.

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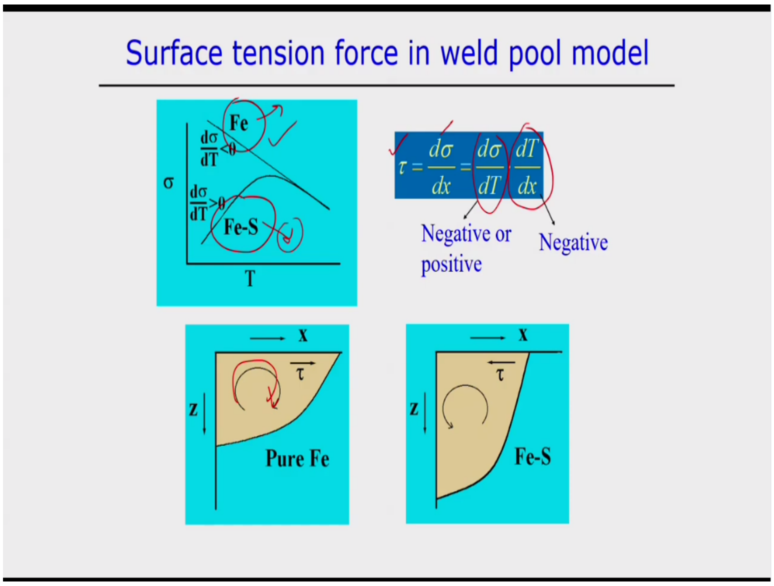

So, here how we can model the surface tension force we can see the shear stress the surface is the gradient in terms of temperature coefficients, see that is negative or positive or this is temperature gradient which depends the shear stress is basically depends on the negative or positive temperature coefficients of the surface tension and the gradient okay and this is the temperature gradient.

So, if it is negative that is we say that there is the surface active elements is not in the weld pool but if it is positive then we can say there existence of the surface active elements. So, based on that we can define we should consider whether we should take positive value or negative value by considering whether there is any surface active elements or not. See there is a change of the temperature coefficient of the surface elements.

It is pure iron system that means if there is no presence of any surface active elements in Fe-S system, this is the typical nature of the surface tension force with respect to temperature and because this is the pure iron, here is the iron and sulphur. So, that means there is presence of the surface active elements in this case. So, how fluid flow it influence the pattern, shear stress is basically acting to outward directions and it nature the flow of the material like this in case of pure iron.

So, that in this case, we can expect the depth of penetration is low and with this high but other cases shear stress just change the sign this directions and presence of the surface active elements and it actually modify the weld pool, the depth of penetration with this is very less.

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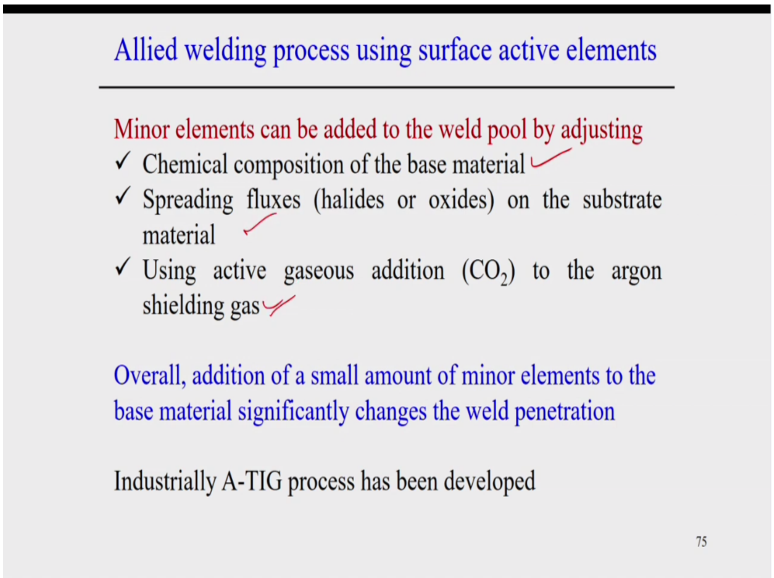

So, actually the theoretical aspect of the presence of surface active elements is based on that there is a development of the allied welding processes using the surface active elements because one advantage is that surface active elements it actually increases the depth of penetration, so in conventional welding process, specifically in arc welding processes if it is possible to incorporate the effect of the surface active elements, then it is possible to enhance the weld depth of penetration.

So, that is the advantage of using the surface active elements, so without knowing what is the adverse effect of the other material properties during the welding process but what are the ways the surface active elements can act and based on that different welding process can be developed just using the concept of the surface active elements. One may be the chemical composition of the base metal.

So, for example, presence of surface active elements already incorporated in the base material. Other options may be using the kind of fluxes, halides or oxides just simply putting a coating on the substrate material and that actually this oxides or oxygen or sulphur presence in that coating that actually act as a surface active element. Other option that using the active gas, so we just mixing CO2 gas with the shielding gas.

So, through the surface active elements maybe supplied simply through the application of the modifying the shielding gas, so that is other way to supply the surface active elements to the domain. Overall, addition of the small amount of the minor elements basically to the base element significantly changes the weld penetration and that we can say this is the effect of the surface active elements and based on that activated TIG process has been developed.

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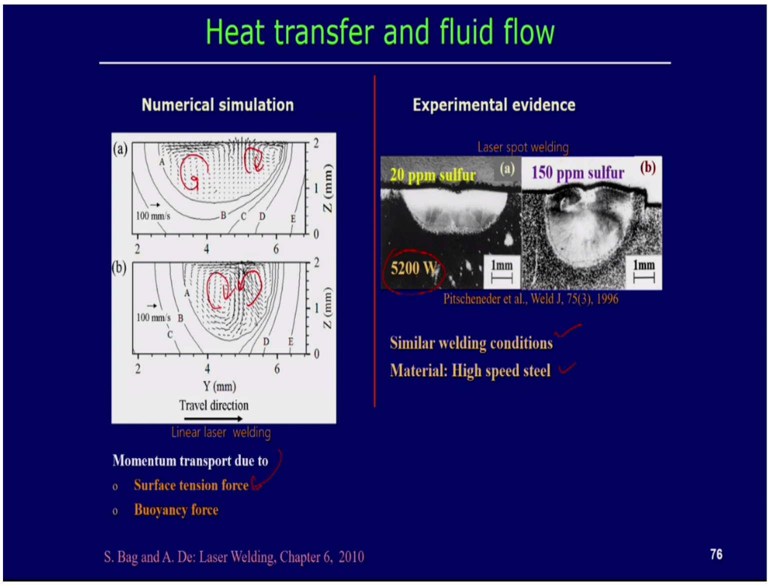

Now, we can see that experimentally what is the effect of the surface active elements, so experimental evidence is a kind of laser spot welding and similar welding conditions, for example power=5200 watt, similar welding conditions and material equal to high speed steel. If you see these two cases, there is a huge change in the weld profile. First cases only having 20 ppm sulphur, second one is the 150 ppm sulphur.

So, first one we see the weld profile widened but depth of penetration is low and second one if you see depth of penetration is actually very high and width is basically narrow but we follow in these cases the similar welding process, similar material has been followed, all parameters are same but there is a change and this change actually happens due to presence of the surface active elements.

So, that quantities although it is very small but that influence the weld profile, so that it is experimental evidence, so similar kind of the phenomena can be of possible to capture if you try to do the fluid flow analysis of the weld pool. Here numerical simulation we can see that similar conditions situation, see the numerical fluid flow analysis, see the flow pattern, material flow is in that directions but here it is see it is in opposite direction in these two cases.

First case is small amount of the sulphur present in the material; second case is huge amount of the sorry 150 ppm amount sulphur present in the second case. Then, we are getting the depth of penetration is high. So, in terms of the modeling approach we can analyze the effect of the surface active elements just by incorporating the effect of the material flow in the modelling approach, not only on the heat conduction based model may not able to predict this kind of the behaviour.

But when you try to incorporate the material flow in the modelling approach, then this phenomena actually is basically introduced through the mainly the surface tension force because buoyancy forces as usual will be acting in the fluid flow in the other driving forces, electromagnetic force also there driving forces but we need to modify the surface tension force model if we try to incorporate the effect of the surface active elements.

And accordingly if we do the fluid flow analysis on these cases, then we will be able to predict the weld pool shape and size. So, that is the importance of the fluid flow specifically in the modelling approach of fusion welding processes. So, thank you very much for your kind attention.