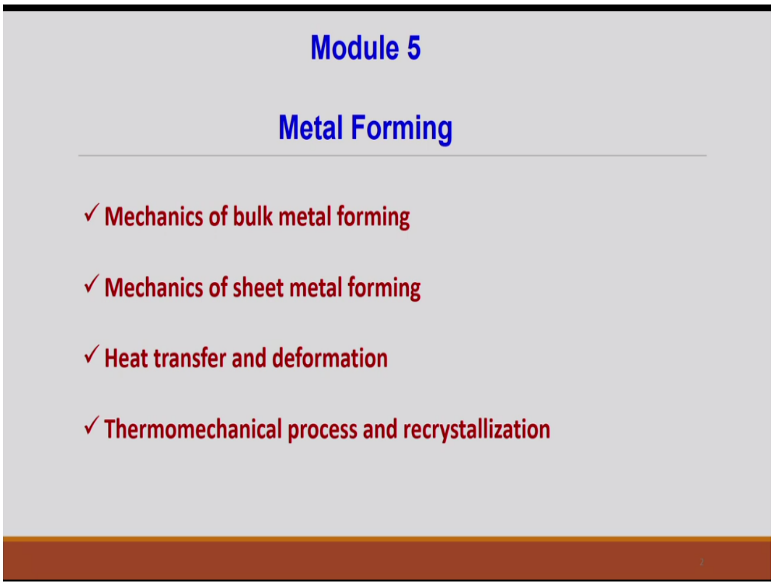
Mathematical Modeling of Manufacturing Processes
Swarup Bag
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 14
Mechanics of Bulk Metal Forming

Hello everybody. Today, I will discuss the different module that is called metal forming process.

(Refer Slide Time: 00:37)



In this module, will try to cover the 4 basic elements, we can see them here. The mechanics of bulk metal forming processes, then mechanics of sheet metal forming processes, what are the deformation and heat transfer mechanism here, temperature development during this metal forming process and finally will look into the most relevant to the metal forming process, the recrystallization behaviour that actually decides the final microstructure of a manufactured component.

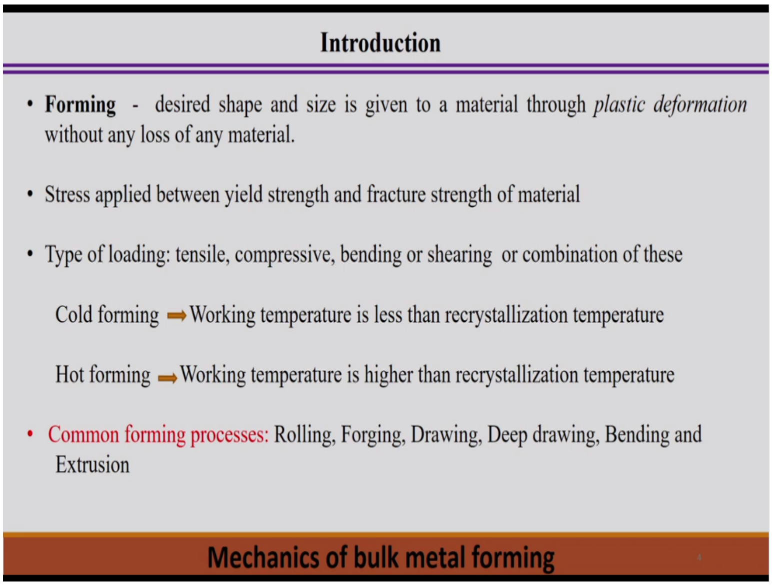
And that manufacture component is done by different types of the metal forming process. So, first thing is that what do we understand by the metal forming process? It is a very simple in terms of the physical aspect the metal forming processes, there is a piece of material if we apply some kind of the load, so then the material will deform and thus if it is controlled in different way that means whether we are applying kind of compressive load, we can apply kind of tensile load, shear loading.

So, depending upon that the different names of the different metal forming actually comes into the picture. The mechanism of metal forming is something like that it is more relevant to the deformation that we represent in terms of the strain, strain rate these things and of course the relation between the stress and strain and finally is the recrystallization mechanism which is also a function of the temperature strain and strain rate.

And then recrystallization brings into the picture here because of that this that final microstructure of the product is very much important in a metal forming processes and that is why thermomechanical conditions is responsible to bring the different types of the recrystallization behaviour and of course it depends on the what type of materials were handling.

So, will look in that the intention is not to discuss in details about the different metal forming processes rather it is more intended to discuss about the physical phenomena happening during in general in metal forming process or is very specific to, it is a bulk material and what may be the difference if we consider the sheet metal and of course after that some heat transfer and deformation. Now, look into first the mechanics of bulk metal forming processes.

(Refer Slide Time: 03:19)



Forming, we get the desire shape and size is one kind of manufacturing process which is of course different therefore basic manufacturing process casting, machining, welding and one of them is the forming. So, we get the desired shape and size in the material and the material is normally so plastic deformation happens.

But of course in this case when there is a plastic deformation happens that we change the shape from initial shape of the workpiece to final shape through plastic deformation and of course in these cases there is no loss of material which we normally found in the machining processes or metal cutting processes. So, that is the one advantage, there is no loss of the material but of course the product, quality of the product depends on what control we are deforming the material and what rate all this matters here.

So, stress applied definitely if there is a question of plastic deformation, so when there is a stress applied such that it must cross the yield strength and of course it should not go beyond the fracture strength of a particular material. So, plastic deformation behaviour in between the yield strength and the fracture strength is the basic point of analysis to understand the mechanism of metal forming processes.

Now, when you try to deform any particular one shape to some other desired shape then it is possible to apply the different types of the loading. For example, tensile loading, compressive loading, bending or shearing or combination of all these things. So, based on this, the different names of the metal forming processes actually come out. Cold forming is the one; we can differentiate between the two categories of the metal forming process.

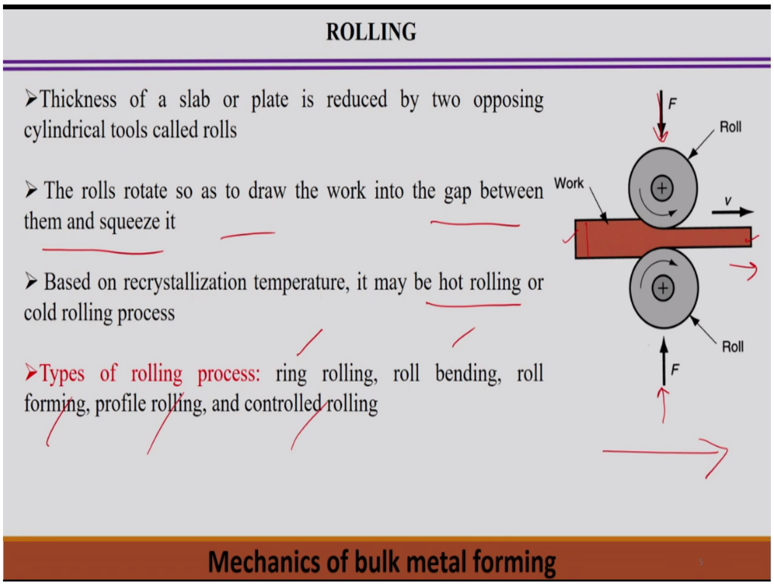
One is the cold forming; another is the hot forming process. So, cold forming, we say that deformation process normally happens at the room temperature. So, there is no added temperature during this process and of course the definition is maybe we can say the working temperature is less than that of the recrystallization temperature almost in room temperature what deformation process is happening that is called the cold forming process.

But in hot forming process, the working temperature is higher than the recrystallization temperature, so relatively high temperature, it will be easy, the metal becomes softens and then at the same time we apply the mechanical loading to the particular piece of metal and then hot forming normally happens. So, definitely the recrystallization behaviour is more relevant to the hot forming process.

So, in most of the cases, hot deformation process we can expect kind of the (()) (06:06) recrystallization, of course it depends on the type of the material and alloy system, we are considering for the manufacture component. The common forming processes that we try to look into that. Basic elements of this common forming processes that is rolling, forging, drawing, of course deep drawing, painting and extrusion.

So, out of this is a very common forming processes but of course deep drawing process bending normally we keep under the category of the sheet metal forming. So, this is we are discussing about the bulk metal forming processes, so as a big bulk metal forming processes which is definitely different from the sheet metal forming processes in terms of the dimensionality of the stress, applied stress or in generated stress during the process.

(Refer Slide Time: 07:03)



Now, first one of the metal forming process is the rolling process. It is a mechanism, very simple process, so very thick material, thick slab or plate is reduced by less thickness and by using some to roller and the cylindrical tools we use and that tool is normally call the roll. So, in this picture, we can see this is a role and of course this applied force is there and such that this is the thickness of the material, this thickness of the material is reduced to some other thickness of the material and this is called the simple process is called the rolling process.

So, in these cases, the rolls rotate such that it rotates in one particular direction such that to draw the workpiece from one this thing from one side and workpiece actually flow from this side to that side. So, to draw the workpiece from one side and we reduce the gap between the two rolls such that the thickness of the final product is decided by the gap of the roll between these two.

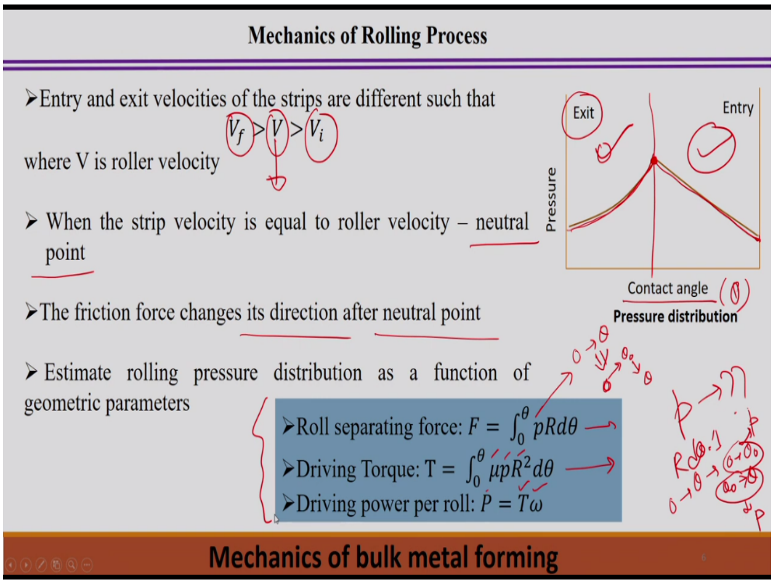
And the gap with is simply squeezed by the roll, squeeze the work material by this using this roll and we get the product just simply reduce the thickness of the higher thickness to lower thickness. Now, based on the recrystallization temperature definitely we need to differentiate between the hot and cold forming processes whether it can be hot rolling process or it can be cold rolling process.

Both can be possible depending upon the type and nature of the material okay. Now, there are other types of the rolling processes. For example, there may be ring rolling, roll rolling, roll forming, profile rolling and controlled rolling, all this the different types of the rolling processes but we are not supposed to discuss that in detail that what are the difference type of the rolling processes but in general will discuss the mechanism of the rolling process in this case.

Now, mechanics of the rolling process is such that that we observe that definitely in this figure that of course using this roller, this velocity e of the entrance velocity of the metal between this roller and roller also rotating. So, some kind of the friction will be there in between this workpiece and roll in that contact and of course this we can say this is the contact angle some theta.

Of course, this contact angle is typically very small and of course it depends on what is the reduction of the thickness we are looking for, based on that the contact angle may be less or may be high. So, this is the contact angle, so of course we assume this is symmetric with respect to this line but the entry velocity here and exit velocity Vf should be different because if we maintain the continuity equation soon of course there thickness maybe less in these cases, so velocity must be different between entry and exit.

(Refer Slide Time: 10:28)



Now entry and exit velocities of the strips are different such that Vf is a final velocity or we can say the exit velocity and V is the velocity which is equivalent to the roller velocity. So, some point and Vi is the exit velocity. So, within this contact angle with that zone, the entry velocity and exit velocity are different and of course the exit velocity is less because there is a reduction of the thickness of the shape and entry velocity is more.

But in between there is one velocity point, there must be which is one particular point, this velocity of the strip must be equal to the linear velocity of the roller. So, strip velocity is equal to the roller velocity that is called the neutral point so in that point but beyond the neutral point there must be some kind of the relative velocity between the strip velocity or some difference in terms of velocity roller velocity or the strip velocity.

But of course since this velocity is different with respect to the neutral point, so in this case the friction force also changes before and after the neutral point. So, if we look into the pressure distribution or stress distribution we can say executed by the roller and between the contact angle, so here contact angle for example theta that which in contact, so in certain point the pressure actually increases up to the neutral point.

So, neutral point is that up to the neutral point sorry this is the exit point, so it is increasing up to the neutral point and of course again it is decreasing up to the entry point. So, this is the typical nature of the pressure and of course between if you take into two different zone since frictional force changes the sign because of the difference in the velocity between the roller and the strip with reference to the neutral point.

So, therefore there is a change of the pressure distribution and of course there is a change of the frictional force acting between the roller and the workpiece material and of course all this normally happens, all this phenomena happens in between at the within the contact angle. So, friction force actually changes its direction after the neutral point. So, with reference to the neutral point, friction force directions are different.

If we look into this one zone entry zone and exit zone, so friction force changes but it is possible to estimate the rolling pressure distribution as a function of the geometric parameters. So, we can estimate the roll separating force. Here, we can see these two rolls are there and of course it is squeezing from higher thickness to the lower thickness value. So, there must be some reaction forces will be acting on the roller.

So, then this roller separating force can be estimated like that if we know the pressure distribution, with pressure distribution in terms of the other parameters, then we can easily estimate the roll separating force. So, F can be estimated between 0 to theta and p is the pressure distribution and of course some elements we take the elemental area and of course with direction if you take the unit dimension then Rd theta*1 is the elemental area.

And pressure into that elemental area that represent the elemental force and if we integrate the elemental force then we can estimate the roll separating force. So, of course this p is exclusively defined with reference to the neutral point. So, we can express in such a way that this instead of 0 to theta, we can say that 0 to theta i, say 0 to theta 0, theta 0 neutral point and then theta exit is the exit angle.

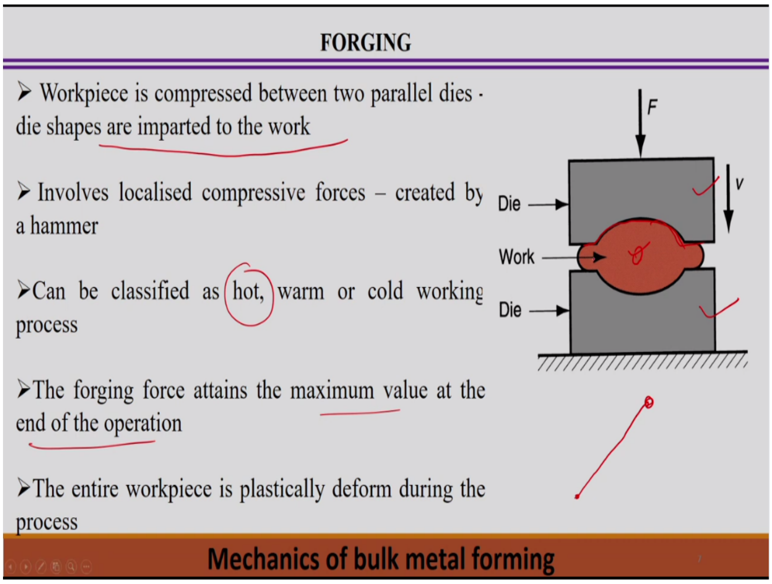
So, that means 0 to theta oh sorry 0 to theta so then we can say that it is from 0 to theta 0 and then theta 0 to theta. So, that we divide 0 to theta into these two parts then we need to know explicitly what is the pressure distribution function between 0 to theta 0 and of course and the other part between theta 0 to theta. So, then we can estimate the roll separating force. Similarly, once we estimate the roll separating force, we can estimate the driving torque T.

T can be estimated by this thing, so mu is the coefficients of the friction and then p is the pressure distribution and R is basically radius of the roller and d theta is the elemental angle. So, in this case, we can estimate what is the value of the driving torque and similarly in these cases also basically the frictional torque and then friction torque similar elemental way we can estimate the frictional torque.

And of course, in these cases 0 to theta can also be divided into two parts 0 to theta 0 and then theta 0 to theta and we should know what is the pressure distribution for this range and of course the other range what is the pressure distribution. If we know the function in the functional form of pressure distribution, we can easily estimate by integrating this, what is the driving torque during the rolling process.

So, once we estimate the driving torque then we can estimate the driving power P=torque*the omega, omega is the rotational speed of the roller. So, I am not discussing explicitly how to estimate the pressure, there may be some assumption to estimate the pressure distribution but of course if we know the pressure distribution of the contact between the roller and the workpiece, then we can if we know this value then definitely can estimate to all this parameters during the rolling process.

(Refer Slide Time: 17:01)



Now, coming to the next metal forming process that is called the forging process, say in forging process also similar phenomena in the sense that workpiece is there and we apply the force and of course workpiece is compressed basically between the two parallel die or maybe this is one die and this is another die and it can be with certain velocity.

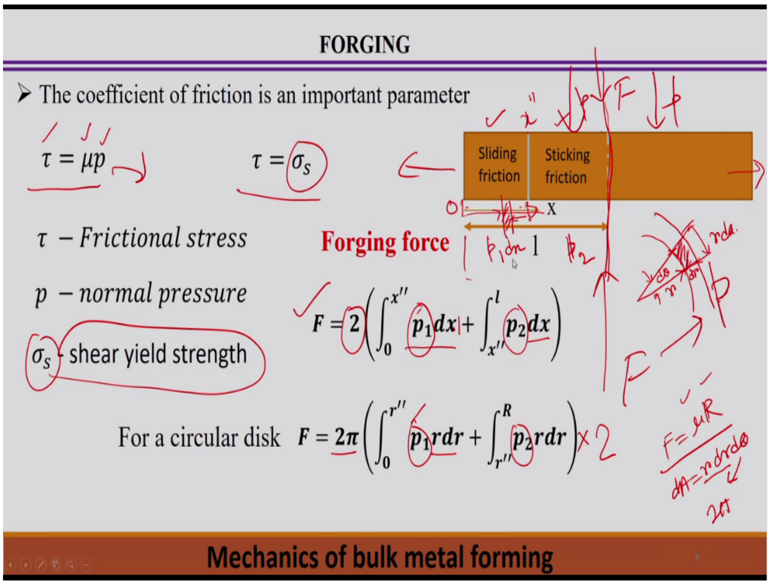
We just simply compressive it to get the desired shape according to the shape of the die and imparted to the work so that it is applying a force and this is the workpiece material and the shape of this is depending upon the shape of the die. So, here also involves localized compressive stress forces created by hammer. So, this force can be created by a hammer or we can uniformly apply the load also.

But it can also be divided into this hot forming, warm forming or cold forming depending upon the working temperature we are handling all these things. So, of course in this case, the forging force attains the maximum value at the end of the operation. So, if we apply the load gradually, so it is gradually increasing the load and we reach the maximum value at the end of the operation.

So, there is a change of the force may also happen and of course if we want to do that compressive load and it can be also dynamic in nature that applied load so but with the application of the load, the entire workpiece is actually in plastic deformation happens during this process. So, similar thing which is common features in case of the rolling process also. So, in this case, once we apply the load, we get the desire shape and size.

Then, what are the different, how we can explain the mathematical form or to estimate the different parameters in the forging process we can look into that aspects.

(Refer Slide Time: 19:10)



But before explaining the forging operations, here also coefficients of the friction play an important role and of course it is the important parameter and what are the frictional forces acting and the resistance where the friction also happens during this process, so it is interesting to know that frictional stress, how we can estimate the frictional stress, the two things are there.

We know the frictional stress here is the coefficient of the friction and the applied pressure and this is the frictional stress or of course this frictional stress can be directly to the shear yield strength also sigma s, remember this sigma s is defined as the yield strength in shear, shear yield strength which is different from the normal yield strength value but we can apply this condition to estimate the frictional stress and the two different formulation.

And if we look into that suppose this is the centre and symmetric application of the load and expand this direction, the material expand in this direction sorry with respect to this is the origin and with respect to the origin this is x-axis and suppose with respect to the centre point this is the total length l. Now, it expands outward with respect to the centre, it expands with these things.

But in these cases, two frictional conditions actually exist during this process; one is the sliding frictions that outside here the sliding friction and here is the sticking friction. Sticking friction means there is shearing happens with respect to the material. So, when there is shearing happen with respect to material in that case is the frictional stress is equivalent to the sheared yield strength value.

So, that is why one condition in which part the sticking friction exist and of course that sticking friction exist up to certain distance and sliding friction exist up to this distance and of course we can say this point is suppose x double dot and this is 0, so 0 to x double dot that sliding friction exist and from x double dot up to l distance l the sticking friction exist. So, if we identify the zone of sticking friction and the sliding friction accordingly we can put the frictional stress estimation.

So, frictional stress we simply use the Coulomb's law of friction and from there we can estimate the frictional force. So, frictional force now F=mu*normal force, reaction mu*the normal force. In similar way, frictional stress=coefficient of the friction in the normal stress value. So that normal stress is here is the p, basically the p is acting the normal stress value and other cases the frictional stress is equivalent to the shear yield strength value.

Now, once we know decide the conditions then we can estimate what are the forging forces is applied because forging forces is applied this F and that distributed force, the force is applied but which is the force corresponding pressure=p. So, now once we know in between suppose we assume the p1 is the pressure distribution in this zone and p2 is the pressure distribution of this zone.

Of course, there are other things we need to apply to estimate the p1 and p2 explicitly as the function of the other parameters but that we are not discussing here but if we know the pressure distribution p1 and p2 then we can estimate the forging force also. So, forging force can also be estimated like that, so p1*dx, dx is the elemental length at a distance x. So, suppose this is the elemental length l at a distance x, the length is dx.

So, once we know the elemental length dx and p1 and the other dimension we have taken as 1, so that is dx*1 actually so unit dimension in other direction such that the pressure*area that represents the force and if we integrate over the elemental area 0 to x double dot and that with this p1 is condition for the sliding friction condition. So, we can explicitly express p1 by looking into that frictional stress value or other parameters value we can explicitly express p1 as a function of other parameters.

Similarly, if we know the p2 exist between these distances, so if we know the distribution of the p2 and dx and that range of the integration will be x double dot this point up to l point. So, 2 is multiplied if we considering the both sides then multiplied by the 2. So, then this is the expression of the forging force and of course in this case is definitely we need to know the distribution of p1 and p2.

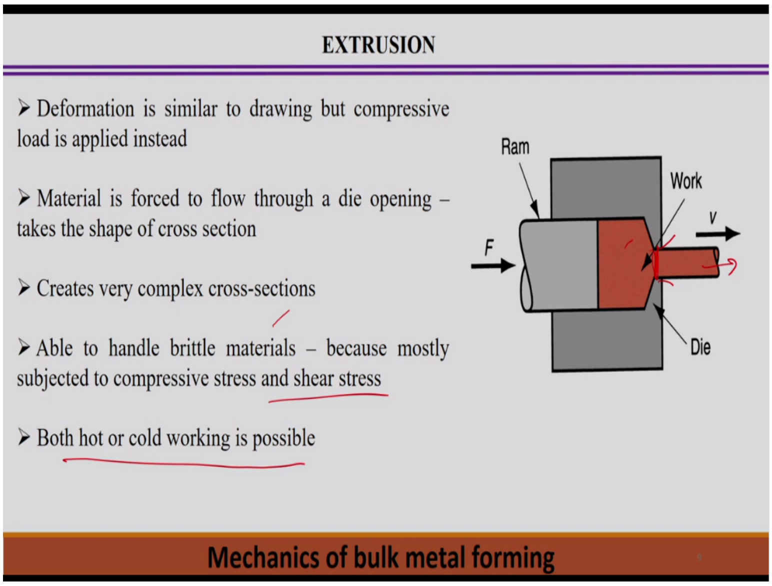
Then, we can easily estimate the forging force for this process. Similarly, if it is a circular disc, since circular discs, we have the radial symmetry in the circular disc, so in this case, we need to modify these things. So, in circular disc, if we consider the circular disc and if we take one element for example elemental area angle d theta and this area at a distance r, so this is equal to rd theta.

So, area equal to so this elemental radial distance=dr, so this is rd theta and this is dr. So, rdr d theta is the dA elemental area=rdr d theta this is the elemental area. Now, it is a circular symmetry, so we can d theta means the theta can be go theta, if we integrate d theta then it becomes 2 pi, so that is why the 2 pi will come here and it is because of this symmetry and p1 is the pressure distribution and remains rdr and that is the rdr.

So, then similar if you put the sliding friction condition and sticking friction zone if we define it for a cylindrical circular disc, then we can estimate the particular radial distance is r double dot exist the sliding friction and beyond r double dot to r there existence of the sticking friction and there if you know distribution of p1 and p2 and from there we can estimate the force, forging force requirement in these cases.

So, definitely forging force requirement in these cases but if you consider both the dies contact between the workpiece and the die surface, both the surfaces may be you need to multiply by 2 similarly what we did here, what we have done here. So, this way we can estimate the total forging force in case of the forging process.

(Refer Slide Time: 26:50)



So, there are other metal forming process also that is called the extrusion process. So, extrusion process is simply there is a ram and the workpiece material. We just keep the compressive load by the ram to the workpiece material and this squeeze here and then it is the worked or deformed material actually comes out from this through this die. So, that is the basic mechanism of the extrusion process.

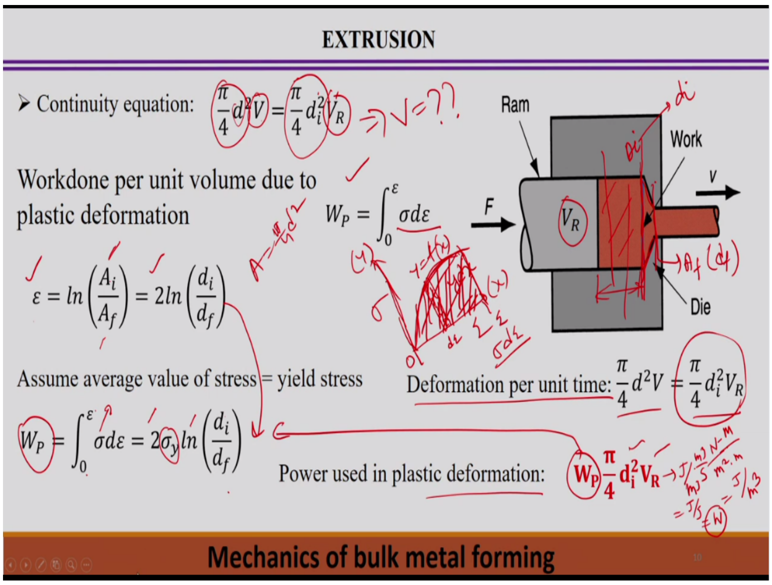
So, deformation is actually similar to the drawing process but in these cases definitely there is application of the compressive load in this case, compressive load is applied here. So, material is actually forced to flow through a die opening, so this is die opening and this is die opening through this die opening material is forced to flow through this die opening and takes the shape of this product depends on the what is the cross-section at this point.

So, the shape of the cross-section decided by what we are considering this separate cross-section. So, definitely very complex cross-sectional shape can also be created by using this extrusion process and the other advantage of the extrusion process means it is actually able to handle the brittle material because brittle material we know the deformation, may be elongation of the brittle material is very less.

So, in these cases, able to handle brittle material because most of the cases is subjected to compressive, so deformation behaviour is better if material is subjected to compressive as compared to the tensile loading and of course there is a possible too because most of the in these cases, the extrusion process, specifically in the extrusion process since we apply the compressive load and then it is advantageous to use extrusion process for brittle material as compared to the other metal forming processes.

Of course, similarly in these cases also both hot and cold working processes is also possible here also.

(Refer Slide Time: 28:52)



Now, we can do the simple calculation in the extrusion process. Here first is that looking into the continuity equation, so continuity equation in the sense suppose up to this point is the this part up to this material component, this is the material and this is the ram velocity is VR and ram is applied the force F to the workpiece material and then the final product comes out with the velocity V.

Now, if we assume that this behaves this zone actually up to that conical zone when that behave like a rigid body and what are the deformation normally happens? If there is a change of the cross-section from the cylinder, during this conical shape, so gradually reaction of the cross-section that actually enforces to reduce the plastic deformation of the material and that actually comes out through the die opening.

Now, definitely if we follow at any section, any cross-section if you consider, it should maintain the continuity equation. Continuity equation now if we apply the continuity equation between this point and maybe this point between these two, then use the continuity equation pi/4 disc area and then velocity, velocity of the which has come out through the die. So, that actually indicates the volume flow rate during this metal forming process we can say also.

Similarly, that should be equal to the pi/4 di square that means what was the initial diameter of the component or we can say that in these cases, the diameter of the ram we have used here di because diameter ram is normally well defined. So, that parameters if we know and of course the ram velocity if we define based on that if we know this value, we can estimate the other values also.

And if we decide the d then we can easily estimate from here what is the value of the velocity of the final product here in these cases. So, that continuity equation, from the continuity equation we can easily estimate what is the velocity V here. Now, work done per unit volume during if we assume there completely plastic deformation happens during this forming process or during this extrusion process then we can estimate what is the work done per unit volume.

So, work done per unit volume if we look into normally in the stress-strain diagram, if we take any elemental d epsilon for example, the area of this element is basically sigma d epsilon, it is a kind of elemental area if it is equivalent to x-axis and opposite is y-axis. So, the area of this and if we suppose if y is a function of x, this is representation of this curve. So, then this elemental area can be represented that ydx.

Now, if we estimate the total area, then integration ydx so similar thing has been followed here also to estimate the integration ydx also that means sigma is the stress value and d epsilon is the elemental strain and integration suppose this is the range, suppose this is 0 and suppose up to that point epsilon is the strain. So, that is the limit of the integration 0 to epsilon and if we estimate this and we can estimate the work done.

But this work done is basically per unit volume because if we know the work done per unit volume, the stress. Suppose in SI units stress is Newton per meter square and strain is dimensionless or you can say that Newton meter by into meter. So, Newton meter=joule meter cube. So, basically here it represents the work done per unit volume.

So, when we estimate the work done per unit volume, the interpretation of this work done per unit volume of a typical stress-strain curve is represented by this area actually that is between these two what we have highlighted if we know the functional form of the stress as a function of strain. Now, plastic work done can be easily estimated and if you know the other parameters so sigma let us look into the other way also.

Now, what maybe the strain, how we can estimate the strain, this is true strain, actually this is true strain, this is true strain we know that true strain is the engineering strain we estimate change of length with respect to the initial length but here in this thing is the logarithm of it is an initial because initial area of the cross-section here and because in this is a symmetric component and radially symmetric.

So, the instead of length the cross-section is more important here. So, it is a reduction of the one single cross-section to another cross-section. So, that is why the in logarithm ln logarithm of the initial cross-section area/final but here we have taken the initial/final because initial cross-section was more and is easiest to the other final cross-section is less. So, that is why initial/final.

So, if we convert it that area=pi/4 d square so then rest of the area is the function of di/df square and if we look into that 2 ln di/df. So, that this Ai is basically area of the cross-section here initial and here is the Af final cross-section area. So, that it is a squeezing within this conical zone and does gives out from the die. So, from here that means here Ai di change here Af means basically change to df the diameter of the final product.

So, (()) (35:21) twice ln di/df is basically the true strain here and now if we assume the average value of the stress, here if we assume the average value of the stress is equivalent to the yield stress value, then we can easily estimate what is the plastic work done during this process because when we say the plastic work done it needs to overcome the yield point value, so yield stress value.

And this if sigma is replaced by the yield stress value sigma y and if we know that d epsilon integration 0 to epsilon is basically epsilon and then that is 2 epsilon=2 ln di/df because 0 to epsilon limit is the epsilon here, now look and estimate the typically we roughly estimate that what is the plastic work done during this deformation process.

Now, deformation per unit time can also be, deformation per unit time also is from the continuity equation we can know the deformation per unit time is basically the volume flow rate which is the pi/4 di square V=pi/4 di square/VR is the same. So, that is also we can say deformation per unit time. Now, power used in this plastically deforms the material, so power used in plastic deformation.

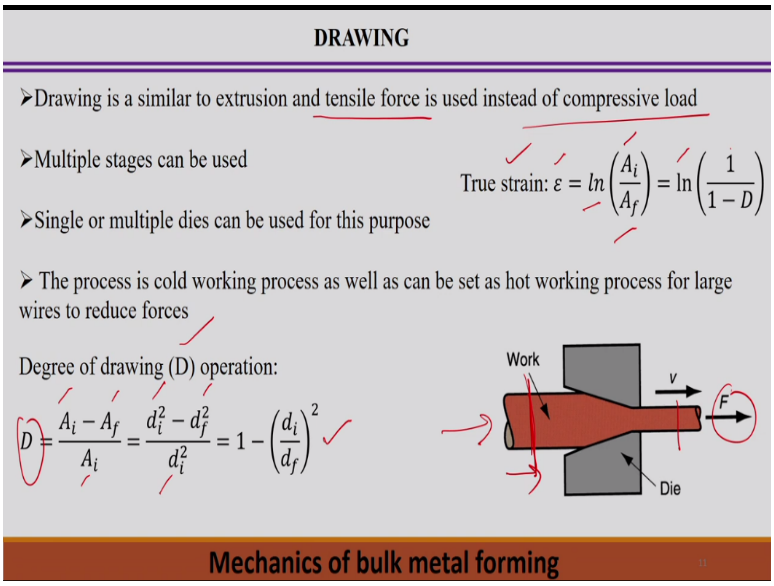
So, here we can estimate the WP, we can easily estimate here the plastic work done and that work done is per unit volume and if we multiply the volume flow rate that means it indicates in SI unit the units will be joule per meter cube and volume flow rate is meter cube per second which is equal to joule per second. So, joule per second is represented by the work. So, that is when power used, power can be estimated here if you know the WP is replaced by this expression.

And pi/4 di square VR if we know all this parameter we can easily estimate the power used in plastic deformation. So, this estimation is the approximate estimation the plastic work done but there are so many assumptions here also. First assumption is that is not necessary that the steady state will be only the yield stress during this process. So, the actual steady state can be different and is not may be single dimensional steady state.

Stress compression will be different, you need to know what are the principle stresses accordingly we can estimate requirement of this power that expression can be more accurately estimated and second part is that of course we are assuming the energy but we are assuming only energy expand for plastic deformation here also but we are not counting the, of course there must be some elastic recovery will also be there.

But we are not counting that elastic recovery part in this process; we are assuming that is completely plastic deformation during this process. So, this way we can roughly estimate the power used in plastic deformation in extrusion process.

(Refer Slide Time: 38:27)



Similarly, in case of drawing process also, drawing is also similar kind of the extrusion process but tensile force is used instead of the compressive load. Here, we use the tensile load and instead of the compressive. Look into this figure, so in extrusion process, we used the compressive load but here in the drawing process, there is a thick diameter of that is very high and it is when we are given here the tensile load, so diameter is reduced from this big diameter to less diameter.

Of course, the velocity, the entry and exit velocity will be different to maintain the continuity and here the force is applied is a tensile force instead of the compressive force like extrusion process, so that is the difference. Of course, mostly drawing process can be done in multistage condition; multistage means not the reduction of the diameter not in a single stage may be multiple stages we can follow.

Because the reduction ratio cannot be very high during this process, so there may be other kind of problem arise also but of course in drawing process we can estimate true strain also. In the true strain, similarly the true strain we already estimated the logarithm of the initial and the final so from here which is ln 1/1-D. Of course, we come to that point that what is D, so D the degree of the drawing operation.

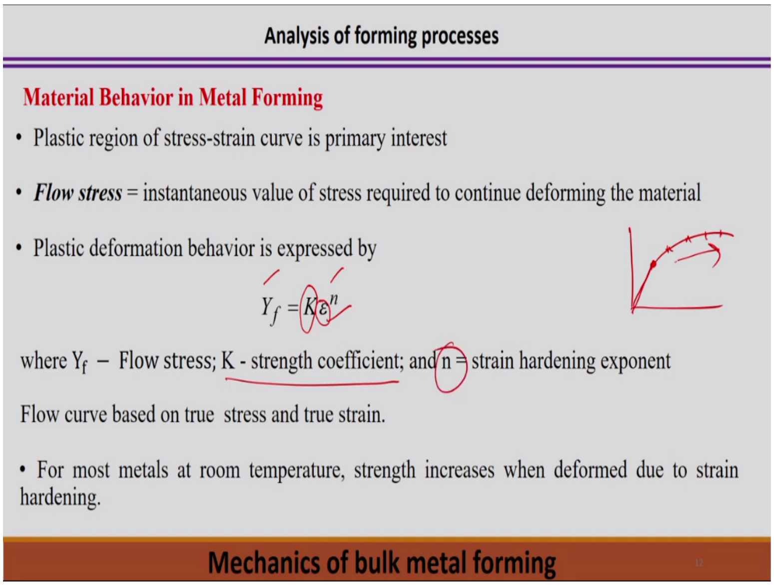
Degree of the drawing operation is simply that what is the reduction in the cross-section, so that is initial-final the reduction and with respect to the initial, so that can be represented in terms of the diameter also di square because Ai is proportional to the diameter square, di square-df square/di square which is equal to 1-di/df square. So, this is called the degree of the drawing operation.

This is important parameter in the drawing process, we can easily estimate. So, if we know the degree of drawing operation, we can estimate the true strain also 1-1/D. So, all this parameter we measure where we have, the purpose is to decide should we go for the reduction of the cross-section in the drawing process in a single stage or we should follow the multiple stages.

And the each and every stage, there must be some limitation in terms of the degree of the drawing in individual steps and accordingly we can design also drawing process to using these parameters. Of course, single or multiple die can also be used for this purpose, so maybe in sequentially there may be other die can also be used with the reduced diameter. The process is cold working process normally.

But at the same time, it can be set as a hot working process and specifically for the large wears to reduce the forces. So, this is the drawing process.

(Refer Slide Time: 41:18)



Now, we come to that, now we try to analyze the different forming processes from the material behaviour in the metal forming process. So, of course when we try to analyze the behaviour of the metal forming process, the plastic region of the stress-strain is more important because most of the engineering materials the elastic part of a stress-strain diagram is very small actually.

So, we can neglect that elastic part and we can work with the only the plastic zone in the stress-strain curve. Of course, the flow stress value, in the metal forming process, the flow stress is one important parameter and it can be defined the instantaneous value of the stress to continue the plastic deformation of the material. So, that amount of the stress required to continue the plastic deformation that can be considered as a flow stress value which is almost equal to or may be less than the yield stress value.

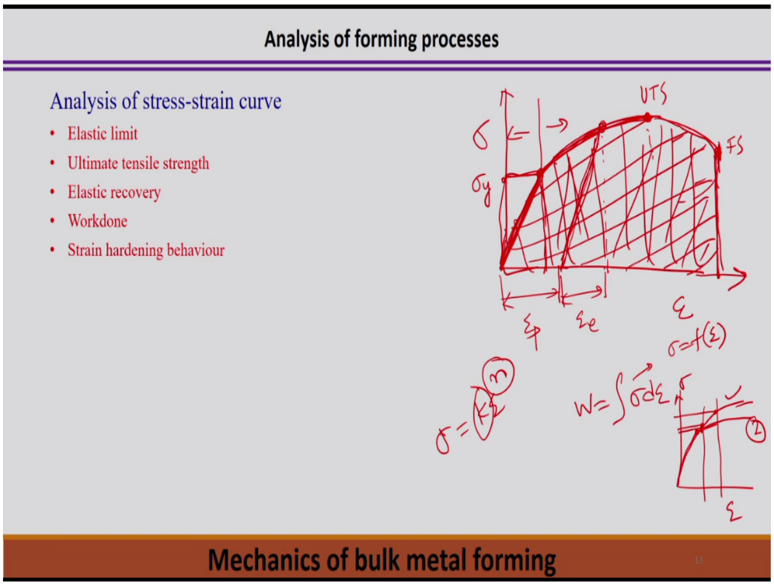
But of course, this flow stress value to some extent depends on what is the strain hardening effect of this particular material. Most of the plastic deformation of the engineering materials specifically steel can be represented like that. The flow stress is as a function of K constant and this is the strain true strain and n is some constant. So, here K is called the strength coefficients and n is called the strain hardening exponent.

So, depending upon the material behaviour during the deformation process, the flow stress can be predicted using this simple expression where K is the strength coefficient it depends on the material and of course n also depends on the type of the material but n is called specifically the strain hardening exponent and which is the strain. Now, flow curve actually based on the true stress and true strain, based on the true stress and true strain flow curve can be defined.

But for most of the metals at room temperature actually strength increases when deformed due to the strain hardening. So, most of the case we can see this is the yield strain value, elastic and then increment of the strain value gradually. So, this increment of the strain is basically because of the strain hardening effect of a particular material.

But that strain hardening effect is actually represented mathematically by the exponent n, the strain hardening exponent. So, that n can be different for different types of the materials.

(Refer Slide Time: 43:43)



Now, before analysis of forming processes, so the analysis of the stress-strain curve is important in these cases. So, if we look into typical stress-strain curve for example in particular material metallic material it looks like elastic, so this is elastic part or we can say this is the yield point that yield point normally decides the transition from the elastic to plastic components.

And we can say this point is the ultimate tensile strength and this is fracture strength if fracture happens, if you deform further the fracture happens. Now, we know usually know this is the elastic limit, so strength for that so elastic limit within that limit we follow by the Hooke's Law the stress is proportional to the strain but mathematically we represent stress is proportional to the strain and this is a linear, this is a straight line basically and the linear line.

So, that representation of the elastic component from a stress-strain diagram of a particular material. So, now of course the slope of this curve after initial slope of this curve that represents the Young's modulus E and once you know the elastic limit, then the ultimate tensile strength so ultimate tensile strength at this point when we reach the maximum value of the tensile strength and then normally from that point.

That is making starts the reduction of the cross-section area of the sample and finally it breaks if we apply further load during this further application of the load when there is a mechanical wave of this material. Now, define the ultimate tensile strength and elastic recovery can also be defined in these cases because within this and there is if you remove the load at a particular point, then it may not come back to this point, so it will come back to this point.

So, this point we define in such a way that it comes back to this point so that which is parallel to this initial this slope. So, this part is basically the elastic recovery, elastic recovery strain and this strain (()) (46:15) is the plastic strain because this zone is plastic zone and this is the elastic components elastic zone but this amount is called the elastic recovery. So, that if material behaves like elastoplastically if we assume the behaviour of the material is elastoplastic in nature.

So, definitely if application of the load and at any point the removal of the load from this material then some amount of the elastic recovery will be there but from a typical stress-strain diagram, we can simply estimate what is elastic recovery and what is the plastic deformation part, we can easily estimate this component. Then, work done and if you look into the ultimate tensile strength or may be fracture strength.

So, if we know this fracture strength up to this zone basically if we neglect the elastic recovery part and this zone actually represents the toughness, the energy required or energy absorbed before the fracture of the sample, that energy is basically the fracture the toughness, what is the measurement of the toughness from a simple tensile stress-strain diagram? So, the area actually represents the measurement of the toughness for a particular material.

So, definitely sigma this area represent this thing, so this area what is the work done to fracture the sample at this point that is the amount of the representation of the toughness mathematically. Of course, if we know the stress, function of strain, if we put it and then we can easily estimate which is equal to the work done and of course we already discussed that work done, this represent the work done per unit volume.

And then if we integrate over the range then it also representation of what is the toughness of a particular material. Now, strain hardening behaviour also can be represented like that after yield strength further yielding happens, deformation happens, there is increment of the strength, so that increment of the strength is because the material is having the strain hardening effect.

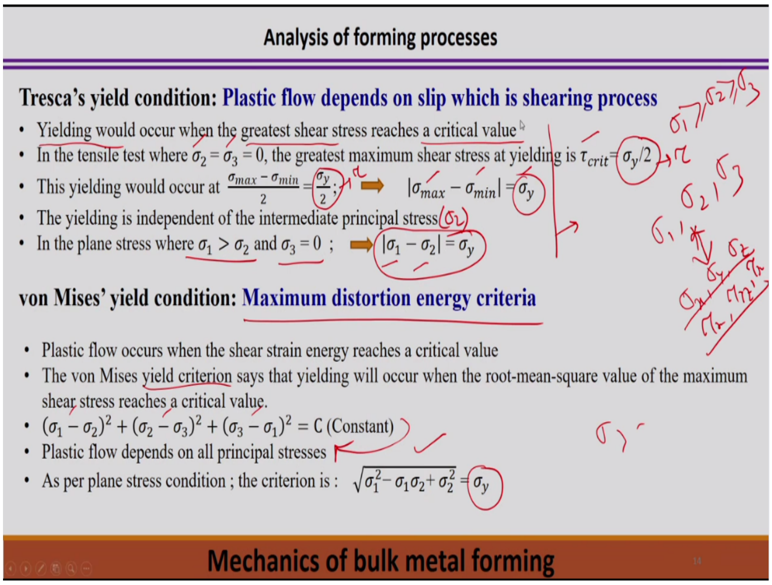
So, this strain hardening effect can be different, so in certain material the increment of the strength with further straining is not much in certain material, the strain hardening can be very big for example, so this to up to this elastic but one material this material strain hardening effect is more, that means particular if you strain, this is the strain and stress so if you strain deform the material then increment of the strength from this point to this point there is increment of the strength.

So, this is the increment of the strength corresponding to but for the second material if we further deform the material, so increment of the strength this point to that point, so that strength increment is very low with further deformation. So, that means for the second material the strain hardening effect is less as compared to the first material.

So, that is why when you represent the flow stress value, this n is basically we can fix the value of the n depending upon the strain hardening effect of a particular material. So, n should be different for different types of the material. Of course, K will be different for other types of the material but that K is nothing to do with the strain hardening effect here. So, this is the typical stress-strain diagram of a particular material.

And that we get all information from the stress-strain diagram and that may be utilized in the analysis of the metal forming process.

(Refer Slide Time: 50:13)



Now, we look into other aspect that means in metal forming process with deform but deformation the very first slide I have mentioned that during the deformation the deformation normally happens in between the yield strength and the fracture strength. So, we have to ensure the strength such that strength should be that deformation is such that it should not cross the fracture strength for a particular metal forming process.

So, to understand that we need to go through some kind of the failure criteria that can be decided on the different failure criteria during the plastic deformation what were the material fails or what situation, what conditions, on what yielding condition we can decide such that material can fails also. Now, with the application of the load in metal forming process, we can represent the state of the stress.

So, that state of the stress may be 6 components normally having 6 normal stress components and 6 shear stress components and from this normal and shear stress components, we can estimate the principle stresses. So, in the principle stresses most of the cases sigma 1, sigma 2, sigma 3 and that principle stresses we can estimate from any state of the stress that state of the stress is subjected to xy, yz, zx.

So, 3 normal stress and normally in symmetric cases 3 shear stress components. So, that is represented in terms of the 3 principle stress components. So, if we assume that 3 principle stress components is such that sigma 1>=sigma 2>=sigma 3. The state of the 3 principle stresses we can estimate in such a way that sigma 1>=sigma 2 and sigma 3.

So, it is during the metal forming process what are the different loading conditions happens, there is a tensile, compressing, bending combination of all these things finally represents the state of the stress within the material point. It is in terms of the 3 stress components, sigma x, sigma y and sigma z, tau xy, tau yz, tau zx these 6 components. These 6 components can be represented in terms of the principle stress components, sigma 1, sigma 2 and sigma 3.

And once we estimate this principle stresses then we can decide the different failure criteria in these cases. So, first is the Tresca's yield conditions, in these cases, the plastic flow depends on the slip which normally if we look into the deformation behaviour of the material and the microscopically normally happens the plastic deformation happens over the slip planes and because of the slip.

So, in that sense the Tresca's yield condition, they assume that plastic flow depends on the slip and that actually the slip normally happens due to the shearing acting so during the shearing process. So, based on that the yielding that criteria can be decided like that yielding would occur when the greatest shear stress reaches a critical value. So, in these cases, the yielding will occur and the greatest shear stress value reaches a critical value.

Now, if we into the yield conditions here but of course we are not discussing here the fracture conditions rather we are discussing the yield condition here. So, suppose in tensile testing if sigma 2 sigma 3=0 so therefore the maximum shear stress at yielding can be like that the sigma y/2 rather sigma y is measured the normal yield strength value and of course sigma y/2 is basically the yield strength value but shear yield strength value.

So, maximum shear stress at yielding will be there, so this is the critical value. Thus, yielding will occur, the sigma max/2 that is we can say the maximum value of the shear stress and the sigma minimum/2 that is the minimum value of the shear stress, difference between these two is equal to the critical value of the yield stress value. So, then sigma max-sigma min/2=sigma y/2, this is equal to the shear yield strength value.

Now, otherwise sigma max-sigma min mod=sigma y but remember this sigma y is equal to not the shear yield strength value rather we can normal yield strength value of a particular material, so that is the criteria. So, difference between these two should maintain equal to some critical value. So, of course in this criteria, we can see there is no role of the intermediate stress value.

Because in this case the yield, when you decide this yield criteria in this case, we are not considering if there is any role of the intermediate stress value sigma 2. So, in a plane stress if we assume the sigma 1>sigma 2 in that cases and sigma 3=0 then sigma 1-sigma 2=sigma y this is the Tresca's yield conditions and normally we follow during the deformation process. So, apart from that which is mostly yield conditions?

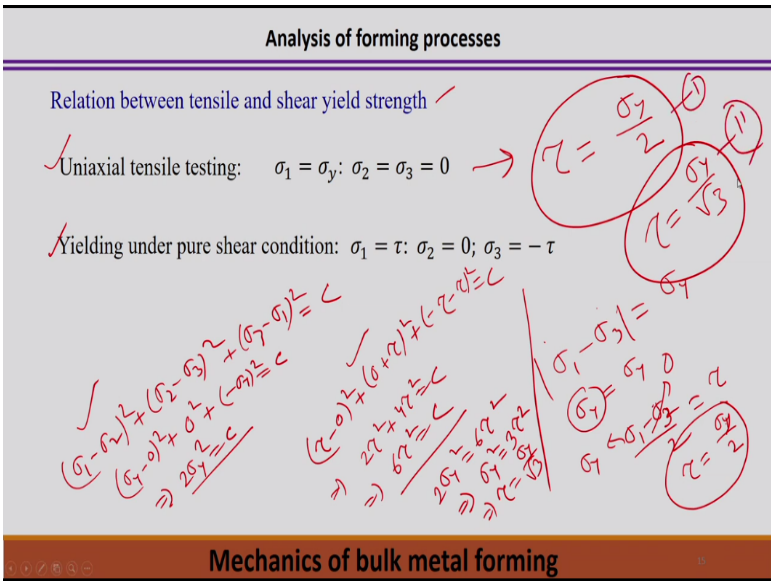
Actually, in this case the main drawback of this condition is that that intermediate stress value having no role to decide the yield condition in this case. Second case, the von Mises yield condition also, in this case it follows the maximum distortion energy criteria. In this case, plastic flow occurs when the shear strain energy reaches a critical value then only plastic deformation or plastic flow occurs.

Based on that the von Mises yield criteria says that yielding will occur but in other way also the yield criteria says that yielding will occur when the root-mean-square value of the maximum shear stress reaches a critical value. So, it mathematically represents like that in terms of the principle stresses sigma 1-sigma 2 square+sigma 2-sigma 3 square+sigma 3-sigma 3 is equal to some constant value, constant C here.

But of course this condition if you see that the plastic flow during the deformation process, it depends on all principle stresses but of course if we reduce in the plane stress conditions and if we say sigma 3=0 then we can reduce at this criteria. Now, in these cases, we assume the sigma y means here we can assume the sigma y is the yield strength of a particular, normal yield strength of a particular material.

But of course we will try to look into the relation between normal yield strength and the shear yield strength value under these two cases.

(Refer Slide Time: 56:49)



So, relation between the tensile and the shear yield strength value, so in case of the uniaxial tensile testing if we put uniaxial tensile testing sigma 1=sigma y so in this case say we assume that sigma 1-sigma 2 equal to this is cases but if because sigma 1 is the maximum, sigma 3 is the minimum and sigma y here and sigma 1=sigma y here and uniaxial tensile testing sigma 3=0 and of course in these cases it equal to the sigma yield strength of a particular material. So, here the constant becomes the sigma y.

If we put that here sigma max-sigma min/2=sigma y/2 here also we can see equal to the sigma max/2 is the yield strength value in this case. So, if we sorry sigma 3, so in these cases sigma 3=0 uniaxial tensile testing, then shear tau equal to basically sigma 1/2 or if we replace sigma 1 by sigma y so it is basically sigma y/2. So, that means in Tresca's yield conditions actually in these cases, the shear yield strength is equal to the normal yield strength value/2.

But if we look into that von Mises yield condition, so here we can see that sigma 1-sigma 2 square+sigma 2-sigma 3 square+sigma 3-sigma 1 square=some constant. In these cases, if we put for uniaxial tensile testing value, suppose we have the data for uniaxial tensile testing, so in these cases, sigma 1=sigma y so sigma y-0 sigma 2=0+sigma 2 sigma 3=0+sigma 3-sigma 1=-sigma y=C.

So, in this case, we are getting sigma y square+2, sigma y square=C. Now, if we put this is the yielding under pure shear conditions, the same thing if you put the shear condition, the sigma 1=tau, sigma 2=0+sigma 2=0-sigma 3=+tau+sigma 3-tau-sigma 1=C. So, in this case, we are getting twice tau square+4 tau square=C that means we are getting 6 tau square=C.

So, that means for von Mises yield conditions, we are putting, we are trying to, we are putting one is the uniaxial tensile testing data and other cases we are having the pure shear condition data and we can see that this is the case and this is the constant, so we can say that sigma y square=6 tau square this corresponds to the sigma y square=3 tau square or we can say that tau=sigma y/root 3.

So, that means this is the case in case of Tresca's conditions, the relation between the normal yield strength and shear yield strength value but in case of von Mises yield condition, we put the shear stress can be represented by sigma y/root 3. So, this is the two cases we can find out the relation between the normal stress and the shear stress. So, thank you very much for your kind attention. Next topic will be the sheet metal forming, I will try to discuss. Thank you.