

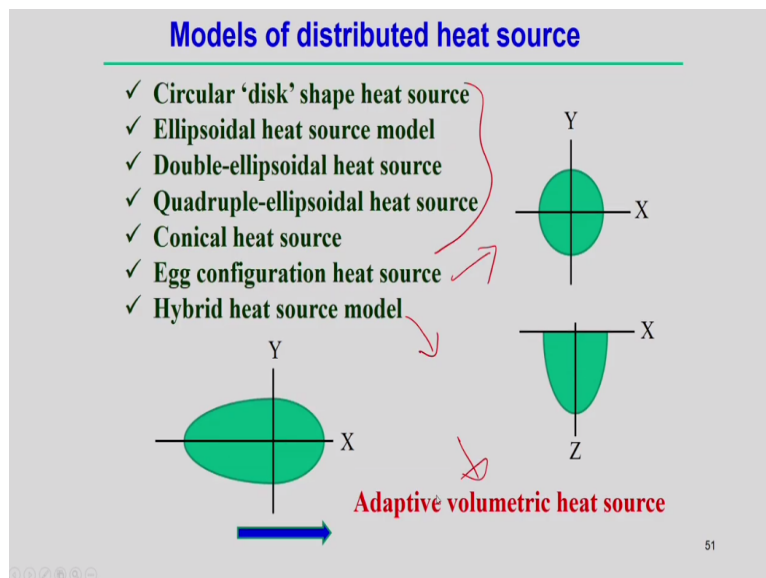
Mathematical Modeling of Manufacturing Processes
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Lecture - 21
Heat Source Model in Fusion Welding

Now, we will discuss about the models of distributed heat source we have so far discussed the different point line heat source that is relevant to the different types of the welding processes but in actual more realistic welding process, the distributed heat source is most suitable, so in that sense we will try to look into the different types of the heat source model. Heat source model means how basically we represent the different types of the heat source used in welding process.

For example, in arc welding process, in laser welding process, in electron beam welding process or sometimes when they try to do the welding using some kind of the dissimilar types of the material. So, in all these cases, the representation of the heat source must be different. So, let us look into that how we can represent all these kinds of distributed heat source.

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First is the circular disk shape heat source, this is the most simplified heat source, it is like that only. There is when arc has been created over the workpiece material; it is focused on a circular area and specifically in case of spot welding that means if we do not use any kind of the moving heat source. So, in that case circular disk type shape of the heat source can be used if you look further.

If the heat source or maybe arc is created over the workpiece but that it is not projected exactly on the normal to the workpiece surface, in that case the heat source can be represented in more the using assumptions that it creates some kind of the ellipsoidal heat source model. That means ellipsoidal heat source model is differently not only on the surface but it is a kind of volumetric heat source.

That means heat energy is distributed over the volume. Apart from that if we use the in case of not stationary heat source rather if you use in case of moving heat source problem, in that case double ellipsoidal volumetric heat source is the more suitable. Similarly, quadruple-ellipsoidal heat source model is more suitable in case of joining of the dissimilar kind of materials along with the moving heat source.

Even conical heat source is suitable in cases for example maybe keyhole mode welding, maybe laser welding process or maybe keyhole mode plasma arc welding process, conical shape the heat source can be used. Apart from that there is a development of the other heat source model that is egg configuration heat source model and of course hybrid heat source model.

So, all this kind of heat source model actually developed depending upon the realizing the situation during the different types of the welding process or different combination of the materials but in general if you look into all this type of the distributed heat source model, one thing is that we try to represent this under some regular geometric shape. It can be over the circular shape or circle or can be over the volumetric shape.

But at the same time, how the heat intensity distributes over this volumetric shape that is also another, that also can be defined in some kind of distribution parameters or we assume certain kind of the distribution just looking into the type of the welding process. So, in that sense that egg configuration heat source model or if you say in general all types of the heat source model, the two things are important.

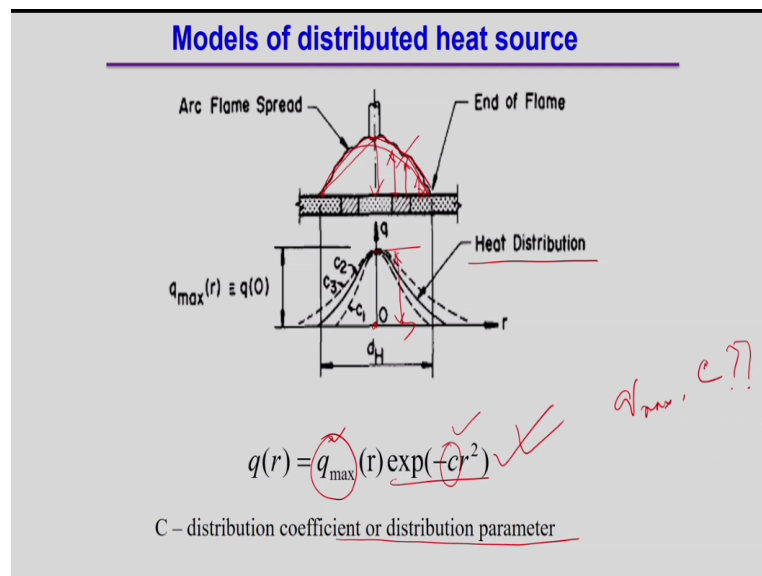
One is the geometric shape and another is the distribution. Distribution means how intensity distributed over the geometric shape. Even we discuss maybe hybrid heat source model are not extensively but in general in principle hybrid heat source modeling also developed by

combining two different types of the geometric shape. For example, certain part on the top surface of the workpiece is following the ellipsoidal or double ellipsoidal kind of the heat source and bottom part is following the kind of conical heat source.

All depends that actual welding process, types of the welding process which material power intensity were using for all these purposes. To represents this situation, we normally use different mathematical models of the different distributed heat source. Apart from that, there is other aspect that is the adaptive volumetric heat source and in that case adaptive volumetric heat source also developed.

Because there is some limitation of all this kind of the heat source model, we need to predefine the dimension of the heat sources, geometric dimension. For example, in case of ellipsoidal heat source model, we need to define all the semi-axes length before start of the simulation but adaptive volumetric heat source actually overcomes this kind of limitation, will discuss in details in the respective slide.

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Now, how we models the distributed heat source, we will try to represents that the distributed heat source in mathematical sense, if you look into the actual process for example if you see there is an arc welding process, it creates arcs this kind of distribution and normally if you observe the shape of the arc is like that, it means that at center the intensity is very high and outward periphery towards the boundary of the arcs the intensity, this indicates the intensity is gradually decreasing.

So, this type of intensity or we can say the distribution of the heat flux, we can say the distribution of the heat flux that actually better described this kind of equation that is called the Gaussian type of heat distribution model. So, if you see the Gaussian distribution how it represents the q_{max} , q_r , r is the radial distance of the particular, so radial distance we measure with respect.

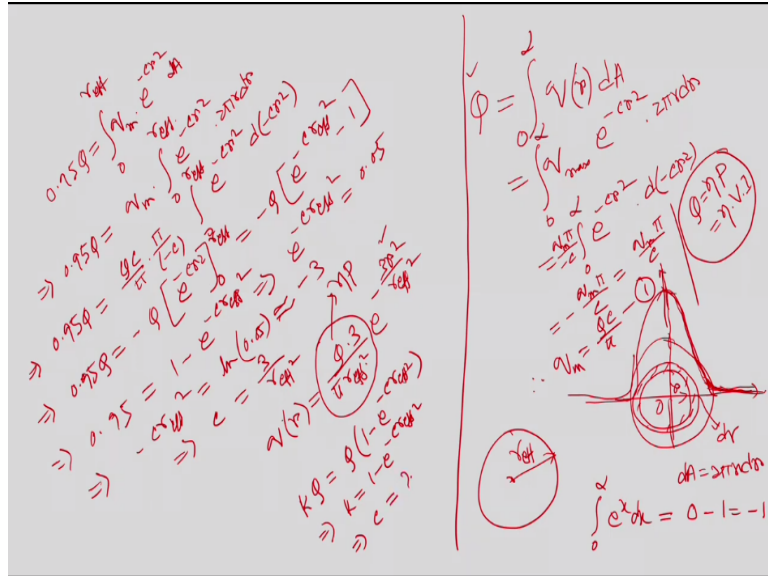
Suppose this is the center point and we measure with respect to the center point the radial distance at radial distance r , the heat intensity is expressed like that q_{max} that is a maximum intensity defined exactly at the center point. So, this actually indicates the maximum intensity and it also depends on the, at a radial distance, it define as radial distance r and then exponentially decaying with respect to the maximum intensity this using this term $c \cdot r^2$.

Actually, c is constant here and c you can say the distribution coefficients or it is called the distribution parameters and r is the variable is the radial distance we measure. So, if you look into the heat distribution what is the influence of the parameters. So, q_{max} we define the maximum heat intensity in a particular point that acts exactly at the center of the heat source and then the distribution actually follows depending upon what is the value of the distribution parameter C .

If you see, if you change the distribution parameters then accordingly the arc may be very steep or arc may be wider depending upon the what is the value of distribution parameter we choose but definitely in this expression if we see that this is the unknown parameters Q_{max} we say and this is another C is the other unknown parameter. Now, how we can estimate this q_{max} , the maximum intensity at the center point and what is the value of distribution parameter c if we follow this Gaussian type of the heat distribution equation.

We will try to look into that. So, we express that what is the amount of the heat generation can be written like that.

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For example, if we see what is the total heat Q is exactly deposited on the workpiece surface that can be represented like that say q which is a function variable r and suppose over the elemental area dA . So, if we project like this suppose this is the circular area over which arc has been created and at the same time if we try to say the intensity distribution is the maximum at the center point and of course this is not exactly touch.

If we follow this equation, it actually asymptotically converge to this axis and this type of equation it is obvious from this, this is the nature of the Gaussian distribution equation. So, in this case $Q = q r \cdot dA$. Here if you see that $q r$ can represent the q max and e to the power $-c \cdot r$ square dA , dA is the, this is the center point and if we consider at a radial distance r and that elemental area at a radial distance r is basically the thickness is dr .

So, dA is basically twice $\pi r \cdot dr$. So, here we can say twice $\pi r \cdot dr$ and integration but if you see estimate now Q actually represents the what is the total amount of the heat deposited to the workpiece and other way we are making equal to what is the total amount of energy actually transferred to the workpiece based on the heat intensity distribution as well as the area over which the heat actually heat energy actually falls to the workpiece.

So, in this case, we put the limit from 0 to infinity because infinity in the sense that this curve intensity actually not exactly met with the boundary rather it tends to, it do not converge to the axis, so in that sense we put the 0 to infinity. Now, it is 0 to infinity you see and here if you can estimate that 0 to infinity q max outside and we can represent $-c \cdot r$ square d of $-c$ square and of course this we adjust q max $\cdot \pi / -c$.

So, it represents that $-\frac{q_m \pi}{c}$ and then we can look into the integration in other way that 0 to infinity $e^{-c} dx$ can be written as $\frac{1}{c}$ that means $\frac{1}{c}$. So, here it becomes $\frac{q_m \pi}{c}$. So, from here we can find out that the maximum intensity is $\frac{Qc}{\pi}$ but still we can estimate the maximum heat intensity just looking into what if we assume Q is known but of course Q can be calculated from other way also.

Q can be estimated that which is energy (J) (13:18) that can be equal to the laser power for example laser power directly we can measure it but it corresponds to the efficiency term or Q can be in case of arc welding, the efficiency what we measure in the volt and ampere current with the machines that represents the Q . So, Q can be measurable quantity of course if efficiency is defined.

So, in that sense, the maximum intensity depends actually distribution parameter as well as what is a power supplied, actual power supplied to the workpiece or maybe send the energy supplied to the workplace. So, but still this equation say this is equation number 1, there is one c still unknown in this case but how we can estimate the value of the c . If we look into other way also, suppose the energy force and that can be estimated over the circular arc and suppose this is the center point and this distance is basically effective radial distance.

So, we can say that effective radius of the arc for example if we define the boundary and accordingly if we define the effective radius of the arc and suppose it is defined. So, in that case, we can say that Q can also be calculated in other way that we say that 95% of energy is defined and that is defined over the effective radius r and then in that case it can be estimated that $Q_{max} = \frac{Q}{\pi} \int_0^r e^{-cr} dA$.

So, within the boundary total 95% of the energy force and remaining 5% outside of the boundary because this nature of the Gaussian distribution because it never converse to this axis. So, if you assume this, therefore here you can estimate the $0.95Q$ is equal to similar way we can estimate that q_m integration 0 to r effective $e^{-cr} dA$ twice πr , $0.95Q$ that is equal to and q_m already we have estimated the $q_m = \frac{Qc}{\pi}$.

And of course $\frac{Q}{\pi c}$ has come out here and we can say 0 to r effective $e^{-cr} dA$ twice πr , $0.95Q$ that is equal to and q_m already we have estimated the $q_m = \frac{Qc}{\pi}$.

integration e to the power $-cr$ square 0 to r effective, it indicates that $-Q$ e to the power $-cr$ effective square -1 that indicates $0.95=1-e$ to the power $-c*r$ effective square. We do further so e to the power $-cr$ effective square $=0.05$.

If you take logarithm both side, then $-cr$ effective square $=0.05$ logarithm that is equivalent to -3 . It indicates that $c=3/r$ effective square. So, this way we can estimate if we follow the Gaussian distribution disc heat source model and that because it is about the surface, then we can represent the distribution equation $q_{max}=Q_c/Q_c=3/r$ effective square $*\pi$ and e to the power $-c3r$ square $/r$ effective square.

So, this as the complete distribution equation where all parameters are known to us, of course if we see the maximum intensity actually that represents the heat flux that means heat energy over per unit area and that unit area which is $3Q/\pi r$ effective square that is a maximum intensity $*e$ to the power $-3 r$ square $/r$ effective. Here r is the variable and r effective is a different.

So, that means suppose in arc welding process we are using only on the surface heat flux model. So, in that case, this parameter if you know Q actually represents the power, already discussed the Q can be this, are the equations of the Q . If you know the power supply from the laser and if we multiply by the efficiencies that is the effective value of the Q and that we represent the Q in that way.

So, Q can be represents that efficiency $*power$ p in that way also or Q can be represented by efficiency $*voltage*amps$ that is current and then this defines the distribution equation following the Gaussian distribution. Of course, if you look into this expression that c actually defines $c=3/r$ effective square but this c distribution parameter is defined if we assume the 95% of the energy force within the effective arc radius.

But if we change this parameter, if we say no instead of 95% say fraction K actually deposited within the, if we assume the fraction K is deposited within the effective radius. So, in that case, we can find out the KQ equal to that $Q*1-e*e$ to the power $-cr$ effective square. So, we generalize value of the thing so K can be represented like that $1-e$ to the power $-cr$ effective square. So, from here we can represent c in terms of the K value.

It means that c can be the distribution parameter actually depends what fraction or percentage of the value we are considering the energy within the effective radius. It can be also different value but what may be the different value that we can estimate by this assumption instead of taking 95%, we can take the some other value and we can estimate what is the value of the C corresponding to that.

So, this is about the surface heat flux distribution following the Gaussian distribution. We can use this heat flux distribution equation and probably we can develop any kind of the thermal simulation model in case of the fusion welding process. Now, what will happen but one assumption this was this is actually we are using in case of the stationary heat source that means heat source is not moving.

But if heat source moves then whatever we can represents that the heat source distribution or heat flux density distribution. Of course, I have written here the ellipsoidal heat source model, this ellipsoidal heat source, we start with the ellipsoidal heat source model and of course it is a volumetric distribution rather the surface distribution what I discussed earlier.

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Ellipsoidal heat source model

$w(x,y,z) = w_m \cdot e^{-Ax^2} \cdot e^{-By^2} \cdot e^{-Cz^2}$
 A, B, C
 $Q = \int_{-a}^{+a} \int_{-b}^{+b} \int_{-c}^{+c} w_m \cdot e^{-Ax^2} \cdot e^{-By^2} \cdot e^{-Cz^2} dx dy dz$
 $= 4w_m \int_0^a \int_0^b \int_0^c e^{-Ax^2} \cdot e^{-By^2} \cdot e^{-Cz^2} dx dy dz$
 $= 4w_m \int_0^a e^{-Ax^2} dx \int_0^b e^{-By^2} dy \int_0^c e^{-Cz^2} dz$
 $= 4w_m \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{1}{\sqrt{A}} \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{1}{\sqrt{B}} \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{1}{\sqrt{C}} = \frac{\pi\sqrt{\pi} w_m}{2\sqrt{ABC}}$
 $w_m = \frac{2Q\sqrt{ABC}}{\pi\sqrt{\pi}}$
 $\int_0^{\infty} e^{-Ax^2} dx = \frac{\sqrt{\pi}}{2} \frac{1}{\sqrt{A}}$

So, suppose the heat density distribution is something like that it is distributed over an ellipsoid and suppose this is axis x , y and suppose this is axis z and this define the ellipsoid, so further axis length say a , semi axis length along the x -axis is a , semi axis length along y -axis it is b and in z direction the semi axis length is suppose c and this of course this is the geometric shape of an ellipsoidal with the geometric parameters.

That means the semi axis length a, b and c. Now, we assume that heat density distribution of course it is a volumetric density distribution. So, we can say that q_x , q_y and q_z = the maximum intensity and e^{-Ax^2} along x-axis along y-axis e^{-By^2} and along z-axis e^{-Cz^2} . It also follows the Gaussian distribution but here A, B, C is actually the distribution parameters.

And it follow the Gaussian distribution over these things, so Q_{maximum} is the volumetric heat flux intensity at the center that means maximum value so at the center. Suppose, this is the defined center A. Now, here we can see we follow the similar kind of strategy and we can find out what is the value of maximum intensity and what is the value of A, B, C this distribution parameters.

So, let us look into that, in case of volumetric heat source and if we assume the geometric shape as the ellipsoidal, so we can say this is ellipsoidal heat source model, how we can estimate all these parameters. So, suppose the total energy force within this geometric shape of course it is distributed over the volume and that is represented that energy equal to Q. So, the Q can be represented that $e^{-Ax^2 - By^2 - Cz^2}$ and over the element dx, dy and dz .

And of course, if we see the axis length, it is along x-axis, it extends from $-\infty$ to $+\infty$, along y-axis it extends to $-\infty$ to $+\infty$ but along z-axis it is 0 to infinity. So, this equivalent to that 4 that means symmetric nature 0 to infinity, 0 to infinity and 0 to infinity $e^{-Ax^2 - By^2 - Cz^2} dx dy dz$. Now, we assume that distribution parameter actually independent along x, y and z directions.

So, we can write this equation say $\int_0^{\infty} e^{-Ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{A}}$ $\int_0^{\infty} e^{-By^2} dy = \frac{\sqrt{\pi}}{2\sqrt{B}}$ $\int_0^{\infty} e^{-Cz^2} dz = \frac{\sqrt{\pi}}{2\sqrt{C}}$. So, we can simplify this equation, we can if you know the indefinite integral and if you look at this expression that $\int_0^{\infty} e^{-Ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{A}}$. So, this indefinite integral you can use this, we can use this formula here.

We can say that $\frac{\sqrt{\pi}}{2\sqrt{A}}$ correspond to this term, second root $\frac{\sqrt{\pi}}{2\sqrt{B}}$ and root $\frac{\sqrt{\pi}}{2\sqrt{C}}$. So, that is equal to $\frac{\sqrt{\pi}}{2} \sqrt{\frac{1}{ABC}}$. So, similar way we can find out that of course this should be a component by the one term that is maximum intensity. So, into

qm, so from here we can find out what is the maximum intensity value here. So, qm can be represented like that twice Q root over ABC/pi root pi.

So, here also ABC are unknown and of course to find the actual value of the maximum intensity, we need to know what are the distribution parameters.

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Now, distribution parameters can be estimated in other way also that if you see the intensity we represent in this way that this along x-axis is the boundary of the arc a, 0, 0 that means x, y, z; y=0, z=0 and x=a. The intensity is in terms of maximum intensity of only 5% of the maximum intensity that means 0.05. So, that means if we look into this expression that actually at the boundary the intensity is only 5% of the maximum intensity.

If we follow this assumption, then we can say that q at the boundary along x-axis equal to this value maximum intensity*0.05 that means 5% of the maximum intensity. So, here if we see that it is equal to qm*e to the power -A*a square remaining term will be equal to I think will be equal to 0 sorry will be equal to 1. So, that is equal to 0.05 into qm, so from here we can find out that e to the power -Aa square=0.05.

So, here take the logarithm both sides, so -A*a square (()) (29:58) logarithm of 0.05=-3 so approximately. So, A equal to basically 1 by sorry 3/a square. So, the distribution parameters can be estimated like that equal to 3/a square because in this case definitely the ellipsoidal heat source model, the geometric parameters AB should be defined then we can estimate what is the distribution parameter.

So, of course here we assume that at the boundary only, at the boundary the intensity, heat flux intensity is only 5% of the maximum intensity. If we change these assumptions in that case, the distribution parameters expression can be different, maybe this 3 value can be some other value depending upon whether what you can say -2 only 2% or 1% or maybe more 10% falls at the boundary, based on that we can find out the distribution parameter.

Similarly, along y-axis and z-axis, we can roughly estimate the B is not exactly equal to 3 but approximately $3/b^2$ square, similarly C the distribution parameter is $3/c^2$ square. That means in terms of the geometric parameters of these things. So, finally we can write the heat flux density distribution that q_m . So let us see what is the expression for Q maximum intensity, we can estimate the maximum intensity here.

If we see twice Q root over $ABC \pi$ root pi, so here twice Q root over ABC that means $3 \sqrt{3/\pi} \sqrt{abc}$. Now, we can say the intensity distribution, so $6 \sqrt{3} Q/\pi \sqrt{abc} e^{-3x^2/a^2 - 3y^2/b^2 - 3z^2/c^2}$. So, these are the heat flux density distribution that is distributed over the volume. Of course, this maximum intensity is defined the volumetric heat flux density.

That means the heat flux is defined over the per unit volume and the distribution see that all parameters are defined and of course in this case if you see the x, y and z are the variables, so when we try to define the Cartesian coordinate system, we assuming the ellipsoidal heat source model and of course if we go back to that picture, see in this case we assume that A, B and C are the axis of the length.

Actually, A, B, C can be defined depending upon the welding process and of course most of the cases the parameters are defined from the experimental measurement of the weld dimension. In other sense, all this ABC parameter has to be predefined, then only we can estimate the heat flux and we can implement this heat flux density distribution in case of the thermal simulation process.

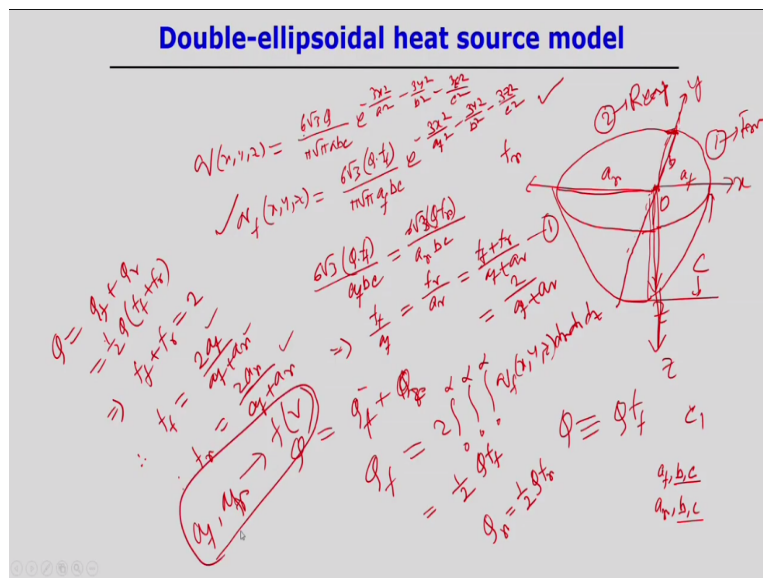
But of course this ellipsoidal heat source model still it is applicable in case of stationary heat source and if arc is tilted or such that it creates this kind of the weld pool profile can be created this kind of profile which is equivalent to the ellipsoidal in shape but apart from that

in actual practice when you try to look into that if the heat source moves in particular direction then what we represent the volumetric heat flux or volumetric, how we can define the volumetric heat source model?

So, in this case, the most suitable or most widely used the heat source model in welding process simulation that is double ellipsoidal heat source model. So, double ellipsoidal heat source model, it is nothing but the merging of the part of the two different ellipsoids and then we create the double ellipsoidal heat source model to represent the situation when there is a moving heat source.

Then, the heat source moves, arc moves in a particular velocity and of course the shape and size of this double ellipsoidal volumetric heat source model depends it should be a function of the welding velocity but geometrically how we can define like that.

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So for example heat source moving suppose x direction, so in this case this is ellipsoid 1 here the parameters are like that for example a_1 is the semi axis length along x-axis and say suppose this is b and along the z-axis it is c for example and this is the other part remaining part, this another part of the ellipsoid which we merge this thing second ellipsoid, here also we having that this common parameter.

But these parameters are different, this is the a_1 and of course the other parameter B and C parameters are same and of course in this case it follows the C_1 continuity. That means slope at this point is the same, although 1 and 2 are the two different ellipsoid having the geometric

dimension, first ellipsoidal having the geometric dimension a , b and c . Second ellipsoidal model, here the geometric parameter a_r , b and c .

So, b and c these two parameters are common for both the cases such that it maintains the c_1 continuity. That means slope at this point is not different, it is the same. So, this kind of heat source model we generally say the double ellipsoidal heat source model in welding simulation and of course why we have taken the different value of a_f and a_r to just simply represents the temperature gradient in the front and the rear part of a particular arc may not be the same.

And it happens when the arc actually moves in a particular direction, so this is the way to represent the heat source but if you look into how we can estimate based on the energy balance over this ellipsoidal volume and what is the supply power or energy from the certain kind of the heat source, I will merge it in the similar way we can do calculate for what we have done in case of the ellipsoidal heat source model.

But of course in this case, we can utilize the expression of the single ellipsoidal heat source model. So, what we got in single ellipsoidal heat source model q_x , q_y , and q_z that is equal to maximum intensity we estimated the $\frac{6\sqrt{3Q}}{\pi\sqrt{abc}}$ and this happens e to the power $-\frac{3x^2}{a^2} - \frac{3y^2}{b^2} - \frac{3z^2}{c^2}$. So, this is the expression with this thing for ellipsoidal.

Similar expression we will be using here but little modification, so two modifications we can do. First is that for example we will consider suppose r_1 is the front we can say the front ellipsoidal and r_2 is the rear ellipsoidal. So, front and rear these two cases we say that front intensity distribution in case of ellipsoidal x , y , z that is equal to here we modify Q just simply $Q \cdot f_f$ and of course in this case $\pi\sqrt{\pi}$.

And we modify a because here the a parameter different say a_f , b and c are the same, e to the power $-\frac{3x^2}{a_f^2} - \frac{3y^2}{b^2} - \frac{3z^2}{c^2}$. So, here we incorporate $Q \cdot f_f$, actually f_f accounts the fractional heat deposited in the front part which should be different from what was in fractionally deposited in the rear part. So, instead of Q we just represent in this case Q is equivalent to $Q \cdot f_f$.

So, we just compatibility of the heat intensity we can find out what is the value of f_f and similar way we can say the fractional heat deposited in the rear part that can also be represent in the f_r and similar kind of expression we can found out and this indicates, this expression actually indicates that the heat intensive distribution in the front part of the ellipse. Similarly, accordingly we can find out what is the heat intensity distribution for the rear part of the ellipse.

But of course at the center point, the intensity should be the same, so if you follow this strategy the intensity should be the same. So, that means in these cases, the $6 \sqrt{3} Q f_f$ the maximum intensity should be the same by $a f b c$ that should be equal to $6 \sqrt{3} Q f_r$ and $a r b c$ should be the same. This indicates the $f_f/a f = f_r/a r$, $f_f + f_r/a f + a r$ like that. Suppose, this is equation 1 we got.

Now, what we are doing, we are trying to find out what this should be the f_f and f_r value in specific to the double ellipsoidal heat source model. Now, if you estimate the value of the total heat intensity Q say Q is the two components total heat input to the domain, it consists of the Q_1 and Q_2 or we can say the Q_f and Q_r , Q_f is the front part, Q_r is the rear part. So, Q_f can be estimated that the front part what is the amount of the energy in the similar way.

But in this case, we can say 0 to infinity 0 to infinity 0 to infinity but $2 * Q_f$ intensity distribution $x, y, z * dx dy$ and dz . So, here if we find out the if you look into if you do the integration in the similar way what we discussed earlier also, so similar strategy if we follow here we will be able to find out, this is equivalent to actually $1/2$ of $Q * f_f$. Similarly, if you look into the Q_r , here we can estimate it should be equal to $1/2$ of $Q * f_r$.

Now, total $Q = Q_f + Q_r$, this is equal to $1/2$ of $Q * f_f + f_r$. It indicates that $f_f + f_r = 2$. So, now if we are using the equation 1, if we use the equation 1 $f_f + f_r = 2$, $2/a f + a r$, so from here we can find out $f_f = \text{twice } a f/a f + a r$. Similarly, f_r we can find out that $a r$, twice $a r$, $a f + a r$. So, it indicates that in case of double ellipsoidal model we follow this we assume that at the center point that in maximum intensity should be the same and that actually increase the, then it gradually varies from both the sides of the ellipsoidal from maximum to some other value.

So, that we accounts we use the similar strategy and we use the expression for the ellipsoidal heat source model and then using the similar expression we can found out that f_f and f_r

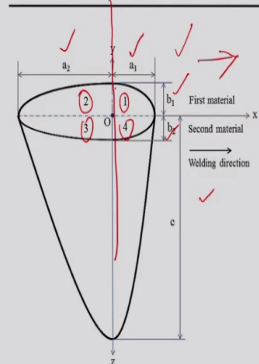
expression in terms of the geometric parameters of the double ellipsoidal. So, actually here all the geometric parameters need to know a_f and a_r and then based on that we can find out what is the fractional deposit in the front and what is the fractional deposit at the rear part.

And accordingly we can use this expression for the front part, how heat flux density distribution in the front part and what way it varies in the rear part also but of course the a_r and a_f should be defined but in actual problem this defining the a_f and a_r theoretically is not possible. So, most of the cases, we assume some rate ratio or certain value a_f and a_r and based on the assumptions we can find out that what is the fractionally deposited in the front and rear part.

And then we do the simulation all this process but of course the value of a_f and what is the value of a_r , it should be actually a function of the weld velocity but it needs a lot of trials and of course at the same time we need to compare the results with this experimental data and then we can find out what is the value of the a_f and a_r in case of the simulation using the double ellipsoidal heat source model.

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Quadruple-ellipsoidal heat source model



- Non-symmetry in energy distribution due to
 - a moving heat source ✓
 - welding of two different materials having different thermo physical properties ✓
- Double ellipsoidal heat source model accounts moving heat source only ✓
- Double ellipsoidal model is extended to Quadruple ellipsoidal heat source model to accounts non-symmetry energy distribution for dissimilar materials ✓
- Consists of part of four ellipsoids ✓
- Depth of penetration remain same ✓
- Maintain C^1 continuity ✓

Follows
Gaussian distribution

Now, we can shape in the continuation in the similar way, we can shape to the quadruple ellipsoidal heat source model but why it is applicable? So, like double ellipsoidal heat source model instead of taking two ellipsoidal merging in this cases in the four ellipsoidal we can merge and we can say this can be a quadruple ellipsoidal model but practically when it can be used, so this can be used in the specifically that welding of the two different types of materials.

Actually, the thermal properties or other mechanical properties or we can say thermo physical properties of all the two materials are different. So, that will create the weld profile not in symmetric in the both the sides of the ellipsoid. For example, not symmetric with respect to the velocity vector, so suppose look into this picture, here this is the x and this direction the heat source moves and suppose this side is the material 1 and the other side is the material 2.

So, with respect to these velocity vectors and the both the sides, this weld profile may not be the same or should be different and that is very obvious if there is because there is a difference in the thermal properties. So, this kind of difference also there, at the same time since it moves in a particular direction, it is also non-symmetric with respect to this. So, right side and the left side the profile may not be symmetric because of the moving, it is the heat source moves.

So, that means if you consider these two type of the non-symmetric, then it is better to represent the heat source by combining the four ellipsoidal, part of the four ellipsoidal and to make a heat source model that is we have defined, this is a first ellipsoid, second, third and fourth, merge it and here you see the a_1 is the parameter, this side is the a_2 are different, this side b_1 and b_2 are different but c is the same depth is that we consider the same.

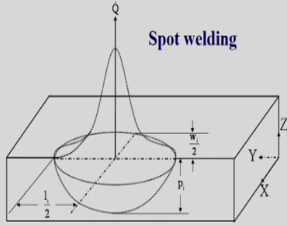
So, then in this physical situation, it is possible to develop the quadruple heat source model when you try to look into the simulation for the dissimilar materials. So, here if you see the non-symmetric in energy distribution, it is a moving heat source and of course different thermo physical properties is reason, so double ellipsoidal heat source model accounts the moving heat source only but quadruple ellipsoidal both moving heat source as well as the different material dissimilar combination of the materials.

And of course consists 4 parts of the ellipsoids and depth of penetration remains same in this cases also we maintain the c_1 continuity to that means slope at the when you merging two different ellipsoids at the point, the slope should be maintained, the slope should not be defined. So, the similar kind of calculation energy density distribution just what we do for double ellipsoidal heat source model.

So, following the similar strategy, we can do all this calculation in case of the quadruple ellipsoidal heat source model.

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Adaptive heat source model



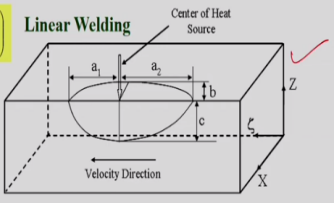
Spot welding

Limitation:
predefinition of heat source parameters

a, b, c

$$\dot{Q}(x, y, z) = \frac{6\sqrt{3}f_{int}P_w\eta_{ad}}{\pi\sqrt{\pi abc}} \exp\left(-\frac{3x^2}{b^2} - \frac{3y^2}{a^2} - \frac{3z^2}{c^2}\right)$$

$a = l_1$
where $b = \frac{w_1}{2}$
 $c = p_1$



Linear Welding

Center of Heat Source

Velocity Direction

Strategy to implementation in FEM based analysis

Now, we come to that point and the adaptive volumetric heat source model but of course before explaining these things what are the limitation of all what we discussed, double ellipsoidal or single ellipsoidal heat source model, the main limitation is that before start of the simulation, that means we need to define what is the value of all these geometric parameters A, B, C all these parameters we need to define.

And then based on that the heat source model is static in nature and we calculate the density distribution and we actually give as an input to the model and then output we get the temperature distribution. So, that is the that conventional strategy we follow but in case of adaptive volumetric heat source, here it can be we can avoid or maybe we can remove that limitation what we did in case of the double ellipsoidal heat source model.

So, how it works for example, we use in spot welding process we start with the these things, we start with the small load step or within the small time step, apply what is the heat flux intensity to the domain and based on that we estimate what is the value of the dimension of the weld pool say depth, penetration and width and accordingly with this dimension, we estimate what is the heat flux intensity.

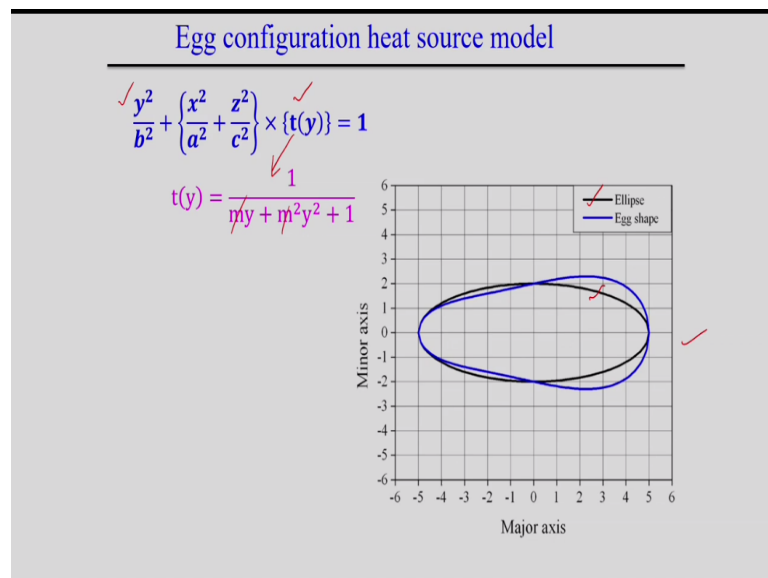
Of course, distribution remains the same but heat flux intensity, maximum intensity vary actually. So, we define the maximum intensity and the domain over which it distributed that

actually vary with respect to the time step as the same in case of linear welding process it actually varies with respect to the load step value. So, that means each and every load step or each and every time step we just keep on updating the value of the geometric parameters a, b and c which is not fixed.

Keeping updated we just take that one step what is the value of a, b, c estimate and next step we implement this a, b, c value to estimate what the heat flux density distribution and maximum intensity. So, gradually it is since this is varying in nature so that is why it is called the adaptive volumetric heat source model and of course this kind of heat source models we need to follow some kind of strategy.

And we need to integrate with the finite element analysis or maybe other kind of commercial numerical techniques; we have to integrate this strategy to get the solution.

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So, apart from the adaptive volumetric heat source, there is a development of the egg configuration heat source, the concept is like that we need to consider that ellipsoidal equation of the what the geometric shape of ellipse and but this term if you modify or introduce some functional form as a function of y, so if you see the y separate and here the functional form as y, then it actually modify the ellipsoidal equations.

And then it looks like a shape of an egg, so like that for example here this figure ellipse figure, equation of ellipse. If we introduce the ty in this equation and if we take some functional form of the ty, of course ty in that way so introducing some value of m and then

there is a shape actually changes and that shape changes and looks like the oval shape or we can say that kind of egg configuration heat source model.

And this is the more other realistic way to define the heat source, the actual shape of the weld pool just what we look and it is possible to implement the flux density distribution, what is geometric shape and all this can be done under the development of the egg configuration heat source model. So, with this heat source model and we can do all this heat source model of course so far we discussed the different types of the heat source model, all are distributed heat source model.

But using this distributed heat source model to get the temperature distribution analytically is really not possible or almost impossible until and unless we do some assumptions but all types of the distributed heat source model is normally we apply in case of the simulation, specifically temperature simulation by following some kind of numerical techniques. For example, finite element or it can be finite volume or it can be finite difference techniques.

But this analysis or maybe discussion of this kind of simulation process in welding, we try to restrict only on the very basic model that means temperature distribution using by some kind of analytical means, we will not follow any kind of the numerical techniques here.