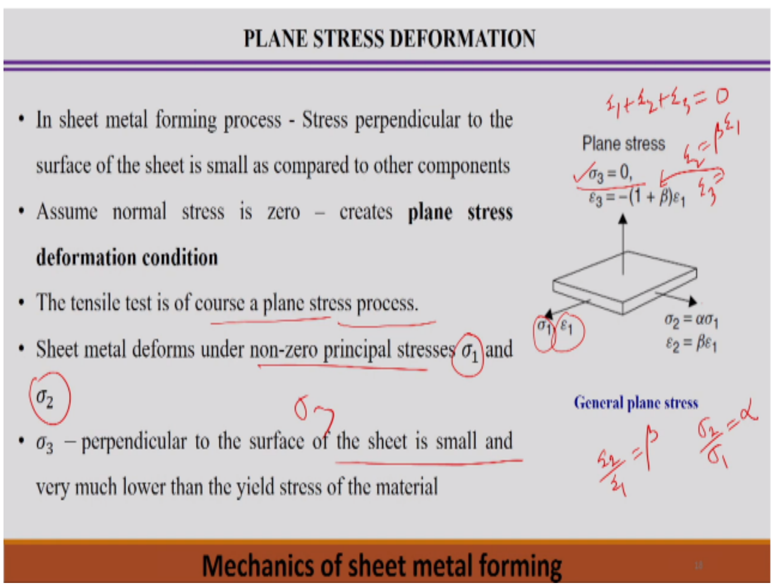
Mathematical Modelling of Manufacturing Processes
Swarup Bag
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture - 15
Mechanics of Sheet Metal Forming-1

So after discussing the bulk metal forming processes, today I will discuss the mechanics of sheet metal forming processes. So mechanics of sheet metal forming process is a little bit different as compared to the bulk material deformation. So bulk metal deformation process, we normally handle the three dimensional stress state and then but in case of sheet metal forming, the two dimensional state of the stress is sufficient to explain the state of the stress or deformation behavior during the sheet metal forming processes. So in sheet metal forming process.

(Refer Slide Time: 01:04)



We can say the normally stress perpendicular to the surface of the sheet is small as compared to the other components and in that cases we can assume the stress is equal to 0 normal to this surfaces and that is called the plane stress deformation condition normally prevails in case of sheet metal forming processes.

So if we look into this picture also we can see the stress state the of course this stress state, we assuming the sigma 1, principal stress and epsilon 1, the strain component and this sigma 2 but we can define the in this cases sigma 2/sigma 1 is actually equal to alpha and epsilon 2/epsilon 1

=beta. So it defined the stress ratio or strain ratio epsilon 2 epsilon 1=beta or sigma 2/sigma 1=alpha.

Such that the sigma 2 is actually acting this direction and sigma 3=0 acting perpendicular to this but it is not necessary in plane stress deformation condition. Sigma 3 we assume is 0 but that does not mean the strain component will be 0, the strain component will be different. If we look into the before and after deformation the total material volume remains the same.

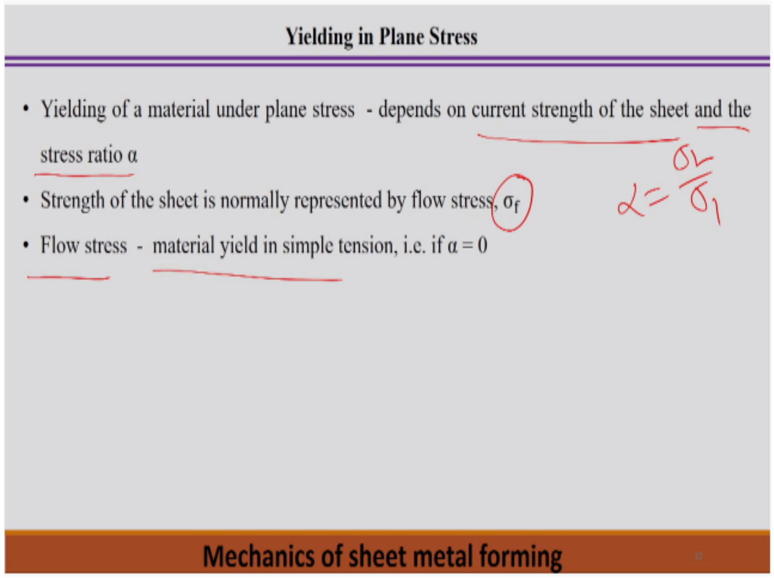
Then we can see that epsilon 1 the strain component, epsilon 2 and epsilon 3, the three component of the strain equal to 0 and that comes from the conservation of the volume before and after deformation. Now it is obvious that if we put epsilon 2=beta*epsilon 1 then epsilon 3 will be coming to this expression that -1+beta*epsilon 1.

So that we can see the plane stress condition deformation situation that the sigma 3=0 and sigma 1 and sigma 2 other principal stresses that is non-0. And similarly but all epsilon 1 epsilon 2 and epsilon 3 all the strain components are non-0 in this case. Now of course in this case the tensile test is of course of a plane stress process we can consider in these cases and sheet metal deform under the non-0 principal stresses that is sigma 1 and sigma 2 these are the non-0.

And of course sigma 3, we assume it is a very small as compared to the other two stress component so that is why we assume as the 0. And of course other way also we can see the sheet is that perpendicular to the sigma 3 it is very much lower as compared to the yield stress value of the particular material. So therefore we neglect the effect of the stress and we assuming the sigma 3=0 in a state of the stress or maybe we can say this is the plane stress deformation condition.

And that this plane stress deformation condition actually prevails in case of sheet metal forming process.

(Refer Slide Time: 04:09)

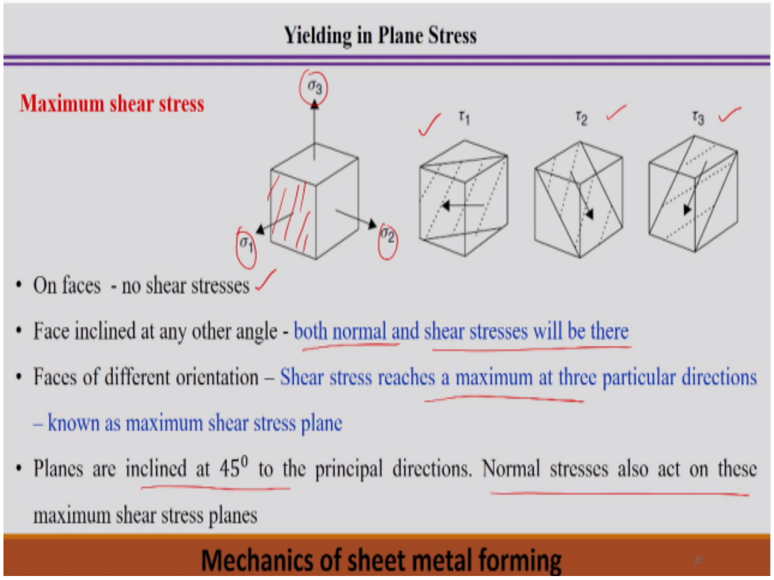


Now yielding in plane stress conditions yielding up a material under plane stress condition depends on the current state of the stress, strength of the material and of course this current strength we are talking about the current strength material because the material we were having some strain hardening effect. So that is why with further straining the strength level actually increases.

And of course it depends on the stress ratio, stress ratio alpha which is defined by sigma 2/sigma 1. Strength of the sheet is normally represented by the flow stress value. Actually in sheet metal forming, we can represents that the strength of a particular sheet in terms of the flow stress value of a particular flow stress we can in general we can define the flow stress value it is the stress required to or continue the plastic deformation in a particular deformation process.

So therefore flow stress is basically material yield point, flow stress which is=material yield in simple tension and of course in that case if alpha=0 that means in that situation we can say it is it can be the material yield in the simple tension. There also flow stress equal to 0 we can assume that.

(Refer Slide Time: 05:31)



Now we try to understand the maximum shear stress and if we look into the a material point. In that particular material point, state of the stress maybe the stress tension sigma X, sigma Y, sigma Z and the shear stress component but we represent this normal stress and shear stress components in terms of the principal stresses. Such that this principal stresses sigma 1, sigma 2 and sigma 3 is acting of a particular element.

Now we can visualize realize the what are the stress state exist when a particular element during the analysis process. So on faces so that means on any faces actually there is no shear stress on these faces that faces means the principal stresses acting normal to that particular faces so that means sigma 1. So corresponding to this all these faces, there is no shear stresses that is the first point.

Second point face inclined at any other angle. So that this face is inclined any other angle then that plane is subjected to both normal stress as well as the shear stress both will be there. Now if we orient the faces in different angle so that means with respect to the faces of different orientation, we can say the stress reaches at a maximum reaches a maximum value at 3 perpendicular directions and that is known as the maximum shear stress plane.

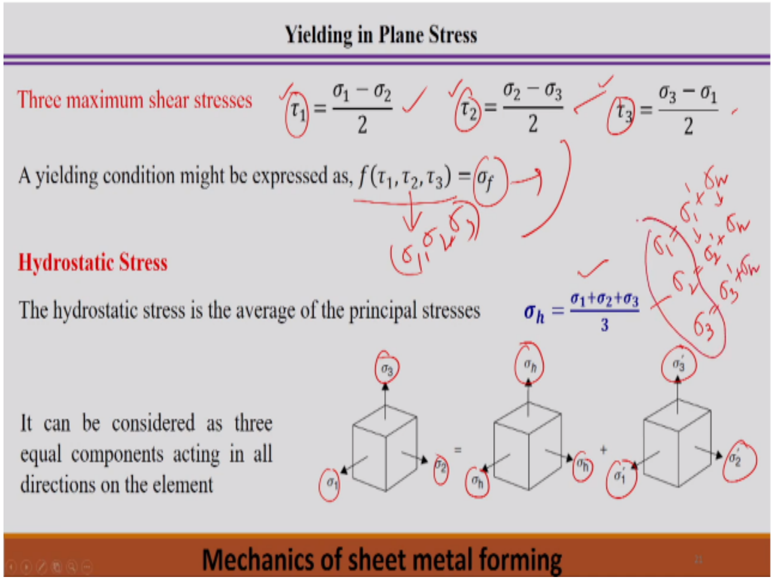
So from the figure also we can see that the plane is indicated by this dotted line and this figure this is one plane and that had the stress reaches the maximum value. Here also we can see another stress another plane and we can here also we can see the another plane. So all this three plane, the shear stress is acting tau 1, tau 2 and until this plane is we call the maximum shear stress plane.

But of course that means at that plane the shear stress will be the maximum. But as compared on the faces shear stress is=0, no shear stress. Now this plane normally makes the inclined angle actually at 45 degree with respect to the principal direction. So that maximum shear stress plane. And normal stresses also act on this maximum normal stress also acts on this maximum shear stress plane.

So this is the typical situation in case of three dimensional stress state and of course all are we explaining with reference to the three principal stresses sigma 1, sigma 2 and sigma 3. Now three maximum shear stresses can be estimated like that tau 1=sigma 1-sigma 2/ 2, tau 2=the different sigma 2-sigma 3/2 and of course tau 3=sigma 3-sigma 1/ 2.

These are the three shear stresses just we define the tau 1, tau 2 in terms of the principal stresses and that we can see that just means with that plane is defined already which makes the 45 degree angle with respect to the principal stress principal direction and this three such planes can be defined by the tau 1, tau 2 and tau 3 are active. Tau 1, tau 2 and tau 3 are the shear stress components.

(Refer Slide Time: 08:57)



Which is expressed like this by this three equation. Now therefore it is possible to represent the yielding condition that means just start the yielding situation can be expressed as a function of this shear stress. So ultimately this shear stress is in terms of the sigma 1, sigma 2 and sigma 3. So it can be also a function of sigma 1, sigma 2 and sigma 3 and that is=the flow stress value.

And that flow stress is basically significant in case of sheet metal forming process. So that yielding criteria in the functional form we can represent that the yielding depends on all these three principal stress components or these three shear stress component maximum shear stresses and then that=some known value sigma f that flow stress value and that flow stress value normally we can evaluate in by using in uniaxial tensile testing.

So this is the yielding condition. Now we try to look into that any state of the stress can be divided into two components, one is the hydrostatic stress component and another is the deviatoric stress component. So suppose this is the state of the stress sigma 1, sigma 2 and sigma 3 are the principal stresses for this particular element. It is consist of the two components this is called the hydrostatic stress.

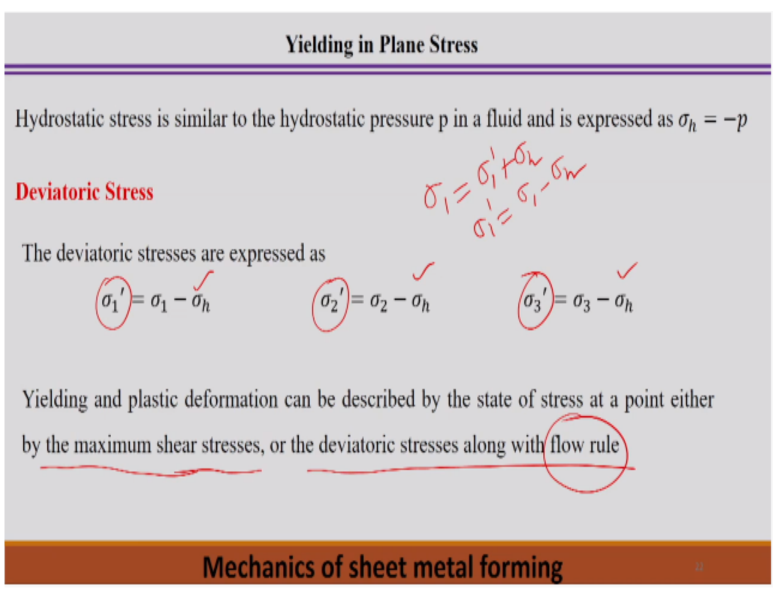
And this hydrostatic stress actually equal in all three direction+sigma 1 dot sigma 2 dot and sigma 3 dot all are the deviatoric stress components. But what we can represent the hydrostatic stress components, the hydrostatic stress component just simply the average of the principal stresses that is sigma 1, sigma 2+sigma 3/3 that is the hydrostatic stress component. So it is considered as three equal components acting in all directions on the element.

So that sigma h sigma h all the three hydrostatic stress components is acting on the element. So therefore with the application of the hydrostatic stress components the size of the element actually changes either it is if it is a compressive stress and we normally found in the fluid mechanics problem. Then in that cases it is compressed there is a reduction in the size and of course if it is tensile in nature this hydrostatic stress component then it will expand.

But it expand equally in all three directions. So therefore it is not subject to it is subjected to only the change in the size not the change of the shape**.** Now that means here we divide this thing sigma 3 that means we can say that sigma 1=sigma 1 dot, this is the hydrostatic stress component=sigma h sorry the sigma 1 is the deviatoric component and sigma h is the hydrostatic stress component.

Similarly, sigma 2 can also be represent that sigma 2 dot which is deviatoric and sigma h hydrostatic. Similarly, sigma 3=sigma 3 dot+sigma h. So that means this state of the stress normally we divide it is possible to divide into this two components one is the hydrostatic another is the deviatoric stress components.

(Refer Slide Time: 12:09)



Therefore, hydrostatic stress is similar to the hydrostatic pressure that means p which is normally used in fluid mechanics and is expressed by sigma h=-p that means we change the sign because hydrostatic pressure in case of fluid mechanics it creates the compressive stress within that element. So that is why sigma h can be represent replaced by -p. There is another stress the deviatoric stress we have already talked about the deviatoric stress components.

The deviatoric stress component sigma 1 is divided into two component one is the deviatoric another is the hydrostatic. So therefore sigma 1 dot deviatoric stress component is=sigma 1 -sigma h. So similarly sigma 1 dot is the deviatoric stress component, sigma 2 is the deviatoric stress component and of course sigma 3 is the deviatoric stress component.

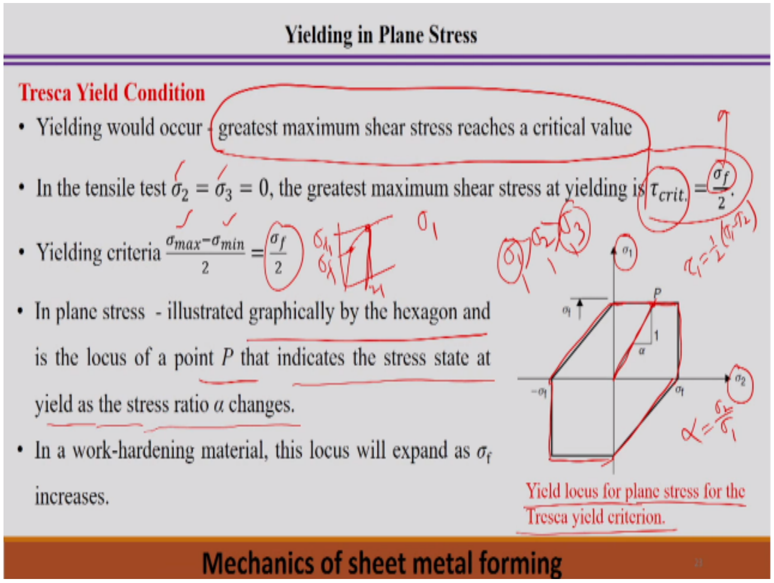
And of course if we look into all the deviatoric stress component that it is actually the principal stress -the similar amount of the hydrostatic stress. Because all these cases the hydrostatic is sigma h that means the same amount. So that means normally when we try to explain the yielding phenomena and plastic deformation, the concept of hydrostatic stress and deviatoric stress components is actually useful to describe the state of the stress at particular point.

So therefore yielding and plastic deformation phenomena can be explained and by considering the state of the stress at particular point either in terms of the by maximum shear stresses we can look into the maximum shear stress situations or in terms of the deviatoric stress components and this of course when we use the deviatoric stress component then to maintain the continuity in the plastic flow or plastic flow during the deformation.

Then we need must apply some kind of the flow rule. So therefore the yielding can be explained or plasticity theory most of the case we try to explain basic elements in either the state of the maximum shear stress at a particular element in terms of that we can represents, we can feel the find the criteria for yielding or we can decide the criteria of yielding by looking into the using the deviatoric stress component constants along with the flow rule.

We will look into that how we can utilize or what is the influence of the deviatoric stress component and hydrostatic stress component in yielding. But of course, we must comment here the that normally plastic yielding and when we try to assume the Von-Mises plasticity model. Therefore, this yielding is actually independent of the hydrostatic stress component. So basically the yielding can be explained by using only on the deviatoric stress component.

(Refer Slide Time: 14:49)



So now Trescas yield condition, first we look into that yielding will occur but according to this theory we are already explained but we just summarize that yield condition is the greatest maximum shear stress reaches the critical value based on that the Trescas yield criteria can be decided. So therefore yielding will occur in this cases. But if we look into other way if we consider how use that Trescas yield condition in particular tensile testing.

In a uniaxial tensile testing definitely we assume the two principal stresses sigma 2 and sigma 3=0 because there is a unidirectional load is applied. So only principal stress is sigma 1 is acting here. So therefore but with the application of the normal stress normal principal stress sigma 1 in a particular direction the shear stress will be the maximum that such that the critical value of the shear stress is basically the half of this normal stress value.

And half if we assume that it during the yielding it just=the flow stress value then it is=the half of sigma f flow stress value. So that is that data we can use we can apply the yield condition in a uniaxial tensile testing. Now yield criteria can be reaches the greatest maximum shear stress so we already explained that shear stress can be the maximum particular plane and here we express the shear stress is sigma 1-sigma 2 or sigma 2-sigma 3 or sigma 3-sigma 1.

So therefore if we look into any one out of these three principal stresses we considered the maximum value of the normal stress value because in this cases we assume the sigma 1 greater than sigma 2 greater than sigma 3. So these are the state of the stress and we defined from the state of the stress particular element the sigma 1, sigma 2, sigma 3 are the principal stresses. But out of this sigma 1 is the maximum and sigma 3 is the minimum.

So therefore maximum value of the stress -minimum value of the stress divided by 2 that represents the shear stress value and that shear stress value greatest critical shear stress value which is also=the half of the flow stress value that we can get from here also. They put it and that creates the yield criteria according to that Trescas yield conditions.

So now we can graphically represents this yield conditions that suppose this is 2 in plane stress condition may be we assuming that there are two components of the stress sigma 1 and sigma 2. So within that plane sigma 1 constituted by sigma 1 and sigma 2. These are the locus of this such that at a particular point P that it is slope is here the this slope is actually defined 1/alpha and that slope is defined by the alpha and that alpha is we know already defined in terms of the stress ratio.

So here we can see the graphically by the hexagon is the graphically by hexagon represents the yield condition the locus of this yielding that means beyond that yielding will at this particular line the yielding will occur when it is subjected to two different principal stresses sigma 1 and sigma 2 and is the locus of the point P that indicates the stress state at yield as the stress ratio alpha changes.

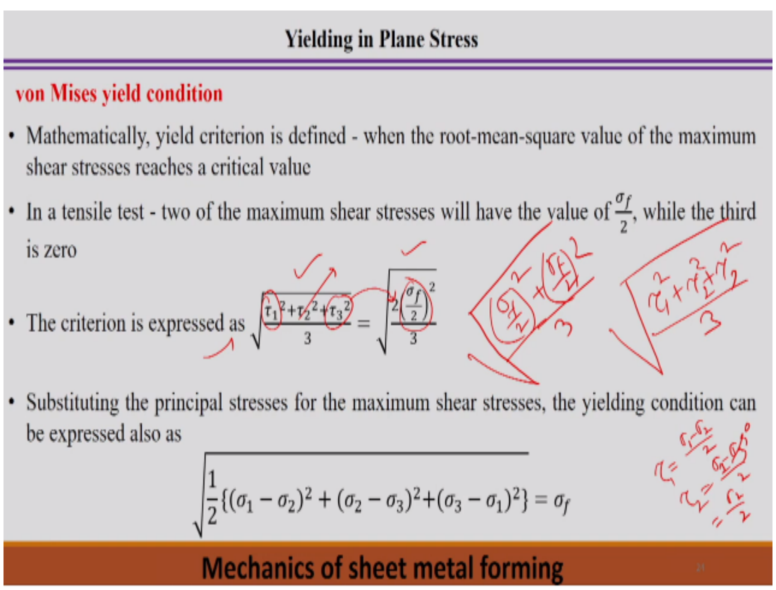
So therefore the stress state actually changes according to the what is the value of alpha. So alpha is basically the stress ratio, alpha we can already defined sigma 1/sigma 1,2. Therefore yielding depends on this yielding can be represent in terms of the stress ratio. So here we can see that from here to here if we move that creates the yielding condition that is correspond to the sigma f value that is critical value when it is cross the critical value then yielding will occur here.

Now in work-hardening material this locus will expand at sigma f increases. But we define the sigma f, sigma f can be represented like it is can be equivalent to the with the first yielding point. So that yielding point of a particular material if suppose this is a yielding point, so this yielding point will actually changes with the further work-hardening, work-hardening after work hardening reached to this point it is a corresponding f1 and corresponding strain.

So therefore during the work-hardening, the state of the stress actually changes so that means sigma f can be a function of the work-hardening coefficients. So based on that, that decides the yielding conditions and basically that work-hardening coefficient decides what way this hexagonal this yield surface actually evolves during the straining due to the working of a particular material.

So this indicates the yields locus for a plane stress for the Trescas yield criteria and we can see that it also actually is a function of the stress ratio alpha.

(Refer Slide Time: 19:56)



Now we come to the other yield conditions yielding condition that is Von Mises yield conditions we have already discussed but here the other way we can represent the Von Mises yield conditions that mathematically yield criteria is defined when the root mean square value of the maximum shear stresses. So roots mean square value of the maximum shear stress reaches a critical value.

So this is the root mean square so that means root mean square tau there are series three components of the shear stress tau 1, tau 2 and tau 3. So root mean square this average mean value and the square of that. This is the root mean square of this maximum shear stresses when that value will you reach some critical value. But how we can define the critical value? so then critical value we can always decide the critical value from the uniaxial tensile testing data.

So in tensile testing data two of the maximum shear stress will have the value of sigma f/2. So we can see the maximum two of this maximum shear stresses will value will be the sigma f/2 while the third is 0. So that means suppose we represents that maximum shear stress will be that sigma 1 -sigma 2/2 tau 1 and for example tau 2=sigma 1-sigma 2-sigma 3/2. If sigma 3=0, third one 0 so it is=sigma 2/2. So that means half of this normal stress value.

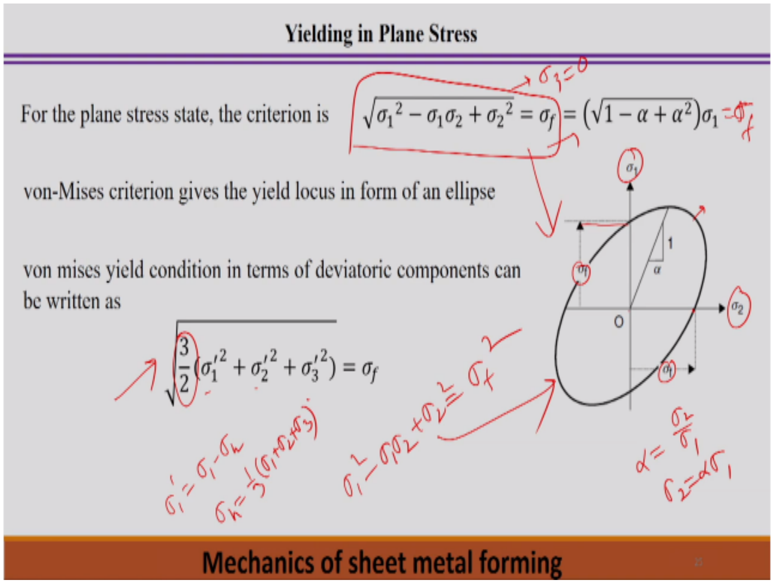
Now we have sigma f/2 while the third is 0. So in that case we can found out that in tensile testing during the tensile testing the two maximum shear stresses will be that if we put the maximum shear stresses value here. We can find out sigma f is the critical value sigma f/2 the critical shear stress value square and of course one cases so from here we can find out that sigma f/2, one cases it will be 0/3 that is the critical root of this and critical value.

And once we reach that critical value in uniaxial tensile testing and we make it equal between these two so therefore criteria can be expressed like this. So this one and this we can said from that critical value. So here basically one of the shear stress component will be 0 and other two having this component critical value and we can make this tau 1=say tau 1 or tau 2 is equivalent to the critical value sigma f/2.

So therefore two terms will be so this is from one, this is from one, two will be from two components and tau is basically equivalent to sigma f/2 by square and 3. So in make this criteria therefore if we put all this value tau 1, tau 2 and tau 3 and in terms of the principal stresses and we can reach the yielding condition can be expressed like that half of sigma 1-sigma 2 square, sigma 2-sigma 3 square, sigma 3-sigma=sigma f. This is the well-known the Von Mises yield criteria.

And of course we can see the yield criteria all the three principal stresses are actually involved to explain the to decide the yield criteria during the deformation process.

(Refer Slide Time: 23:30)



Now for plane stress the criteria can be here we can put simply sigma 3=0, the criteria Von Mises yield criteria can be modified that if putting sigma 3=0. Then it is become sigma 1 square-sigma 1 sigma 2+sigma 2 square=sigma f. This is the Von Mises yield criteria and this is corresponds to the plane stress condition. So now this can be expressed in terms of the stress ratio because alpha is basically sigma 2/sigma 1 or sigma 2=alpha sigma 1.

If we put this value sigma 2=alpha sigma 1 so therefore we can reach root over of 1-alpha+alpha square*sigma 1=sigma f. So this is the yield criteria we can reach in case of the plane stress conditions. So Von Mises this plane stress condition the Von Mises criteria gives the yield locus in terms of the ellipse.

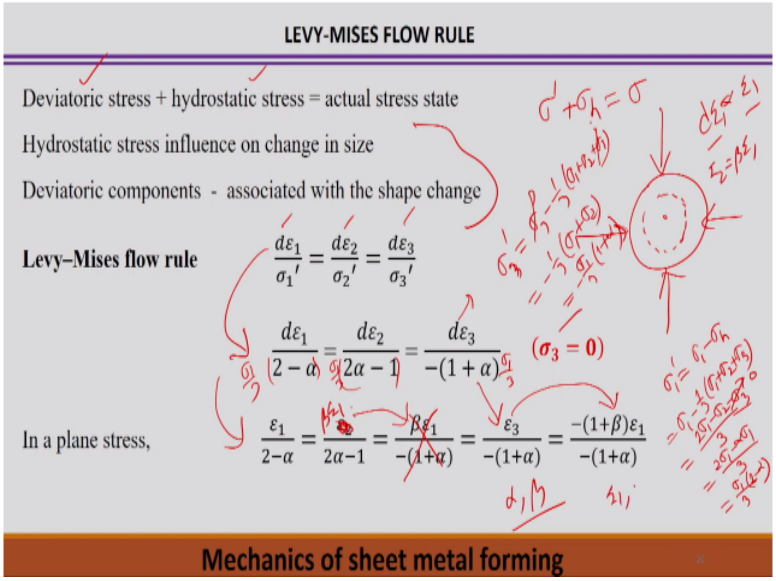
If we plot it actually if we plot it, it is looks like an ellipse because the if we make the square both the sides so basically the equation=sigma 1 square-sigma 1 sigma 2+sigma 2 square=sigma f square. So we plot it then we can get this form over the sigma 1 and sigma 2 axis and such that this point represents the sigma f and from here to here it represents the sigma f and therefore if we can see that alpha is changing with respect to that because alpha is a ratio of the sigma 1 and sigma 2.

So this is the yield locus and this yield locus, so yielding will start at this point. And of course this Von Mises yield criteria one point is that can also be represent in terms of the deviatoric stress components. So deviatoric stress components if we put it sigma 1 dot=sigma 1-sigma h. Then the same criteria 3 dimensional criteria this it can be represented in terms of the deviatoric stress component 3/2.

Remember here it should be 3/2 and which was sigma 1 dot square+sigma 2 dot square all of the deviatoric stress component=sigma f. Now we can go back here, here we can see that 1/ 2 and sigma 1-sigma 2+sigma 2-sigma 3 in terms of the actual principal stress component. But here in terms of the deviatoric stress component. The same equation can be converted to these things if we know the relation and of course here we need to use sigma h=one-third of sigma 1+sigma 2+sigma 3.

The relation we have to use it and we can reach this expression of the Von Mises yield conditions in terms of the deviatoric stress components.

(Refer Slide Time: 26:18)



Now apart from this yield condition, the flow rule is some to explain the plastic behavior and when we try to explain the Levy Mises flow rule here we can see the deviatoric stress+hydrostatic stress that actually=the actual stress state. So similarly deviatoric stress sigma dot+hydrostatic stress sigma h=the actual stress. Now therefore hydrostatic stress influence on change in size.

Therefore, hydrostatic stress that explain that it change suppose the hydrostatic stress is acting this thing if hydrostatic acting these things, so then after that it actually change the size but shape remains the same. So influence of the hydrostatic stress is basically change the size not the shape. But in other way also deviatoric stress components is more associated with the change of the shape rather than change of the size.

So if we try to explain that what is the role of the hydrostatic stress and deviatoric stress component in during the deformation process. We can ideally represent in that way that hydrostatic stress component actually makes the change contribute to the change of the size but deviatoric stress component actually contribute to the change of the shape. So these are the roles of the hydrostatic stress and deviatoric stress component.

But if we know this thing Levy-Mises flow rule is saying the change of the strain component and ratio with the deviatoric stress component is there they are equal actually. So mathematically we can represents the Levy-Mises flow rule d epsilon 1/sigma 1 dot, so this is the incremental this is the incremental strain, but sigma 2 dot=d epsilon 3/sigma 3 dot, that ratio all are equal. Now we can represent from here to here, here just put sigma 3=0.

Therefore, we can say that sigma 1 dot sigma 1 dot is basically sigma 1-sigma h or we can say the sigma 1-one-third of sigma 1+sigma 2+sigma 3. Therefore, we can say that 3-2 twice sigma 1-sigma 2-sigma 3. Now sigma 3 plane stress condition we assume the sigma 3=0. So it is basically twice sigma 1-sigma 2=alpha sigma 1/3. So therefore sigma 1/3 into twice -alpha, we can reach this expression.

Similarly, d epsilon 2/sigma 2 dot, sigma 2 dot similarly we can estimate that twice alpha-1 and d epsilon 3 we can represents that in terms of the sigma 3 dot here. So sigma 3 dot=0. But we can say sorry not sigma 3 dot=0 because sigma 3 dot here=sigma 3 -one-third of sigma 1+sigma 2+sigma 3. But here sigma 3=0 and sigma 3=0. So therefore -one-third sigma 1+sigma 2.

So that means -sigma 1/3 is sigma 2=1+alpha. So sigma 1/3 1+alpha. So everywhere sigma 1/3 will be cancel out. So sigma 1/3 and of course in this cases also sigma 1/3. So all sigma 1/3 will be cancelling each other and then this ratio can be represented like that d epsilon 1/twice -2 -alpha epsilon 3. So this ratio can be that means this flow rule actually we can represents the proportional way but in terms of the stress ratio here also.

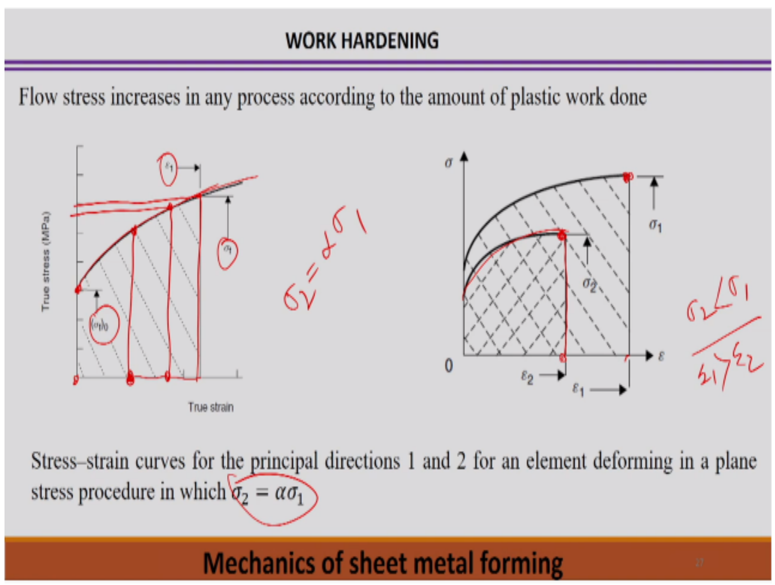
But in case of plane stress of course we can convert it the incremental form so that means in particular situation the incremental form or d epsilon is basically over a certain distance or deformation length. The d epsilon can be proportional to the so d incremental form is basically proportional to the total epsilon. So d epsilon, the incremental form can be replaced by the actual value of the strain.

So that we can represent the epsilon 1/2/twice -alpha epsilon 2/2 of alpha -1 and similarly because d epsilon 3, epsilon 3 okay. So epsilon 3 can be represented in d epsilon 3 can be represented in other way also beta epsilon 1 by this sorry epsilon 3 this one and of course epsilon sorry epsilon 2 can be represented in terms of beta/epsilon 1 and finally we can represent that is equal to -1/alpha 1/beta because this epsilon 3=-1+beta epsilon 1/ -1 -alpha.

So of course this I think this epsilon 2 epsilon 2 can be represented at beta x epsilon 1. So here it should be beta x epsilon 1. So this can be represented by beta*epsilon 1 and that divided by twice alpha–1. So these are the proportionate we can estimate the from one in known quantity to other quantity if we know the alpha and beta and in that cases we represent in that way that all in terms of the epsilon 1 alpha and beta.

Basically these three terminology we are using and we can relate the different variable and that corresponds to the Levy-Mises flow rule during the plastic deformation process.

(Refer Slide Time: 32:26)



Now we try to explain the work-hardening. Of course we have some idea about the work-hardening and we have already explained what is work-hardening during the plastic deformation process. So we start with these things and we relate with respect to the flow stress because in sheet metal forming, the flow stress is more significant or important parameter. So therefore flow stress increases of course flow stress increases at any process according to the amount of the plastic work done.

So if we assume this is the initial value of the flow stress then with the work-hardening that means true strain if we reach the straining the component up to this point so this is the corresponding flow stress value and of course if we reach up further so this is the corresponding flow stress value and of course in this cases this is the corresponding flow stress value. So that means particular strain epsilon 1 sigma f.

So corresponding strain this is the f but and initially it was sigma f0, so that means this actually increment of the flow stress value with the further strain hardening or work-hardening or during the plastic work done that depends on the that what are the this that slope of this curve, the slope of this curve that represents the effective of the strain hardening effect and of course other way also and if a particular two different stress-strain curve for principal directions 1 and 2 of an element deforming in the plane stress procedure in sigma 2=sigma 1.

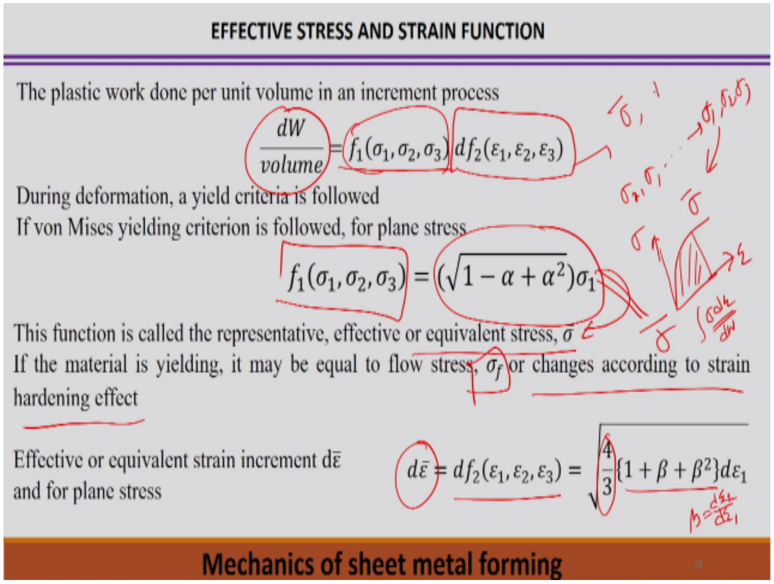
So that always we use this relation sigma 2=alpha x sigma 1. So in other we can express the flow strain work-hardening effect also. So we can see that 1 2 curve with one particular material this is the flow stress value sigma 2 and this is the particular strain at epsilon 2. And of course this is the stress value at a particular strain value epsilon 1.

So that means the material to material and if there is a change of the strain value and if there is a change of the different types of the material the flow stress value can have or material may have must have some kind of the work-hardening effect and because of this work-hardening effect the flow stress value can change with respect to the initial value. Similar analysis we can say that even there is a material is having the strain hardening effect.

The yield point can also be change depending upon the further straining of the particular material so in that is the way so here the flow stress value is the sigma 1, here this process value is sigma 2. But of course graphically we can see that of course sigma 2 is less than sigma 1 and in this case epsilon 1 is greater than epsilon 2.

So that different strains different state of the strain the flow stress value can be different and that difference actually comes because of the work-hardening effect of a particular material.

(Refer Slide Time: 35:15)



Now what is effective stress and strain function and effective stress and strain function is significant or important when we try to look into we try to represents the three-dimensional state of the stress. First if there is a three-dimensional stress states in a particular stress state. We can define in terms of the XYZ Cartesian coordinate system. There are 6 components sigma X, sigma Y, sigma Z, tau XY, tau YZ, and ZX.

So this 6 components can be represented in terms of the three principal stresses sigma 1, sigma X, sigma Y dot dot dot that can be represent as a function of the sigma 1, sigma 2 and sigma 3. Then the state of the principal stresses can also be represent in terms of the one effective stress that is called a sigma bar. So that effective stress may be equivalent to the uniaxial tensile testing data during the deformation of the material.

But what we can look into that estimate the effective stress and strain function. So of course the incremental work done per unit volume incremental work done per unit volume it is a functional form of the stress. Basically for uniaxial tensile stress testing if this is the stress-strain diagram we can represent the work done by this area.

So how we can estimate the area integral from? It is a kind of sigma d epsilon, this is the elemental work done so this is the dw and this dw per unit volume. Similarly, sigma so that is the functional form of the stress and that increment of the strain df2, this is a strain where the strain functional form is as a function of the three principal strain component epsilon 1, epsilon 2 and epsilon 3. So this is the elemental.

So now during deformation yield criteria is followed. So that we can estimate the work done per unit volume but when you try to look into the yielding condition. So therefore the yield we need to define some kind of the yield criteria. This can be stress yield criteria, it can be Von Mises yield criteria. So if we assume the Von Mises yield criteria therefore in Von Mises yield criteria we can see that functional form of this stress component is a in a particular for plane stress condition.

This we can represent root to the bar of 1-alpha+alpha square x sigma 1. That we have already explained these things that Von Mises yield criteria. Now this function is called the representative effective or equivalent stress sigma bar. So therefore this functional form is basically is we can say this equal to the sigma bar and the sigma bar equal to this for a particular.

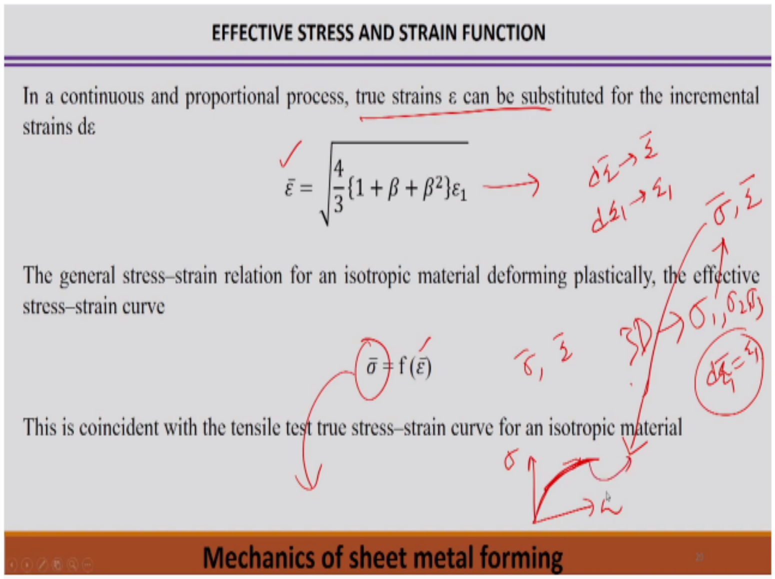
But this functional form is particularly important the effective stress of this expression if we assume the Von Mises yield condition. Then only this functional form can be represented like that. So that is equivalent to the that is equivalent stress or we can say that is the effective stress or representative stress. Now if metal is yielding it may be=the flow stress value definitely sigma f or yield stress value or changes according to the strain hardening effect.

So therefore this either yield stress value or the flow stress value actually changes depending upon the strain hardening effect. So therefore effective or equivalent strain increment can also be used so therefore we can use here the incremental strain form. So therefore the effective strain in the incremental form can also be use the d epsilon.

This is the effective strain and which is d of this and that can be represented in terms of the in terms of the all the principal stress component that comes with the 4/3 root to the bar of 1+beta such that beta=d epsilon 2/d epsilon 1 that is the ratio. So the ratio between the true strain component epsilon 2/3. So therefore if we put this and we can represent in case of the plane stress conditions, the incremental effective strain equal to this.

So all this stress state we represents these two, one is the incremental effective stress or effective strain. These are the two parameter we can use this.

(Refer Slide Time: 39:28)



Now the effective stress value can be represented in actual true strain epsilon in instead of representing in terms of the incremental form because in case of continuous process or in any proportional process. So therefore incremental means it is a small now question is how small is this incremental strain? so therefore this incremental strain form can be is equal to epsilon. The epsilon 1 can be equal to epsilon one over a long range if process is continuous.

In that cases it can be or it is a proportional the strain increment is proportional to the actual strain in that sense that it can be incremental form can be represent in the actual strain. So therefore instead of d epsilon bar, the effective strain d epsilon bar in plane stress condition can be represented like that. Here we just simply represent replace this by epsilon bar or epsilon 1 can be replaced, d epsilon 1 can be replaced by epsilon 1.

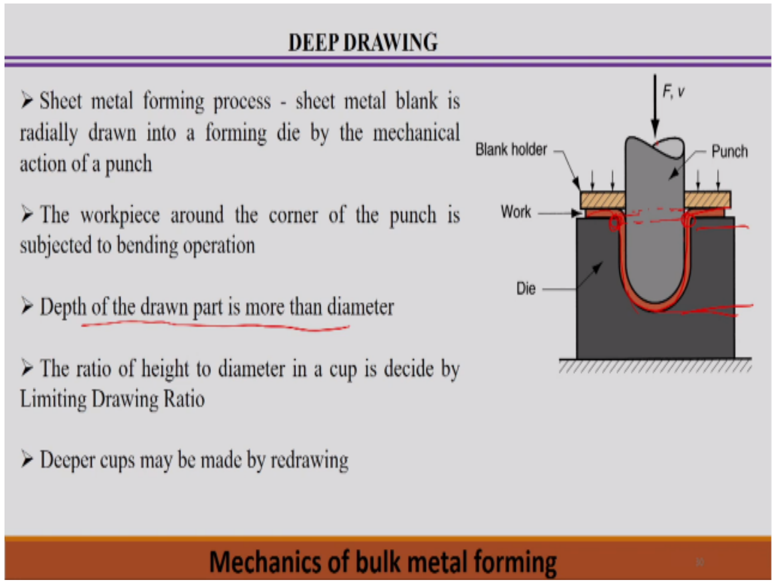
So therefore this is the state of the strain. Now general stress stress relation can be represented in case of the isotropic material when they deform plastically in terms of the effective stress and effective strength. So sigma bar can be represented in terms of the epsilon bar that means sigma bar is the effective stress and epsilon bar is the effective strain and which can be equal to the uniaxial tensile testing.

So therefore once we try to analyze the plastic deformation we try to apply the plasticity theory during the plastic deformation because actual state of the stress can be three-dimensional that three-dimensional stress state it represented sigma 1, sigma 2 and sigma 3. But this sigma 1, sigma 2 and sigma 3 can be represented in terms of the effective stress condition. So that effective stress what way the functional form of the effective stress that depending upon what type of yielding criteria we are following.

So whatever we have discussed in plane stress conditions we actually we have followed in Von Mises yield condition. So functional form will be accordingly. So therefore this stress and then this compare this effective stress and corresponding effective strain that effective strain and effective strain can be compared with respect to the uniaxial tensile testing data. So that is the stress and that is a strain.

So that uniaxial tensile testing data will be there and we can compare accordingly and then we can take the data from the experimental value and then accordingly the plastic theory can be applied during the deformation process by the concept of using the effective stress or effective strain.

(Refer Slide Time: 42:13)



Now we come to that point the sheet metal forming processes. So deep drawing process is one of the sheet metal process we can consider it is one of the sheet metal forming process. So in this sheet metal forming process, we can see that there is s application of the force by the punch and the initial position of the work sheet was there and then punch actually draw this initial state of the initial flat sheet and with a particular shape is taken after depending upon the shape and size of the punch and die.

So that this is called the blank holder and actual work pieces actually deform from the initial flat position to a particular desired shape. So this is in general that is called the deep drawing process. So if deep drawing possess, the application of the force that is the one parameter and the velocity up to what velocity the punch velocity follow such that it can takes the this is the shape of the final shape of the product that means after deep drawing process.

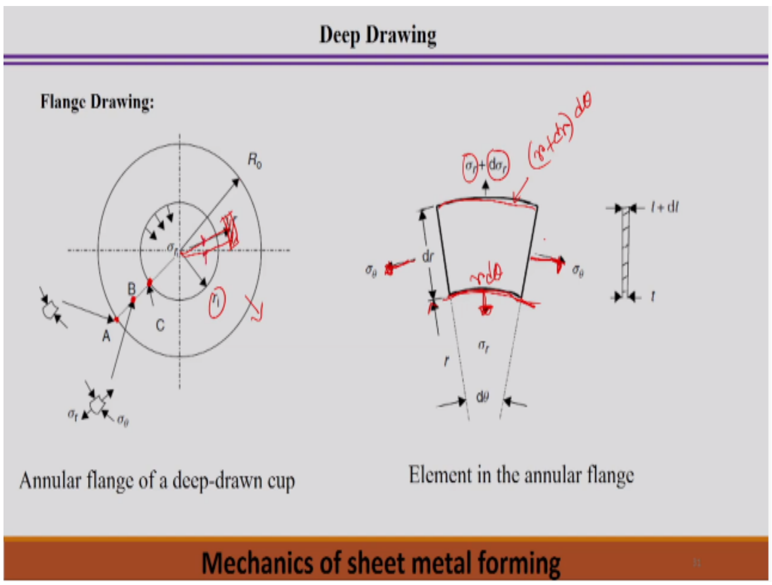
So now we try to look into the mechanics of the deep drawing process. So but before that so sheet metal blank is radially drawn. So in this cases it is radially drawn into a forming die by the mechanical action of the punch. So punch is there and they radially drawn into particular directions. So therefore the workpiece around the corner so around the corner so there around the workpiece around the corner, the punch is subjected to bending operations at this particular point.

And of course depth of drawn depending upon the diameter of the punch. So normally depth of the drawn depth of drawn that means the depth of the drawn is actually is more than the diameter of the punch or basically we can say the more than the diameter of the die. So therefore the ratio of the height to the diameter in a cup is decided by the limiting draw ratio. So but it is not possible to draw in any arbitrary ratio during the deformation process.

There is some limit of this that limit is called limiting drawing ratio and that height of the ratio of the height of the diameter in a cup. That we will try to look into the in a particular situation what may be the limiting draw ratio in deep drawing process. So therefore deeper cups may made by redrawing. So therefore if we look into further deformation process is required in that cases maybe after in the first operation then we can repeat the similar operation.

That is called the redrawing process can also be done. If we want to go for deeper drawing in case of the deep drawing process.

(Refer Slide Time: 44:55)



Now we look into the mechanics of this deep drawing process. So we consider the flange drawing process. So suppose this is the initial shape of the on a radially flat the flange having particular thickness and the punch the in that so the diameter ri is the punch within which there is a punch is acting there and then it is readily drawn to the depth, depth direction and outside up to from here to here it is a diameter R0 and that we define the 3 different points A,B and C.

Now A, B and C and the annular flange of the deep drawing cup, so ABC we can say we pick up the different points so deformation behavior can be different at the particular form or state of the stress will be different at the different points of A, B and C. So A is particularly on the surface, B some intermediate point and C is at that in between the punch and the workpiece in that contact. So C is basically we can see that C point is somehow here in that point. So this is the C point.

And B point is in between and that A point is just outside edge. So this of course we can see the stress of the stress in A is basically the sigma theta is hoop stress kind of acting here. But there is no radial stress along the radial direction there is no this is free from the radial stress if we take any element at point A. Now if we look into that element B this is more general here we can see the radial stress also acting as well as the hoop stress also acting, sigma theta also acting for a particular element.

And of course C we can see that all type of stresses will be there. Now at point B if we take some element and the very element and then we assume the thickness of the sheet is T initial thickness of the sheet was T. Now with this element if we look into for particular element so sigma theta is acting here, here also sigma theta is also acting and radial direction, the radial stress is acting sigma r and over a distance.

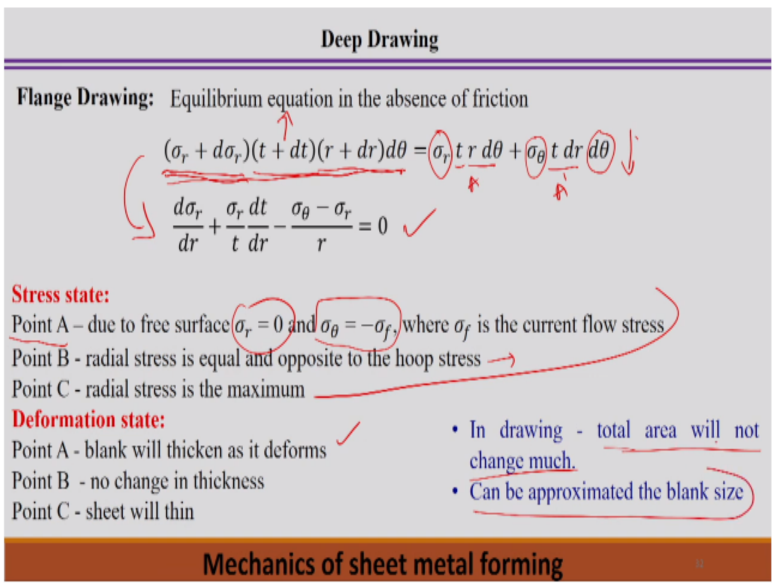
So actually we have taken one element at a distance r. For this at the r this is the thickness of this radially element the is the up to these distance is r up to this is the r and there we can take the elemental length along this radial direction that is dr. So sigma theta is acting both the direction and here sigma r is acting, radial stress is acting during this process and of course this radial stress is varying.

So here we can see the sigma r+d sigma r is the increment of the we assume there is increment of the stress along the radial directions. But of course we assume the constant hoop stress which is sigma theta is acting because there is no variation with respect to this thing. But of course the sigma r dr is also acting. But at the same time this area and this area elemental area are also varying.

At the same time, we assume the at this point the thickness equal to t and this particular point thickness equal to t, the depth directions but at this point there is a change of the thickness t+delta t and we consider all the variable during this process. Now just looking into this element we can do the further calculation we can make the force equilibrium and in this cases by neglecting the friction also force equilibrium.

But if we understand that this elemental form now what is this length, this length can be estimated by equal to r d theta. So this length can be estimated by r+dr x d theta this is the length. Now the angle d theta basically we make an element here at a particular this is the elemental angle that is the d theta.

(Refer Slide Time: 49:14)



So now we will try to analyze this elemental form. So equilibrium equation in the absence of the friction can be like that. So this is the stress value and the direction it is acting then area this is the length r+dr x d theta. So r+dr x d theta, this is the length and the area of the cross section and normal to that here it is t+delta t along that point. So therefore r+d theta x t+delta t, this is the area of the cross section and with normal to that cross section the stress is acting sigma r+d sigma r.

So this is the force is acting this direction. Now what is the making the force balance in the radial direction. So what are the force components acting this direction. So some components of the theta will be there. So therefore we can see if this is the this is the angle d theta. So therefore this elemental angle will be d theta/2. So therefore sigma theta components of the sigma theta along this direction equal to sin theta.

So sin basically sin d theta/2. So sin d theta/2 can be approximated as d theta/2. So similarly here also this component will be there. So here also sigma theta sin d theta/2 so this can be approximated sigma theta d theta/2 here also approximated sigma theta d theta/2. So therefore total force is acting this radially this direction. Sigma r a corresponding area+sigma theta x this+this so this will be the d theta x area.

Whatever the force is acting whatever area but this area will be consider that side. Now this is the one direction that is upward direction radially upward and radially inward that sigma r is the stress value here. Sigma r is the stress value and what is the r d theta is the length and t is the thickness here.

So r d theta is the length and t is the thickness. So therefore this is the area A so then this is the force similarly sigma theta d theta is the stress component but t dr so this is the thickness we assume the uniform thickness this is the dr so that area A dot equal to t is the thickness and this is the length dr over which the sigma theta was acting. So therefore t is the area A dot. So therefore this is the outward direction force and this is the inward direction force component.

We make this and we can explain this differential equation d sigma r/dr+sigma r/t dt/ dr-=0. So this kind of differential equation we can form making the force balance radially in the deep drawing process. Now we can clearly observe that different point stress state, we have already discussed the point A just outside of the flange here it is free from surface stress so therefore sigma r=0.

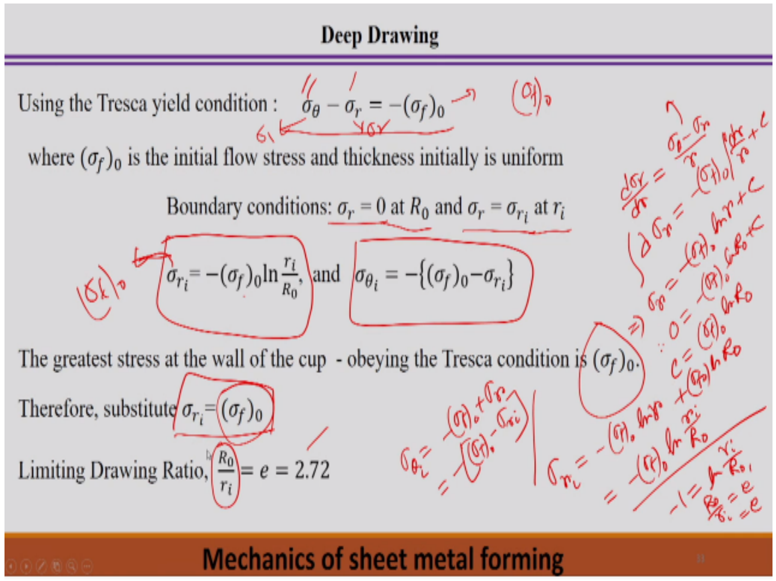
And of course in this case there exists sigma theta is also there but sigma theta is acting in such way that sigma theta equal to minus of the flow stress value or sigma is the current flow stress at this particular point. So at point A is acting. So now at point B radial stress is equal to and opposite to the hoop stress value. At point B we can see the stress state, point B the hoop stress value will be there, sigma theta will be there and that radially acting the inward.

Point C radial stress is the maximum. At point C we can see the radial stress is maximum because there is a bending actually happens at the point C. So at this point the radial stress will be the maximum. So these are the state of the stress 3 states and then if we look into the deformation state also at point A blank will thicken as it deforms so at the outside edge of point A blank will be thicken at this point edge blank will be thicken because there is a hoop stress is acting at this point and it is free from the radial stress value.

So therefore outside is blank will be thicken. Next point B, no change in the thickness because at point B there is no change of the thickness because equally it is subjected to both radial stress as well as the hoop stress is acting there. At point C sheet will be thin because at the point C there is a thick sheet will be thickened because bending is acting at this point C. So these are the typical state of the stress.

But in general in drawing process total area will not change much therefore it can be approximated the blank size. So that means since total area will not change much. So we can easily estimate approximately estimate what is the blank size before drawing operation.

(Refer Slide Time: 54:46)



Now if we look using the Trescas yield conditioned we try to apply in the deep drawing process. So we can see that these two maximum stress values sigma theta -sigma r but here the state of the initial stress value sigma f0 is the initial flow stress value at this point where it is initial flow stress value and thickness initially is uniform. If we assume that and this we can get this Trescas suppose this is the one maximum value this is another maximum, two maximum state of the stress and equal to the sigma 1 -sigma 2 equal to the critical stress value.

So that critical stress value equal to the flow stress value in this cases we can apply easily here. Now boundary conditions can also be applied and this particular expression and if we look into that. So here this equation if we see that there is no change in the at point B there is no change in the this is the more generalized form of the equation differential equation making the force balance during the deep drawing process.

Of course by using neglecting the effect of the friction also. Now if we assume at point B there is no change in the thickness. So therefore this equation can be modified as d sigma r/dr -sigma theta -sigma r/r=0. Because this term can be neglected because there is no change in the thickness if we assume that. Now if this from this equation we can find out that putting the boundary condition sigma r=0.

So the equation is like that, d sigma r/dr=sigma theta -sigma r/r. This is the differential equation and particular point at any element at the point B. So therefore and if we put here if we integrate these things that sigma r=sigma theta sigma r=-sigma f0, if we put that yield condition integration dr/r+c and that sigma theta -sigma r we just putting that Trescas yield condition according to the yield criteria we can put the -sigma f0.

So therefore sigma r=-sigma f0 x lnr+C. Now boundary condition at sigma at outside at point A basically in that cases sigma r=0. Because there is no it is free from the radial stress value at a distance r0. So then if we put this boundary condition then it is like that sigma r=0 -sigma f0 lnR0+C. So C is basically sigma f0 lnR0. So that is the constant.

Now this equation can be modified that sigma r putting this constant value -sigma f0 ln small r+sigma f0 lnr0. So that means -sigma f0 ln if we take the minus common then r/R0, this is the equation. Now once we get this equation then if we put another boundary condition sigma r=sigma ri at r=ri, sigma r=ri. So therefore at this inside at point C basically at this point sigma r=sigma ri so at r=ri.

So suppose at this point sigma r=sigma ri at r=ri. If we put the another boundary condition, then we can reach that sigma ri equal to just simply replace sigma ri at r=ri. So this is the sigma r=-sigma f0 ri/ R0. So that is the expression we can reach the radial stress value at a particular point we can estimate and similarly once we know these things and of course from here from the yield condition that sigma theta=-sigma f0+sigma r.

So then we can put -sigma f0 -sigma r, sigma r=sigma ri so at r=ri theta i. That particular position sigma ri sigma theta i at a distance. Then we can estimate the both of the state of the stress. We can roughly estimate it. So therefore the greatest stress at the wall of the cup can observe we can find out that wall of the cup obeying the Trescas yield condition is sigma f0 that is the initial value of the flow stress.

So that is the maximum value of the stress and that comes from the Trescas condition. So therefore this is the greatest amount of the stress and that acting during this drying process therefore if we therefore substitute sigma ri equal to because replace sigma ri=sigma rf0 because this sigma f0 is the maximum value of the stress in this cases. So we want to see replace sigma ri=sigma f0.

And we can found out that from this equation sigma ri basically sigma ri=sigma f0 if we replace because this is the value of the maximum value. Then we can find out that -1=lnri/R0. So from here we can say that R0/ri=e to the power 1=e, e means 2.72. So therefore that R0/ri. So that R0/ri. We can see that R0 means this is the value of R0. So this is the maximum distance and ri this is the ri value.

So that R0/ri is physically interfere the limiting drawing ratio and that limiting drawing ratio can be estimated as e value, e means=the numerically it is equal to 2.72. So in that way we can estimate the limiting drawing ratio in the deep drawing process.